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Kinetic-theory approach to quark-gluon plasma oscillations

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The difficulties in the unique definition of oscillations of a plasma with a non-Abelian interaction are considered. The recently formulated kinetic theory of the quark-gluon plasma in the semiclassical limit is presented and discussed with particular attention to the gluon sector of the theory. The transport equations are linearized around the global equilibrium and the chromoelectric permeability tensor is found. The dispersion relations of the plasma oscillations are discussed and the rate of oscillation damping is estimated.

I. INTRODUCTION

The quark-gluon plasma oscillations have been studied for a long time in the framework of finite-temperature (FT) QCD (for a review see Ref. 1 and references therein). Recently, however, doubts have arisen[2,3] over how to define the oscillations of the system with a non-Abelian gauge interaction. To understand the problem let me recall the case of the electromagnetic plasma, where the oscillations are defined as solutions of the sourceless Maxwell equations in a plasma medium, which for the Fourier-transformed fields read

$$\mathbf{k}\cdot\mathbf{D}(k)=0,\quad \mathbf{k}\cdot\mathbf{B}(k)=0\ ,$$
$$\mathbf{k}\times\mathbf{E}(k)=\omega\mathbf{B}(k),\quad \mathbf{k}\times\mathbf{B}(k)=-\omega\mathbf{D}(k)\ , \tag{1}$$

where $k=(\omega,\mathbf{k})$; $\mathbf{E}(\mathbf{D})$ is the electric field (induction) and $\mathbf{B}$ is the magnetic induction. The magnetic field $\mathbf{H}$ is not present in Eqs. (1) since, as explained in, e.g., Refs. 4 and 5, the magnetic permeability tensor can be put equal to unity if one considers plasma oscillations. Introducing the electric permeability tensor defined as

$$D^{\alpha}(k)=\epsilon^{\alpha\beta}(k)E^{\beta}(k) \tag{2}$$

($\alpha,\beta,\gamma=1,2,3$ denote space axes), one finds that Eqs. (1) are automatically solved if

$$\epsilon_{\rm L}(k)=0,\quad \epsilon_{\rm T}(k)=\mathbf{k}^2/\omega^2\ , \tag{3}$$

where the longitudinal and transversal parts of the permeability tensor are defined for an isotropic medium as

$$\epsilon^{\alpha\beta}=\epsilon_{\rm T}(\delta^{\alpha\beta}-k^{\alpha}k^{\beta}/\mathbf{k}^2)+\epsilon_{\rm L}k^{\alpha}k^{\beta}/\mathbf{k}^2\ . \tag{4}$$

Therefore, to find the plasma oscillations, or more precisely, the dispersion relations of the oscillations, one has to find the electric permeability tensor and then solve Eqs. (3).

The tensor can be found by means of the kinetic theory of the plasma (see, e.g., Refs. 5 and 6) or one can use the powerful apparatus of FT QED (see, e.g., Ref. 7 for the nonrelativistic calculations and Ref. 8 for the relativistic ones). In the latter case the permeability tensor can be expressed through the photon propagator and the dispersion relations occur as the poles of it.

The question arises of how to define the oscillations of the quark-gluon plasma. In the approach presented in Ref. 1, which I call standard, the dispersion relations are determined by the positions of the gluon propagator singularities.

The starting point of the background-field method[2] is the definition of the permeability tensor as a coefficient in front of the term quadratic in the chromoelectric field in the so-called effective action. The permeability tensor is expressed through the gluon propagator and the dispersion relations are defined according to Eqs. (3). However, these relations correspond to oscillatory solutions of linearized field equations similar to those of electrodynamics [Eqs. (1)].

In the linear-response approach[3] one finds the permeability tensor which, by definition, describes the reaction of the system to the external field. The dispersion relations are given by zeros of the permeability tensor, which is expressed not only by the gluon propagator (two-point Green's function) but by a combination of two-, three-, and four-point gluon Green's functions.

Most of the computational results of these methods are the same: however, there are some differences even at the qualitative level. In particular, the background-field method[2] provides a negative damping of the plasma oscillations (the amplitude of the wave increases in time), while the linear-response analysis[3] gives a positive damping. All the methods are faced with the problem of the gauge dependence of the results, which is treated in a different manner in each case.

The aim of this paper is to consider the quark-gluon plasma oscillations from a point of view which is different from that of FT QCD. Namely, the recently formulated gauge-covariant kinetic theory of the quark-gluon plasma[9–12] is used here. In fact, the oscillations of the quark plasma (the system of quarks and antiquarks interacting via non-Abelian classical potentials) and those of quarkless plasma have been considered separately in Refs. 13 and 12, respectively. In this paper I join both approaches, reformulating them to elucidate peculiarities of the problem. In particular, I have found a new, more transparent form of the transport equations of gluons. Instead of the four-index gluon distribution function used in Refs. 11 and 12, I have introduced a two-index function with a simple physical interpretation. Then the transport equations of quarks and of gluons are formally identical. The damping mechanisms of oscillations, which have not been studied in Refs. 12 and 13, are also discussed here. Finally, I analyze the physical meaning of the oscillations and I arrive at the conclusion that the meaning is clear only for the *linearized* QCD, where, in particular, the color current is conserved, not only covariantly conserved.

