APPLICATIONS OF TRANSPORT THEORY TO QUARK-GLUON PLASMA*

Stanisław Mrówczyński**

Institut für Theoretische Physik
Universität Regensburg
Postfach 397, D-8400 REGENSBURG, West Germany

ABSTRACT: Applications of transport theory methods in quark-gluon plasma physics are reviewed. At first, the basic transport equations and parton distribution functions are presented and briefly discussed. Then, I consider the *locally colorless* plasma, where the dynamical content of QCD enters only through the parton-parton scattering cross sections. The hydrodynamics of such a plasma and transport coefficients are discussed. Further, the response of the *colored* plasma to a chromodynamic field is considered. The knowledge of the response function allows to study plasma oscillations around any quasi-stable *colorless* state, in particular, around the global thermodynamical equilibrium. Then, I discuss the hydrodynamic description of the *colored* plasma and the respective kinetic coefficients. At the end a very brief review of other transport theory applications to quark-gluon plasma is given.

March 1989

---

* Published in *Quark-Gluon Plasma* edited by R. Hwa, World Scientific, Singapore, 1990.

** Permanent address: Institute for Nuclear Studies, ul. Hoża 69, PL-00-681 WARSAW, Poland

1. INTRODUCTION

The quantum chromodynamics (QCD) with quarks and gluons as fundamental constituents is recognized as the dynamical theory, or at least, as a candidate of such a theory for the strong interactions (see *e.g.* [Ynd83]). The quark-gluon plasma (QGP) - a macroscopic system of deconfined quarks and gluons, appears as a many body aspect of QCD, and the existence of QGP in the early Universe, or perhaps in the compact stellar objects, is, in fact, unavoidable consequence of QCD. This explains the interest in studies of QGP, in particular, in those done in the framework of transport theory.

The transport, or kinetic, theory provides a framework to consider systems out of thermodynamical equilibrium. Although the theory was initiated more than 100 years ago - Boltzmann derived his famous equation in 1872, the theory is still under vital development. Application of the Boltzmann's ideas to systems which are relativistic and of quantum nature is faced with difficulties which have been overcome only partly till now. For a review see the monography [Gro80]. In the case of the quark-gluon plasma specific difficulties appear due to the non-Abelian character of the dynamics which governs the system. In spite of this, the transport theory approach to QGP is in fast progress and some interesting results have been found already.

Because the generation of QGP is supposed to proceed in relativistic heavy-ion collisions, there is a *practical* aspect of the studies on non-equilibrium QGP. The point is that the life time of the plasma produced, if indeed produced, in these collisions is not much longer than the characteristic time scale of parton processes*. Therefore QGP can achieve, in the best case, only a quasi-equilibrium state, and the studies of nonequilibrium phenomena are of importance to discriminate the characteristic features of QGP produced in laboratory experiments.

The aim of this article is to give a systematic presentation of applications of the kinetic theory methods in the studies of QGP. I start the review with the brief discussion of kinetic equations and of distribution functions of quarks and gluons (Sec. 2). The problem of derivation of transport equations is completely omitted here since it is the topic of the article [Elz89]. The discussion of applications of the transport theory to QGP splits into two branches. The first one concerns the phenomena of *locally colorless* plasma.** The dynamical content of QCD enters here only through the cross-sections of parton-parton interactions. Consequently, the discussion is of rather general character and it fits, with minor modifications, to any relativistic system. I consider the hydrodynamic limit of the kinetic theory and the transport coefficients (Secs. 3 and 4).

The characteristic features of QGP appear when the plasma is not locally colorless and consequently, it interacts with the chromodynamic mean field. The response of the plasma to this field is discussed in Secs. 5 and 6. Then, I analyze the oscillations around the

---

* The word *parton* is used as a common name of quarks and gluons.

** I call the plasma *locally colorless* if the color four-current vanishes at each space-time point. It differs from the terminology used in the electron-ion plasma physics, where the plasma is called *locally neutral* if the electric charge (zero component of electromagnetic four-current) is everywhere zero.

global thermodynamical equilibrium state (Sec. 7) and in a two-stream system (Sec. 8). In the later case, specific plasma instabilities occur. The hydrodynamic description of the *colored* plasma and the respective transport coefficients are considered in Sec. 9 and 10. At the end, in Sec. 11 the final remarks are collected.

Presenting the QGP transport theory I try to avoid model dependent concepts concerning the mechanism of confinement, or of nucleus-nucleus interactions. However, a very crucial assumption is made that the plasma is *perturbative, i.e.* that the partons weakly interact with each other due to the smallness of the QCD coupling constant. Indeed, because of the asymptotic freedom, QGP becomes *perturbative* at temperatures much greater than the QCD scale parameter Λ, see *e.g.* [Kal84]. Since Λ is of order 200 MeV [Ynd83], the temperature, at which the coupling constant is small, are, at least, of order 1 GeV. However, one believes that many results obtained in the framework of *perturbative* QCD can be extrapolated to the *nonperturbative* regime.

I do not discuss the numerical values of parameters and relevance of the results for QGP from heavy-ion collisions. All these problems are very important for the experimental studies of QGP, however they are still a matter of debates and controversies. Because of the scope of the review I do not touch several *hot* topics of the QGP physics, where the transport theory methods have been successfully applied, see Sec. 11.

In the whole article the units are used, where $c = k = \hbar = 1$, the metric tensor is diagonal and $g_{00} = -g_{11} = -g_{22} = -g_{33} = 1$.

2. THE DISTRIBUTION FUNCTIONS AND TRANSPORT EQUATIONS

The (anti-)quark distribution function $f(p,x)$ $\big(\bar f(p,x)\big)$ is a hermitian $N \times N$ matrix in color space (for a $SU(N)$ color group) with p denoting quark four-momentum and x space-time position [Hei83, Win84, Elz86a]. The function transforms under local gauge transformations as

$$f(p,x) \rightarrow U(x) f(p,x) U^\dagger(x) \; . \tag{2.1a}$$

The color indices are suppressed everywhere. The gluon distribution function [Elz86b] is a hermitian $(N^2-1) \times (N^2-1)$ matrix [Mro89] and it transforms as

$$G(p,x) \rightarrow M(x) G(p,x) M^\dagger(x) \; , \tag{2.1b}$$

where

$$M_{ab}(x) = Tr\big[\tau_a U(x) \tau_b U^\dagger(x)\big]$$

with $\tau_a,\ a = 1,...,N^2-1$ being the $SU(N)$ group generators.

One sees that, in contrast to the distribution functions known from the physics of atomic gases, the distribution functions of quarks and gluons have not simple probabilistic interpretation due to the gauge dependence. It is however not surprising if one realizes that the question of probability to find, let me say, a red quark with momentm p in space point x is not physical since the color of a quark can be changed by means of a gauge transformation.

It follows from the transformation laws (2.1) that the traces of the distribution functions are gauge independent, and consequently they can have familiar probabilistic interpretation. Indeed, the question of probability to find a quark of any color with a four-momentumin p in a space-time point x is of physical character since this question is gauge independent.

Quantities which are color (gauge) independent, like the baryon current or the energy momentum tensor, are expressed only through the traces of distribution functions. In other words, the distribution functions summed over colors enter these quantities. The baryon current reads

$$b^\mu(x) = \int \frac{d^3p}{(2\pi)^3 E} p^\mu \big[Tr f(p,x) - Tr \bar{f}(p,x)\big], \tag{2.2}$$

where $p \equiv p^\mu = (E, \mathbf{p})$. The energy-momentum tensor is expressed as follows

$$t^{\mu\nu}(x) = \int \frac{d^3p}{(2\pi)^3 E} p^\mu p^\nu \big[Tr f(p,x) + Tr \bar{f}(p,x) + Tr G(p,x)\big]. \tag{2.3}$$

To simplify the notation I write the expressions as if quarks and gluons are of the same mass, or massless. The modifications due to the mass difference are trivial.

The color current, which is a gauge dependent quantity, is expressed not only through the traces of distribution functions and it reads*

$$\begin{aligned} j^\mu(x) = -\frac{1}{2} g \int \frac{d^3p}{(2\pi)^3 E} p^\mu \Big[f(p,x) - \bar{f}(p,x) - \frac{1}{N} Tr\big[f(p,x) - \bar{f}(p,x)\big] + \\ + 2i\tau_a f_{abc} G_{bc}(p,x) \Big] , \end{aligned} \tag{2.4}$$

where g is the coupling constant and f_{abc} is the structure constant of the $SU(N)$ group.

The quarks and gluons are treated as spinless. Additionally, I consider the quarks of one flavour only. However, if the plasma is in equilibrium with respect to spin and quark flavours, these quantum numbers can be treated as nondistinguishable internal degrees of freedom of partons. As seen from Eqs. 2.2 - 2.4 the partons are also assumed to satisfy the mass-shell constraints. In principle, the mass-shell equations can include medium corrections, and consequently the quark and gluon masses can differ from those of the *perturbative* vacuum.

The distribution functions of quarks and gluons satisfy the following set of transport equations [Hei83, Win84, Elz86a, Elz86b, Elz88, Mro89]

$$p^\mu \mathcal{D}_\mu f(p,x) + g p^\mu \frac{\partial}{\partial p_\nu} \frac{1}{2} \{F_{\mu\nu}(x), f(p,x)\} = C[f, \bar{f}, G] , \tag{2.5a}$$

* For the reader convenience I apply in this article the sign convention which is used in [Elz89]. Therefore, the most of signs differ from those which appear in my previous works.

$$p^\mu \mathcal{D}_\mu \bar{f}(p,x) - g p^\mu \frac{\partial}{\partial p_\nu} \frac{1}{2} \{F_{\mu\nu}(x), \bar{f}(p,x)\} = \bar{C}[f,\bar{f},G] \;, \tag{2.5b}$$

$$p^\mu \widetilde{\mathcal{D}}_\mu G(p,x) + g p^\mu \frac{\partial}{\partial p_\nu} \frac{1}{2} \{\mathcal{F}_{\mu\nu}(x), G(p,x)\} = C_g[f,\bar{f},G] \;, \tag{2.5c}$$

where $\{...,...\}$ denotes the anicommutator; $\mathcal{D}_\mu$ and $\widetilde{\mathcal{D}}_\mu$ are the covariant derivatives which act as

$$\mathcal{D}_\mu = \partial_\mu - ig[A_\mu(x), ...] \;, \qquad \widetilde{\mathcal{D}}_\mu = \partial_\mu - ig[\mathcal{A}_\mu(x), ...] \;,$$

where A_μ and $\mathcal{A}_\mu$ are the mean-field four-potentials defined as

$$A^\mu(x) = A^\mu_a(x)\tau_a \;, \qquad \mathcal{A}^\mu_{ab}(x) = -i f_{abc} A^\mu_c(x) \;.$$

$F_{\mu\nu}$ and $\mathcal{F}_{\mu\nu}$ are the mean-field stress tensors defined analogously to the four-potentials. The mean-field is generated by the color current of quarks and gluons (2.4) and the respective equation is

$$\mathcal{D}_\mu F^{\mu\nu}(x) = j^\nu(x) \;. \tag{2.6}$$

C, $\bar{C}$ and C_g are the collisions terms which equal zero in the collisionless limit *i.e.* when the plasma evolution is dominated by the mean-field effects*. The collision terms of QGP kinetic equations have not been derived yet, however it has been argued [Mro87b, Mro88d] that these terms should be formally similar to those of the so-called Waldmann-Snider equations describing system of spinning particles [Gro80]. In the applications of Eqs. 2.5 the collision terms in the relaxation time approximation have been used. Then, these terms read

$$C = \nu p_\mu u^\mu \Big(f^{eq}(p,x) - f(p,x) \Big) \;, \tag{2.7a}$$

$$\bar{C} = \bar{\nu} p_\mu u^\mu \Big(\bar{f}^{eq}(p,x) - \bar{f}(p,x) \Big) \;, \tag{2.7b}$$

$$C_g = \nu_g p_\mu u^\mu \Big(G^{eq}(p,x) - G(p,x) \Big) \;, \tag{2.7c}$$

where ν is the equilibration rate parameter (the inverse relaxation time), which is usually identified with the particle inverse free flight time (as we will see this identification is not always correct); u^μ is the hydrodynamic four-velocity, which in the plasma rest frame is (1,0,0,0); $f^{eq}(p,x)$ is the local thermodynamical equilibrium distribution function. In the case of global equilibrium, the distribution function is proportional to the unit matrix in the color space *i.e.* it can be expressed as

$$f^{eq}_{ij}(p) = \delta_{ij} n^{eq}(p) \;, \quad i,j = 1,...,N \;, \tag{2.8}$$

where $n^{eq}(p)$ is the Fermi-Dirac equilibrium distribution function. The respective quantities corresponding to antiquarks and gluons are of analogous, to the quark case, meaning.

---

* This occurs when the characteristic mean-field frequency is much greater than the parton collision frequency.

The collision terms (2.7) make the distribution functions evolve towards the local thermodynamical equilibrium with the characteristic time equal ν^{-1}.

