Chromohydrodynamic approach to the unstable quark-gluon plasma

Cristina Manuel[*]

Institut d'Estudis Espacials de Catalunya (IEEC) and Instituto de Ciencias del Espacio (CSIC),
Campus Universitat Autònoma de Barcelona, Facultat de Ciències, Torre C5 E-08193 Bellaterra (Barcelona), Spain

Stanisław Mrówczyński[†]

Sołtan Institute for Nuclear Studies, ul. Hoża 69, PL - 00-681 Warsaw, Poland
and Institute of Physics, Świętokrzyska Academy, ul. Świętokrzyska 15, PL - 25-406 Kielce, Poland

We derive hydrodynamic-like equations that are applicable to short-time-scale color phenomena in the quark-gluon plasma. The equations are solved in the linear response approximation, and the gluon polarization tensor is derived. As an application, we study the collective modes in a two-stream system and find plasma instabilities when the fluid velocity is larger than the speed of sound in the plasma. The chromohydrodynamic approach, discussed here in detail, should be considered as simpler over other approaches and well-designed for numerical studies of the dynamics of an unstable quark-gluon plasma.

I. INTRODUCTION

Bulk features of electromagnetic plasmas are usually studied by means of fluid equations [1]. To get more detailed information, one refers to kinetic theory [1]. Since the fluid equations are noticeably simpler than the kinetic ones, the hydrodynamic approach is also frequently used in numerical simulations of the plasma evolution, studies of the nonlinear dynamics, etc. [1]. The situation in the field of quark-gluon plasma studies is rather different. Although chromohydrodynamics equations were discussed by several authors over a long period of time [2–12], they were not carefully studied. The field of their applicability was not established and very few important results were obtained by means of them. Consequently, the chromohydrodynamics has not attracted much attention. Instead, field theory diagrammatic methods have been successfully applied to reveal equilibrium properties of the quark-gluon plasma [13], while transport theory [14–16] proved efficient in the nonequilibrium studies. In particular, the important role of instabilities in the quark-gluon plasma evolution was clarified within the kinetic theory approach, see the review [17]. The kinetic equations were also a basis of extensive numerical simulations of the unstable QCD plasma [18–24]. The early stage of relativistic heavy-ion collisions, when the quark-gluon system is produced, was effectively studied using methods of classical field theory, see e.g. the review [25] and very recent publications [26,27].

Inspired by the success of hydrodynamic methods in the electromagnetic plasma, we discuss the approach to be applied to the quark-gluon plasma. Before going to the main subject of our study, however, a very important point has to be clarified. Real hydrodynamics deals with systems which are in local equilibrium, and thus it is only applicable at sufficiently long time scales. The continuity and the Euler or Navier-Stokes equations are supplemented by the equation of state to form a complete set of equations. The equations can be derived from kinetic theory, using the distribution function of local equilibrium, which by definition maximizes the entropy density, and thus the function cancels the collision terms of the transport equations. Such a real chromohydrodynamics was derived in [10] where the state of local equilibrium was found, using the collision term of the Waldman-Snider form. The chromohydrodynamics has occurred trivial in the sense that although the local equilibrium can be colorful, all color components of the plasma move with the same hydrodynamic velocity. Therefore, chromodynamic effects disappear entirely once the system is neutralized. It actually happens even before the local equilibrium is achieved [28]. Thus, there is no QCD analog of the magneto-hydrodynamics which is well known in the electromagnetic plasma. The magneto-hydrodynamic regime appears due to a large difference of electron and ion masses which effectively slows down a mutual equilibration of electrons and ions. Therefore, at a relatively long time scale one deals with the charged electron fluid in local equilibrium which moves in a passive background of positive ions.

Since the hydrodynamic equations express the macroscopic conservation laws, the equations hold not only for systems in local equilibrium but for systems out of equilibrium as well. In particular, the equations can be applied at time scales significantly shorter than that of local equilibration. At such a short time scale the collision terms of the transport equations can be neglected. However, extra assumptions are then needed to close the set of equations, as

―――――
[*]Electronic address: cmanuel@ieec.uab.es
[†]Electronic address: mrow@fuw.edu.pl

the (equilibrium) equation of state cannot be used. In the electromagnetic plasma physics, several methods to close the system of equations were worked out.