In fact, I have added no new results concerning the quark-gluon plasma behavior to those from Refs. 1–3. However, I have rederived these results in a completely different way, shedding new light on them. I have developed a formalism which can be used to study oscillations around any quasistable state of the plasma, not only the global thermodynamical equilibrium state.

The paper is organized as follows. In Sec. II, I present the kinetic theory with particular attention to the gluon transport equations. The linearized theory is studied in Sec. III, while Sec. IV is devoted to the discussion of the linearized QCD in the plasma medium and of the dispersion relations. In Sec. V the damping rate is estimated, and finally, the conclusions are collected in Sec. VI.

II. TRANSPORT THEORY OF THE QUARK-GLUON PLASMA

A. Quark sector

The quark [antiquark] distribution function $f(p,x)$ $[\tilde{f}(p,x)]$ is an $N\times N$ matrix in color space [for an SU(N)-color group] which transforms under local gauge transformations as

$$f(p,x)\rightarrow U(x)f(p,x)U^{\dagger}(x)\ , \tag{5}$$

where the color indices are suppressed. Quantities which are color (gauge) independent, such as the baryon current or energy-momentum tensor, are expressed through the traces of $f(p,x)$ and $\tilde{f}(p,x)$. The quark-color current is

$$j_q^{\mu}(x)=\frac{g}{2}\int\frac{d^3p}{(2\pi)^3E}p^{\mu}\left[f(p,x)-\tilde{f}(p,x)-\frac{1}{N}\text{Tr}[f(p,x)-\tilde{f}(p,x)]\right]\ , \tag{6}$$

where g is the coupling constant. The units are used where $c=\hbar=k=1$ and the metric is $(1,-1,-1,-1)$. One sees that $j^{\mu}(x)$ transforms under gauge transformations as its QCD analog, i.e., according to Eq. (5).

$f(p,x)$ and $\tilde{f}(p,x)$ satisfy the kinetic equations[9,10]

$$p_{\mu}D^{\mu}f(p,x)-gp^{\mu}\frac{\partial}{\partial p_{\nu}}\tfrac{1}{2}\{F_{\mu\nu}(x),f(p,x)\}=C\ , \tag{7a}$$

$$p_{\mu}D^{\mu}\tilde{f}(p,x)+gp^{\mu}\frac{\partial}{\partial p_{\nu}}\tfrac{1}{2}\{F_{\mu\nu}(x),\tilde{f}(p,x)\}=\tilde{C}\ , \tag{7b}$$

where $p\equiv p^{\mu}=(E,\mathbf{p})$, $x=(t,\mathbf{x})$ are the four-momentum and four-position, respectively. D^{μ} is the covariant derivative in the adjoint representation which acts as $\partial_{\mu}+ig[A_{\mu},\ldots]$; $A_{\mu}(x)$ and $F_{\mu\nu}(x)$ are the chromodynamic mean-field four-potential and stress tensor, respectively, expressed as the Lie-algebra elements

$$A_{\mu}=A_{\mu}^{a}\tau^{a},\quad F_{\mu\nu}=F_{\mu\nu}^{a}\tau^{a},\quad a=1,2,\ldots,N^2-1\ ,$$

where τ^a are the SU(N) group generators; $\{\ldots,\ldots\}$ denotes the anticommutator; C, $\tilde{C}$ represent the collision terms which for the collisionless plasma equal zero. In this paper I use the simplest form of C and $\tilde{C}$, known as the collision terms in the relaxation-time approximation, which read

$$C=\nu p^{\mu}u_{\mu}[f^{\text{eq}}(p)-f(p,x)]\ , \tag{8a}$$

$$\tilde{C}=\tilde{\nu}p^{\mu}u_{\mu}[\tilde{f}^{\text{eq}}(p)-\tilde{f}(p,x)]\ , \tag{8b}$$

where ν is the equilibration rate parameter discussed in Sec. V and u^{μ} is the hydrodynamic velocity which describes the motion of the plasma as a whole [in the plasma rest frame $u^{\mu}=(1,0,0,0)$]. $f^{\text{eq}}(p)$ is the equilibrium distribution function

$$f_{ij}^{\text{eq}}(p)=\delta_{ij}n(p),\quad \tilde{f}_{ij}^{\text{eq}}(p)=\delta_{ij}\tilde{n}(p),\quad i,j=1,\ldots,N\ ,$$

where i,j are the color quark indices and $n(p)$ $[\tilde{n}(p)]$ is the Fermi-Dirac equilibrium distribution function.

More realistic collision terms for the quark transport equations have been discussed in Ref. 14.