Let me note that the set of transport equations (2.5, 2.6) is covariant with respect to the gauge transformations (2.1). The equilibrium distribution functions of the form (2.8) are gauge invariant, and they give zero color current (2.4).

The validity of the kinetic equations (2.5) is discussed in [Elz89], here I only mention that these equations describe the evolution of QGP in the semiclassical limit. In spite of this limitation the content of these transport equations is, as we will see later, quite rich.

Let me note here that the quark transport equations (2.5a, 2.5b) have been analytically solved [Bia84, Bia85, Bia 88b] imposing boost-invariant constraints [Bjo83]*. The solutions have been then used to discuss the variety of problems of QGP, for review see [Bia89].

Variants of the QGP transport theory, other than the presented above, have been also considered in the literature. An approach, where the central role play the equations describing the evolution of momentum moments of distribution functions have been discussed in [Car87]. Transport equations different than (2.5) have been advocated in [Sil85]. *Effective* kinetic theory models, which incorporate some features of QCD at a phenomenological level, have been studied in [Lee86, Miz88].

3. THE COLORLESS PLASMA AND IDEAL HYDRODYNAMICS

When the plasma is *locally colorless* (the distribution functions are proportional to the unit matrices in the color space), there is no color current and we expect that there is zero mean field ($F^{\mu\nu}(x) = 0$). Then, taking the trace of Eqs. 2.5 one finds

$$p^\mu \partial_\mu q(p,x) = c[q,\bar{q},g] \ , \tag{3.1a}$$

$$p^\mu \partial_\mu \bar{q}(p,x) = \bar{c}[q,\bar{q},g] \ , \tag{3.1b}$$

$$p^\mu \partial_\mu g(p,x) = c_g[q,\bar{q},g] \ , \tag{3.1c}$$

where

$$q(p,x) = Trf(p,x) \ , \qquad \bar{q}(p,x) = Tr\bar{f}(p,x) \ , \qquad g(p,x) = TrG(p,x) \tag{3.2a}$$

and

$$c[q,\bar{q},g] = TrC[f,\bar{f},G] \ , \quad \bar{c}[q,\bar{q},g] = Tr\bar{C}[f,\bar{f},G] \ , \quad c_g[q,\bar{q},g] = TrC_g[f,\bar{f},G] \ . \tag{3.2b}$$

Because the trace of a commutator is zero, there are normal derivatives instead of covariant ones in (3.1).

---

* The boost-invariant hydrodynamics has been earlier studied in [Gor78].

Calculating the transport coefficients I will use the collision terms in the relaxation time approximation *i.e.*

$$c = \nu p_\mu u^\mu \Big(q^{eq}(p,x) - q(p,x) \Big) \; , \quad \bar{c} = \bar{\nu} p_\mu u^\mu \Big(\bar{q}^{eq}(p,x) - \bar{q}(p,x) \Big) \; ,$$

$$c_g = \nu_g p_\mu u^\mu \Big(g^{eq}(p,x) - g(p,x) \Big) \; . \tag{3.3}$$

In the case of *colorless* plasma, one expects that the collision terms are of the standard Boltzmann-like form, see *e.g.* [Gro80]. If the effects of quantum statistics are taken into account one gets the collision terms of the Nordheim-Uehling-Uhlenbeck form [Nor28, Uhl33]. Such collision terms are briefly considered in [Hos85, Cha85] in the context of QGP. Since the explicit expressions of these collision terms are not considered in this article I do not write them down. Let me only note that any collision terms have to satisfy the relations

$$\int \frac{d^3p}{(2\pi)^3 E} \{ c[q,\bar{q},g] - \bar{c}[q,\bar{q},g] \} = 0 \; , \tag{3.4a}$$

$$\int \frac{d^3p}{(2\pi)^3 E} p^\mu \{ c[q,\bar{q},g] + \bar{c}[q,\bar{q},g] + c_g[q,\bar{q},g] \} = 0 \tag{3.4b}$$

in order to be consistent with the baryon number and energy-momentum conservation:

$$\partial_\mu b^\mu(x) = 0 \; , \tag{3.5}$$

$$\partial_\mu t^{\mu\nu}(x) = 0 \; . \tag{3.6}$$

Let me define the entropy four-flux of the *colorless* plasma as (see *e.g.* [Bal75])

$$\begin{aligned} s^\mu(x) = \int \frac{d^3p}{(2\pi)^3 E} p^\mu \{ &-[(1 - q(p,x)) \ln(1 - q(p,x)) + q(p,x) \ln q(p,x)] + \\ &-[(1 - \bar{q}(p,x)) \ln(1 - \bar{q}(p,x)) + \bar{q}(p,x) \ln \bar{q}(p,x)] + \\ &+[(1 + g(p,x)) \ln(1 + g(p,x)) - g(p,x) \ln g(p,x)] \} \; , \end{aligned} \tag{3.7}$$

where the quantum statistics of quarks and gluons have been taken into account.

Due to the second principle of thermodynamics one expects $\partial_\mu s^\mu(x) \geq 0$. To prove this relation, which is known as H-theorem in kinetic theory, the explicit form of collision terms is needed. Indeed,

$$\partial_\mu s^\mu(x) = -\int \frac{d^3p}{(2\pi)^3 E} \Bigg[c \ln\Big[\frac{q(p,x)}{1 - q(p,x)} \Big] + \bar{c} \ln\Big[\frac{\bar{q}(p,x)}{1 - \bar{q}(p,x)} \Big] + c_g \ln\Big[\frac{g(p,x)}{1 + g(p,x)} \Big] \Bigg] \; . \tag{3.8}$$

With the Nordheim-Uehling-Uhlenbeck collisions terms (see *e.g.* [Hos85]) one checks following the standard procedure [Gro80, Bal75] that $\partial_\mu s^\mu(x)$ from (3.8) is indeed nonnegative.

When the system reaches local equilibrium $\partial_\mu s^\mu(x) = 0$, therefore this equation defines the local equilibrium distribution functions. In principle, the form of the local equilibrium function can not be determined as long the collision terms are unknown. However, assuming the conservation laws (3.5, 3.6), or equivalently the relations (3.4), one shows that $\partial_\mu s^\mu(x) = 0$ when the entropy four-flux (3.7) is calculated with the following distribution functions

$$q^{eq}(p,x) = \Big[\exp\big(\beta^\mu(x)p_\mu - \beta(x)\mu(x)\big) + 1\Big]^{-1} , \tag{3.9a}$$

$$\bar{q}^{eq}(p,x) = \Big[\exp\big(\beta^\mu(x)p_\mu + \beta(x)\mu(x)\big) + 1\Big]^{-1} , \tag{3.9b}$$

$$g^{eq}(p,x) = \Big[\exp\big(\beta^\mu(x)p_\mu\big) - 1\Big]^{-1} , \tag{3.9c}$$

where $\beta^\mu(x) = \beta(x)u^\mu(x)$, $\beta(x) = T^{-1}(x)$; $T(x)$, $u^\mu(x)$, $\mu(x)$ are identified with the local temperature, the hydrodynamic velocity and the local quark chemical potential, respectively.

Considering the hydrodynamics of *colored* plasma (Sec. IX) we will see that the problem of determination of local equilibrium distribution functions is far not academic and not as trivial as it looks here.

The equations (3.5, 3.6) with the baryon current and the energy-momentum tensor calculated with the local equilibrium distribution functions (3.9) constitute the set of hydrodynamic equations of ideal quark-gluon liquid. Following [Gro80] one easily finds

$$b^\mu(x) = b(x)u^\mu(x) , \tag{3.10}$$

$$t^{\mu\nu}(x) = \big[U(x) + P(x)\big]u^\mu(x)u^\nu(x) - P(x)g^{\mu\nu} , \tag{3.11}$$

where $b(x)$, $U(x)$ and $P(x)$ are the baryon density, the energy density and the pressure, which are expressed by the well-known ideal gas formulas.

Substituting (3.10, 3.11) to (3.5, 3.6) one can get several forms of relativistic hydrodynamic equations of ideal fluid, see *e.g.* [Lan63]. In particular, splitting the derivative ∂^μ into its components parallel (convective) and orthogonal to u^μ as

$$\partial^\mu = u^\mu D + \nabla^\mu ,$$

where

$$D = u_\mu \partial^\mu , \qquad \nabla^\mu = \Delta^{\mu\nu}\partial_\nu , \qquad \Delta^{\mu\nu} = g^{\mu\nu} - u^\mu u^\nu ,$$

one finds [Hos85]

$$DU(x) + \Big(U(x) + P(x)\Big)\nabla_\mu u^\mu(x) = 0 , \tag{3.12}$$

$$\Big(U(x) + P(x)\Big)Du^\mu(x) - \nabla^\mu P = 0, \tag{3.13}$$

$$Db(x) + b(x)\nabla_\mu u^\mu(x) = 0 . \tag{3.14}$$

Using the thermodynamic identities, the following relations can be also derived [Hos85]

$$\frac{DT}{T} = -\left(\frac{\partial P}{\partial U}\right)_b \nabla_\mu u^\mu , \tag{3.15}$$

$$TD\left(\frac{\mu}{T}\right) = -\left(\frac{\partial P}{\partial b}\right)_U \nabla_\mu u^\mu \ . \tag{3.16}$$

These relations will be used to calculate transport coefficients.

4. THE VISCOUS LIQUID AND TRANSPORT COEFFICIENTS

While $C[q^{eq}, \bar{q}^{eq}, g^{eq}] = 0$, one sees that $p^\mu \partial_\mu q^{eq}(p,x) \neq 0$ for nonuniform T, u^μ, or μ. Therefore the local equilibrium distribution functions are not solutions of the transport equations (3.1). Writing $q(p,x) = q^{eq}(p,x) + \delta q(p,x)$ and analogous expressions for antiquark and gluon distribution functions, the transport equations (3.1) imply that

$$\delta q(p,x) = -\big(\nu p^\lambda u_\lambda\big)^{-1} p^\mu \partial_\mu q^{eq}(p,x) \ , \tag{4.1a}$$

$$\delta \bar{q}(p,x) = -\big(\bar{\nu} p^\lambda u_\lambda\big)^{-1} p^\mu \partial_\mu \bar{q}^{eq}(p,x) \ , \tag{4.1b}$$

$$\delta g(p,x) = -\big(\nu_g p^\lambda u_\lambda\big)^{-1} p^\mu \partial_\mu g^{eq}(p,x) \ , \tag{4.1c}$$

where δq, $\delta\bar{q}$, δg and their gradients have been assumed much smaller than, repectively, q^{eq}, $\bar{q}^{eq}$, g^{eq} and their gradients.

Calculating the baryon current and the energy-momentum tensor with $q = q^{eq} + \delta q$, $\bar{q} = \bar{q}^{eq} + \delta\bar{q}$ and $g = g^{eq} + \delta g$ and comparing with (3.10, 3.11), one finds additional dissipative terms. Specifically,

$$b^\mu(x) = b(x) u^\mu(x) + b^\mu_{dis}(x) \ , \tag{4.2}$$

$$t^{\mu\nu}(x) = \big[U(x) + P(x)\big] u^\mu(x) u^\nu(x) - P(x) g^{\mu\nu} + t^{\mu\nu}_{dis}(x) \ , \tag{4.3}$$

where

$$b^\mu_{dis}(x) = \int \frac{d^3p}{(2\pi)^3 E} p^\mu \{\delta q(p,x) - \delta\bar{q}(p,x)\} \ , \tag{4.4}$$

$$t^{\mu\nu}_{dis}(x) = \int \frac{d^3p}{(2\pi)^3 E} p^\mu p^\nu \{\delta q(p,x) + \delta\bar{q}(p,x) + \delta g(p,x)\} \ . \tag{4.5}$$

The equations of hydrodynamics are still those of (3.5, 3.6), however the dissipative terms from (4.2, 4.3) have to be taken into account. Their structure depends on the definition of what constitutes the local rest frame of the fluid *i.e.* on the definition of hydrodynamic four-velocity. One natural proposition made by Eckart [Eck40] associates the hydrodynamic velocity with the current, in the case of QGP - the baryon current. Namely,

$$u^\mu(x) = \frac{b^\mu(x)}{b^\nu(x) u_\nu(x)} \ . \tag{4.6}$$

If the plasma is baryonless, Eq. 4.6 is obviously ill defined. Another definition is the one by Landau and Lifshitz [Lan63], which associates the hydrodynamic velocity with the momentum flow:

$$u^\mu(x) = \frac{u_\nu(x) t^{\nu\mu}(x)}{u_\sigma(x) u_\lambda(x) t^{\sigma\lambda}(x)} \ . \tag{4.7}$$

There are possible other definitions interpolating between (4.6) and (4.7) [Isr79]. With the definition (4.7), the requirements that the dissipative terms are of the first order in gradients and that the entropy increases with time lead to [Lan63]

$$b^{\mu}_{dis} = \kappa\Big(\frac{bT}{U+P}\Big)^2 \nabla^{\mu}\Big(\frac{\mu}{T}\Big) \ , \tag{4.8}$$

$$t^{\mu\nu}_{dis} = \eta(\nabla^{\mu}u^{\nu} + \nabla^{\nu}u^{\mu} - \frac{2}{3}\Delta^{\mu\nu}\nabla_{\sigma}u^{\sigma}) + \zeta\Delta^{\mu\nu}\nabla_{\sigma}u^{\sigma} \ , \tag{4.9}$$

where κ, η and ζ are the transport coefficients of heat conductivity, shear viscosity and bulk viscosity. The general discussion of the dissipative term form can be found in [Isr79].

