Whitening of the Quark-Gluon Plasma

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- **inter-particle collisions & local equilibrium**
- **collective effects – diffusion vs. conductivity**

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The problem

The total color charge of QGP is <u>zero</u> but the local (macroscopic) color charges are initially <u>non-zero</u>.

$$\int d^3x \, \rho^a(x) = 0$$

$$\rho^a(x) \neq 0$$

Global equilibrium ⟺ **Maximum entropy** → $\rho^a(x) = 0$

How the system becomes locally neutral?

$$\rho^a(x) = 0$$

Transport theory

Transport equations for QGP

fundamental

$$\left(p_\mu D^\mu - g p^\mu F_{\mu\nu}\partial_p^\nu\right) Q(p,x) = C$$ quarks

$$\left(p_\mu D^\mu + g p^\mu F_{\mu\nu}\partial_p^\nu\right) \overline{Q}(p,x) = \overline{C}$$ antiquarks

adjoint

$$\left(p_\mu \mathcal{D}^\mu - g p^\mu \mathcal{F}_{\mu\nu}\partial_p^\nu\right) G(p,x) = C_g$$ gluons

Entropy flow

classical statistics

$$S^\mu(x) = -\int dp \frac{p^\mu}{E} \mathrm{Tr}[\mathrm{Q}\ln Q + \overline{Q}\ln\overline{Q} + G\ln G]$$

local equilibrium ⟺ maximum entropy ⟺ $\partial^\mu S_\mu = 0$

Entropy production

$$[\partial_p^\mu Q, Q] = 0 \longrightarrow \partial_p^\mu \ln Q = Q^{-1} \partial_p^\mu Q$$

$$\partial_\mu S^\mu = -\int dp \, \mathrm{Tr}[C \ln Q + \overline{C} \ln \overline{Q} + C_g \ln G] = 0$$

Collision invariants

baryon charge conservation

$$\partial_\mu b^\mu = 0 \quad \Rightarrow \quad \int dp \, \mathrm{Tr}[C - \overline{C}] = 0$$

energy-momentum conservation

$$\partial_\mu T^{\mu\nu} = 0 \quad \Rightarrow \quad \int dp \, p^\nu \mathrm{Tr}[C + \overline{C} + C_g] = 0$$

color charge conservation

$$D_\mu j^\mu = 0 \quad \Rightarrow \quad \int dp \, (C - \overline{C} + 2\tau^a \mathrm{Tr}[T_a C_g]) = 0$$

Local Equilibrium from collision invariants

Local equilibrium distribution functions

$$Q^{\mathrm{eq}} = \exp[-\beta(u^\mu p_\mu - \mu_b - \tilde{\mu})]$$

$$\overline{Q}^{\mathrm{eq}} = \exp[-\beta(u^\mu p_\mu + \mu_b + \tilde{\mu})]$$

$$G^{\mathrm{eq}} = \exp[-\beta(u^\mu p_\mu - \tilde{\mu}_g)]$$

$$\tilde{\mu}_g = 2T_a \mathrm{Tr}[\tau^a \tilde{\mu}]$$

$\tilde{\mu}$ - color chemical potential $N_c \times N_c$ matrix

Local equilibrium is colorful !

local rest frame $u^{\mu} = (1,0,0,0)$ → $\mathbf{j} = 0$ color current

but

color density

$$\rho(x) \neq 0 \quad \text{if} \quad \tilde{\mu}(x) \neq 0$$

$$\rho(x) = -g \int dp \left[Q - \overline{Q} - \frac{1}{N_c} \mathrm{Tr}[Q - \overline{Q}] + 2\tau^a \mathrm{Tr}[T_a G] \right]$$

$$\tilde{\mu}(x) \neq 0 \quad \text{does NOT imply} \quad \int d^3x \, \rho(x) \neq 0$$

Local equilibrium from $C = 0$

Entropy production

$$\partial_\mu S^\mu = -\int dp\, \mathrm{Tr}[C \ln Q + \overline{C} \ln \overline{Q} + C_g \ln G] = 0$$

What is the role of collision dynamics?