The above transport equations are written for spinless quarks of one flavor only. However, if the plasma is in equilibrium with respect to spin and flavor, both quantum numbers can be treated as indistinguishable internal degrees of freedom of quarks.

Equations (7) with $C=\tilde{C}=0$ form the semiclassical limit of the full kinetic equations, derived in Ref. 10, of quarks interacting with the non-Abelian classical mean field. They are formally very similar to those of an electron-positron plasma.

B. Gluon sector

The transport theory of gluons has been studied in Refs. 11 and 12, where the gluon distribution function has been the four-color-index matrix $G_{ijkl}^{\mu\nu}(p,x)$ with $i,j,k,l=1,2,\ldots,N$ satisfiying a kinetic equation which, even in the semiclassical limit, has been of rather complicated form. Our first task is to rewrite the equations from Ref. 12 in another representation in which they are formally very similar to those of quarks.

I assume, as in Ref. 12, that the system is equilibrated with respect to the spin of gluons. Then

$$G^{\mu\nu}(p,x)=p^{\mu}p^{\nu}G(p,x)\ . \tag{9}$$

Further, I introduce the distribution function $\mathcal{G}_{ab}(p,x)$, which is an $(N^2-1)\times(N^2-1)$ Hermitian matrix, defined as

$$G_{ijkl}(p,x)=\mathcal{G}_{ab}(p,x)\tau_{ij}^{a}\tau_{kl}^{b}\ .$$

The diagonal elements of $\mathcal{G}_{ab}$ should be interpreted as the distribution functions of gluons of each kind. $\mathcal{G}_{ab}$ transforms under a local gauge transformation as

$$\mathcal{G}(p,x)\rightarrow M(x)\mathcal{G}(p,x)M^{\dagger}(x)\ ,$$

where

$$M_{ab}(x)=\text{Tr}[\tau_a U(x)\tau_b U^{\dagger}(x)]\ ,$$

with $U(x)$ being the respective transformation operator in the fundamental representation.

Using the distribution function $\mathcal{G}_{ab}$ with the structure of the Lorentz indices given by Eq. (9), the transport equation from Ref. 12 can be rewritten as (for details see Appendix A)

$$p_\mu \mathcal{D}^\mu \mathcal{G}(p,x) - gp^\mu \frac{\partial}{\partial p_\nu} \tfrac{1}{2}\{\mathcal{F}_{\mu\nu}(x), \mathcal{G}(p,x)\} = C_g \ , \quad (10)$$

where $\mathcal{D}^\mu = \partial^\mu - ig[\mathcal{A}^\mu, \ldots]$; $\mathcal{A}^\mu$ and $\mathcal{F}^{\mu\nu}$ are the $(N^2-1)\times(N^2-1)$ Hermitian matrices defined as

$$\mathcal{A}^\mu_{ab}(x) = if_{abc}A^\mu_c(x), \quad \mathcal{F}^{\mu\nu}_{ab}(x) = if_{abc}F^{\mu\nu}_c(x) \ ,$$

where f_{abc} is the SU(N) group structure constant. As previously, I choose the collision term in the relaxation-time approximation, i.e.,

$$C_g = \nu_g p^\mu u_\mu [\mathcal{G}^{\rm eq}(p) - \mathcal{G}(p,x)] \ , \quad (11)$$

where

$$\mathcal{G}^{\rm eq}_{ab}(p) = n_g(p)\delta_{ab} \ .$$

It is amusing that Eqs. (7) and (10) are formally identical.

To make the set of equations (7) and (10) complete one has to add an equation describing the self-consistent generation of the chromodynamic mean field: namely,

$$D_\mu F^{\mu\nu}(x) = j^\nu(x) \equiv j^\nu_q(x) + j^\nu_g(x) \ , \quad (12)$$

where the quark current is given by Eq. (6) and the gluon current under the assumption (9) reads[12]

$$j^\mu_g(x) = ig \int \frac{d^3p}{(2\pi)^3 E} p^\mu \tau^a f_{acb} \mathcal{G}_{bc}(p,x) \ . \quad (13)$$

As needed the current (13) is Hermitian. One can easily check that the set of transport equations (7) and (10) with (12) is gauge covariant.

III. LINEARIZED TRANSPORT THEORY

Let us consider a plasma which is close to the global equilibrium. Then the distribution functions can be expressed as

$$f_{ij}(p,x) = n(p)\delta_{ij} + \delta f_{ij}(p,x) \ ,$$

$$f_{\bar{i}\bar{j}}(p,x) = \bar{n}(p)\delta_{ij} + \delta \bar{f}_{ij}(p,x) \ ,$$

$$\mathcal{G}_{ab}(p,x) = n_g(p)\delta_{ab} + \delta\mathcal{G}_{ab}(p,x) \ .$$

The functions describing the deviation from the equilibrium are assumed much smaller than the respective equilibrium functions, and the same is assumed for the momentum gradients of these functions.