The calculation of the transport coefficients is more, or less straightforward in the relaxation time approximation and it has been done for a one component gas in [And74]. In the context of QGP the problem has been discussed in [Hos85, Dan85, Gav85, Cha85 and Czy86]. In my presentation I follow [Hos85]. At first, one evaluates b^{μ}_{dis} and $t^{\mu\nu}_{dis}$ substituting (4.1) into (4.4, 4.5). The convective derivatives of T, μ and u^{μ}, which are absent in (4.8, 4.9), arising from $\partial^{\mu}q^{eq}$, $\partial^{\mu}\bar{q}^{eq}$ and $\partial^{\mu}g^{eq}$ are eliminated by means of (3.15, 3.16) (the ideal hydrodynamics (3.12 - 3.14) is assumed), and finally one finds [Hos85]

$$\begin{aligned} b^{\mu}_{dis}(x) = T^{-1}\int \frac{d^3p}{(2\pi)^3E}p^{\mu}\Bigg[& \big[\frac{1}{\nu}q^{eq}(1-q^{eq}) - \frac{1}{\bar{\nu}}\bar{q}^{eq}(1-\bar{q}^{eq})\big] \\ & \times\big[p^{\sigma}u_{\sigma}\Big(\frac{\partial P}{\partial U}\Big)_b \nabla_{\lambda}u^{\lambda} + p_{\rho}X^{\rho} + \frac{p^{\sigma}p^{\lambda}}{p^{\nu}u_{\nu}}\nabla_{\sigma}u_{\lambda}\big] + \\ & + \big[\frac{1}{\nu}q^{eq}(1-q^{eq}) + \frac{1}{\bar{\nu}}\bar{q}^{eq}(1-\bar{q}^{eq})\big] \\ & \times\big[\Big(\frac{\partial P}{\partial b}\Big)_U \nabla_{\lambda}u^{\lambda} - \frac{U+P}{b}\frac{p_{\nu}}{p^{\sigma}u_{\sigma}}X^{\nu}\big]\Bigg] \ , \end{aligned} \tag{4.10}$$

$$\begin{aligned} t^{\mu\nu}_{dis}(x) = T^{-1}\int \frac{d^3p}{(2\pi)^3E}p^{\mu}p^{\nu}\Bigg[& \big[\frac{1}{\nu}q^{eq}(1-q^{eq}) + \frac{1}{\bar{\nu}}\bar{q}^{eq}(1-\bar{q}^{eq}) + \frac{1}{\nu_g}g^{eq}(1+g^{eq})\big] \\ & \times\big[p^{\sigma}u_{\sigma}\Big(\frac{\partial P}{\partial U}\Big)_b \nabla_{\lambda}u^{\lambda} + p_{\rho}X^{\rho} + \frac{p^{\sigma}p^{\lambda}}{p^{\nu}u_{\nu}}\nabla_{\sigma}u_{\lambda}\big] + \\ & + \big[\frac{1}{\nu}q^{eq}(1-q^{eq}) - \frac{1}{\bar{\nu}}\bar{q}^{eq}(1-\bar{q}^{eq})\big] \\ & \times\big[\Big(\frac{\partial P}{\partial b}\Big)_U \nabla_{\lambda}u^{\lambda} - \frac{U+P}{b}\frac{p_{\rho}}{p^{\sigma}u_{\sigma}}X^{\rho}\big]\Bigg] \ , \end{aligned} \tag{4.11}$$

where

$$X^{\mu} = \frac{\nabla^{\mu}P}{U+P} - \frac{\nabla^{\mu}T}{T} = \frac{bT}{U+p}\nabla^{\mu}\Big(\frac{\mu}{T}\Big) \ .$$

Further, we consider the space components of b^{μ}_{dis} and $t^{\mu\nu}_{dis}$. Going to the Landau-Lifshitz local rest frame of the fluid and comparing the result with (4.8, 4.9) one gets [Hos85]

$$\kappa = \frac{1}{3T^2}\int \frac{d^3p}{(2\pi)^3}\mathbf{p}^2\Big[\frac{1}{\nu}q^{eq}(1-q^{eq})\big(1-\frac{U+P}{bE}\big)^2+ \\ +\frac{1}{\bar{\nu}}\bar{q}^{eq}(1-\bar{q}^{eq})\big(1+\frac{U+P}{bE}\big)^2+\frac{1}{\nu}g^{eq}(1+g^{eq})\Big] \,, \tag{4.12}$$

$$\zeta = \frac{1}{3T}\int \frac{d^3p}{(2\pi)^3}\Bigg[\Big[\frac{1}{\nu}q^{eq}(1-q^{eq})+\frac{1}{\bar{\nu}}\bar{q}^{eq}(1-\bar{q}^{eq})+\frac{1}{\nu_g}g^{eq}(1+g^{eq})\Big] \\ \times\Big[\frac{\mathbf{p}^4}{3E^2}-\Big(\frac{\partial P}{\partial U}\Big)_b\mathbf{p}^2\Big]+ \\ -\Big[\frac{1}{\nu}q^{eq}(1-q^{eq})-\frac{1}{\bar{\nu}}\bar{q}^{eq}(1-\bar{q}^{eq})\Big]\Big(\frac{\partial P}{\partial b}\Big)_U\Bigg] \,, \tag{4.13}$$

$$\eta = \frac{1}{15T}\int \frac{d^3p}{(2\pi)^3}\frac{\mathbf{p}^4}{E^2}\Big[\frac{1}{\nu}q^{eq}(1-q^{eq})+\frac{1}{\bar{\nu}}\bar{q}^{eq}(1-\bar{q}^{eq})+\frac{1}{\nu_g}g^{eq}(1+g^{eq})\Big] \,. \tag{4.14}$$

For illustration let me consider the quarkless plasma ($q(p,x)=\bar{q}(p,x)=0$). Since the thermal gluons are assumed massles* $\big(\frac{\partial P}{\partial U}\big)_b = 1/3$. Additionally, $\big(\frac{\partial P}{\partial b}\big)_U = 0$. Then, one finds from Eqs. 4.12 - 4.13 [Hos85]

$$\kappa = \frac{4\pi^2}{45}(N^2-1)\frac{1}{\nu_g}T^3 \,, \qquad \zeta = 0 \,, \qquad \eta = \frac{4\pi^2}{225}(N^2-1)\frac{1}{\nu_g}T^4 \,. \tag{4.15}$$

To estimate the numerical values of the kinetic coefficients one has to express the equilibration rate parameters ν, $\bar{\nu}$ and ν_g through the QCD parameters and through the thermodynamical characteristics of QGP. Usually ν is identified with the inverse mean free flight time. Then, assuming that the plasma particles move with the velocity of light (the plasma is ultrarelativistic) one finds

$$\nu = \rho\sigma_t^{qq} + \bar{\rho}\sigma_t^{q\bar{q}} + \rho_g\sigma_t^{qg} \,,$$

$$\bar{\nu} = \rho\sigma_t^{\bar{q}q} + \bar{\rho}\sigma_t^{\bar{q}\bar{q}} + \rho_g\sigma_t^{\bar{q}g} \,,$$

$$\nu_g = \rho\sigma_t^{gq} + \bar{\rho}\sigma_t^{g\bar{q}} + \rho_g\sigma_t^{gg} \,,$$

where ρ, $\bar{\rho}$ and ρ_g are the densities of quarks, antiquarks and gluons, respectively; σ_t^{ab} is the so-called transport cross section [Lif81] of interaction of a parton a with with a parton b,

$$\sigma_t^{ab} = \int d\Omega \frac{d\sigma^{ab}}{d\Omega}\sin^2\Theta \,. \tag{4.16}$$

* In fact, the dispersion relation of gluons in the plasma medium differs from that in the *perturbative* vacuum, and gluons are not exactly massless.

The $\sin^2 \Omega$ weight in (4.16) damps the role of small-angle scattering which proceed with large cross sections giving small contribution to the transport processes because of smallness of momentum transfer.

The *perturbative* QCD gives the Rutherford-like cross section of parton-parton scattering *i.e.* at small angle scattering, the cross section behaves as $\sin^{-4} \Theta/2$. Therefore, in spite of the $\sin^2 \Theta$ weight, the integral (4.16) is logarythmically divergent.

One easily finds that

$$\sigma_t \sim \alpha_s^2 \ln(1/\Theta_{min})\, T^{-2} \ ,$$

where α_s is the QCD couplig constant ($\alpha_s = g^2/4\pi$) and Θ_{min} is the small-angle cut-off, which can be estimated as follows. The Coulomb forces are screened in the plasma at the distance - the Dedye length, of order $1/gT$ (see Eq. 7.13), and consequently, the minimal momentum transfer in parton-parton scattering is of order gT. Because the characteristic parton momentum is T, $\Theta_{min} \sim 1/g$. Taking into account that $\rho \sim T^3$ for massless partons, one finally finds that

$$\nu \sim \alpha_s^2 T \ln 1/\alpha_s \ . \tag{4.17}$$

The above reasoning is however not quite correct. Considering the scattering of the relativistic partons one has to take into account, except Coulomb chromoelectric interaction, the chromomagnetic interaction, which also gives the Rutherford singularity. However, in contrast to the chromoelectric field, the chromomagnetic field is not screened in the *perturbative* QCD plasma. Therefore the minimal scattering angle is zero, the transport cross section is infinite and the kinetic coefficients vanish. The problem has been recognized in [Dan85], where the magnetic screening due to a non-perturbative mechanism in QGP has been assumed.

It has been argued in a very recent paper [Bay88], that the Rutherford singularity is removed due to the screening of magnetic field of small but nonzero frequency. It appears because of a dynamical mechanism analogous to that one responsible for the Landau damping of plasma oscillations.

The transport coefficienets of QGP has been also considered from the point of view different than that presented here. Using the *effective* kinetic theory of quasiparticles - *colorless* quarks [Lee86], the transport coefficients have been calculated in [Cal86]. Another *effective* approach have been used in [Miz88]. Kubo-type formulas have been applied to the problem in [Hor87a, Hor87b]. The possibility to extract the transport coefficients from Monte Carlo lattice calculations has been discussed in [Sch86, Kar87].

The more general discussion of the kinetic coefficients of relativistic systems can be found in [Gro80].

5. THE COLORED PLASMA NEAR QUASI-STABLE COLORLESS STATE

In this section I consider, following [Mro89], the plasma which is *colored* but close to the quasi-stable *colorless* homogenous state. Then, the distribution functions can be written as

$$f_{ij}(p,x) = n(p)\delta_{ij} + \delta f_{ij}(p,x) \;, \tag{5.1a}$$

$$\bar{f}_{ij}(p,x) = \bar{n}(p)\delta_{ij} + \delta \bar{f}_{ij}(p,x) \;, \tag{5.1b}$$

$$G_{ab}(p,x) = n_g(p)\delta_{ab} + \delta G_{ab}(p,x) \;. \tag{5.1c}$$

The functions describing the deviation from the *colorless* sate are assumed much smaller that the respective *colorless* fuctions, and the same is assumed for the momentum gradients of these fuctions.

Substituting the functions (5.1) in (2.4) one gets

$$\begin{aligned} j^\mu(x) = -\frac{1}{2} g \int \frac{d^3p}{(2\pi)^3 E} p^\mu \Big[\delta f(p,x) - \delta \bar{f}(p,x) - \frac{1}{N} Tr\big[\delta f(p,x) - \delta \bar{f}(p,x)\big] + \\ + 2i\tau_a f_{abc} \delta G_{bc}(p,x)\Big] \;. \end{aligned} \tag{5.2}$$

One sees that the current occurs due to the deviation of the system from the *colorless* state. When the system approches this state there is no current and one expects that there is no mean field. Therefore I linearize Eq. 2.6 with respect to the four potential to the form

$$\partial_\mu F^{\mu\nu}(x) = j^\nu(x) \tag{5.3}$$

with $F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$. It should be stressed here that the linearization procedure does not cancel all non-Abelian effects, since gluons contribute to the color current (5.2). Therefore the gluon-gluon coupling, which is of essentially non-Abelian character is included in a specific way. Let me also note that in the linearized theory the color current is conserved (due to antisymmetry of $F^{\mu\nu}$) *i.e.* $\partial_\mu j^\mu = 0$.

