How to distinguish coalescence from thermal production of light nuclei

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with **Sylwia Bazak & Patrycja Słoń**

Background

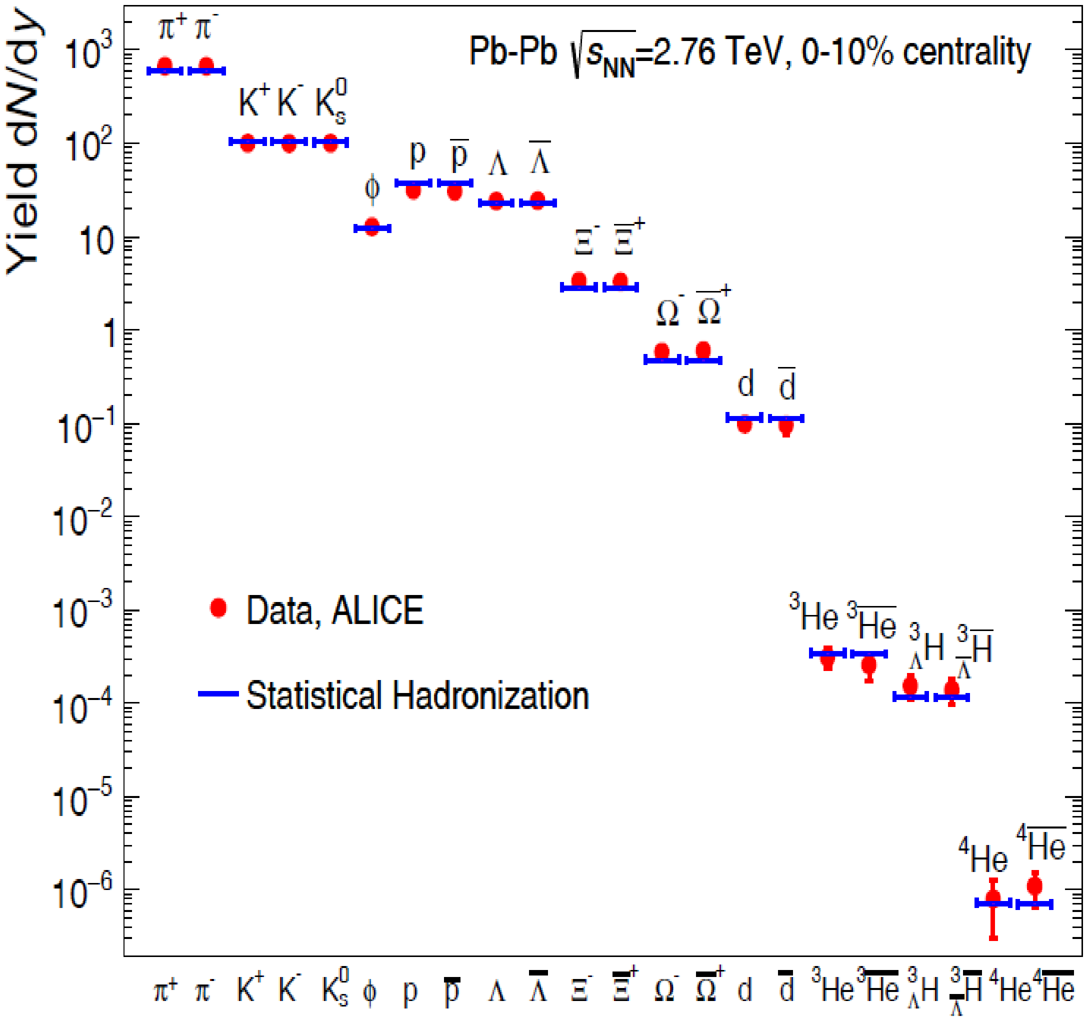
Conventional wisdom

- Production of light nuclei in relativistic heavy-ion collisions is well understood as a process of final state interactions.
- Coalescence models work well in a broad collision energy range.

News from RHIC & LHC

- Production of $^{2}\mathrm{H}$, $^{2}\overline{\mathrm{H}}$, $^{3}\mathrm{H}$, $^{3}\overline{\mathrm{H}}$, $^{3}\mathrm{He}$, $^{3}\overline{\mathrm{He}}$, $^{4}\mathrm{He}$, $^{4}\overline{\mathrm{He}}$, ${}^{3}_{\Lambda}\mathrm{H}$, ${}^{3}_{\bar{\Lambda}}\overline{\mathrm{H}}$ is observed in midrpidity.
- Matter-antimatter symmetry is seen.
- Thermal model properly describes yields of light nuclei.

Thermal model prediction



baryonless fireball

$$\text{Yield} \sim g\, e^{-\frac{m}{T}}$$

$$T = 156 \text{ MeV}$$

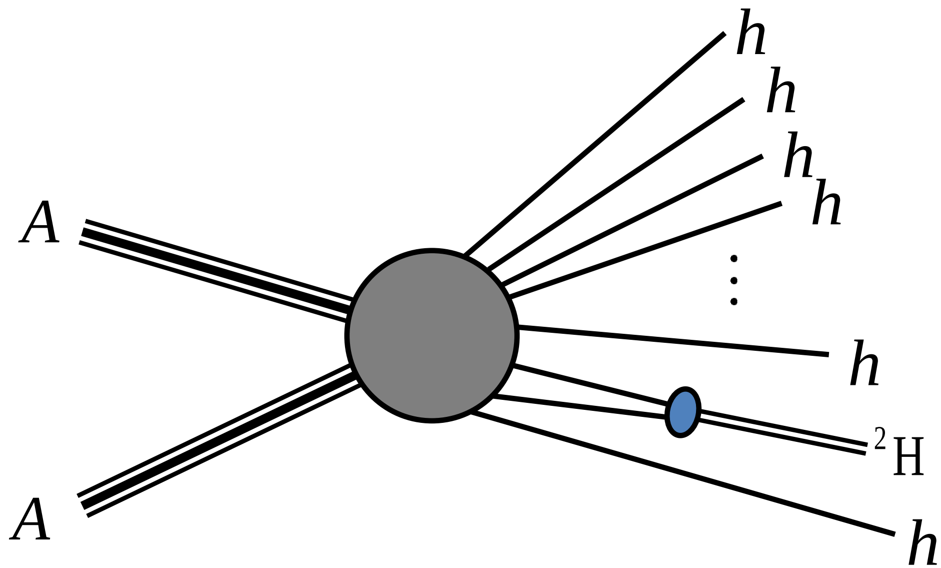
A. Andronic, P. Braun-Munzinger, K. Redlich and J. Stachel, Nature **561**, 321 (2018)

Can light nuclei exist in a fireball?

- Interparticle spacing in a hadron gas is about 1.5 fm at T = 156 MeV.
- Root mean square radius of a deuteron is 2.0 fm.
- Binding energy of a deuteron is ε_B = 2.2 MeV.
- A characteristic time of deuteron formation is $1/\varepsilon_B$ = 100 fm/c.
- A hadron gas at T = 156 MeV is essentially a classical system.

Snowflakes in hell ?
or
Snowflakes from hell ?