Substituting the above distribution functions in Eqs. (6) and (13) one gets the color current

$$j^\mu(x) = \frac{g}{2} \int \frac{d^3p}{(2\pi)^3 E} p^\mu [\delta f(p,x) - \delta\bar{f}(p,x) + 2i\tau^a f_{acb}\delta\mathcal{G}_{bc}(p,x)] \ . \quad (14)$$

I have omitted the term proportional to $\mathrm{Tr}(\delta f - \delta\bar{f})$ since, as it follows from Eq. (16) ($\mathrm{Tr}F_{\mu\nu}=0$), this term gives zero contribution to the color current. One sees that the current occurs due to deviation of the system from equilibrium. In equilibrium there is no current and no mean field. Therefore the field generation equation (12) can be linearized for the quasiequilibrium plasma (with respect to the four-potential) to the form

$$\partial_\mu F^{\mu\nu}(x) = j^\nu(x)$$

with $F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$. One sees that in the linearized theory the color current is conserved (due to antisymmetry of $F^{\mu\nu}$), i.e., $\partial_\mu j^\mu(x)$.

Now I substitute the quasiequilibrium distribution functions to the transport equations (7) and (10) with the collision terms (8) and (11). Linearizing the equations with respect to δf, $\delta\bar{f}$, and $\delta\mathcal{G}$, one gets

$$(p_\mu\partial^\mu + \nu p_\mu u^\mu)\delta f(p,x) - gp^\mu F_{\mu\lambda}(x)\frac{\partial}{\partial p_\lambda} f^{\rm eq}(p) = 0 \ ,$$

$$(p_\mu\partial^\mu + \bar{\nu} p_\mu u^\mu)\delta \bar{f}(p,x) + gp^\mu F_{\mu\lambda}(x)\frac{\partial}{\partial p_\lambda} \bar{f}^{\rm eq}(p) = 0 \ , \quad (15)$$

$$(p_\mu\partial^\mu + \nu_g p_\mu u^\mu)\delta \mathcal{G}(p,x) - gp^\mu \mathcal{F}_{\mu\lambda}(x)\frac{\partial}{\partial p_\lambda} \mathcal{G}^{\rm eq}(p) = 0 \ .$$

It should be remembered that $A_\mu(x)$ is of order of $\delta f(p,x)$. Treating the chromodynamic mean field as an external one, Eqs. (15) can be easily solved:

$$\delta f(p,x) = g\int d^4x' \Delta^\nu_p(x-x') p^\mu F_{\mu\lambda}(x') \frac{\partial}{\partial p_\lambda} f^{\rm eq}(p) \ ,$$

$$\delta \bar{f}(p,x) = -g\int d^4x' \Delta^{\bar{\nu}}_p(x-x') p^\mu F_{\mu\lambda}(x') \frac{\partial}{\partial p_\lambda} \bar{f}^{\rm eq}(p) \ , \quad (16)$$

$$\delta \mathcal{G}(p,x) = g\int d^4x' \Delta^{\nu_g}_p(x-x') p^\mu \mathcal{F}_{\mu\lambda}(x') \frac{\partial}{\partial p_\lambda} \mathcal{G}^{\rm eq}(p) \ ,$$

where $\Delta^\nu_p(x)$ is the Green's function of the kinetic operator with the collision term in the relaxation-time approximation:

$$\Delta^\nu_p(x) \equiv E^{-1}\Theta(t)e^{-\nu' t}\delta^{(3)}(\mathbf{x} - \mathbf{v}t) \ ,$$

where $\mathbf{v} = \mathbf{p}/E$ and $\nu' \equiv \nu p^\mu u_\mu / E$; in the plasma rest frame $\nu' = \nu$.

Substituting (16) in (14) one finds

$$j^\mu(x) = \int d^4x' \sigma^{\mu\rho\lambda}(x-x') F_{\rho\lambda}(x') \quad (17)$$

with the color conductivity tensor

$$\sigma^{\mu\rho\lambda}(x) = \frac{g^2}{2}\int \frac{d^3p}{(2\pi)^3 E} p^\mu p^\rho \left[\Delta^\nu_p(x)\frac{\partial}{\partial p_\lambda} n(p) + \Delta^{\bar{\nu}}_p(x)\frac{\partial}{\partial p_\lambda}\bar{n}(p) + 2N\Delta^{\nu_g}_p(x)\frac{\partial}{\partial p_\lambda} n_g(p) \right] , \quad (18)$$

where the identity $f_{abc}f_{abd} = N\delta_{cd}$ has been used.