In this paper we discuss not a real hydrodynamics describing the quark-gluon plasma in local equilibrium, but the fluid equations which are valid at much shorter time scale. The approach is designed to study temporal evolution of the unstable QCD plasma. In Sec. III the fluid equations are derived from the kinetic theory which is presented in Sec. II. The equations are solved in the linear response approximation in Sec. IV and the polarization tensor is derived. As an example, the collective plasma modes in the two-stream system are analyzed in Sec. V. The paper is closed with a few remarks on the hydrodynamic approach.

Throughout the paper we use the natural units with $c = \hbar = k_B = 1$ and the metric $(1, -1, -1, -1)$.

II. KINETIC THEORY

In this section we briefly present the transport theory of quarks and gluons [14,15] which is used to derive the fluid equations.

The distribution function of quarks $Q(p, x)$, where p is the four-momentum and x is the four-position, is a hermitian $N_c \times N_c$ matrix in color space (for a SU(N_c) color group). The distribution function is gauge dependent and it transforms under a local gauge transformation U as

$$Q(p, x) \rightarrow U(x)Q(p, x)U^\dagger(x). \tag{1}$$

The distribution function of antiquarks, which we denote by $\bar{Q}(p, x)$, is also a hermitian $N_c \times N_c$ matrix in color space it also transforms according to Eq. (1). The distribution function of gluons is a hermitian $(N_c^2 - 1) \times (N_c^2 - 1)$ matrix, which transforms as

$$G(p, x) \rightarrow \mathcal{U}(x)G(p, x)\mathcal{U}^\dagger(x), \tag{2}$$

where

$$\mathcal{U}_{ab}(x) = 2\,\text{Tr}[\tau^a U(x)\tau^b U^\dagger(x)], \tag{3}$$

with τ^a, $a = 1, \ldots, N_c^2 - 1$ being the SU(N_c) group generators in the fundamental representation with $\text{Tr}(\tau^a \tau^b) = \frac{1}{2}\delta^{ab}$.

The four-momentum p, which is the argument of the distribution functions, is not constrained by the mass-shell condition but it is taken into account in the momentum integration measure. We use the notation

$$\int_p \cdots \equiv \int \frac{d^4 p}{(2\pi)^3} 2\Theta(p_0)\delta(p^2), \tag{4}$$

and the color current is expressed in the fundamental representation as

$$j^\mu(x) = -\frac{g}{2} \int_p p^\mu \bigg[Q(p, x) - \bar{Q}(p, x)$$
$$- \frac{1}{N_c} \text{Tr}[Q(p, x) - \bar{Q}(p, x)]$$
$$+ 2\tau^a \text{Tr}[T^a G(p, x)] \bigg], \tag{5}$$

where g is the QCD coupling constant. A sum over helicities, two per particle, and over quark flavors N_f is understood in Eq. (5), even though it is not explicitly written down. The SU(N_c) generators in the adjoint representation are expressed through the structure constants $T_{bc}^a = -if_{abc}$, and are normalized as $\text{Tr}[T^a T^b] = N_c\delta^{ab}$. The current can be decomposed as $j^\mu(x) = j_a^\mu(x)\tau^a$ with $j_a^\mu(x) = 2\,\text{Tr}(\tau_a j^\mu(x))$.

The distribution functions of quarks, antiquarks and gluons satisfy the transport equations:

$$p^\mu D_\mu Q(p, x) + \frac{g}{2} p^\mu\{F_{\mu\nu}(x), \partial_p^\nu Q(p, x)\} = C[Q, \bar{Q}, G],$$
$$\tag{6}$$

$$p^\mu D_\mu \bar{Q}(p, x) - \frac{g}{2} p^\mu\{F_{\mu\nu}(x), \partial_p^\nu \bar{Q}(p, x)\} = \bar{C}[Q, \bar{Q}, G],$$
$$\tag{7}$$

$$p^\mu \mathcal{D}_\mu G(p, x) + \frac{g}{2} p^\mu\{\mathcal{F}_{\mu\nu}(x), \partial_p^\nu G(p, x)\} = C_g[Q, \bar{Q}, G],$$
$$\tag{8}$$