Collision terms

Waldmann-Snider collision term

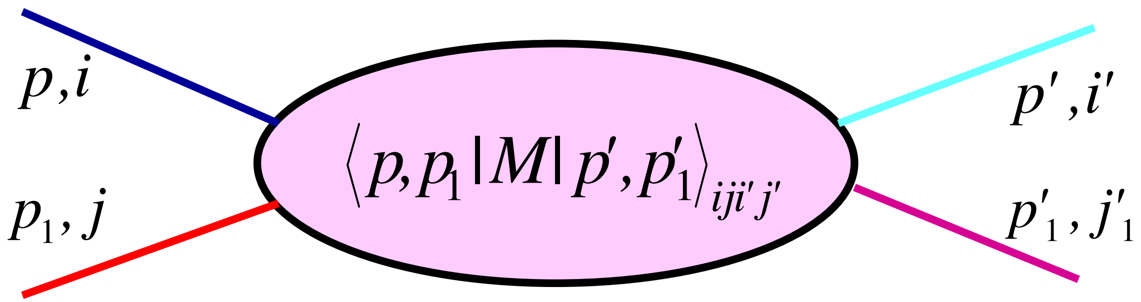
gain

loss

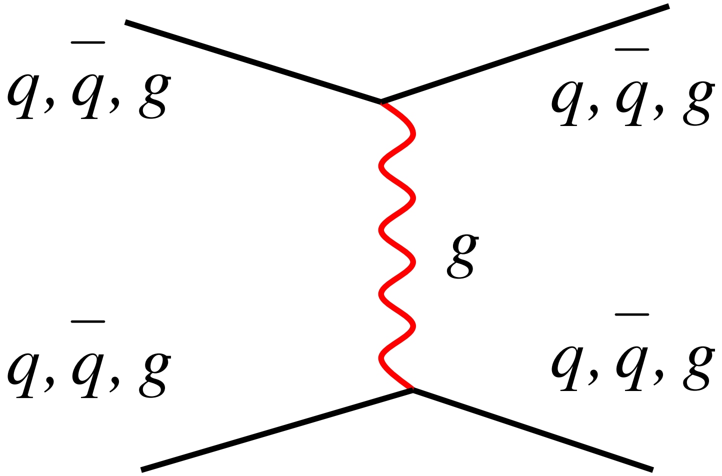
$$C_{qq} = \mathrm{Tr}_1 \int dp_1 dp' dp'_1 [$$
$$\times \langle p, p_1 | M | p', p'_1 \rangle Q' Q'_1 \langle p', p'_1 | M^* | p', p'_1 \rangle$$
$$- \tfrac{1}{2} (\langle p, p_1 | M | p', p'_1 \rangle \langle p', p'_1 | M^* | p', p'_1 \rangle Q Q_1 + \mathrm{h.c.})]$$

$$Q \equiv Q(x,p), \quad Q_1 \equiv Q(x,p_1), \quad Q' \equiv Q(x,p'), \quad Q'_1 \equiv Q(x,p'_1),$$

quark-quark scattering



The fastest collisions



in vacuum

$$\langle p, p_1 | M | p', p'_1 \rangle \underset{t \to 0}{\propto} \frac{1}{t}$$

quark-quark scattering

$$\langle p, p_1 | M | p', p'_1 \rangle_{iji'j'} \propto \tau^a_{ii'} \tau^a_{jj'}$$

Local Equilibrium from $C = 0$

$$Q \equiv Q(x,p), \quad Q_1 \equiv Q(x,p_1), \quad Q' \equiv Q(x,p'), \quad Q'_1 \equiv Q(x,p'_1),$$

quark-quark scattering

$$C_{qq} = 0$$

$$p + p_1 = p' + p'_1$$

$$(\mathrm{Tr}[Q']Q'_1 - \mathrm{Tr}[Q]Q_1)$$
$$-\frac{1}{N_c^2}(Q'\,\mathrm{Tr}[Q'_1] - Q\,\mathrm{Tr}[Q_1])$$
$$-\frac{1}{N_c}(\{Q', Q'_1\} - \{Q, Q_1\}) = 0$$