Now I substitute the distribution functions (5.1) to the transport equations (2.5) with the collision terms (2.7). Linearizing the equations with respect to δf, $\delta \bar{f}$ and δG, one gets

$$\Big(p^\mu \partial_\mu + \nu p_\mu u^\mu\Big)\delta f(p,x) = -g p^\mu F_{\mu\nu}(x) \frac{\partial}{\partial p_\nu} n(p) + \nu p_\mu u^\mu \big(n^{eq}(p) - n(p)\big) \;, \tag{5.4a}$$

$$\Big(p^\mu \partial_\mu + \bar{\nu} p_\mu u^\mu\Big)\delta \bar{f}(p,x) = g p^\mu F_{\mu\nu}(x) \frac{\partial}{\partial p_\nu} \bar{n}(p) + \bar{\nu} p_\mu u^\mu \big(\bar{n}^{eq}(p) - \bar{n}(p)\big) \;, \tag{5.4b}$$

$$\Big(p^\mu \partial_\mu + \nu_g p_\mu u^\mu\Big)\delta G(p,x) = -g p^\mu \mathcal{F}_{\mu\nu}(x) \frac{\partial}{\partial p_\nu} n_g(p) + \nu_g p_\mu u^\mu \big(n_g^{eq}(p) - n_g(p)\big) \;. \tag{5.4c}$$

Performing the linearization one should remember that A^μ is of order of δf.

Treating the chromodynamic field as an external one, Eqs. 5.4 can be easily solved

$$\delta f(p,x) = -g \int d^4x' \Delta_p(x-x') \Big[p^\mu F_{\mu\nu}(x') \frac{\partial}{\partial p_\nu} n(p) - \nu p_\mu u^\mu \big(n^{eq}(p) - n(p)\big)\Big] \;, \tag{5.5a}$$

$$\delta \bar{f}(p,x) = g \int d^4x' \Delta_p(x-x') \Big[p^\mu F_{\mu\nu}(x') \frac{\partial}{\partial p_\nu} \bar{n}(p) + \bar{\nu} p_\mu u^\mu \big(\bar{n}^{eq}(p) - \bar{n}(p)\big)\Big] \;, \tag{5.5b}$$

$$\delta G(p,x) = -g \int d^4x' \Delta_p(x-x') \Big[p^\mu \mathcal{F}_{\mu\nu}(x') \frac{\partial}{\partial p_\nu} n_g(p) - \nu_g p_\mu u^\mu \big(n_g^{eq}(p) - n_g(p) \big) \Big] \;, \quad (5.5c)$$

where $\Delta_p(x)$ is the Green function of the kinetic operator with the collision term in the relaxation time approximation,

$$\Delta_p(x) = E^{-1} \Theta(t) e^{-\nu' t} \delta^{(3)}(\mathbf{x} - \mathbf{v}t) \;,$$

where t is the zero component of x, $x^\mu \equiv (t, \mathbf{x})$, $\mathbf{v} \equiv \mathbf{p}/E$ and $\nu' \equiv \nu p^\mu u_\mu$; in the plasma rest frame $\nu' = \nu$.

Substituting the solutions (5.5) in Eq. 5.2 and performing the Fourier transformation with respect to x-variable we get

$$j^\mu(k) = \sigma^{\mu\rho\lambda}(k) F_{\rho\lambda}(k) \tag{5.6}$$

with the color conductivity tensor expressed as

$$\sigma^{\mu\rho\lambda}(k) = i\frac{g^2}{2} \int \frac{d^3p}{(2\pi)^3 E} p^\mu p^\rho \Big[\Big(p^\sigma (k_\sigma + i\nu u_\sigma) \Big)^{-1} \frac{\partial n(p)}{\partial p_\lambda} + \\ + \Big(p^\sigma (k_\sigma + i\bar{\nu} u_\sigma) \Big)^{-1} \frac{\partial \bar{n}(p)}{\partial p_\lambda} + 2N \Big(p^\sigma (k_\sigma + i\nu_g u_\sigma) \Big)^{-1} \frac{\partial n_g(p)}{\partial p_\lambda} \Big] \;. \tag{5.7}$$

If the plasma *colorless* state is isotropic, which is the case for global equilibrium, one finds that $\sigma^{\mu\rho\lambda}(k) = \sigma^{\mu\rho}(k) u^\lambda$ and Eq. 5.6 gets more familiar form of the Ohm law, which in the plasma rest frame reads

$$j^\alpha(k) = \sigma^{\alpha\beta}(k) E^\beta(k) \;, \tag{5.8}$$

where the indices $\alpha, \beta, \gamma = 1, 2, 3$ label the space axes and $E^\alpha(k)$ is the α-component of the chromoelectric vector.

The conductivity tensor describes the responce of the QGP to the chromodynamic field. In the approximation used it is a color scalar (no color indices), or one can say that this tensor is proportional to the unit matrix in the color space. In Secs. 6, 7 and 8 we will extract the information about QGP contained in $\sigma^{\mu\rho\lambda}(k)$. The conductivity in the static limit ($k \to 0$) is also discussed in Sec. 10.

6. THE LINEARIZED QCD IN A MEDIUM

Let me introduce, as in electrodynamics, the polarization vector $\mathbf{P}(x)$ defined as

$$\mathrm{div}\mathbf{P}(x) = -\rho(x) \;, \quad \frac{\partial}{\partial t}\mathbf{P}(x) = \mathbf{j}(x) \;, \tag{6.1}$$

where ρ and $\mathbf{j}$ are the time-like and space-like components, respectively, of the color four-current, $j^\mu = (\rho, \mathbf{j})$. The definition (6.1) is self-consistent, only when the color current is conserved, not covariantly conserved. This just the case of *linearized* QCD.

Further, I define the chromoelectric induction vector $\mathbf{D}(x)$,

$$\mathbf{D}(x) = \mathbf{E}(x) + \mathbf{P}(x) \tag{6.2}$$

and the chromoelectric permeabilitty tensor, which relates the Fourier transformed $\mathbf{D}$ and $\mathbf{E}$ fields,

$$D^\alpha(k) = \epsilon^{\alpha\beta}(k) E^\beta(k) \ . \tag{6.3}$$

In this definition the permeability tensor is a color scalar (no color indices) since the conductivity tensor (5.7) is a color scalar.

Using the definitions (6.1, 6.2, 6.3) one easily finds that

$$\epsilon^{\alpha\beta}(k) = \delta^{\alpha\beta} - \frac{i}{\omega}\sigma^{\alpha 0 \beta}(k) - \frac{i}{\omega^2}\Big[k^\gamma \sigma^{\alpha\beta\gamma}(k) - k^\gamma \sigma^{\alpha\gamma\beta}(k)\Big] \tag{6.4}$$

with $\sigma^{\alpha\gamma\beta}(k)$ given by Eq. 5.7 ; ω is the time-like component of wave four-vector, $k^\mu \equiv (\omega, \mathbf{k})$. For the isotropic plasma the two last terms in Eq. 6.4 vanish, because such plasma does not interact with the mean chromomagnetic field.

The permeability tensor determines the chromodynamic properties of a medium. In particular, the spectrum of excitations, which are called plasmons in the *quantum* language, or the plasma oscillations in the *classical* one.

Because the equation of *linearized* QCD coincide (up to the trivial matrix structure) with those of electrodynamics, the dispersion relations of the plasma oscillations, or of plasmons, are those of electrodynamics and they read [Lan60, Sil61]

$$\det \mid \mathbf{k}^2 \delta^{\alpha\beta} - k^\alpha k^\beta - \omega^2 \epsilon^{\alpha\beta}(k) \mid = 0 \ . \tag{6.5}$$

The relation (6.5) gets simpler form for the isotropic plasma. Namely,

$$\epsilon_L(k) = 0 \ , \quad \epsilon_T(k) = \mathbf{k}^2/\omega^2 \ , \tag{6.6}$$

where the longitudinal and transversal parts of the permeability tensor are defined as

$$\epsilon^{\alpha\beta}(k) = \epsilon_T(k)\Big(\delta^{\alpha\beta} - k^\alpha k^\beta/\mathbf{k}^2\Big) + \epsilon_L(k) \ k^\alpha k^\beta/\mathbf{k}^2 \ . \tag{6.7}$$

The dispersion relations have transparent meaning in the case of electrodynamics. They are defined in such a way, that the plane wave with $\omega(\mathbf{k})$, which is the solution of dispersion equation (6.5), is automatically the solution of sourceless Maxwell equations in a medium. In other words, the solution of the dispersion relation defines the wave which can propagate in the medium, or using the *quantum* language, the dispersion relations determine the connection of energy and momentum of a quasiparticle - the plasmon in the case of plasma, which can exist in the medium.

The situation with QCD is analogous, however solutions of the dispersion equation (6.5) correspond to solutions only of the *linearized* QCD field equations, not the full QCD equations. If we go beyond this linear approach the correspondence of the dispersion relation solutions and those of the field equations is unknown.

There are three classes of solutions of Eq. 6.5. Those with purely real ω are stable - the wave amplitude is constant with time. If the frequency has negative imaginary part, the oscillations are damped - the amplitude decreases in time. Of particular interest are the solution with positive $\text{Im}\omega$ corresponding to the so-called plasma instabilities - the oscillations, the amplitude of which increases in time.

The permeability tensor in the static limit ($\omega \to 0$) provide the information about the plasma behaviour in constant fields. One easily finds [Lif81, Sil61] that the chromoelectric potential of the static point-like source behaves as

$$A_o(\mathbf{x}) = \frac{g}{4\pi \mid \mathbf{x} \mid} \exp(-m_D \mid \mathbf{x} \mid) \; ,$$

where m_D is the Debye mass - the inverse screening length, if the chromodielectric tensor is of the form

$$\epsilon_L(\omega = 0, \mathbf{k}) = 1 + \frac{m_D^2}{\mathbf{k}^2} \; . \tag{6.8}$$

7. THE OSCILLATIONS AROUND GLOBAL EQUILIBRIUM

Let me cosider a QGP near global thermodynamical equilibrium. Substituting the explicit form of the equilibrium distribution functions, Fermi-Dirac for (massless) quarks and Bose-Einstein for gluons, into (5.7) one finds by means of Eq. 6.4 the following expression of the chromoelectric permeability of the baryonless plasma in its rest frame

$$\begin{aligned} \epsilon^{\alpha\beta}(k) = \delta^{\alpha\beta} - \frac{2g^2}{\omega T} \int \frac{d^3p}{(2\pi)^3} v^\alpha v^\beta \Big[N_f \Big(\omega - \mathbf{kv} + i\nu\Big)^{-1} \Big(e^{p/T} + 1\Big)^{-2} + \\ + 2N \Big(\omega - \mathbf{kv} + i\nu_g\Big)^{-1} \Big(e^{p/T} - 1\Big)^{-2} \Big] e^{p/T} \; , \end{aligned} \tag{7.1}$$

where N_f is the number of flavours, N is the number of colors and p is the length of the vector momentum here ($p \equiv \mid \mathbf{p} \mid$). One should note that the numbers of quarks and of antiquarks are equal in the baryonless plasma, and consequently $\nu = \bar{\nu}$. With no difficulties, the permeability tensor (7.1) can be split into logitudinal and transversal parts according to Eq. 6.7.

In the case of collisionless ($\nu = \nu_g = 0$) plasma of massless particles, the dielectric function (7.1) can be calculated analytically and the results are

$$\epsilon_L = 1 + \frac{3\omega_o^2}{k^2} \left[1 - \frac{\omega}{2k} \Big[\ln \mid \frac{k+\omega}{k-\omega} \mid - i\pi\Theta(k-\omega) \Big] \right] \; , \tag{7.2}$$

$$\epsilon_T = 1 - \frac{3\omega_o^2}{2k^2} \left[1 - \Big(\frac{\omega}{2k} - \frac{k}{2\omega} \Big) \Big[\ln \mid \frac{k+\omega}{k-\omega} \mid - i\pi\Theta(k-\omega) \Big] \right] \; , \tag{7.3}$$

where $k \equiv\mid \mathbf{k} \mid$, ω_o is the plasma frequency and

$$\omega_o^2 = \frac{g^2 T^2 (N_f + 2N)}{18} \; . \tag{7.4}$$

One sees that for $\omega > k$ the dielectric functions (7.2, 7.3) are purely real - there are no dissipative processes, in particular the Landau damping [Lif81] is absent (see below).

Substituting (7.2, 7.3) into (6.6) one finds the following dispersion relations of the longitudinal (the chromoelectric field is parallel to the wave vector) and transverse (the chromoelectric field is perpendicular to the wave vector) modes

A) longitudinal mode

$$\omega^2 = \omega_o^2 + \frac{3}{5} k^2 \; , \qquad \omega_o \gg k \; , \tag{7.5a}$$

$$\omega^2 = k^2 \Big(1 + 4 \exp(-2 - 2k^2/3\omega_o^2) \Big) \; , \qquad \omega_o \ll k \; ; \tag{7.5b}$$

B) transverse mode

$$\omega^2 = \omega_o^2 + \frac{6}{5} k^2 \; , \qquad \omega_o \gg k \; , \tag{7.6a}$$

$$\omega^2 = \frac{3}{2} \omega_o^2 + k^2 \; , \qquad \omega_o \ll k \; . \tag{7.6b}$$

Because the longitudinal and transverse oscillations are time-like ($\omega^2 > k^2$), the phase velocity of the waves is greater than the velocity of light. (The possibility of the space-like longitudinal oscillations in QGP has been discussed in [Sil88].) For this reason the Landau damping is absent. Let me remind that the Landau damping is a collisionless transfer of energy from the wave to plasma particles, the velocity of which is equal to the wave phase velocity [Lif81].