where $\{\ldots, \ldots\}$ denotes the anticommutator and ∂_p^ν the four-momentum derivative; the covariant derivatives D_μ and $\mathcal{D}_\mu$ act as

$$D_\mu = \partial_\mu - ig[A_\mu(x), \ldots],$$
$$\mathcal{D}_\mu = \partial_\mu - ig[\mathcal{A}_\mu(x), \ldots],$$

with A_μ and $\mathcal{A}_\mu$ being four-potentials in the fundamental and adjoint representations, respectively:

$$A^\mu(x) = A_a^\mu(x)\tau^a, \qquad \mathcal{A}^\mu(x) = T^a A_a^\mu(x).$$

The strength tensor in the fundamental representation is $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - ig[A_\mu, A_\nu]$, while $\mathcal{F}_{\mu\nu}$ denotes the field strength tensor in the adjoint representation. C, $\bar{C}$ and C_g represent the collision terms which are neglected in our further considerations, as we are interested in short-time-scale phenomena which are "faster" than the collisions.

The transport equations are supplemented by the Yang-Mills equation describing generation of the gauge field

$$D_\mu F^{\mu\nu}(x) = j^\nu(x), \tag{9}$$

where the color current is given by Eq. (5).

III. FLUID EQUATIONS

We assume here that there are several streams in the plasma system, and then the distribution function of, say, quarks $Q(p, x)$ can be decomposed as $Q(p, x) = \sum_\alpha Q_\alpha(p, x)$ where the index α labels the streams. Since we are interested in the short-time-scale phenomena—"faster" than collisions—the complete quark distribution function $Q(p, x)$ as well as the distribution function of every stream $Q_\alpha(p, x)$ satisfy the collisionless transport equation. The streams interact with each other only via the mean-filed which is generated in a self-consistent way by the current where every stream contributes. We note that the decomposition of the distribution function $Q(p, x)$ into streams is, in principle, not unique. However, if the stream distribution functions $Q_\alpha(p, x)$ solve the transport equations combined with the field generation equation, the complete distribution function solves the transport equation and the field generation equation as well.

Further analysis is limited to quarks but inclusion of antiquarks and gluons is straightforward. Integrating the collisionless transport Eq. (6) satisfied by Q_α over the four-momentum with the integration measure (4), one finds the covariant continuity equation

$$D_\mu n_\alpha^\mu = 0, \tag{10}$$

where n_α^μ is $N_c \times N_c$ matrix defined as

$$n_\alpha^\mu(x) \equiv \int_p p^\mu Q_\alpha(p, x). \tag{11}$$

The four-flow n_α^μ transforms under gauge transformations as the quark distribution function, i.e. according to Eq. (1).

Multiplying the transport Eq. (6) by the four-momentum and integrating the product with the measure (4), we get

$$D_\mu T_\alpha^{\mu\nu} - \frac{g}{2}\{F_\mu{}^\nu, n_\alpha^\mu\} = 0, \tag{12}$$

where the energy-momentum tensor is

$$T_\alpha^{\mu\nu}(x) \equiv \int_p p^\mu p^\nu Q_\alpha(p, x). \tag{13}$$

We further postulate that n_α^μ and $T_\alpha^{\mu\nu}$ have the form of an ideal fluid i.e.

$$n_\alpha^\mu(x) = n_\alpha(x) u_\alpha^\mu(x), \tag{14}$$

$$T_\alpha^{\mu\nu}(x) = \frac{1}{2}(\epsilon_\alpha(x) + p_\alpha(x))\{u_\alpha^\mu(x), u_\alpha^\nu(x)\} - p_\alpha(x) g^{\mu\nu}, \tag{15}$$

where the hydrodynamic velocity u_α^μ is, as n_α, ϵ_α and p_α, a $N_c \times N_c$ matrix. The anticommutator of u_α^μ and u_α^ν is present in Eq. (15) to guarantee the symmetry of $T_\alpha^{\mu\nu}$ with respect to $\mu \leftrightarrow \nu$ which is evident in the definition (13).