Performing the Fourier transformation with respect to the x variable, Eq. (17) and (18) can be rewritten as

$$j^\mu(k) = \sigma^{\mu\rho\lambda}(k) F_{\rho\lambda}(k) \ , \quad (19)$$

where

$$\sigma^{\mu\rho\lambda}(k)=i\frac{g^2}{2}\int\frac{d^3p}{(2\pi)^3E}p^\mu p^\rho\left[\frac{1}{p^\sigma(k_\sigma+i\nu u_\sigma)}\frac{\partial}{\partial p_\lambda}n(p)+\frac{1}{p^\sigma(k_\sigma+i\tilde{\nu}u_\sigma)}\frac{\partial}{\partial p_\lambda}\bar{n}(p)+2N\frac{1}{p^\sigma(k_\sigma+i\nu_g u_\sigma)}\frac{\partial}{\partial p_\lambda}n_g(p)\right]. \tag{20}$$

When the plasma equilibrium state is isotropic, which is obviously the case of global equilibrium, one finds that the structure of Lorentz indices of the conductivity is

$$\sigma^{\mu\rho\lambda}(k)=\sigma^{\mu\rho}(k)u^\lambda .$$

Then Eq. (19) gets a more familiar form, which in the plasma rest frame $[u^\mu=(1,0,0,0)]$ reads

$$j^\alpha(k)=\sigma^{\alpha\beta}(k)E^\beta(k) ,$$

where **E** is the chromoelectric vector.

One should note that the conductivity tensor is a scalar quantity in color space (no color indices). Equivalently, one can say that the conductivity is proportional to the unit matrix in color space.

IV. LINEARIZED QCD IN THE PLASMA MEDIUM

Let us introduce, as in electrodynamics, the polarization vector $\mathbf{P}(x)$ defined as

$$\mathrm{div}\mathbf{P}(x)=-\rho(x),\quad \frac{\partial}{\partial t}\mathbf{P}(x)=\mathbf{j}(x) , \tag{21}$$

where ρ and $\mathbf{j}$ are the timelike and spacelike components, respectively, of the color four-current, $j^\mu=(\rho,\mathbf{j})$. It should be stressed that the definition (21) is self-consistent if the color four-current is conserved (not only covariantly conserved). This is just the case of the linearized QCD.

Further, I define the chromoelectric induction vector $\mathbf{D}(x)$,

$$\mathbf{D}(x)=\mathbf{E}(x)+\mathbf{P}(x) , \tag{22}$$

and the chromelectric permeability tensor, which relates the Fourier-transformed **D** and **E** fields:

$$D^\alpha(k)=\epsilon^{\alpha\beta}(k)E^\beta(k) . \tag{23}$$

In this definition the permeability tensor is a color scalar (no color indices) since the conductivity tensor in (18) and (20) is a color scalar. Using the definitions (23), (22), and (21) one easily finds that

$$\epsilon^{\alpha\beta}(k)=\delta^{\alpha\beta}-i\sigma^{\alpha0\beta}(k)/\omega$$

with $\sigma^{\alpha0\beta}(k)$ given in Eqs. (19) and (20); ω is the timelike component of k, $k^\mu=(\omega,\mathbf{k})$.

Substituting the explicit form of the equilibrium distribution functions, Fermi-Dirac for (massless) quarks and Bose-Einstein for gluons, one gets

$$\epsilon^{\alpha\beta}(k)=\delta^{\alpha\beta}-\frac{2g^2}{\omega T}\int\frac{d^3p}{(2\pi)^3}v^\alpha v^\beta[N_f(\omega-\mathbf{k}\cdot\mathbf{v}-i\nu)^{-1}(e^{p/T}+1)^{-2}+N(\omega-\mathbf{k}\cdot\mathbf{v}+i\nu_g)^{-1}(e^{p/T}-1)^{-2}]e^{p/T} , \tag{24}$$

where N_f is the number of flavors, p is the length of the vector momentum, i.e., $p\equiv|\mathbf{p}|$ here (I hope that this change of notation will not lead to confusion) and T is the temperature. The above formula is valid in the reference frame where the plasma as a whole is at rest. One should note that for a baryonless plasma the numbers of quarks and antiquarks are equal to one another and $\nu=\tilde{\nu}$.

Because the equations of linearized QCD coincide (up to the trivial matrix structure) with those of electrodynamics, the dispersion relations are defined by Eqs. (3) and (4). The relations are, of course, very similar to those of the electrodynamic plasma; see, e.g., Refs. 5 and 6. They also agree with the results of FT QCD calculations[1-3] in the one-loop approximation.

For illustration I consider the long-wave limit of the oscillations. Calculating the integral in Eq. (24) for $\omega\gg|\mathbf{k}|$ and $\omega\gg\nu(\nu_g)$, one finds the following dispersion relations.