Final state interaction – conventional approach to production of light nuclei



energy scale
of particle production
- particle's mass

>>

energy scale
of formation of a nucleus
- binding energy

fast process

slow process

S.T. Butler & C.A. Pearson, Phys. Rev. **129**, 836 (1963)
A. Schwarzschild & C. Zupancic, Phys. Rev. **129**, 854 (1963)

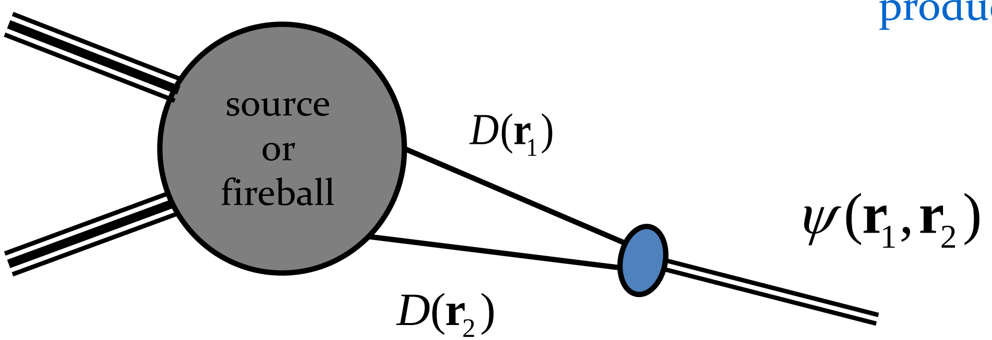
Factorization of production of nucleons and formation of a nucleus

Deuteron production cross section

$$\frac{d\sigma^D}{d^3\mathbf{P}_D} = W \frac{d\sigma^{np}}{d^3\mathbf{p}_n d^3\mathbf{p}_p} \qquad \frac{1}{2}\mathbf{P}_D = \mathbf{p}_n = \mathbf{p}_p$$

deuteron formation

production of *np* pair



spin factor

$$W = \frac{3}{4}(2\pi)^3 \int d^3\mathbf{r}_1 d^3\mathbf{r}_2\, D(\mathbf{r}_1) D(\mathbf{r}_2) \left|\psi(\mathbf{r}_1, \mathbf{r}_2)\right|^2$$

H. Sato and K. Yazaki, Phys. Lett. B **98**, 153 (1981)

Deuteron formation rate vs. n-p correlation

$$\psi(\mathbf{r}_1,\mathbf{r}_2) = e^{i\mathbf{P}\cdot\mathbf{R}}\varphi(\mathbf{r})$$

$$\mathbf{R} \equiv \frac{1}{2}(\mathbf{r}_1 + \mathbf{r}_2), \qquad \mathbf{r} \equiv \mathbf{r}_1 - \mathbf{r}_2$$

$$\boxed{W = \frac{3}{4}(2\pi)^3 \int d^3\mathbf{r}\, D_r(\mathbf{r}) \left|\varphi(\mathbf{r})\right|^2}$$

$$D_r(\mathbf{r}) \equiv \int d^3\mathbf{R}\, D\left(\mathbf{R} - \frac{1}{2}\mathbf{r}\right) D\left(\mathbf{R} + \frac{1}{2}\mathbf{r}\right)$$

n-*p* – correlation function

$$\boxed{C(\mathbf{q}) = \int d^3\mathbf{r}\, D_r(\mathbf{r}) \left|\varphi_{\mathbf{q}}(\mathbf{r})\right|^2}$$

$\varphi(\mathbf{r})$ – wave function of a bound state

$\varphi_{\mathbf{q}}(\mathbf{r})$ – wave function of a scattering state

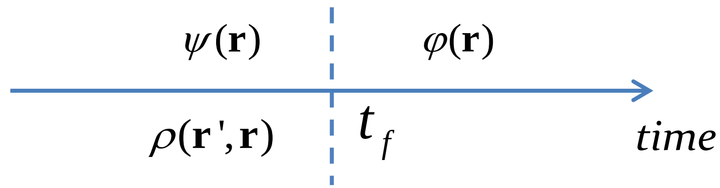
If emission time included

$$R_s \rightarrow \sqrt{R_s^2 + v^2\tau^2}$$

St. Mrówczyński, Phys. Lett. B **277**, 43 (1992)

Quantum-mechanical meaning of the formation rate formula

Sudden approximation



Transition matrix element

$$W = \left| \int d^3\mathbf{r}\, \psi^*(\mathbf{r})\, \varphi(\mathbf{r}) \right|^2 = \int d^3\mathbf{r}\, d^3\mathbf{r}'\, \varphi^*(\mathbf{r}')\, \underbrace{\psi(\mathbf{r}')\psi^*(\mathbf{r})}_{\rho(\mathbf{r}',\mathbf{r})}\, \varphi(\mathbf{r})$$

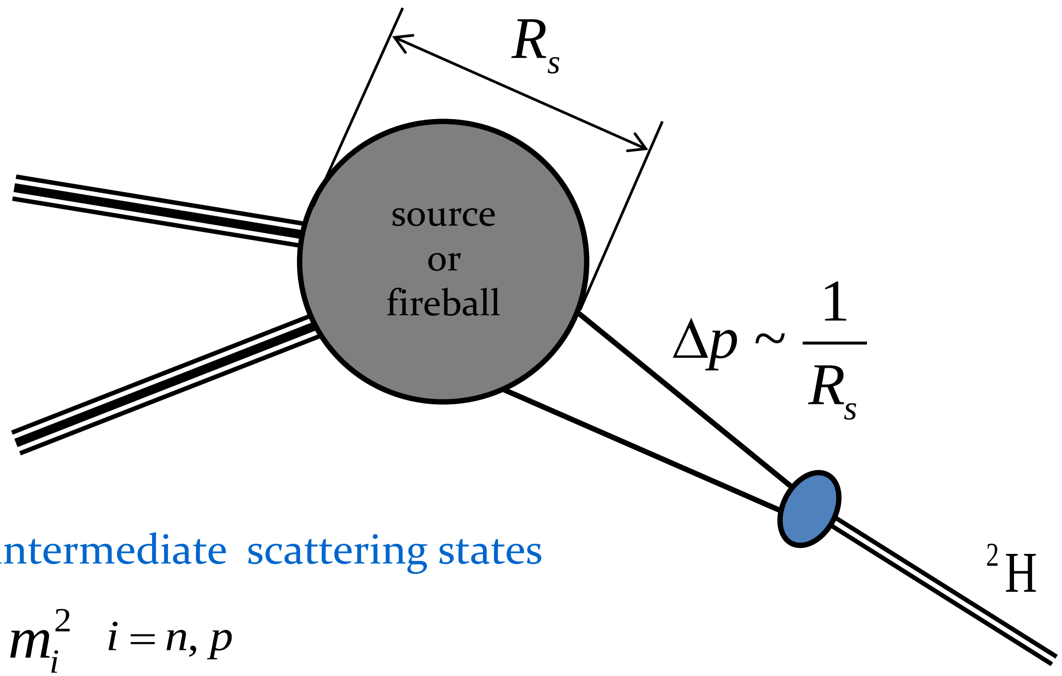
density matrix

$$W = \int d^3\mathbf{r}\, d^3\mathbf{r}'\; \varphi^*(\mathbf{r}')\, \rho(\mathbf{r}',\mathbf{r})\, \varphi(\mathbf{r})$$