Since Eqs. (14) and (15) are of crucial importance for our further considerations, let us discuss them in more

detail. There are two aspects of the ideal-fluid form of n_α^μ and $T_\alpha^{\mu\nu}$ to be justified: the Lorentz structure and the matrix structure. We start the discussion with the Lorentz structure. Equations (14) and (15) assume that n_α^μ and $T_\alpha^{\mu\nu}$ can be expressed as products of the Lorentz scalars n_α, ϵ_α and p_α and the four-vector u_α^μ. Such a Lorentz structure appears when the distribution function $Q_\alpha(p, x)$, which gives n_α^μ and $T_\alpha^{\mu\nu}$ through Eqs. (11) and (13), is isotropic in the local rest frame where $u_\alpha^\mu = (1, 0, 0, 0)$. One does not need to assume that the function is of local equilibrium form, the isotropy is a sufficient condition. And we note that the isotropy of the momentum distribution, required Eqs. (14) and (15), is not a serious limitation because we actually consider not a single stream but a superposition of several streams. And then, one can easily model an arbitrary momentum distribution as a sum of several momentum distributions, everyone isotropic in its Lorentz rest frame. Thus, the Lorentz structure expressed by Eqs. (14) and (15) seem to be well justified.

The problem of the matrix structure of Eqs. (14) and (15) is more complicated. We first note that n_α^μ and $T_\alpha^{\mu\nu}$ have to be matrix value functions as the distribution function $Q_\alpha(p, x)$ which defines them through Eqs. (11) and (13) is a matrix in a color space. The question is whether n_α, ϵ_α, p_α and u_α^μ have to be all matrix quantities. When the system under consideration is colorless, these quantities can be treated as scalars in the color space. However, as explicitly shown in Sec. IV, even small perturbations of these quantities induced by a color field are of matrix value. Therefore, n_α, ϵ_α, p_α and u_α^μ have to be all treated as matrices.

When n_α^μ and $T_\alpha^{\mu\nu}$ are expressed through n_α, ϵ_α, p_α and u_α^μ, one faces the problem of ordering the matrices. One can define n_α^μ in three ways: $n_\alpha^\mu = n_\alpha u_\alpha^\mu$, $n_\alpha^\mu = u_\alpha^\mu n_\alpha$ and $n_\alpha^\mu = \{n_\alpha, u_\alpha^\mu\}/2$ where $\{\ldots, \ldots\}$ is the anticommutator. The three different definitions obviously lead to three different fluid equations coming from $D_\mu n_\alpha^\mu = 0$. We first note that the problem of ordering does not appear when small perturbations of colorless state are studied because the linearized equations are all the same, as follows from considerations presented in Sec. IV. The problem of ordering is also absent for systems, which are significantly colorful, if the matrices belong to the Cartan subalgebra of $SU(N_c)$, as then they commute with each other. Unfortunately, we have not found a general solution of the ordering problem. Numerical simulations will be very helpful to clarify whether the three formulations are equivalent to each other.

In the case of an Abelian plasma, the relativistic version of the Euler equation is obtained from Eq. (12) by removing from it, following Landau and Lifshitz [29], the part which is parallel to u_α^μ. An analogous procedure is not possible for the non-Abelian plasma because the matrices n_α, u_α^μ, and u_α^ν, in general, do not commute with each other. Thus, one has to work directly with Eqs. (10) and

(12) with n_α^μ and $T_\alpha^{\mu\nu}$ of the form (14) and (15). The equations have to be supplemented by the Yang-Mills Eq. (9) with the color current of the form

$$j^\mu(x) = -\frac{g}{2} \sum_\alpha \left(n_\alpha u_\alpha^\mu - \frac{1}{N_c} \operatorname{Tr}[n_\alpha u_\alpha^\mu] \right), \qquad (16)$$

where only the quark contribution is taken into account.

The fluid Eqs. (10) and (12) do not form a closed set of equations even when the chromodynamic field is treated as an external one. There are five matrix equations ($\nu = 0, 1, 2, 3$) and six unknown matrix functions: $n_\alpha(x)$, $\epsilon_\alpha(x)$, $p_\alpha(x)$ and three components of the four-velocity $u_\alpha^\mu(x)$. We note that the constraint $u_\alpha^\mu(x)u_{\alpha\mu}(x) = 1$ is imposed.