$$C_{q\bar{q}} = C_{qg} = C_{\bar{q}g} = C_{gg} = 0$$

Local equilibrium distribution functions

$$\tilde{\mu}_g = 2T_a \mathrm{Tr}[\tau^a \tilde{\mu}]$$

$$Q^{\mathrm{eq}} = \exp[-\beta(u^\mu p_\mu - \mu - \tilde{\mu})]$$

$$\overline{Q}^{\mathrm{eq}} = \exp[-\beta(u^\mu p_\mu - \overline{\mu} + \tilde{\mu})]$$

$$G^{\mathrm{eq}} = \exp[-\beta(u^\mu p_\mu - \mu_g - \tilde{\mu}_g)]$$

Subdominant processes

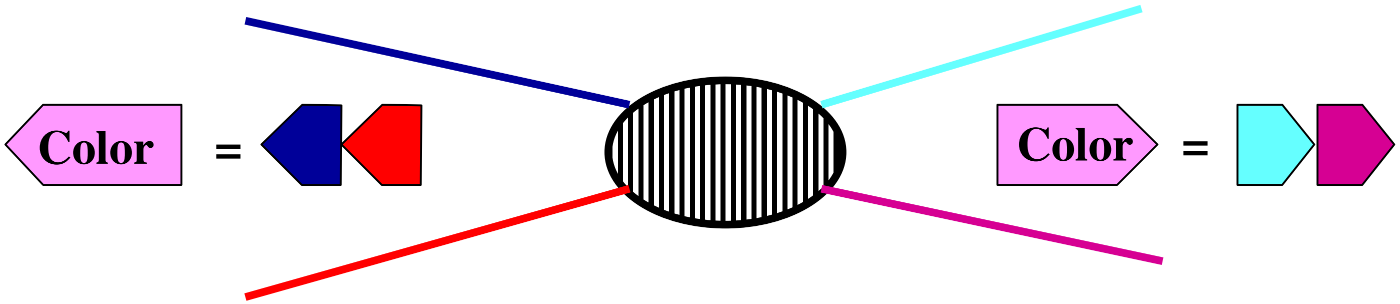
$$q\,\bar{q} \leftrightarrow gg \quad \Leftrightarrow \quad \mu + \bar{\mu} = \mu_g$$

$$gg \leftrightarrow ggg \quad \Leftrightarrow \quad 2\mu_g = \mu_g$$

$$\mu = -\bar{\mu} = \mu_b, \quad \mu_g = 0$$

Local equilibrium is colorful !

Parton-parton collisions do not neutralize local color charges!



$$D_\mu j^\mu(x) = 0$$

Collisions only redistribute colors among momentum modes.

Whitening

Q: How the system becomes locally neutral?

$$\rho^a(x) \neq 0 \quad \overset{?}{\Rightarrow} \quad \rho^a(x) = 0$$

A: Due to the collective currents caused by diffusion and conductivity.

Neutralization of electron-ion plasma

diffusion

conductivity

$$\mathbf{j} = -d\,\nabla\rho + \sigma\,\mathbf{E}$$

Charge conservation

$$\frac{\partial\rho}{\partial t} + \nabla\cdot\mathbf{j} = 0$$

Gauss law

$$\nabla\cdot\mathbf{E} = \rho$$

$$\left(\frac{\partial}{\partial t} - d\nabla^2 + \sigma\right)\rho(x) = 0$$

Neutralization of electron-ion plasma

$$\left(\frac{\partial}{\partial t} - d\nabla^2 + \sigma\right)\rho(x) = 0$$

Solution

Exponential decay

$$\rho(x) = \int dk\, e^{-i\mathbf{k}\mathbf{r} - (\sigma + d\mathbf{k}^2)t} \rho_0(\mathbf{k})$$

$$dk \equiv \frac{d^3k}{(2\pi)^3}$$

Initial condition

$$\rho(t = 0, \mathbf{x}) = \rho_0(\mathbf{x}) = \int dk\;\; e^{-i\mathbf{k}\mathbf{r}} \rho_0(\mathbf{k})$$