If density matrix is diagonal (random phase approximation)

$$\rho(\mathbf{r}',\mathbf{r}) = D(\mathbf{r})\, \delta^{(3)}(\mathbf{r}'-\mathbf{r}) \qquad \Rightarrow \qquad \boxed{W = \int d^3\mathbf{r}\, D(\mathbf{r}) \left| \varphi(\mathbf{r}) \right|^2}$$

Energy-momentum conservation



Nucleons are intermediate scattering states

$$E_i^2 - \mathbf{p}_i^2 \neq m_i^2 \quad i = n, p$$

Energy-momentum conservation

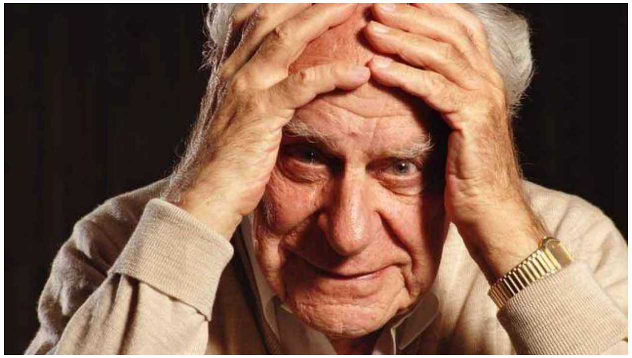
$$\begin{cases} \mathbf{p}_p + \mathbf{p}_n = \mathbf{p}_D \\ E_p + E_n = E_D \end{cases}$$

St. Mrówczyński, J. Phys. G **11**, 1087 (1987)

Thermal vs. coalescence model

- The two models usually give quantitatively similar predictions.

- How to falsify one of the models experimentally?



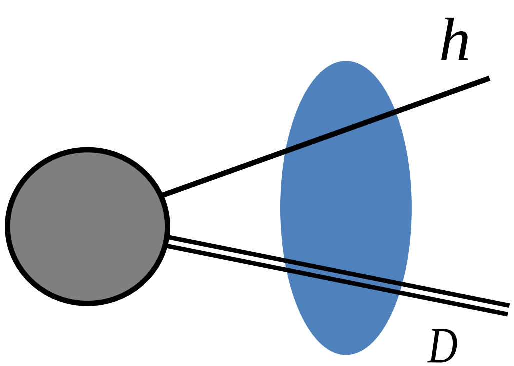
Karl Popper 1902-1994

St. Mrówczyński, Acta Phys. Pol. B **48**, 707 (2017)

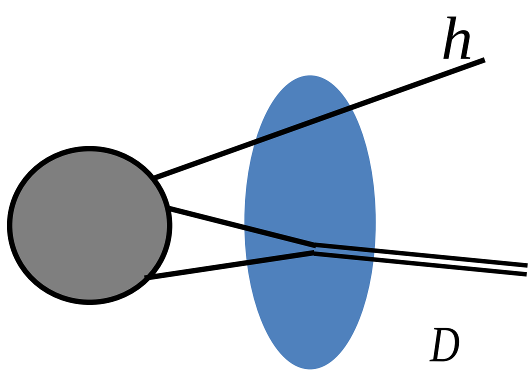
The first idea: *h*-*D* correlations

- Hadron-deuteron correlations carry information about a source of deuterons.

- A measurement of *p*-*D* & *p*-*p* correlation functions can falsify the thermal or coalescence model.



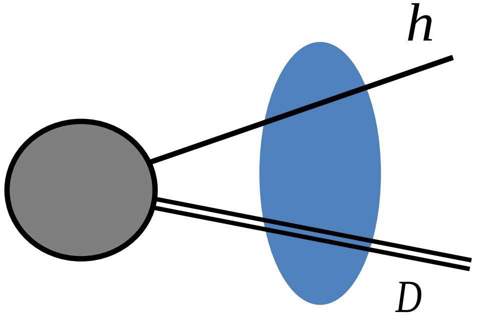
or



St. Mrówczyński & P. Słoń, arXiv:1904.08320

Hadron-deuteron correlation function

1) Deuteron is treated as an elementary particle



Experimental definition

$$\frac{dN_{hD}}{d\mathbf{p}_h \, d\mathbf{p}_D} = R(\mathbf{p}_h, \mathbf{p}_D) \, \frac{dN_h}{d\mathbf{p}_h} \, \frac{dN_D}{d\mathbf{p}_D}$$

Theoretical formula

$$R(\mathbf{p}_h, \mathbf{p}_D) = \int d^3 r_h \, d^3 r_D \, \underbrace{D(\mathbf{r}_h) \, D(\mathbf{r}_D)}_{\text{distribution of emission points}} \, \underbrace{\left| \psi(\mathbf{r}_h, \mathbf{r}_D) \right|^2}_{h\text{-}D \text{ wave function}}$$

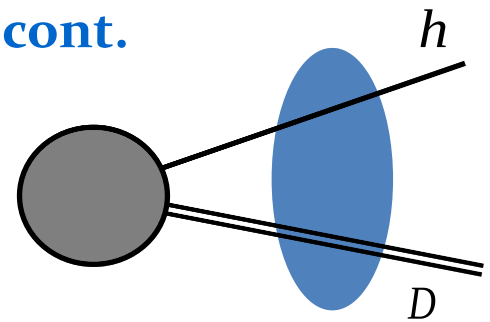
distribution
of emission points

h-*D* wave function

S.E. Koonin, Phys. Lett. B **70**, 43 (1977)
R. Lednicky and V.L. Lyuboshitz, Yad. Fiz. **35**, 1316 (1982)

Hadron-deuteron correlation function

1) Deuteron is treated as an elementary particle cont.



Separation of CM and relative motion

$$\begin{cases} \mathbf{R} \equiv \dfrac{m_D \mathbf{r}_D + m_h \mathbf{r}_h}{m_D + m_h} \\ \mathbf{r} \equiv \mathbf{r}_D - \mathbf{r}_h \end{cases}$$

$$\psi(\mathbf{r}_h, \mathbf{r}_D) = e^{i\mathbf{PR}} \phi_{\mathbf{q}}(\mathbf{r})$$

$$\boxed{R(\mathbf{q}) = \int d^3 r \; D_r(\mathbf{r}) \left| \phi_{\mathbf{q}}(\mathbf{r}) \right|^2}$$

„Relative” source function

$$D_r(\mathbf{r}) \equiv \int d^3 R \; D\left(\mathbf{R} - \frac{m_D}{m_D + m_h}\mathbf{r}\right) D\left(\mathbf{R} + \frac{m_h}{m_D + m_h}\mathbf{r}\right) = \left(\frac{1}{4\pi R_s^2}\right)^{3/2} \exp\left(-\frac{\mathbf{r}^2}{4R_s^2}\right)$$

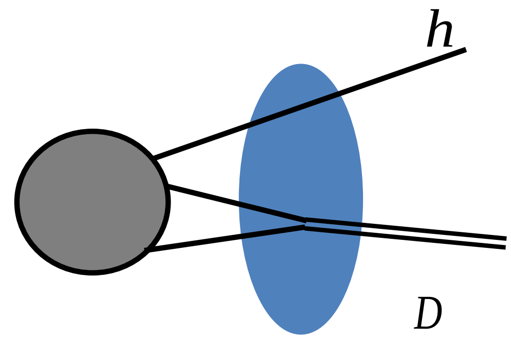
$$D(\mathbf{r}) = \left(\frac{1}{2\pi R_s^2}\right)^{3/2} \exp\left(-\frac{\mathbf{r}^2}{2R_s^2}\right)$$