The oscillations of the collisionless QGP around global equilibrium have been studied by means of the transport theory based on Eqs. 2.5 in several papers [Mro87a, Elz87, Bia88, Mro89]. The problem has been also discussed using a specific variant of the QGP theory with *classical* color [Hei83, Hei85a] in [Hei85b, Hei86]. In the above presentation I have followed [Mro89].

The dispersion relations for the collisionless plasma (7.5, 7.6) agree, up to the lowest order in the coupling constant, with those found in the finite-temperature QCD by means of the one-loop approximation, see *e.g.* [Wel82, Kal84, Han87, Hei87].

Let me now cosider the dielectric function with nonzero equilibration rates. One easily evaluates the integral (7.1) when $\omega \gg k$, $\omega \gg \nu$ and $\omega \gg \nu_g$. Then, we find the following dispersion relations [Mro89]:

A) longitudinal mode

$$\omega^2 = \omega_o^2 - \zeta^2 + \frac{3}{4}\phi^2 + \frac{3}{5}k^2 \; , \qquad \gamma = \frac{1}{2}\phi \; , \tag{7.7}$$

B) transverse mode

$$\omega^2 = \omega_o^2 - \zeta^2 + \frac{3}{4}\phi^2 + \frac{6}{5}k^2 \;, \qquad \gamma = \frac{1}{2}\phi \;, \tag{7.8}$$

where ω and γ denote the real and imaginary part, respectively, of the complex frequency, *i.e.* I performed the substitution $\omega \rightarrow \omega - i\gamma$; ϕ and ζ are parameters related to the equilibration rates,

$$\phi = \nu \frac{N_f}{N_f + 2N} + \nu_g \frac{2N}{N_f + 2N} \;, \tag{7.9a}$$

$$\zeta^2 = \nu^2 \frac{N_f}{N_f + 2N} + \nu_g^2 \frac{2N}{N_f + 2N} \;. \tag{7.9b}$$

One sees that, when compared with the colisionless plasma (Eqs. 7.5, 7.6), the frequency of the oscillations is smaller and that the oscillations are damped. To find the numerical value of the damping rate - the plasma oscillation decrement γ, the equilibration rates (ν and ν_g) have to be estimated.

If ν or ν_g is identified with the mean free flight time, as it has been done in Sec. 4, the equilibration rate is of order $g^4 \ln 1/g$. However, in the relativistic plasma, there is another mechanism of damping different than binary parton collisions. It is the plasmon decays into quark-antiquark or gluon-gluon pairs. The first process is very similar to the plasmon decay into electron-positron pair known from ultrarelativistic electrodynamic plasma, while the second one, which occurs due to the three-gluon coupling, is characteristic for non-Abelian interactions. The plasmon decay is, in another language, particle-antiparticle pair generation from vacuum due to the mean (oscillatory) field.

The plasmon decay width, in the lowest order of the perturbation expansion in the coupling constant, is proportional to g^2, however the plasma frequency ω_o, which is of order g, enters the formula and more detailed analysis is needed to find the order of the width. It is easy to observe that, even in the limit of massless quarks, the decay into gluons is much more probable than the decay into quarks [Hei87, Mro89]. The argument is as follows. If one considers the decay of plasmon of zero momentum into (massless) quarks or gluons, the phase-space volume of the final state is proportional to the factor

$$\Big(1 \mp n^{eq}(\omega_o/2)\Big)^2 \;, \tag{7.10}$$

where the upper sign is for quark decay, while the lower one for gluon decay. It appears because of the quantum *repulsion* of fermions and of the *attraction* of bosons in the momenum space. Since the plasma frequency (7.4) in the perturbative plasma is much smaller than the temperature, the factor (6.12) can be expanded as

$$\Big(1 - n^{eq}(\omega_o/2)\Big)^2 \cong 1/4 + \omega_o/8T \;,$$

$$\Big(1 + n^{eq}(\omega_o/2)\Big)^2 \cong 4T^2/\omega_o^2 \;.$$

Therefore it is seen that the decay into gluons is more probable than the decay into quarks by a factor of order g^{-2}, what has been observed in [Hei87].

Using the standard rules of finite-temperature field theory, one easily finds (see [Mro89]) the width of the zero-momentum plasmon decay into gluons

$$\Gamma_d = \frac{g^2 N}{2^4 3\pi}\omega_o\Big(1 + n^{eq}(\omega_o/2)\Big)^2 \cong \frac{gNT}{2^{3/2}\pi(N_f + 2N)^{1/2}} \;,$$

which is the same for longitudinal and transverse plasmons. The radiation gauge has been used in the calculations.

Γ_d cannot be identified with the plasmon equilibration rate Γ, because, in addition to the plasmon decays, there are also plasmon formation processes. As shown in [Wel83], see also [Hei87], the formation rate Γ_f is related to Γ_d as

$$\Gamma_f = \exp(-\omega_o/T)\Gamma_d \cong (1 - \omega_o/T)\Gamma_d \;.$$

Since the equilibration rate of plasmon (as boson) $\Gamma = \Gamma_d - \Gamma_f$ [Wel83], one finds

$$\Gamma \cong \frac{g^2 NT}{12\pi} \;. \tag{7.11}$$

This result has been found in [Hei87], where Γ has been evaluated as an imaginary part of polarization tensor. Let me note that Γ_d and Γ_f are of order of g, while Γ is of order of g^2. This means that the plasmon decay and formation rates cancel one another in the lowest order of g.

One should note that the plasmon decay width is not a Lorentz scalar, since there is a prefered reference frame - the rest frame of the termostat. Therefore the result (7.11) is valid only for zero-momentum plasmons, or approximately for long-wave plasmons.

There is a delicate question whether Γ can be identified with ν_g. Since the momentum distribution of gluons from the long-wave plasmon decays is rather far from thermal equilibrium one, parton collisions (with the cross section of order $g^4 \ln 1/g$) are needed to equilibrate the system. However, the gluons from plasmon decay locally neutralize the plasma and consequently damp the oscillations. Therefore if one considers the plasma oscillations, it seems reasonable to identify ν_g with Γ.

Substituting ν_g equal Γ from Eq. 7.11 and $\nu = 0$ (as explained above $\nu_g \gg \nu$ for the perturbative plasma) in Eq. 6.11, one finds the decrement of the plasma oscillation damping

$$\gamma \cong \frac{g^2}{12\pi}\frac{N^2}{N_f + 2N}\,T \;. \tag{7.12}$$

The characteristic feature of Eq. 7.12 is the fact that the damping rate depends on the number of quark flavours although $\nu = 0$. This seems in agreement with the physical intuition. When the number of quark flavours is increased the inertia of the system is also increased, and consequently the time needed to damp the oscillations is longer. However Eq. 7.12 disagrees (by a factor $2N/(N_f + 2N)$) with the result from [Hei87], where γ equals Γ which is given by Eq. 7.11. Therefore, the damping decrement is independent of the number of quark flavours. The only way to reproduce this result in the approach presented is to assume that $\nu = \nu_g = \Gamma$. However it is hard to understand this assumption on physical grounds. Probably, the problem can not be resolved as long as the collision terms of the QGP transport equations are not derived.

In the kinetic-theory approach discussed here, the plasma oscillations around global thermodynamical equilibrium are damped, what, in fact, is built in the approach. Due to the positiveness of the equilibration rate parameters, the plasma always goes to the global thermodynamical equilibrium described by the distribution function (2.8). Therefore the considerations presented here do not contribute to the controversy (see [Lop85, Han87, Kob88, Nad88, Par88, Pis88, Boz89, Pro89]) concerning the sign of the damping decrement of plasma oscillations and the possible instability of the perturbative QCD vacuum at finite temperature.

Comparing the chromodielectric tensor (7.1) in the static limit with Eq. 6.8, one finds the screening mass of the collisionless plasma, which is

$$m_D^2 = 3\omega_o^2 \; . \tag{7.13}$$

Because the parton density is $\sim T^3$, one finds from Eq. 7.13 that the number of partons in the Debye sphere (the sphere of the radius equal to the screening length) is $\sim 1/g^3$. It is much greater than unity if the plasma is *perturbative i.e.* the coupling constant is small. In fact, a big parton number in the Debye sphere justifies the use of the mean-field concept to describe QGP.

Let me also mention here that the ultrarelativistic *perturbative* plasma is simultaneously *ideal i.e.* the average parton interaction energy, which is $\sim g^2/ < r >$ with $< r >\sim T^{-1}$ being the average interparticle distance, is much smaller than the parton thermal energy which equals $\sim T$.

In the case of nonrelativistic electron plasma, the plasma frequency and screening length, respectively, read (see *e.g.* [Lif81, Sil61])

$$\omega_o^2 = e^2 \frac{n_e}{m_e} \; , \qquad\qquad m_D^2 = e^2 \frac{n_e}{T} \; , \tag{7.14}$$

where n_e is the electron density and m_e is the electron mass*. In this case there are two independent from one another thermodynamic quantities - the electron density and temperature, which determine two plasma parameters - the plasma frequency and screening length. (For ultrarelativistic plasma there is only one thermodynamic quantity - the temperature.) As seen, the smallness of the coupling constant does not guarantee that the nonrelativistic plasma is *ideal*. It occurs that the plasma is *ideal* when the number of electrons in the Debye sphere is big, i.e. when

$$\frac{T^{3/2}}{e^3 n_e^{1/2}} \gg 1 \; .$$

---

* I use the units, where the fine structure constant $\alpha = e^2/4\pi$. In the Gauss units, which are traditionally used the elctron-ion plasma physics, $\alpha = e^2$.

8. OSCILLATIONS IN THE TWO-STREAM SYSTEM

When the plasma state deviates from global thermodynamical equilibrium the instabilities can occur. According to the terminology from the electron-ion plasma [Has75], the instabilities caused by the system inhomogenity are called macroscopic, while those, which appear when the momentum distribution of the plasma particles differs from the equilibrium one, are known as the microscopic instabilities. In this section I discuss, following [Mro88c], the particular example of the instability of the second type. However, the considerations can be adopted for other physical situations. The so-called *pinch* instability, which is of the first type, has been briefly considered in [Mro88c].

Let me consider two colliding streams of QGP. The streems are assumed infinite in space and homogenous. The densities of both streams in their rest frames are equal to one another. It is also assumed that the thermal energy of plasma particle is much smaller than the particle energy related to the stream collective motion. Then, the quark distribution function reads

$$n(p) = (2\pi)^3 \rho \Big[\delta^{(3)}(\mathbf{p} - \mathbf{q}) + \delta^{(3)}(\mathbf{p} + \mathbf{q}) \Big] \; , \tag{8.1}$$

where $N\rho$ has to be interpreted as the quark density of the stream in the reference frame, where the stream velocities are opposite. The form of the distribution functions of antiquarks and gluons is analogous, however the gluon density is $(N^2-1)\rho_g$.

Substituting the distribution functions (8.1) in the expression of the chromoelectric permeability tensor of anisotropic plasma (found from Eqs. 6.4, 5.7), which is

$$\begin{aligned} \epsilon^{\alpha\beta}(k) = \delta^{\alpha\beta} + \frac{g^2}{2\omega} \int \frac{d^3p}{(2\pi)^3} v^\alpha \Big[\Big(\omega - \mathbf{kv} + i\nu\Big)^{-1} \frac{\partial n(p)}{\partial p_\gamma} + \Big(\omega - \mathbf{kv} + i\bar\nu\Big)^{-1} \frac{\partial \bar n(p)}{\partial p_\gamma} + \\ + 2N \Big(\omega - \mathbf{kv} + i\nu_g\Big)^{-1} \frac{\partial n_g(p)}{\partial p_\gamma} \Big] \Big[\Big(1 - \frac{\mathbf{kv}}{\omega}\Big)\delta^{\gamma\beta} + \frac{k^\gamma v^\beta}{\omega} \Big] \; , \end{aligned} \tag{8.2}$$

one gets

$$\epsilon^{xx}(k) = \epsilon^{yy}(k) = 1 - \frac{\omega_p^2}{\omega^2} \; ,$$

$$\epsilon^{xy}(k) = \epsilon^{yx}(k) = 0 \; ,$$

$$\epsilon^{xz}(k) = \epsilon^{zx}(k) = -\frac{\omega_p^2}{\omega^2} \frac{k_x k_z u^2}{\omega^2 - k_z^2 u^2} \; ,$$

$$\epsilon^{yz}(k) = \epsilon^{zy}(k) = -\frac{\omega_p^2}{\omega^2} \frac{k_y k_z u^2}{\omega^2 - k_z^2 u^2} \; ,$$

$$\epsilon^{zz}(k) = 1 - \frac{\omega_p^2}{\omega^2} - \frac{\omega_p^2}{\omega^2} u^2 \frac{\omega^2 + k_z^2 u^2}{(\omega^2 - k_z^2 u^2)^2} (k_x^2 + k_y^2) - \frac{\omega_p^2}{\omega^2} \frac{m^2}{E^2} k_z^2 u^2 \frac{3\omega^2 - k_z^2 u^2}{(\omega^2 - k_z^2 u^2)^2} \; , \tag{8.3}$$

where the plasma has been assumed collisionless and the vector $\mathbf{q}$ has been chosen parallel to the z-axis; $\omega_p^2 = g^2(\rho + \bar\rho + 2N\rho_g)/E$ is the plasma frequency, $E = (m^2 + \mathbf{q}^2)^{1/2}$ is the energy of particle in the stream and m is the particle thermal energy, which is assumed to be identical for quarks and gluons. (If one considers the streams of massive particles with the temperatures much smaller than the particle mass, m is just the particle mass.) Finally, $u = \mid \mathbf{q} \mid / E$ is the stream velocity.