(A) Longitudinal mode

$$\omega^2=\omega_0^2-\xi^2+\tfrac{3}{4}\phi^2+\tfrac{3}{5}k^2,\quad \gamma=\tfrac{1}{2}\phi . \tag{25a}$$

(B) Transverse mode

$$\omega^2=\omega_0^2-\xi^2+\tfrac{3}{4}\phi^2+\tfrac{6}{5}k^2 ,\quad \gamma=\tfrac{1}{2}\phi . \tag{25b}$$

ω and γ denote the real and imaginary part, respectively, of the complex frequency, i.e., I performed the substitution $\omega\to\omega-i\gamma$; ω_0 is the plasma frequency and

$$\omega_0^2=\frac{g^2T^2(N_f+2N)}{18} . \tag{26}$$

ϕ and ξ parameters are related to ν and ν_g as

$$\phi=\nu\frac{N_f}{N_f+2N}+\nu_g\frac{2N}{N_f+2N} , \tag{27a}$$

$$\xi^2=\nu^2\frac{N_f}{N_f+2N}+\nu_g^2\frac{2N}{N_f+2N} . \tag{27b}$$

The presence of the nonzero imaginary part of the frequency makes the amplitude of the oscillations decreasing in time as $e^{-\gamma t}$. To find the value of γ, one has to estimate the equilibration rates ν and ν_g. This is done in the next section.

V. DAMPING MECHANISMS OF THE PLASMA OSCILLATIONS

Usually the equilibration rate parameter ν, which is present in the relaxation-time-approximation collision term, is identified with the inverse particle mean free flight time τ estimated from the formula

$$\tau^{-1}=\sigma_t \rho ,$$

where σ_t is the transport cross section and ρ is particle density. Then, ν is of order of $g^4 \ln g^{-2}$ (Ref. 15) for the perturbative QCD plasma.

However, there is also another damping mechanism—plasmon decay into quark-antiquark or gluon-gluon pairs. The first process is very similar to the plasmon decay into electron-positron pairs known from the ultrarelativistic electrodynamic plasma, while the second one, which occurs due to the three-gluon coupling, is characteristic for non-Abelian interactions. The plasmon decay is, in another language, a particle-antiparticle pair generation from a vacuum due to the mean (oscillatory) field. The plasmon decay width is, in the lowest order, proportional to g^2. However, the plasma frequency ω_0, which is of order of g, enters the formula, and more detailed considerations are needed to find the order of the width. It is easy to observe that, even in the limit of massless quarks, the decay into gluons is much more probable than the decay into quarks. The argument is as follows. If one considers the decay of plasmon of zero momentum into (massless) quarks or gluons, the phase-space volume of the final state is proportional to the factor

$$[1 \mp n(\omega_0/2)]^2 , \tag{28}$$

where the upper sign is for fermions, while the lower one is for bosons; $n(E)$ is the Fermi-Dirac or Bose-Einstein distribution, respectively. Because the plasma frequency (26) in perturbative plasma is much smaller than the temperature, the factor (28) can be expanded as

$$[1 \mp n(\omega_0/2)]^2 \simeq \begin{cases} \frac{1}{4}+\omega_0/8T & \text{for fermions,} \\ 4T^2/\omega_0^2 & \text{for bosons .} \end{cases}$$

Therefore it is seen that the decay into gluons is more probable than the decay into quarks by a factor of order of g^{-2} (Ref. 3).

Using the standard rules of FT field theory, one easily calculates (see Appendix B) the width of the zero-momentum plasmon decay into gluons

$$\Gamma_d = \frac{g^2 N}{2^4 3\pi}\omega_0[1+n(\omega_0/2)]^2 \simeq \frac{gN}{2^{3/2}\pi(N_f+2N)^{1/2}}T , \tag{29}$$

which is the same for longitudinal and transverse plasmons.

Γ_d cannot be identified with the plasmon equilibration rate Γ since, in addition to the plasmon decays, there are also plasmon formation processes. As shown by Weldon[16] (see also Ref. 3), the formation rate Γ_f is related to Γ_d as

$$\Gamma_f = \exp(-\omega_0/T)\Gamma_d \simeq (1-\omega_0/T)\Gamma_d .$$

Since the equilibration rate of plasmon (as boson) $\Gamma=\Gamma_d-\Gamma_f$ (Ref. 16), one finds

$$\Gamma \simeq \frac{g^2 N}{12\pi}T , \tag{30}$$

which agrees (up to the additionally introduced coefficient $\frac{1}{2}$ in Ref. 3) with the result of Ref. 3, where Γ has been expressed through the imaginary part of the polarization tensor. The agreement is not surprising since I have used, as in Ref. 3, the radiation gauge ($A^0(x)=0$, $\partial^i A^i(x)=0$).

Let me note that Γ_d and Γ_f are of order of g, while Γ is of order of g^2. This means that the plasmon decay and formation processes cancel one another in the lowest order of g. One should also observe that the plasmon decay width is not a Lorentz-invariant quantity, since there is a preferred reference frame—the rest frame of the thermostat. Therefore the result (30) is valid for zero-momentum plasmons, or approximately for long-wave plasmons only.