There are several ways to close the system. In the case of real hydrodynamics, which describes the system in local thermodynamic equilibrium, one adds the equation of state. Although the system under consideration is not in equilibrium, we can still add a relation analogous to the equation of state. The point is that the energy-momentum tensor (13) of a weakly interacting gas of massless quarks is traceless ($T_{\alpha\mu}^\mu(x) = 0$) because $p^2 = 0$. Then, Eq. (15) provides the desired relation

$$\epsilon_\alpha(x) = 3p_\alpha(x) \qquad (17)$$

due to the constraint $u_\alpha^\mu(x)u_{\alpha\mu}(x) = 1$. In fact, the complete energy-momentum tensor of any conformal theory is traceless but quantum effects, as the trace anomaly, can spoil conformal invariance. The phenomenon of mass generation in a medium modifies Eq. (17) but the effect is small in the perturbative regime studied here. We also note that $\epsilon_\alpha(x)$ and $p_\alpha(x)$ are *not* the energy density and pressure of the equilibrium system but the matrices in color space which in equilibrium become the energy density and pressure, respectively.

Another method to close the system of equations is to neglect the gradients of pressure ($p_\alpha(x)$), assuming that the system's dynamics is dominated by a self-consistently generated chromodynamic field. In the following section, where the equations are solved in the linear response approximation, we use both methods to close the system.

Our analysis is limited to quarks but, as already mentioned, inclusion of antiquarks and gluons is straightforward. Since the distribution functions of quarks, antiquarks and gluons of every stream are assumed to obey the collisionless transport equation, we have a separated set of fluid equations for quarks, antiquarks and gluons of every stream. The equations are coupled only through the chromodynamic mean field. The quarks belong to the fundamental representation of the $SU(N_c)$ group and thus, the hydrodynamic quantities $n_\alpha(x)$, $\epsilon_\alpha(x)$, $p_\alpha(x)$ and $u_\alpha^\mu(x)$ are $N_c \times N_c$ matrices. Antiquarks can be treated as in kinetic theory that is as belonging to the fundamental representation even so they belong, strictly speaking, to the transposed fundamental representation. Since gluons belong to

the adjoint representation, the hydrodynamic quantities, and consequently the fluid equations, are $(N_c^2 - 1) \times (N_c^2 - 1)$ matrices.

IV. LINEAR RESPONSE ANALYSIS

In this section the hydrodynamic Eqs. (10) and (12) are linearized around an stationary, homogeneous and colorless state described by $\bar{n}$, $\bar{\epsilon}$, $\bar{p}$ and $\bar{u}^\mu$. We mostly skip here the index α to simplify the notation. The index is restored in the very final formulas.

Because every stream is assumed to be colorless, the matrices $\bar{n}$, $\bar{\epsilon}$, $\bar{p}$ and $\bar{u}^\mu$ are proportional to the unit matrix in the color space. Thus,

$$\bar{n}\bar{u}^\mu - \frac{1}{N_c} \operatorname{Tr}[\bar{n}\bar{u}^\mu] = 0, \qquad (18)$$

which means that the color four-current of every stream vanishes. The quantities of interest are decomposed as

$$n(x) = \bar{n} + \delta n(x), \qquad \epsilon(x) = \bar{\epsilon} + \delta\epsilon(x), \qquad (19)$$

$$p(x) = \bar{p} + \delta p(x), \qquad u^\mu(x) = \bar{u}^\mu + \delta u^\mu(x). \qquad (20)$$

Since the state described by $\bar{n}$, $\bar{\epsilon}$, $\bar{p}$ and $\bar{u}^\mu$ is assumed to be stationary, homogeneous and colorless, we have

$$D^\mu \bar{n} = 0, \qquad D^\mu \bar{\epsilon} = 0, \qquad D^\mu \bar{p} = 0, \qquad D^\mu \bar{u}^\nu = 0. \qquad (21)$$

And because we consider only small deviations from the stationary, homogeneous and colorless state, the following conditions are obeyed

$$\bar{n} \gg \delta n, \qquad \bar{\epsilon} \gg \delta\epsilon, \qquad \bar{p} \gg \delta p, \qquad \bar{u}^\mu \gg \delta u^\mu. \qquad (22)$$

Actually, δn, $\delta\epsilon$, δp and δu^μ should be diagonalized to be comparable to the $\bar{n}$, $\bar{\epsilon}$, $\bar{p}$ and $\bar{u}^\mu$.