Treating the plasma in the stream as a baryonless ideal gas of massless quarks and gluons, the parton thermal energy and the densities can be expressed through the plasma temperature as $m = 3T$ and $\rho^0 + \bar{\rho}^0 + 2N\rho_g^0 = \pi^{-2}(3N_f + 4N)\zeta(3)T^3$, where the index 0 labels the densities in the stream rest frame. Then, the plasma frequency reads

$$\omega_p^2 = \frac{g^2}{3\pi^2}(3N_f + 4N)\zeta(3)T^2 \ . \tag{8.4}$$

Substituting the chromodielectric tensor (8.3) into (6.5) one finds the dispersion relations. Since we are interested in the relativistic streams, $u = 1$ in the further considerations. To simplify the analysis, let me consider two specific cases.

A) Oscillations along the stream axis

Only the z-component of the wave vector is nonzero ($\mathbf{k} = (0, 0, k)$). Then, Eq. 6.5 is of the form

$$(k^2 - \omega^2\epsilon^{xx})(k^2 - \omega^2\epsilon^{yy})\epsilon^{zz} = 0 \ .$$

There are two solutions ($k^2 - \omega^2\epsilon^{xx} = k^2 - \omega^2\epsilon^{yy} = 0$), related to the transverse modes (the chromoelectric field is perpendicular to the wave vector),

$$\omega^2 = \omega_p^2 + k^2 \ , \tag{8.5}$$

which are stable, and there is one solution corresponding to the longitudinal mode ($\epsilon^{zz} = 0$). For ultrarelativistic streams ($E \gg m$) the fourth term of ϵ^{zz} from (8.3) can be neglected (except $\omega^2 = k^2$) and the dispersion relation reads

$$\omega^2 = \omega_p^2 \ . \tag{8.6}$$

Therefore, the longitudinal mode, as the transvers ones, is stable. It is interesting to note that the analogous longitudinal mode for nonrelativistic cold streams ($m = E$) is unstable [Has75].

B) Oscillations perpendicular to the stream axis

I choose the wave vector along the x-axis ($\mathbf{k} = (k, 0, 0)$). For this case Eq. 6.5 reads

$$\epsilon^{xx}(k^2 - \omega^2\epsilon^{yy})(k^2 - \omega^2\epsilon^{zz}) = 0 \ .$$

The dispersion relation of the longitudinal mode coincides with (8.6) and the one of the transverse mode with the chromoelectric field along the y-axis has the form (8.5). Both modes are stable. For the transverse mode with the chromoelectric field along the z-axis, one finds two solutions for the equation $k^2 - \omega^2 \epsilon^{zz} = 0$,

$$\omega_\pm^2 = \frac{1}{2}\Big[\omega_p^2 + k^2 \pm \Big((\omega_p^2 + k^2)^2 + 4\omega_p^2 k^2\Big)^{1/2}\Big] . \tag{8.7}$$

One sees that $\omega_+^2 \geq 0$ and $\omega_-^2 \leq 0$. Therefore, the modes represented by ω_+ are stable. On the other hand, the frequency of the modes related to the ω_- solutions is pure imaginary. The mode with the negative Imω is damped, and with the positive Imω, which is called the filamentation mode [Dav83] is unstable.

In the further discussion I concentrate on the filamentation mode, which in the context of QGP has been considered for the first time in [Pok88]. The physical picture of filamentation is the following. A density fluctuation of the initially homogenous streams occurs. When the density gradient is nonzero in the direction perpendicular to the beam, the fluctuation increases in time, and finally the colliding streams are split into filaments of transversal size equal to the half-wave-length of the initial fluctuation. The color currents are of the opposite sign in the neighbouring filaments.

Let me consider time τ of developement of the instability, which equals $1/\mathrm{Im}\omega_-$. One sees from Eq. 8.7 that the absolute value of ω_-^2 increases with k^2. If $k^2 \gg \omega_p^2$ one finds

$$\omega_-^2 \cong -\, \omega_p^2(1 - \omega_p^2/k^2) \; .$$

Therefore the maximal negative value of ω_-^2 is $-\omega_p^2$. In this way one finds the minimal time of the instability developement $\tau_{min} = \omega_p^{-1}$, which occurs occurs for $k^2 \gg \omega_p^2$.

The oscillations of QGP in the two-stream system have been discussed in [Hei84, Sil86, Pok88, Mro88c]. The analysis presented in this section can be easily adopted for a plasma system different than the two-stream one.

9. THE IDEAL CHROMOHYDRODYNAMICS

Chromohydrodynamics describes a hydrodynamic evolution of *colored* QGP interacting with the self-consistently generated chromodynamic field.

The equations of chromohydrodynamics are, as in the case of *colorless* plasma, contained in the conservation law equations. These are the baryon current conservation (3.5), the color current covariant conservation

$$\mathcal{D}^\mu j_\mu(x) = 0 \tag{9.1}$$

where the color current is defined by Eq. 2.5, and the energy-momentum conservation which for the *colored* plasma reads

$$\partial_\mu t^{\mu\nu}(x) = 2Tr\Big[F^{\nu\sigma}(x) j_\sigma(x)\Big] \; . \tag{9.2}$$

The color current conservation implies the relation, which can be written as

$$\int \frac{d^3p}{(2\pi)^3E}\{C[f,\bar{f},G] - \bar{C}[f,\bar{f},G] + 2ig\tau_a f_{abc} C_g^{bc}[f,\bar{f},G]\} = 0 \; . \tag{9.3}$$

where I have taken into account the relation (3.5). The energy-momentum conservation (9.2) leads to Eq. 3.4b.

To convert Eqs. 9.1, 9.2 into the equations of ideal chromohydrodynamics one has to calculate the energy-momentum tensor and color current with the local equilibrium distribution functions of *colored* plasma.

As usually, the local equilibrium state is assumed to maximize the entropy. I define the entropy four-flux as in [Dyr87] *i.e.*

$$\begin{aligned} S^\mu(x) = -Tr \int \frac{d^3p}{(2\pi)^3E} p^\mu \{ f(p,x)(\ln f(p,x) - 1) + \bar{f}(p,x)(\ln(\bar{f}(p,x) - 1) + \\ + G(p,x)(\ln G(p,x) + 1)\} \; . \end{aligned} \tag{9.4}$$

To simplify the discussion in this section I neglect the quantum statistics of quarks and gluons, which has been taken into account in Eq. 3.7. The respective modifications are very simple to implement. Let me note here that the definition (9.4) is gauge invariant.

Assuming that the distribution functions satisfy the transport equations (2.5) one finds the entropy production*

$$\partial_\mu S^\mu(x) = -Tr \int \frac{d^3p}{(2\pi)^3E} \Big[C \ln f(p,x) + \bar{C} \ln \bar{f}(p,x) + C_g \ln G(p,x) \Big] \; . \tag{9.5}$$

To derive Eq. 9.5 one should observe that

$$Tr\Big([A^\mu, f] \ln f \Big) = 0 \; .$$

Such terms appear because of the presence of covariant derivatives in the transport equations. I have also assumed that the distribution functions vanish for infinite momenta, then the mean field does not contribute to the entropy production.

The equation $\partial_\mu S^\mu = 0$ is of very complicated structure, and obviously one can not find the general solution of it not knowing the collisions terms. In the case of the standard Boltzmann equation [Gro80] one considers two classes of solutions. The first one appears due to the conservation laws *i.e.* due to the relations analogous to (3.4, 9.3). The second class of the solutions makes the collision terms equal zero. And it is of priniciple importance that the both classes are identical to one another. As we will see below it can be not the case of QGP transport equations.

---

* I am grateful to A. Dyrek and W. Florkowski for fruitful correspondence concerning this point.

9.1 Chromohydrodynamics with a color-scalar hydrodynamic velocity

Let me first consider the first class of solutions. Assuming the validity of relations (3.4, 9.3) one finds that the equation $\partial_\mu S^\mu = 0$ is solved if

$$f^{eq}(p,x) = \exp\big(-\beta^\mu(x)p_\mu + \beta(x)\tilde{\mu}(x)\big) \;, \tag{9.6a}$$

$$\bar{f}^{eq}(p,x) = \exp\big(-\beta^\mu(x)p_\mu - \beta(x)\tilde{\mu}(x)\big) \;, \tag{9.6b}$$

$$G^{eq}(p,x) = \exp\big(-\beta^\mu(x)p_\mu + \beta(x)\widetilde{\mu}_g(x)\big) \;. \tag{9.6c}$$

where the color chemical potentials $\widetilde{\mu}$ and $\widetilde{\mu}_g$ are the hermitian matrices $N \times N$ for quarks and $(N^2-1)\times(N^2-1)$ for gluons, and

$$\widetilde{\mu}_g^{ab} = 2igf_{abc}Tr[\widetilde{\mu}\tau_c] \;;$$

β^μ , as previously, is the color scalar equal u^μ/T. I denote with a tilde those matrix quantities which are usually scalars.

Substituting the functions (9.6) into Eqs. 2.2, 2.3 and 2.4, one gets, respectively,

$$b^\mu(x) = b(x)u^\mu(x) \;, \tag{9.7}$$

$$t^{\mu\nu}(x) = \big[U(x)+P(x)\big]u^\mu(x)u^\nu(x) - P(x)g^{\mu\nu} \;, \tag{9.8}$$

$$j^\mu(x) = \rho(x)u^\mu(x) \;. \tag{9.9}$$

In fact, the structure of expressions (9.7, 9.8, 9.10) follows from the simple observation that there is only one four-vector - the hydrodynamic four-velocity and one tensor - the metric tensor, at our disposal.

Because all color components of QGP evolve with the same hydrodynamic velocity the chromohydrodynamic equations which follow from (9.7-9.9) are trivial in this sense that the right-hand-side of Eq. 9.2 vanishes in the plasma rest frame ($Tr[F^{\mu\nu}(x)j_\mu(x)]u_\nu(x) = 0$ when $j^\mu(x) = \rho(x)u^\mu(x)$).

On the basis of the kinetic theory with *classical* color [Hei83, Hei85b], chromohydrodynamic equations of essentially the same content as those considered above, have been derived in [Hol84].

9.2 Chromohydrodynamics with a color-matrix hydrodynamic velocity

Since the collision terms of the QGP tranport equation are unkown, one can not be sure that the functions (9.6) are the most general solutions of the equation $\partial_\mu S^\mu = 0$. Therefore, let me now speculate on the class of solutions of this equation, for which $C = \bar{C} = C_g = 0$.

In the case of the standard Boltzmann equation, the collision term vanishes when

$$f(x,p_1)f(x,p_2) = f(x,p_1\prime)f(x,p_2\prime) \ , \tag{9.10}$$

where

$$p_1 + p_2 = p_1\prime + p_2\prime \ .$$

It seems reasonable to assume that the QGP collision terms vanish when the (matrix) distribution functions satisfy the relations analogous to (9.10). Since it is not clear how to generalize the condition (9.10) for quark-gluon scattering, let me consider here the *quark* plasma, *i.e.* the system of quarks and antiquarks with no thermal (nonvirtual) gluons. Then, I assume that $C = \bar{C} = 0$, when the distribution functions satisfy the condition (9.10) and

$$f(x,p_1)\bar{f}(x,p_2) = f(x,p_1\prime)\bar{f}(x,p_2\prime) \ , \tag{9.11}$$

$$\bar{f}(x,p_1)\bar{f}(x,p_2) = \bar{f}(x,p_1\prime)\bar{f}(x,p_2\prime) \ . \tag{9.12}$$

Using the standard arguments [Gro80] one finds that the relations (9.10 - 9.12) are satisfied if

$$f^{eq}(p,x) = \exp\big(- \frac{\widetilde{u}^{\mu}(x)p_{\mu} - \widetilde{\mu}(x)}{T(x)}\big) \ , \tag{9.13a}$$

$$\bar{f}^{eq}(p,x) = \exp\big(- \frac{\widetilde{u}^{\mu}(x)p_{\mu} + \widetilde{\mu}(x)}{T(x)}\big) \ , \tag{9.13b}$$

where the hydrodynamic velocity $\widetilde{u}^{\mu}(x)$ and the chemical potential $\widetilde{\mu}(x)$ are now $N \times N$ hermitian matrices. These matrices have to commute with each other in order to satisfy the relations (9.10 - 9.12). Because the distribution functions are gauge dependent, the same holds for the matrices $\widetilde{u}^{\mu}(x)$ and $\widetilde{\mu}(x)$. Assuming that these matrices transform under gauge transformation as tensors *i.e.* according to (2.1), one finds that the local distribution functions transform as it is required by Eq. 2.1.