Hadron-deuteron correlation function

2) Deuteron is treated as a bound state of neutron and proton

Experimental definition

$$\frac{dN_{hD}}{d\mathbf{p}_h d\mathbf{p}_D} = R(\mathbf{p}_h,\mathbf{p}_D)\, W_D \frac{dN_h}{d\mathbf{p}_h} \frac{dN_n}{d\mathbf{p}_n} \frac{dN_p}{d\mathbf{p}_p}$$



Theoretical formula

$$R(\mathbf{p}_h,\mathbf{p}_D) = \frac{1}{W_D}\int d^3r_h\, d^3r_n\, d^3r_p\, D(\mathbf{r}_h)\, D(\mathbf{r}_n)\, D(\mathbf{r}_p) \left|\psi_{hD}(\mathbf{r}_h,\mathbf{r}_n,\mathbf{r}_p)\right|^2$$

Deuteron formation rate $\quad \dfrac{dN_D}{d\mathbf{p}_D} = W_D \dfrac{dN_n}{d\mathbf{p}_n} \dfrac{dN_p}{d\mathbf{p}_p} \qquad \frac{1}{2}\mathbf{P}_D = \mathbf{p}_n = \mathbf{p}_p$

$$W_D = \frac{3}{4}(2\pi)^3 \int d^3\mathbf{r}_n d^3\mathbf{r}_p\, D(\mathbf{r}_n) D(\mathbf{r}_p) \left|\psi_D(\mathbf{r}_n,\mathbf{r}_p)\right|^2 = \frac{3}{4}(2\pi)^3 \int d^3r_{np}\, D_r(\mathbf{r}_{np}) \left|\phi_D(\mathbf{r}_{np})\right|^2$$

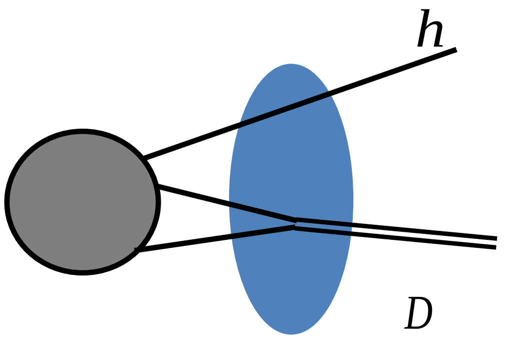
spin factor

$$\psi_D(\mathbf{r}_n,\mathbf{r}_p) = e^{i\mathbf{P}\mathbf{R}} \phi_D(\mathbf{r}_{np})$$

Hadron-deuteron correlation function

2) Deuteron is treated as a bound state of neutron and proton cont

Separation of CM and relative motion



$$\begin{cases} \mathbf{R} \equiv \dfrac{m_p \mathbf{r}_p + m_n \mathbf{r}_n + m_h \mathbf{r}_h}{m_p + m_n + m_h} \\ \mathbf{r}_{np} \equiv \mathbf{r}_p - \mathbf{r}_n \\ \mathbf{r} \equiv \mathbf{r}_h - \dfrac{m_p \mathbf{r}_p + m_n \mathbf{r}_n}{m_p + m_n} \end{cases}$$

$$\psi(\mathbf{r}_h, \mathbf{r}_n, \mathbf{r}_p) = e^{i\mathbf{P}\mathbf{R}} \phi_{\mathbf{q}}(\mathbf{r}) \varphi_D(\mathbf{r}_{np})$$

$$R(\mathbf{q}) = \frac{1}{W_D} \int d^3R\, d^3r_{np}\, d^3r\, D(\mathbf{r}_h) D(\mathbf{r}_n)\, D(\mathbf{r}_p) \left|\phi_{\mathbf{q}}(\mathbf{r})\right|^2 \left|\varphi_D(\mathbf{r}_{np})\right|^2$$

For Gaussian source

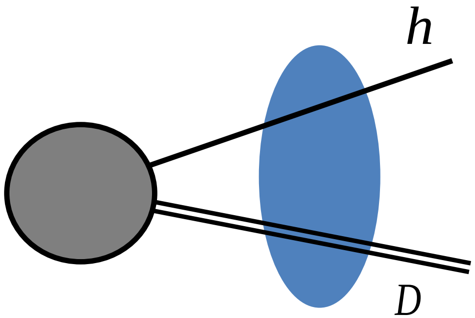
$$\boxed{R(\mathbf{q}) = \int d^3r\, D_{3r}(\mathbf{r}) \left|\phi_{\mathbf{q}}(\mathbf{r})\right|^2}$$

$$D_{3r}(\mathbf{r}) = \left(\frac{1}{3\pi R^2}\right)^{3/2} \exp\left(-\frac{\mathbf{r}^2}{3R^2}\right)$$

Thermal vs. coalescence model

Thermal model

$$R(\mathbf{q}) = \int d^3r\, D_r(\mathbf{r}) \left|\phi_{\mathbf{q}}(\mathbf{r})\right|^2$$



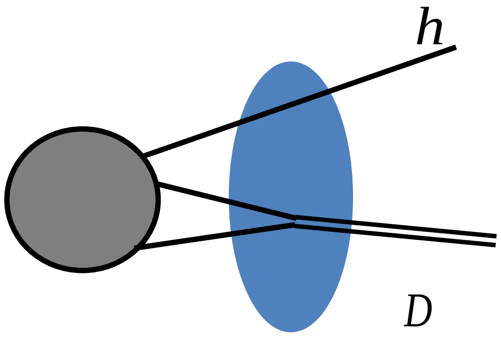
$$D_r(\mathbf{r}) = \left(\frac{1}{4\pi R^2}\right)^{3/2} \exp\left(-\frac{\mathbf{r}^2}{4R^2}\right)$$

$$D(\mathbf{r}) = \left(\frac{1}{2\pi R^2}\right)^{3/2} \exp\left(-\frac{\mathbf{r}^2}{2R^2}\right)$$

$$D_{3r}(\mathbf{r}) = \left(\frac{1}{3\pi R^2}\right)^{3/2} \exp\left(-\frac{\mathbf{r}^2}{3R^2}\right)$$

Coalescence model

$$R(\mathbf{q}) = \int d^3r\, D_{3r}(\mathbf{r}) \left|\phi_{\mathbf{q}}(\mathbf{r})\right|^2$$



h-D correlation function

The wave function in scattering asymptotic state

$$\phi_{\mathbf{q}}(\mathbf{r}) = e^{i\mathbf{qr}} + f(\mathbf{q})\frac{e^{iqr}}{r}$$