There is a delicate question of whether Γ can be identified with ν_g. Since gluons from plasmon decay are not of thermal equilibrium distribution, parton collisions (the cross sections which are of order of $g^4 \ln g^{-2}$) are needed to equilibrate the system. However, the gluons from plasmon decay locally neutralize the plasma and consequently damp the oscillations. Therefore if one considers the plasma oscillations, which make the plasma locally colored, ν_g can be identified with Γ. On the other hand, if one studies, say, the viscosity of the locally neutral plasma, ν_g obviously differs from Γ.

What is the value of the quark equilibration parameter ν? Since the plasmon decays into quarks, and binary collisions involving quarks are at least of order g^4, it seems reasonable to assume that $\nu_g \gg \nu$ for the perturbative plasma. Then, substituting $\nu_g=\Gamma$ and $\nu=0$ in Eqs. (25) and (27), one finds the decrement of the plasma oscillation damping

$$\gamma \simeq \frac{g^2}{12\pi}\frac{N^2}{N_f+2N}T . \tag{31}$$

The characteristic feature of Eq. (31) is the fact that the damping rate depends on the number of quark flavors, although $\nu=0$. This result seems in agreement with physical intuition. When the number of quark flavors is increased the inertia of the system is also increased and consequently the time needed to damp the oscillation is longer. However, Eq. (30) disagrees [by a factor $2N/(N_f+2N)$] with the result from Ref. 3, where γ equals Γ given by Eq. (30) (Ref. 17). Therefore the damping decrement is independent of the number of quark flavors. The only way to reproduce the result from Ref. 3 in the framework of the approach discussed here is to assume that $\nu_g=\nu=\Gamma$. However, it is hard to understand this assumption on physical grounds. Probably the problem could not be resolved as long as the collision terms of the transport equations (7) and (10) are not derived.

VI. CONCLUSIONS

The analysis of quark-gluon plasma oscillations presented here looks much more trivial than the one based on FT QCD (Refs. 1–3). The kinetic approach is based on the semiclassical equations, and the problem is simplified to a linear one strongly resembling the electrodynamic plasma. It should be stressed that the linearization procedure does not lead to the cancellation of all non-Abelian effects, since gluons contribute to the color current which generates the chromodynamic mean field. Therefore the gluon-gluon interactions, which are of essentially non-Abelian character, are included in a specific way. This is seen formally by the presence of the structure constants f_{abc} in Eq. (14), and also supported by comparing with FT QCD where the gluonic contribution to ω_0 arises from the gluon loop involving the three-gluon vertex.

In the approach discussed, the chromoelectric polarization vector, or equivalently the chromoelectric induction, is defined as in electrodynamics. Let me recall that this demands conservation of the color current. Then the plasma oscillations correspond, as in QED, to the oscillatory solutions of the (linearized) field equations. In the case of nonlinear approaches the correspondence between the dispersion relations and solutions of QCD field equations is not established. Therefore, in my opinion, the physical meaning of the plasma oscillations is clear at present only for the linearized QCD.

The dispersion relations (25) agree with those from Refs. 1–3, where they have been found in the one-loop approximation. To estimate the damping decrement I have used arguments beyond the kinetic approach. In principle, it is possible to study the oscillation damping in the framework of the transport theory; however, the particle production processes should be included in the kinetic equations. The first steps in this direction have been recently done.[12,18]

It should be also noted that the kinetic-theory approach can be applied to study the oscillations around any quasistable state of the plasma, not only around the global thermodynamical equilibrium. In my very recent paper,[19] the same approach (with simple modifications) has been used to discuss the instabilities of the system of two streams of quark-gluon plasma.

Note added in proof. The quark-gluon plasma oscillations around global equilibrium have been very recently studied [A. Bialas and W. Czyz, Ann. Phys. (N.Y.) **187**, 97 (1988)] using the kinetic-theory method proposed in Ref. 12.

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APPENDIX A

The starting point for the derivation of Eq. (10) is Eq. (1.9) from Ref. 12, which reads

$$P^\sigma \tilde{D}_\sigma G_{\mu\nu} = \tfrac{1}{2} g p^\sigma \partial_p^\tau \{[F_{\tau\sigma}, G_{\mu\nu}]_R + [G_{\mu\nu}, F_{\tau\sigma}]_L\} - g\{[F_{\sigma\mu}, G^\sigma{}_\nu]_R - [G_{\mu\sigma}, F^\sigma{}_\nu]_L\} \ . \qquad (A1)$$

The notation is as in Ref. 12. The above equation has been derived in the collisionless limit; therefore there is no collision term. Comparing to the original Eq. (1.9) from Ref. 12, I have neglected in Eq. (A1) the second-gradient terms, which are corrections to the classical limit.