The chromohydrodynamics which emerges from the local equilibrium distribution functions (9.13) is nontrivial, when compared with that considered earlier, because the different color components of *quark* plasma evolve with different hydrodynamic velocitites - the hydrodynamic velocity is a matrix in the color space. Therefore, it will be considered, following [Mro88b], in more details.

Because the collision terms vanish with the distribution fucntions (9.13) one finds adding the transport equation (2.5a) to (2.5b)

$$\mathcal{D}_{\mu}T^{\mu\nu}(x) = \frac{g}{2}\{F^{\sigma\nu}(x), N_{\sigma}(x)\} \ , \tag{9.14}$$

where

$$T^{\mu\nu}(x) = \int \frac{d^3p}{(2\pi)^3 E} p^{\mu}p^{\nu}\big(f(p,x) + \bar{f}(p,x)\big) \tag{9.15}$$

and

$$N^{\mu}(x) = \int \frac{d^3p}{(2\pi)^3 E} p^{\mu}\big(f(p,x) - \bar{f}(p,x)\big). \tag{9.16}$$

Subsituting (9.13) into (9.15) and (9.16) one finds

$$T^{\mu\nu}(x) = \left[\widetilde{U}(x) + \widetilde{P}(x)\right]\widetilde{u}^{\mu}(x)\widetilde{u}^{\nu}(x) - \widetilde{P}(x)g^{\mu\nu} \tag{9.17}$$

and

$$N^{\mu}(x) = N(x)\widetilde{u}^{\mu}(x) \ , \tag{9.18}$$

where $\widetilde{U}(x)$ are $\widetilde{P}(x)$ the matrix quantities. It looks curious at first sight that the pressure is a gauge dependent quantity. However one should note that the mechanical pressure is $P = Tr\widetilde{P}$ and it is gauge independent. Therefore, as in the case of the mixture of ideal gases, the pressure is a sum of terms related to the mixture components.

Because of the baryon current conservation ($b^{\mu}(x) = \frac{1}{3}TrN^{\mu}$) and due to the covariant conservation of the color current one observes that

$$\mathcal{D}^{\mu}N_{\mu} = 0 \ . \tag{9.19}$$

The equation describing the mean field generation reads

$$\mathcal{D}^{\mu}F_{\mu\nu} = -g\Big[N_{\nu}(x) - \frac{1}{N}b_{\nu}(x)\Big] \ . \tag{9.20}$$

Eqs. 9.14, 9.19, 9.20 with Eqs. 9.17, 9.18 form the gauge-covariant set of the chromohydrodynamic equations of an ideal *quark* plasma. To make the set complete one has to add the baryon current conservation, the well known ideal gas equation of state and the equation expressing the isoentropic character of an ideal fluid motion.

The equations 9.14, 9.19 and 9.20 can be essentially simplified by the proper choice of a gauge. As quoted previously the matrices $\widetilde{u}^{\mu}$ and $\widetilde{\mu}$, which are hermitian, transform under local gauge transformations according to Eq. 2.1. Therefore they can be diagonalized simltaneously (because they commute with one another) by means of a gauge transformation. This is just our gauge condition. Further, one finds that having diagonal $N^{\mu}(x)$, Eq. 9.19 is decomposed into differential equations where the diagonal components of $A^{\mu}(x)$ enter, and into algebraic equations with the off-diagonal components of the four-potential. Then, it follows from these algebraic equations that the off-diagonal elements of $A^{\mu}(x)$ have to vanish. Therefore all matrix quantities which enter the equations are diagonal.

If we introduce the indices i, j which run over the diagonal components of all quantities of interest, the equations (9.14, 9.19, 9.20) can be rewritten as (the summation over repeated i, j indices is not implied here)

$$\partial_{\mu}T_i^{\mu\nu}(x) = gF_i^{\sigma\nu}(x)N_{\sigma i}(x) \ , \tag{9.21}$$

$$\partial_{\mu}N_i^{\mu}(x) = 0 \ . \tag{9.22}$$

$$\partial_{\mu}F_i^{\mu\nu} = -g\Big[N_i^{\nu}(x) - \frac{1}{N}b^{\nu}(x)\Big] \ , \tag{9.23}$$

where

$$T_i^{\mu\nu}(x) = \left[\widetilde{U}_i(x) + \widetilde{P}_i(x)\right]\widetilde{u}_i^{\mu}(x)\widetilde{u}_i^{\nu}(x) - \widetilde{P}_i(x)g^{\mu\nu} \tag{9.24}$$

and

$$N_i^{\mu}(x) = N_i(x)\widetilde{u}_i^{\mu}(x) \ . \tag{9.25}$$

Because the field stress tensor is traceless, not all equations from Eq. 9.23 are independent from one another. The evolution of each color component seems, on the basis of Eqs. 9.21, 9.22, independent from one another. Although, the field generation equation (9.23) *mixes* the components since the quarks of all colors contribute to the baryon current present in *l.h.s* of Eq. 9.23. Let me also observe that the non-Abelian effects have disappeared in the equations (9.21 - 9.23) due to the *diagonal* gauge choice.

For better understanding of the problems discussed in this section, let me briefly consider the hydrodynamics of electromagnetic plasma [Kli82]. In the case of electron-ion plasma, the ion and the electron component are weakly coupled. This occurs because of the big difference of masses of electrons and of ions. Therefore, each component achieves local equilibrium and then, the system goes to the global thermodynamical equilibrium. Because of the mass difference the local temperatures and hydrodynamic velocities of the electron and ion components are different from each other until the system achieves the global equilibrium. Consequently, the ideal hydrodynamic equations of electron-ion plasma are nontrivial.

The case of the electron-positron plasma (with zero global charge) seems to be quite different than that of electron-ion plasma. There is no decoupling of electron and positron componets. Therefore the ideal hydrodynamics of such plasma should be analogous to the first trivial variant of chromohydrodynamics considered above.

As long the collision terms of QGP transport equations are not derived, the unique chromohydrodynamics can not formulated.

10. THE VISCOUS CHROMOHYDRODYNAMICS AND COLOR CONDUCTIVITY

Because of the ambiguities with the determination of local equilibrium distribution functions of the *colored* plasma, let me assume that the state of QGP is close to the *colorless* local equilibrium one. (I call such plasma *quasicolorless*). Then, the distribution functions can be written as

$$f_{ij}(p,x) = n^{eq}(p,x)\delta_{ij} + \delta f_{ij}(p,x) \ , \tag{10.1a}$$

$$\bar{f}_{ij}(p,x) = \bar{n}^{eq}(p,x)\delta_{ij} + \delta \bar{f}_{ij}(p,x) \ , \tag{10.1b}$$

$$G_{ab}(p,x) = n_g^{eq}(p,x)\delta_{ab} + \delta G_{ab}(p,x) \ , \tag{10.1c}$$

where it is assumed that the functions describing the deviation from the equilibrium and their (space and momentum) gradients are much smaller than the equilibrium functions and their gradients, respectively.

One easily solves the transport equations with the distribution functions of the form (10.1), and the solutions read

$$\delta f(p,x) = -(\nu p_\mu u^\mu)^{-1} \Big[p^\mu \partial_\mu + g p^\mu F_{\mu\nu}(x) \frac{\partial}{\partial p_\nu} \Big] n^{eq}(p,x) \;, \tag{10.2a}$$

$$\delta \bar{f}(p,x) = -(\bar{\nu} p_\mu u^\mu)^{-1} \Big[p^\mu \partial_\mu - g p^\mu F_{\mu\nu}(x) \frac{\partial}{\partial p_\nu} \Big] \bar{n}^{eq}(p,x) \;, \tag{10.2b}$$

$$\delta G(p,x) = -(\nu_g p_\mu u^\mu)^{-1} \Big[p^\mu \partial_\mu + g p^\mu \mathcal{F}_{\mu\nu}(x) \frac{\partial}{\partial p_\nu} \Big] n_g^{eq}(p,x) \;. \tag{10.2c}$$

Let me note here that when the plasma oscillations have been considered (Sec. 5), the space-time gradients of the equilibrium functions have been zero (the equilibrium state has been assumed homogenous and time independent) and, the space-time gradients of the deviation-from-equilibrium functions have been of major importance for these considerations. In the solutions (10.2), the space-time gradients of the deviation-from-equilibrium functions have been neglected, which is justified if the mean field slowly varies in the space-time.

Calculating the baryon current (2.2) and the energy-momentum tensor with the functions (10.1, 10.2) one reproduces the results of Sec. 4, where the *colorless* plasma has been studied. In the case of the color current (2.4) one finds

$$j^\mu(x) = \sigma^{\mu\rho}(x) u^\lambda(x) F_{\rho\lambda}(x) \tag{10.3}$$

with the static color conductivity tensor expressed as

$$\begin{aligned} \sigma^{\mu\rho}(x) = -\frac{g^2}{2T(x)} \int \frac{d^3p}{(2\pi)^3 E} \, \frac{p^\mu p^\rho}{p^\sigma u_\sigma} \, \Big[& \frac{1}{\nu} n^{eq}(p,x) \Big(1 - n^{eq}(p,x) \Big) + \\ & + \frac{1}{\bar{\nu}} \bar{n}^{eq}(p,x) \Big(1 - \bar{n}^{eq}(p,x) \Big) + \frac{2N}{\nu_g} n_g^{eq}(p,x) \Big(1 - n_g^{eq}(p,x) \Big) \Big] \;. \end{aligned} \tag{10.4}$$

Because there is only one four-vector - the hydrodynamic velocity, and only one tensor - the metric tensor, at our disposal, the color conductivity can be decomposed as

$$\sigma^{\mu\nu} = \sigma g^{\mu\nu} + \sigma_1 u^\mu u^\nu \;, \tag{10.5}$$

where

$$\begin{aligned} \sigma(x) = -\frac{g^2}{6T(x)} \int \frac{d^3p}{(2\pi)^3 E} \, \frac{p^\mu p_\mu - p^\lambda u_\lambda p^\rho u_\rho}{p^\sigma u_\sigma} \, \Big[& \frac{1}{\nu} n^{eq}(p,x) \Big(1 - n^{eq}(p,x) \Big) + \\ & + \frac{1}{\bar{\nu}} \bar{n}^{eq}(p,x) \Big(1 - \bar{n}^{eq}(p,x) \Big) + \frac{2N}{\nu_g} n_g^{eq}(p,x) \Big(1 - n_g^{eq}(p,x) \Big) \Big] \;. \end{aligned} \tag{10.6}$$

The second term in Eq. 10.5 does not contribute to the color current (due to antisymmetry of the stress tensor) and is neglected in further considerations.

One easily calculates the integral (10.6) in the plasma rest frame, the result for massless partons is

$$\sigma = \omega_o^2 \Big[\frac{1}{\nu} \frac{N_f}{N_f + 2N} + \frac{1}{\nu_g} \frac{2N}{N_f + 2N} \Big] ,$$

where the plasma frequency ω_o is given by Eq. 7.4.

The color conductivity of QGP has been discussed in [Hei86, Czy86, Dyr87 and Mro88a]. In the first two papers the kinetic theory with the *classical* color [Hei83, Hei85b] has been used. In [Mro88a] the static conductivity has been found considering the permeability tensor in the limit $k \to 0$.

The analysis of the transport coefficients presented here is quite simple because the chromodynamic field does not produce dissipative terms of the baryon current, nor of the energy-momentum tensor. On the other hand the color current does not appear in spite of non-zero gradients of u^ν and of μ. Such decoupling occurs only for the *quasicolorless* plasma. The more general case of the *quark* plasma has been studied in [Dyr87] (see also [Czy86]), where the local equilibrium distribution functions have been chosen in the form analogous to (9.6).

At the end of the section let me now write down the set of chromohydrodynamic equations for the *quasicolorless* plasma

$$\partial^\mu b_\mu = 0 ,$$

$$\mathcal{D}^\mu F_{\mu\nu} = \sigma u^\lambda F_{\nu\lambda} ,$$

$$\partial_\mu t^{\mu\nu} = 2\sigma u^\rho Tr \Big[F^{\nu\sigma} F_{\sigma\rho} \Big] .$$

The baryon current and the energy-momentum tensor are as for the *colorless* plasma discussed in Sec. 4.

11. FINAL REMARKS

Presenting the applications of the transport theory to QGP I have tried to consider the *mean stream* of the field avoiding model dependent concepts. Unfortunately, there is a big gap between these rather academic considerations and the questions which are important for experimental studies of QGP from nucleus-nucleus collisions. To study most of these problems one has to assume a definite scenario of high-energy nuclear interactions and to use an extensive phenomenological input to obtain quantitative results. Although the physics of the nucleus-nucleus collisions at high energies is under fast developement there is no common consensus concerning most of the points of interest. Therefore I have resigned from discussing several interesting topics of QGP physics, where the transport theory methods occured very usefull.