The *s*-wave amplidtue

$$f(\mathbf{q}) = -\frac{a}{1-iqa}$$

a – scattering length

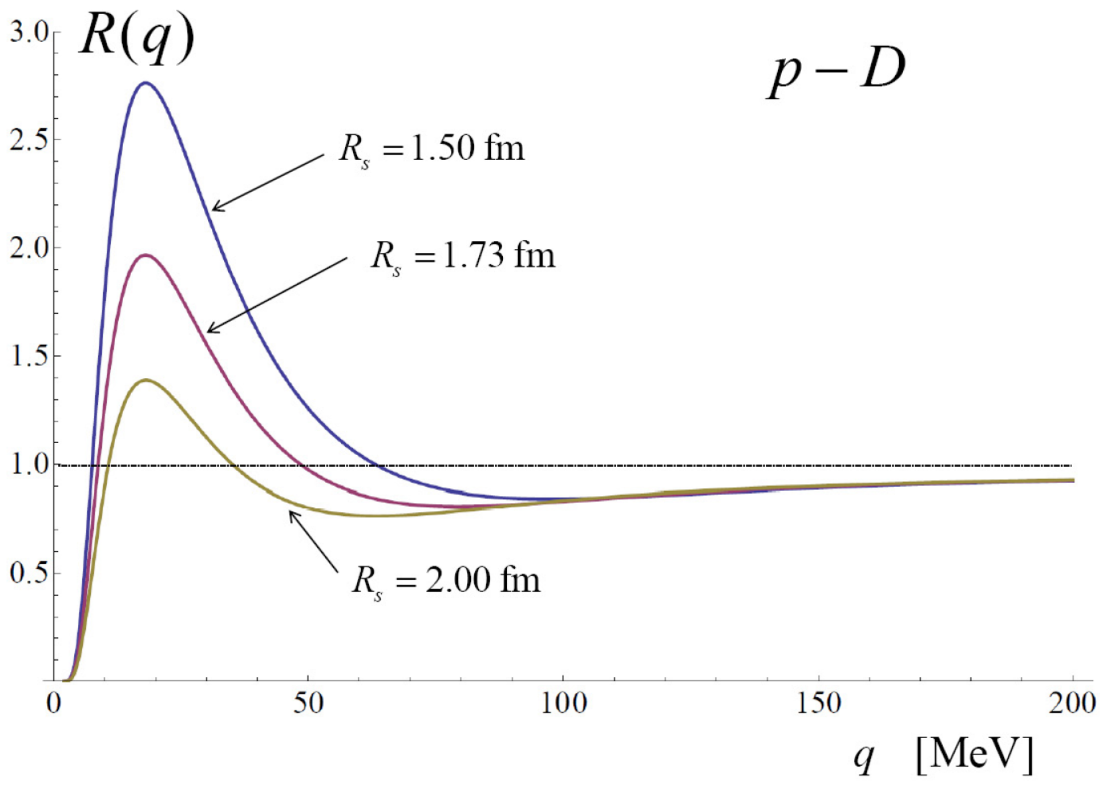
Coulomb interaction via Gamow factor

$$G(q) = \pm\frac{2\pi}{a_B q}\frac{1}{\exp\left(\pm\frac{2\pi}{a_B q}\right)-1}$$

$a_B = \frac{1}{\mu\alpha}$ – Bohr radius

R. Lednicky and V.L. Lyuboshitz, Yad. Fiz. **35**, 1316 (1982)

p-D correlation functions



$$R(q) = \frac{1}{3} R_{1/2}(q) + \frac{2}{3} R_{3/2}(q)$$

$$a_{1/2} = 4.0 \text{ fm}$$

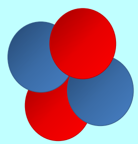
$$a_{3/2} = 11.0 \text{ fm}$$

$$2.00 = \sqrt{\frac{4}{3}} 1.73 = \frac{4}{3} 1.50$$

R_s from *p-D* correlation function vs. R_s from *p-p* correlation function

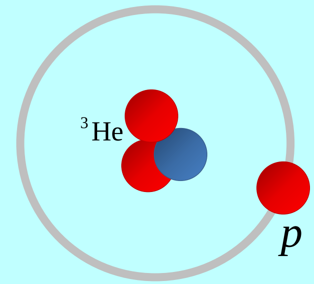
The second idea: ^{4}He vs. ^{4}Li

$^4\mathrm{He}$



$$r_{\mathrm{RMS}} = 1.68\ \mathrm{fm}$$
$$\varepsilon_B = 28.3\ \mathrm{MeV}$$
$$m = 3727.4\ \mathrm{MeV}$$
$$s = 0$$

$^4\mathrm{Li}$



$$^4\mathrm{Li} \to {}^3\mathrm{He} + p$$
$$\Gamma = 6\ \mathrm{MeV}$$
$$m = m_{^3\mathrm{He}} + m_p + 4.1\ \mathrm{MeV}$$
$$m = 3749.7\ \mathrm{MeV}$$
$$s = 2$$

- Thermal model $$\frac{\mathrm{Yield}(^4\mathrm{Li})}{\mathrm{Yield}(^4\mathrm{He})} = \frac{2S_{\mathrm{Li}}+1}{2S_{\mathrm{He}}+1} = 5$$

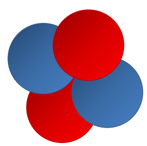
- Coalescence model $$\frac{\mathrm{Yield}(^4\mathrm{Li})}{\mathrm{Yield}(^4\mathrm{He})} = \frac{W_{\mathrm{Li}}}{W_{\mathrm{He}}}$$

S. Bazak & St. Mrówczyński, Mod. Phys. Letters A **33**, 1850142 (2018)
S. Bazak & St. Mrówczyński, arXiv: 2001.11351

Formation rates of ^{4}He & ^{4}Li

$$\boxed{W = g_S g_I (2\pi)^9 \int d^3\mathbf{r}_1 d^3\mathbf{r}_2 \int d^3\mathbf{r}_3 d^3\mathbf{r}_4\, D(\mathbf{r}_1)D(\mathbf{r}_2)D(\mathbf{r}_3)D(\mathbf{r}_4)\left|\psi(\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_3,\mathbf{r}_4)\right|^2}$$

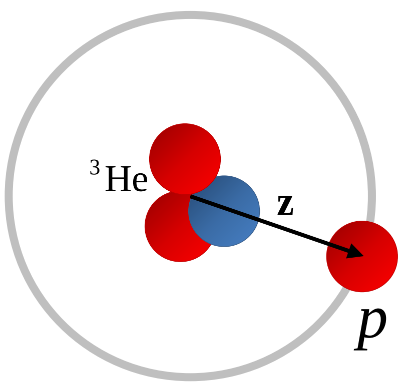
- ^{4}He



$$\mathbf{r}_{ij} \equiv \mathbf{r}_i - \mathbf{r}_j$$

$$\left|\psi_{\mathrm{He}}(\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_3,\mathbf{r}_4)\right|^2 \sim \exp\left[-\alpha(\mathbf{r}_{12}^2+\mathbf{r}_{13}^2+\mathbf{r}_{14}^2+\mathbf{r}_{23}^2+\mathbf{r}_{24}^2+\mathbf{r}_{34}^2)\right]$$

- ^{4}Li



J. C. Bergstrom, Nucl. Phys. A **327**, 458 (1979)

$$\mathbf{z} \equiv \mathbf{r}_4 - \frac{1}{3}(\mathbf{r}_1+\mathbf{r}_2+\mathbf{r}_3)$$

$$\left|\psi_{\mathrm{Li}}(\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_3,\mathbf{r}_4)\right|^2 \sim \exp\left[-\beta(\mathbf{r}_{12}^2+\mathbf{r}_{13}^2+\mathbf{r}_{23}^2)\right]\mathbf{z}^4\exp\left(-\gamma\mathbf{z}^2\right)\left|Y_{lm}(\Omega_{\mathbf{z}})\right|^2$$