For long wavelength modes ($k^2 < \sigma/d$) the conductivity dominates

$$\rho(x) = e^{-\sigma t} \rho_0(\mathbf{x})$$

Transport coefficients d & σ

Transport equations

$$\left(p_\mu D^\mu - g p^\mu F_{\mu\nu} \partial_p^\nu\right) Q(p,x) = C$$

$$\left(p_\mu D^\mu + g p^\mu F_{\mu\nu} \partial_p^\nu\right) \overline{Q}(p,x) = \overline{C}$$

$$\left(p_\mu \mathcal{D}^\mu - g p^\mu \mathcal{F}_{\mu\nu} \partial_p^\nu\right) G(p,x) = C_g$$

Linearization

$$Q = Q^{\text{eq}} + \delta Q, \quad \overline{Q} = \overline{Q}^{\text{eq}} + \delta \overline{Q}, \quad G = G^{\text{eq}} + \delta G$$

$$\left|Q^{\text{eq}}\right| >> \left|\delta Q\right|, \quad \left|D^\mu Q^{\text{eq}}\right| >> \left|D^\mu \delta Q\right|, \quad \left|\nabla_p Q^{\text{eq}}\right| >> \left|\nabla_p \delta Q\right|$$

Transport coefficients d & σ

Linear transport equation

$$(D^0 + \mathbf{v}\mathbf{D})\, Q^{\mathrm{eq}} + \frac{g}{2}\{\mathbf{E}, \nabla_p Q^{\mathrm{eq}}\} = L[\delta Q]$$

Anstaz

1/γ - relaxation time

$$\int dp\ \mathbf{v}\ L[\delta Q] = -\gamma \int dp\ \mathbf{v}\ \delta Q$$

$$dp \equiv \frac{d^3 p}{(2\pi)^3}$$

diffusion

conductivity

$$\mathbf{j}^{\mathrm{a}} = -d\,\mathbf{D}^{\mathrm{ab}} \rho^{\mathrm{b}} + \sigma^{\mathrm{ab}} \mathbf{E}^{\mathrm{b}}$$

$$d = \frac{1}{3\gamma}$$

$$\sigma^{\mathrm{ab}} = \frac{g^2}{3\gamma} \int dp \frac{1}{E_p} \left(\mathrm{Tr}[\{\tau^{\mathrm{a}}, \tau^{\mathrm{b}}\}(Q^{\mathrm{eq}} + \overline{Q}^{\mathrm{eq}})] + \mathrm{Tr}[\{T^{\mathrm{a}}, T^{\mathrm{b}}\} G^{\mathrm{eq}}] \right)$$

Evolution of charge density

$$\mathbf{j}^a = -d\,\mathbf{D}^{ab}\rho^b + \sigma^{ab}\mathbf{E}^b$$

$\mathbf{D} \equiv ig[\mathbf{A},\ldots]$

$a, b = 1, \ldots N_c$

diffusion

conductivity

Current conservation

$$D_\mu j^\mu = 0$$

Gauss law

$$\mathbf{D} \cdot \mathbf{E} = \rho$$

$$(D^0 - d\mathbf{D}^2 + \sigma)\rho(x) = 0$$

$$[\mathbf{D}, \sigma]\mathbf{E} \ll \sigma\mathbf{DE}$$

$$1/g \gg 1$$

Evolution of charge density

$$(D^0 - d\mathbf{D}^2 + \sigma)\rho(x) = 0$$

$$[D^0, \sigma]\rho << \sigma D^0 \rho$$

$1/g >> 1$

$$\rho(x) = e^{-\sigma t} n(x)$$

Exponential decay!

$$(D^0 - d\mathbf{D}^2)n = 0$$

For long wavelength modes ($k^2 < \sigma/D$) the conductivity dominates.

Time scales

Estimates at global equilibrium in perturbative regime

- $t_{\text{hard}} \sim \dfrac{1}{g^4 \ln(1/g) T}$ - scattering at momentum transfer T
  momentum equilibration

- $t_{\text{soft}} \sim \dfrac{1}{g^2 \ln(1/g) T} \sim \dfrac{1}{\gamma}$ - scattering at momentum transfer $g^2 T$
  color redistribution

- $\dfrac{1}{\sigma} \sim \dfrac{\ln(1/g)}{T}$ - whitening

$1/g >> 1$

$$t_{\text{hard}} >> t_{\text{soft}} >> \frac{1}{\sigma}$$

Conclusions

- **QGP becomes locally white due to the conductive currents**
- **Whitening is faster than momentum equilibration**