One of the central question, which has risen in the context of the perspectives to produce QGP in nuclear collisions concerns the time evolution of the generated plasma and, in particular, the thermalization time - the time interval that it would take for the plasma to achieve thermodynamical equilibrium. In the most studies of QGP *e.g.* those based on hydrodynamic calculations (for review see [Bla89a]), the system under consideration is assumed locally equilibrated, although it is not *a priori* impossible that the thermalization is even longer than the plasma life time.

The problem of plasma equilibration has been addressed in several papers. In [Bay84] a kinetic equation with a collision term in the relaxation time approximation has been used. The analogous studies based on the Fokker-Planck equation have been presented in [Cha84, Hwa85]. The role of the parton production due to the strong color field have been discussed in [Kaj85], where a phenomenological source term has been added to the transport equations. For further development of such calculations see [Gat87, Gat88]. Cascade simulations, which provide, in a sense, the numerical solutions of transport equations, have beed used to study the plasma evolution in [Boa86, Ber88].

Because of diagnostics of QGP produced in heavy-ion collisions a great deal of effort has been devoted to the question of time evolution of strange quark density in the plasma. The problem is reviewed in [Koc86, Koc89], see also [Mat86, Bar88, Gaz89].

The transport theory methods have been also applied [Sve88, Gaz89] to the exciting problem of the possible J/ψ particle production suppression in the presence of QGP. For review see [Bla89b].

The kinetic theory seems to provide a natural framework to study nonequilibrium many-body phenomena as those in relativistic nucleus-nucleus collisions. And, in fact, the idea to formulate such an approach was put forward several years ago [Car76, Coo76, Car83]. Therefore, concluding this review let me express the belief that in future the kinetic theory methods will play an important role in the studies of high-energy nucleus-nucleus interactions as, for example, the internuclear cascade models, based on the so-called Vlasov-Uehling-Uhlenbeck kinetic equation, play in the physics of intermediate energy heavy-ion collisions, see *e.g.* [Cas87] and references therein.

Acknowledgements

I am very grateful to Lidia S. Ferreira and Ulrich Heinz for critically reading the manuscript, numerous fruitful discussions and for helpful advices. I would like also to thank to those colleagues whose correspondence helped me to prepare this article. In particular, I am indebted to J.-P. Blaizot, S. Chakraborty, L. McLerran, M. Mizutani, C.J. Pethick, M. Płoszajczak, W. Schoenmaker, V.P. Silin, B. Svetitsky, Ch.G. van Weert and F. Zachariasen.

REFERENCES

[And74] J.L. Anderson and H.R. Witting, Physica, **74** (1974) 466; 489.

[Bal75] R. Balescu, *Equilibrium and Nonequilibrium Statistical Mechanics*, John-Wiley & Sons, New York, 1975.

[Bar88] H.W. Barz, B.L. Friman, J. Knoll and H. Schulz, Nucl. Phys., **A484** (1988) 661.

[Bay84] G. Baym, Phys. Lett., **B138** (1984) 18.

[Bay88] G. Baym, H. Monien and C.J. Pethick, Proc. of Conf. *Quark Matter'88*, Lenox, 1988, in press.

[Ber88] G. Bertsch, M. Gong, L. McLerran, V. Ruuskanen and E. Sarkkinen, Phys. Rev., **D37** (1988) 1202.

[Bia84] A. Białas and W. Czyż, Phys. Rev., **D30** (1984) 2371.

[Bia85] A. Białas and W. Czyż, Z. Phys., **C28** (1985) 255.

[Bia88a] A. Białas and W. Czyż, Ann. Phys., **187** (1988) 97.

[Bia88b] A. Białas, W. Czyż, A. Dyrek and W. Florkowski, Nucl. Phys., **B296**(1988) 611.

[Bia89] A. Białas and W. Czyż, this volume.

[Bjo83] J.D. Bjorken, Phys. Rev., **D27** (1983) 140.

[Bla89a] J.P. Blaizot and P. Ollitrault, this volume.

[Bla89b] J.P. Blaizot, this volume

[Boa86] D. Boal, Phys. Rev., **C33** (1986) 2206.

[Boz89] P. Bożek and M. Płoszajczak, unpublished.

[Cal87] Ch.J. Calkoen, Diploma thesis, Univ. of Amsterdam, Amsterdam, 1987.

[Car76] P. Carruthers and F. Zachariasen, Phys. Rev., **D13** (1976) 950.

[Car83] P. Carruthers and F. Zachariasen, Rev. Mod. Phys., **55** (1983) 245.

[Car87] M.E. Carrington, M.J. Rhoades-Brown and M. Płoszajczak, Phys. Rev.,**D35** (1987) 3981.

[Cas87] W. Cassing, Z. Phys., **A327** (1987) 87; 447; 471; 487.

[Cha84] S. Chakraborty and D. Syam, Lett. Nuovo Cim., **41** (1984) 381.

[Cha85] S. Chakraborty, Pramana J. Phys., **25** (1985) 673.

[Coo76] F. Cooper and M. Feigenbaum, Phys. Rev., **D14** (1976) 583.

[Czy86] W. Czyż and W. Florkowski, Acta Phys. Pol., **B17** (1986) 819.

[Dan85] P. Danielewicz and M. Gyulassy, Phys. Rev., **D31** (1985) 53.

[Dav83] R.C. Davidson, *Foundations of Plasma Physics*, Vol. 1, edited by A.A. Galeev and R.N. Sudan, North-Holland, Amsterdam, 1983.

[Dyr87] A. Dyrek and W. Florkowski, Phys. Rev., **D36** (1987) 2172.

[Eck40] C. Eckart, Phys. Rev., **58** (1940) 919.

[Elz86a] H.-Th. Elze, M. Gyulassy and D. Vasak, Nucl. Phys., **B276** (1986) 706.

[Elz86b] H.-Th. Elze, M. Gyulassy and D. Vasak, Phys. Lett., **B177** (1986) 402.

[Elz87] H.-Th. Elze, Z. Phys., **C38** (1987) 211.

[Elz89] H.-Th. Elze and U. Heinz, this volume.

[Gat87] G. Gatoff, A.K. Kerman and T. Matsui, Phys. Rev., **D36** (1987) 114.

[Gat88] G. Gatoff, A.K. Kerman and D. Vautherin, Phys. Rev., **D38** (1988) 96.

[Gav85] S. Gavin, Nucl. Phys., **A435** (1985) 826.

[Gaz89] M. Gaździcki and St. Mrówczyński, preprint TPR-89-7, Regensburg, 1989.

[Gor78] M.I. Gorenstein, G.M. Zinovjev and Yu. M. Sinyukov, Pis'ma Zh. Eksp. Teor. Fiz., **28** (1978) 371.

[Gro80] S.R. deGroot, W.A. van Leeuwen and Ch. G. van Weert, *Relativistic Kinetic Theory*, North-Holland, Amsterdam, 1980.

[Han87] T.H. Hansson and I. Zahed, Nucl. Phys., **B292** (1987) 725.

[Has75] A. Hasegawa, *Plasma Instabilities and Nonlinear Effects*, Springer, Berlin, 1975.

[Hei83] U. Heinz, Phys. Rev. Lett., **51** (1983) 351.

[Hei84] U. Heinz, Nucl. Phys., **A418** (1984) 603c.

[Hei85a] U. Heinz, Ann. Phys., **161** (1985) 48.

[Hei85b] U. Heinz and P.J. Siemens, Phys. Lett., **B158** (1985) 11.

[Hei86] U. Heinz, Ann. Phys., **168** (1986) 148.

[Hei87] U. Heinz, K. Kajantie and T. Toimela, Ann. Phys., **176** (1987) 218.

[Hol84] D.D. Holm and B.A. Kupershmidt, Phys. Lett., **A105** (1984) 225.

[Hor87a] R. Horsley and W. Schoenmaker, Nucl. Phys., **B280** [FS18] (1987) 716.

[Hor87b] R. Horsley and W. Schoenmaker, Nucl. Phys., **B280** [FS18] (1987) 735.

[Hos85] A. Hosoya and K. Kajantie, Nucl. Phys., **B250** (1985) 666.

[Hwa85] R. Hwa, Phys. Rev., **D32** (1985) 637.

[Isr79] W. Israel and J.M. Stewart, Ann. Phys., **118** (1979) 341.

[Kaj85] K. Kajantie and T. Matsui, Phys. Lett., **B164** (1985) 373.

[Kal84] O.K. Kalashnikov, Fortschritte Phys., **32** (1984) 525.

[Kar87] F. Karsch and H.W. Wyld, Phys. Rev., **D35** (1987) 2518.

[Koc86] P. Koch, B. Mueller and J. Rafelski, Phys. Rep., **142** (1986) 167.

[Koc89] P. Koch, this volume.

[Kli82] Yu. A. Klimontovich, *Statistical Physics*, Nauka, Moscow, 1982 (in Russian).

[Kob88] R. Kobes and G. Kunstatter, Phys. Rev. Lett., **61** (1988) 392.

[Lan60] L.D. Landau and E.M. Lifshitz, *Electrodynamics of Continuous Media*, Pergamon, New York, 1960.

[Lan63] L.D. Landau and E.M. Lifshitz, *Fluid Mechanics*, Pergamon, Oxford, 1963.

[Lee86] M.C.J. Leermakers and Ch.G. van Weert, Physica, **134A** (1986) 577.

[Lif81] E.M. Lifshitz and Pitaevskii, *Physical Kinetics*, Pergamon, New York, 1981.

[Lop85] J. Lopez, J.C. Parikh and P.J. Siemens, Texas A & M preprint, 1985.

[Mat86] T. Matsui, B. Svetitsky and L.D. McLerran , Phys. Rev., **D34** (1986) 783; 2047.

[Miz88] M. Mizutani, S. Muroya and M. Namiki, Phys. Rev., **D37** (1987) 3033.

[Mro87a] St. Mrówczyński, Phys. Lett., **B188** (1987) 127.

[Mro87b] St. Mrówczyński and P. Danielewicz INS report, IPJ/2049/PH, Swierk-Otwock, 1987.

[Mro88a] St. Mrówczyński, Acta Phys. Pol., **B19** (1988) 91.

[Mro88b] St. Mrówczyński, Phys. Lett., **B202** (1988) 568.

[Mro88c] St. Mrówczyński, Phys. Lett., **B214** (1988) 587.

[Mro88d] St. Mrówczyński, Proc. of Workshop *Physics of Intermediate and High-Energy Heavy-Ion Reactions*, Kraków, 1987, edited by M. Kutchera and M. Płoszajczak, World Scientific, 1988.

[Mro88d] St. Mrówczyński, Proc. of 8th Int. Balaton Conf. *Intermediate Energy Nuclear Physics*, Balatonfured, 1987, edited by F. Fodor, Budapest, 1988.

[Mro88e] St. Mrówczyński, Proc. of 19th Int. Conf. *Multiparticle Dynamics*, Arles, 1988, in press.

[Mro89] St. Mrówczyński, Phys. Rev., **D** in press.

[Nad88] S. Nadkarni, Phys. Rev. Lett., **61** (1988) 396.

[Nad88] L.W. Nordheim, Proc. Roy. Soc. (London), **A119** (1928) 689.

[Par88] J.C. Parikh and P.J. Siemens, Phys. Rev., **D37** (1988) 3246.

[Pis88] R.D. Pisarski, Fermilab-Pub-88/123-T, Batavia, 1988.

[Pok88] Yu. E. Pokrovskii and A.V. Selikhov, Pis'ma Zh. Eksp. Teor. Fiz., **47** (1988) 11.

[Pro89] Proceeding of the workshop *Thermal Fields and their Applications*, Cleveland, 1988, Physica, in print.

[Sil61] V.P. Silin and A.A. Ruhadze, *Electrodynamics of Plasma and Plasma-like Media*, Gosatomizdat, Moscow, 1961 (in Russian).

[Sil85] V.P. Silin and V.N. Ursov, Dokl. Akad. Nauk SSSR, **283** (1985) 593.

[Sil86] V.P. Silin and V.N. Ursov, Lebedev institute short reports, N^o1, Moscow, 1986 (in Russian).

[Sil88] V.P. Silin and V.N. Ursov, Lebedev institute short reports, N^o5, Moscow, 1988 (in Russian).

[Sch86] W. Schoenmaker, Phys. Lett., **B182** (1986) 373.

[Sve88] B. Svetitsky, Phys. Rev., **D37** (1988) 2484.

[Ueh33] E.A. Uehling and G.E. Uhlenbeck, Phys. Rev., **43** (1933) 552.

[Wel82] H.A. Weldon, Phys. Rev., **D26** (1982) 1394.

[Wel83] H.A. Weldon, Phys. Rev., **D28** (1983) 2007.

[Win84] J. Winter, J. Phys. (Paris), **45**, C6 (1984) 53.

[Ynd83] F.J. Yndurain, *Quantum Chromodynamics* , Springer, New York, 1983.