Formation rates of ^{4}He & ^{4}Li

$$W = g_S g_I (2\pi)^9 \int d^3\mathbf{r}_1 d^3\mathbf{r}_2 \int d^3\mathbf{r}_3 d^3\mathbf{r}_4\, D(\mathbf{r}_1)D(\mathbf{r}_2)D(\mathbf{r}_3)D(\mathbf{r}_4)\left|\psi(\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_3,\mathbf{r}_4)\right|^2$$

Source function

$$D(\mathbf{r}_i) = \frac{1}{(2\pi R_s^2)^{3/2}} \exp\left(-\frac{\mathbf{r}_i^2}{2R_s^2}\right) \qquad i = 1, 2, 3, 4$$

If emission time included

$$R_s \to \sqrt{R_s^2 + v^2\tau^2}$$

Jacobi variables

$$\begin{cases} \mathbf{R} \equiv \frac{1}{4}(\mathbf{r}_1 + \mathbf{r}_2 + \mathbf{r}_3 + \mathbf{r}_4) \\ \mathbf{x} \equiv \mathbf{r}_2 - \mathbf{r}_1 \\ \mathbf{y} \equiv \mathbf{r}_3 - \frac{1}{2}(\mathbf{r}_1 + \mathbf{r}_2) \\ \mathbf{z} \equiv \mathbf{r}_4 - \frac{1}{3}(\mathbf{r}_1 + \mathbf{r}_2 + \mathbf{r}_3) \end{cases}$$

- $$\mathbf{r}_1^2 + \mathbf{r}_2^2 + \mathbf{r}_3^2 + \mathbf{r}_4^2 = 4\mathbf{R}^2 + \frac{1}{2}\mathbf{x}^2 + \frac{2}{3}\mathbf{y}^2 + \frac{3}{4}\mathbf{z}^2$$
- $$\mathbf{r}_{12}^2 + \mathbf{r}_{13}^2 + \mathbf{r}_{14}^2 + \mathbf{r}_{23}^2 + \mathbf{r}_{24}^2 + \mathbf{r}_{34}^2 = 2\mathbf{x}^2 + \frac{8}{3}\mathbf{y}^2 + 3\mathbf{z}^2$$

  $$\mathbf{r}_{ij} \equiv \mathbf{r}_i - \mathbf{r}_j$$

Fully analytic calculations are possible!

Formation rates of ^{4}He & ^{4}Li

- $$W_{\mathrm{He}} = \frac{\pi^{9/2}}{2^{9/2}} \frac{1}{\left(R_s^2 + R_\alpha^2\right)^{9/2}}$$

- $$W_{\mathrm{Li}} = \frac{3\pi^{9/2}}{2^{11/2}} \begin{pmatrix} \frac{5}{2} \\ 2 \\ 1 \end{pmatrix} \frac{R_s^4}{\left(R_s^2 + \frac{1}{2} R_c^2\right)^3 \left(R_s^2 + \frac{4}{7} R_{\mathrm{Li}}^2 - \frac{3}{7} R_c^2\right)^{7/2}}$$

$$\begin{pmatrix} l = 1 \\ l = 2 \end{pmatrix}$$

Since ^{4}Li is $J^P = 2^-$ then $l = 1$.