I substitute into Eq. (A1) the distribution function in the form

$$G_{\mu\nu} = \mathcal{G}^{ab}_{\mu\nu} \tau^a \otimes \tau^b \ .$$

Then the equation is multiplied by $\tau^c \otimes \tau^d$ and I take the trace, defined as

$$\mathrm{Tr}(\tau^c\tau^a \otimes \tau^d\tau^b) = \mathrm{Tr}(\tau^c\tau^a)\mathrm{Tr}(\tau^d\tau^b) \ .$$

Using the identity Tr $(\tau^a\tau^b) = \frac{1}{2}\delta^{ab}$ one finds the equation

$$p_\sigma \mathcal{D}^\sigma \mathcal{G}_{\mu\nu} + g p^\sigma \frac{\partial}{\partial p_\tau} \frac{1}{2} \{\mathcal{F}_{\tau\sigma}, \mathcal{G}_{\mu\nu}\} = g(\mathcal{F}_{\mu\sigma}\mathcal{G}^\sigma{}_\nu - \mathcal{G}_{\mu\sigma}\mathcal{F}^\sigma{}_\nu) \ . \qquad (A2)$$

The notation is explained under Eq. (10) in the main text.

Further, I assume that $\mathcal{G}_{\mu\nu}$ can be expressed as

$$\mathcal{G}_{\mu\nu}(p,x) = p_\mu p_\nu \mathcal{G}(p,x) \ , \qquad (A3)$$

which corresponds to the equilibrium with respect to the spin degrees of freedom of gluons. The function $\mathcal{G}_{\mu\nu}$ in the form (A3) is substituted into Eq. (A2) and the resulting equation is multiplied by $p^\mu p^\nu$. Assuming that $p_\mu p^\mu$ is infinitesimally small but finite, one finds Eq. (10) with a zero collision term.

APPENDIX B

Using the convention from Ref. 20, the plasmon decay width is expressed as

$$\Gamma_d = \frac{1}{2\omega_0} \frac{1}{2n_s(N^2-1)} \int \frac{d^3p_1}{(2\pi)^3 2E_1} \frac{d^3p_2}{(2\pi)^3 2E_2} (2\pi)^4 \delta^{(4)}(k - p_1 - p_2)[1+n_g(E_1)][1+n_g(E_2)] \sum_{\text{colors}} \sum_{s,s_1,s_2} |M|^2 \ , \qquad (B1)$$

where n_s is the number of polarization states, which is 2 for a transverse plasmon and 1 for a longitudinal one; M is the amplitude of decay of a plasmon (off-shell gluon) of a four-momentum k into two real (transverse) gluons of the four-momenta p_1 and p_2. The amplitude in the lowest order of the coupling constant reads

$$M = \tilde{\epsilon}^{\sigma}(k,s)\{-g f_{abc}[(-k-p_1)_\nu g_{\sigma\mu} + (p_1 - p_2)_\sigma g_{\mu\nu} + (p_2 - k)_\mu g_{\nu\sigma}]\} \times \epsilon^{\dagger\nu}(p_1,s_1)\epsilon^{\dagger\mu}(p_2,s_2) ,$$

where $\tilde{\epsilon}$ and ϵ are the polarization vectors of a plasmon and a gluon, respectively, the expression in the curly brackets is the three-gluon vertex function.[21] For the zero-momentum plasmon ($\mathbf{p}_1 = -\mathbf{p}_2 \equiv \mathbf{p}$, $E_1 = E_2 = \omega_0/2$), one finds

$$M = -2g f_{abc}\tilde{\epsilon}^{\alpha}(k,s)p_\alpha \epsilon^{\dagger\nu}(p_1,s_1)\epsilon^{\dagger}_{\nu}(p_2,s_2) , \tag{B2}$$

where the transversality condition of the real gluons in the temporal axial gauge $[\epsilon^\mu = (0,\epsilon)]$ has been used. The plasmon momentum has been put to zero; however, it should be treated as an infinitesimally small vector to distinguish between the transverse and longitudinal plasmons.

From Eq. (B2) one gets

$$\sum_{\text{colors}} \sum_{s,s_1,s_2} |M|^2 = 2g^2 N(N^2-1)\omega_0^2 \begin{Bmatrix} 1-\cos^2\theta \\ \cos^2\theta \end{Bmatrix} , \tag{B3}$$

where the upper expression in the curly brackets relates to the transverse plasmon, while the lower one relates to the longitudinal plasmon. The angle θ is between **p** and **k** vectors. The summation over plasmon polarization has been performed by means of the formula

$$\sum_s \tilde{\epsilon}_\alpha(k,s)\tilde{\epsilon}^{\dagger}_{\beta}(k,s) = \begin{Bmatrix} \delta_{\alpha\beta} - \mathbf{k}^{-2}k_\alpha k_\beta \\ \mathbf{k}^{-2}k_\alpha k_\beta \end{Bmatrix} .$$

As previously, the upper expression relates to the transverse plasmon and the lower one to the longitudinal plasmon.

Substituting (B3) into (B2) and performing the phase-space integration, one finds

$$\Gamma = \frac{g^2 N}{2^4 3\pi}\omega_0[1 + n_g(\omega_0/2)]^2 .$$

Keeping in mind that $T \gg \omega_0$ and using the explicit expression (26) of ω_0, one finally gets Eq. (30).

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