R_s – root mean square radius of the source

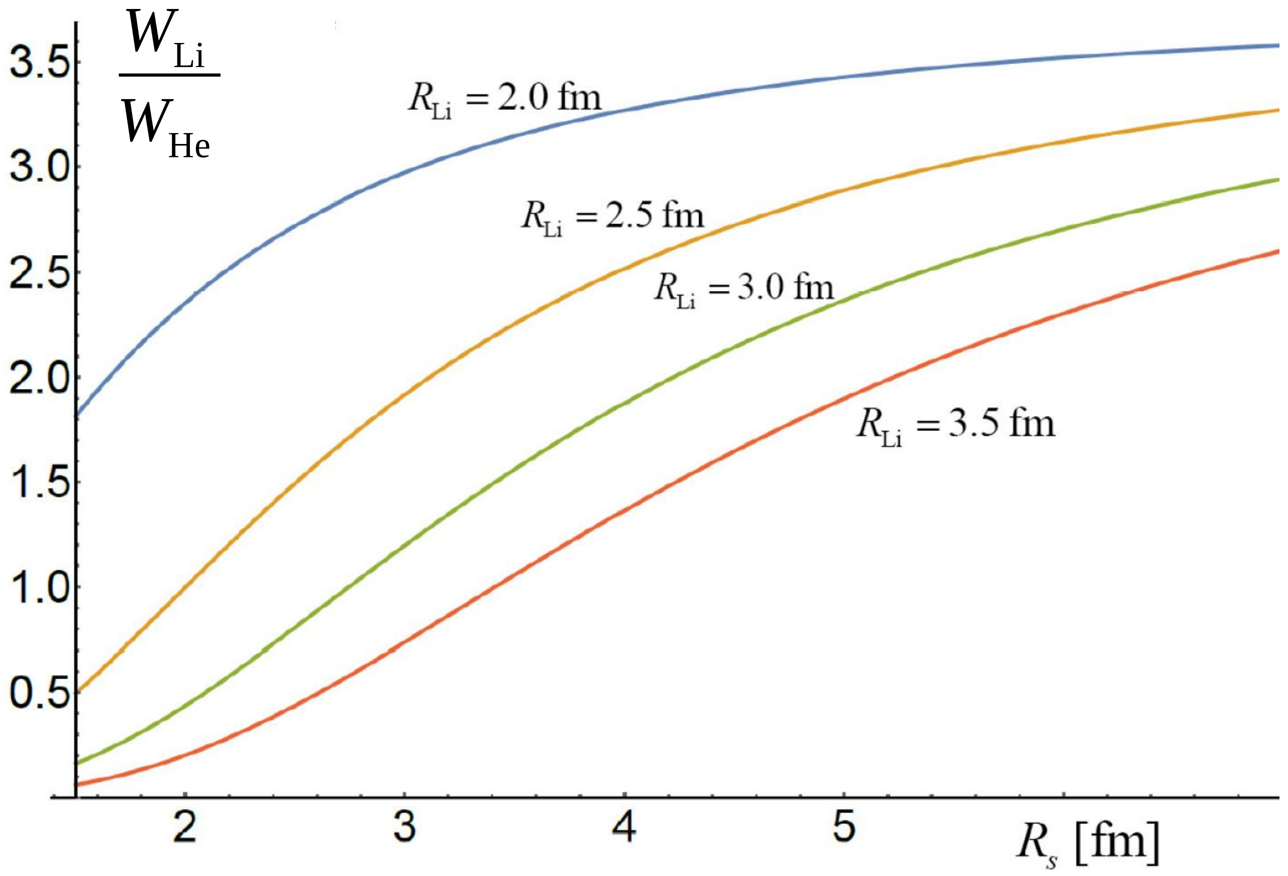
R_α – root mean square radius of ^{4}He

R_{Li} – root mean square radius of ^{4}Li

R_c – root mean square radius of ^{3}He cluster in ^{4}Li

Ratio of yields of ^{4}Li to ^{4}He

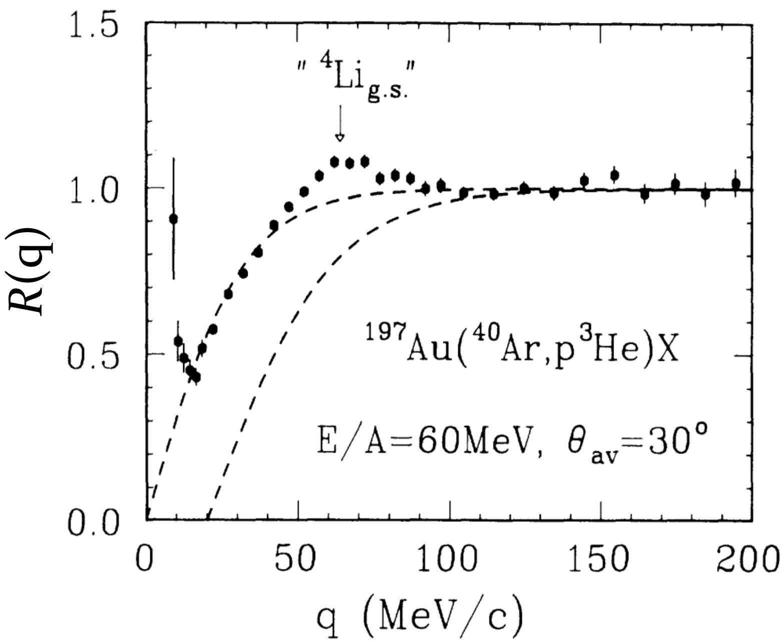
In the thermal model the ratio equals 5.



S. Bazak & St. Mrówczyński, arXiv: 2001.11351

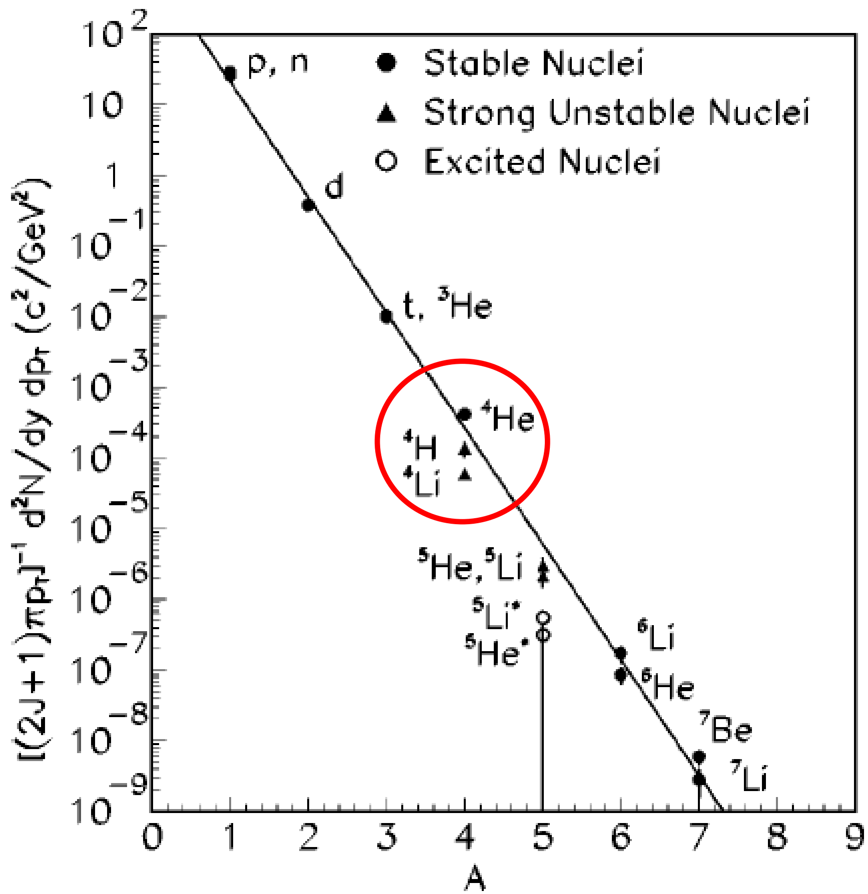
How to observe ^{4}Li?

Measurement of the correlation function of ^{3}He-*p* is needed



J. Pochodzala et al. Phys. Rev. C **35**, 1695 (1987)

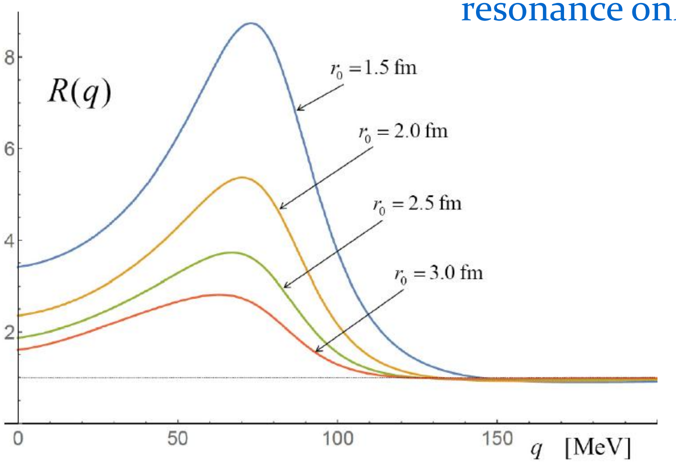
^{197}Au+^{197}Pt @ AGS



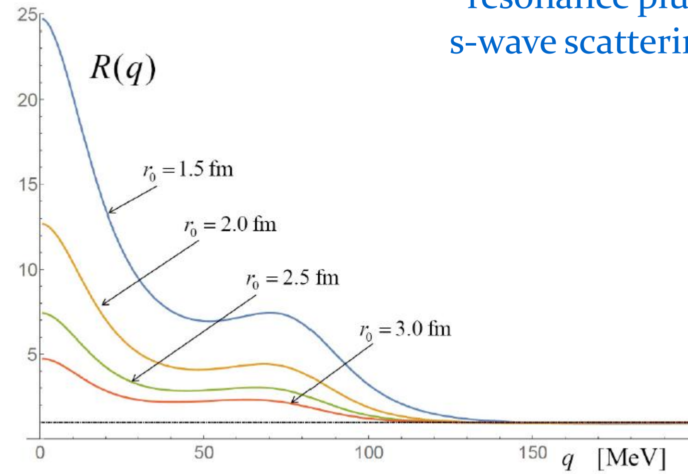
T. A. Armstrong et al. Phys. Rev. C **65**, 014906 (2001)

Correlation function p-^{3}He

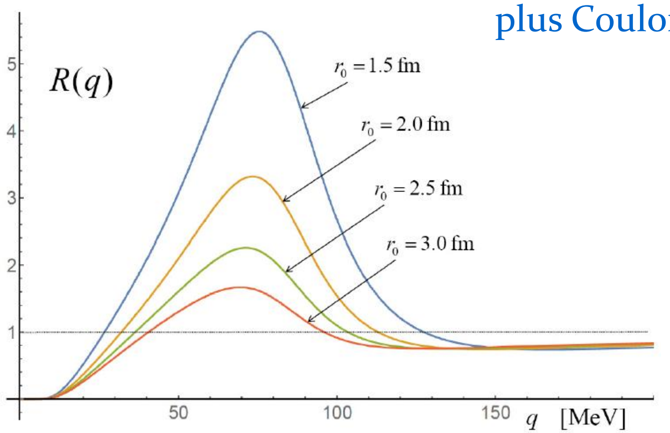
resonance only



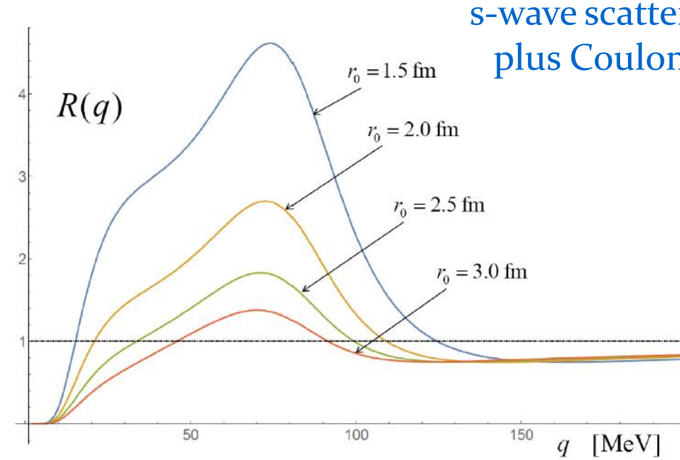
resonance plus s-wave scattering



resonance plus Coulomb



resonance plus s-wave scattering plus Coulomb



S. Bazak & St. Mrówczyński, arXiv: 2001.11351

How to measure yield of ^{4}Li

$$\boxed{\frac{dN_{\mathrm{Li}}}{d\mathbf{p}} = S_R \frac{dN_p}{d\mathbf{p}} \frac{dN_{{}^3\mathrm{He}}}{d\mathbf{p}}}$$

p - momentum per nucleon

$$S_R \equiv \int d^3q \, R_R(\mathbf{q})$$

correlation function where only the ^{4}Li resonance contributes

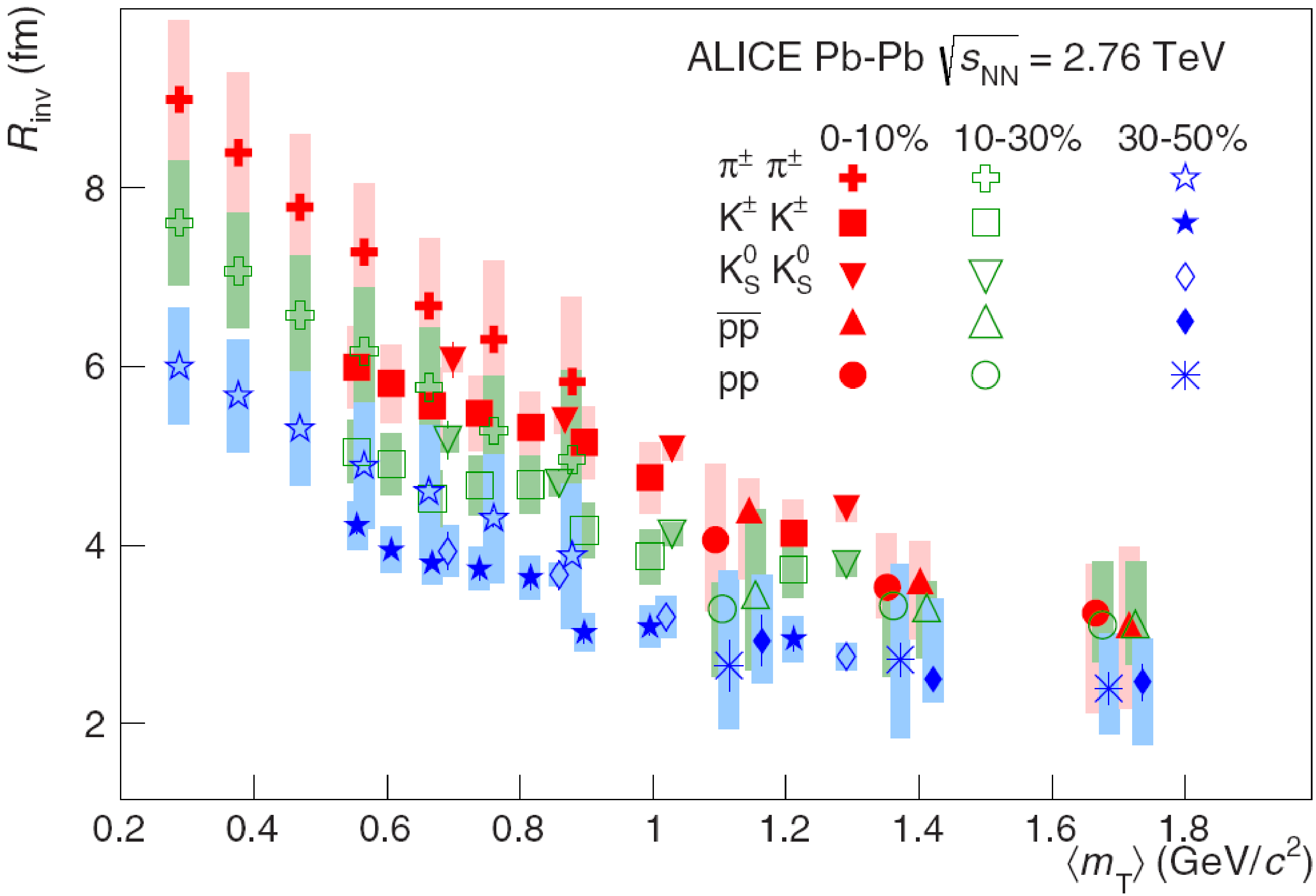
Conclusions

***h*-*D* correlations**

- Hadron-deuteron correlations carry information about source of deuterons.
- Measurement of *h*-*D* & *h*-*p* correlation function can tell us whether deuterons are directly emitted from a fireball like protons or deuterons are formed due to final state interactions.
- *p*-*D* correlation functions show a sufficient sensitivity to a size of particle source to falsify the thermal or coalescence model.

^{4}He vs. ^{4}Li

- The thermal and coalescence models give different predictions on the ratio of yields of ^{4}Li to ^{4}He.
- In the thermal model the ratio of yields is independent of collision centrality.
- In the coalescence model the ratio is maximal for central collisions and rapidly decreases when one goes to peripheral collisions.
- Since ^{4}Li can be observed through the correlation function of ^{3}He-*p*, the correlation needs to be measured.



ALICE Pb-Pb $\sqrt{s_{NN}}$ = 2.76 TeV
0-10%
10-30%
30-50%
$\pi^{\pm}\pi^{\pm}$
$K^{\pm}K^{\pm}$
$K^0_S K^0_S$
$\overline{p}\overline{p}$
pp
R_{inv} (fm)
$\langle m_T \rangle$ (GeV/c^2)
2
4
6
8
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8