Substituting the linearized n^μ and $T^{\mu\nu}$, which are

$$n^\mu = \bar{n}\bar{u}^\mu + \bar{n}\delta u^\mu + \delta n\bar{u}^\mu, \qquad (23)$$

$$T^{\mu\nu} = (\bar{\epsilon} + \bar{p})\bar{u}^\mu\bar{u}^\nu - \bar{p}g^{\mu\nu} + (\delta\epsilon + \delta p)\bar{u}^\mu\bar{u}^\nu$$
$$+ (\bar{\epsilon} + \bar{p})(\bar{u}^\mu\delta u^\nu + \delta u^\mu\bar{u}^\nu) - \delta p g^{\mu\nu}, \qquad (24)$$

into Eqs. (10) and (12), one finds

$$\bar{n}D_\mu\delta u^\mu + (D_\mu\delta n)\bar{u}^\mu = 0, \qquad (25)$$

$$(D_\mu(\delta\epsilon + \delta p))\bar{u}^\mu\bar{u}^\nu + (\bar{\epsilon} + \bar{p})(\bar{u}^\mu(D_\mu\delta u^\nu)$$
$$+ (D_\mu\delta u^\mu)\bar{u}^\nu) - D^\nu\delta p - gF^{\mu\nu}\bar{n}\bar{u}_\mu = 0. \qquad (26)$$

Projecting Eq. (26) on $\bar{u}^\nu$, one finds

$$\bar{u}^\mu D_\mu\delta\epsilon + (\bar{\epsilon} + \bar{p})D_\mu\delta u^\mu = 0. \qquad (27)$$

To derive Eq. (27) one has to observe that $u^\mu u_\mu = 1 = \bar{u}^\mu \bar{u}_\mu$, and consequently, $\bar{u}^\mu \delta u_\mu = \mathcal{O}((\delta u)^2)$ and $\bar{u}^\mu D^\nu \delta u_\mu = \mathcal{O}((\delta u)^2)$.

Acting on Eq. (26) with the projection operator $(g_{\sigma\nu} - \bar{u}_\sigma \bar{u}_\nu)$, one gets the linearized relativistic Euler equation

$$(\bar{\epsilon} + \bar{p})\bar{u}_\mu D^\mu \delta u^\nu - (D^\nu - \bar{u}^\nu \bar{u}_\mu D^\mu)\delta p - g\bar{n}\bar{u}_\mu F^{\mu\nu} = 0. \tag{28}$$

As already mentioned, Eqs. (25), (27), and (28) do not form a closed set of equations. In the following two sections we solve the fluid Eqs. (25), (27), and (28), adopting two different methods to close the system.

A. Pressure gradients neglected

The system is closed when the pressure gradients are neglected, which physically means that the system's dynamics is dominated by the mean field. When the term with the pressure gradient is dropped in Eq. (28), the Eq. (27) effectively decouples from the remaining equations, and one has to solve only two Eqs. (25) and (28) where $\delta\epsilon$ is absent.

In principle, Eqs. (25) and (28) with the pressure term neglected can be formally solved in a gauge covariant manner, using the inverse operator of the covariant derivative. However, we are interested here only in computing the polarization tensor. Thus, we replace the covariant derivatives by the normal ones to fully linearize the equations, and the gauge independence of the result is checked *a posteriori*. After performing the Fourier transformation,

which is defined as

$$f(x) \equiv \int \frac{d^4k}{(2\pi)^4} e^{-ik\cdot x} f(k), \tag{29}$$

Equations (25) and (28) get the form

$$\bar{u}^\mu k_\mu \delta n + \bar{n} k_\mu \delta u^\mu = 0, \tag{30}$$

$$i(\bar{\epsilon} + \bar{p})\bar{u}_\mu k^\mu \delta u^\nu + g\bar{n}\bar{u}_\mu F^{\mu\nu} = 0. \tag{31}$$

Equations (30) and (31) are easily solved providing

$$\delta n = -ig\frac{\bar{n}^2}{\bar{\epsilon} + \bar{p}}\frac{\bar{u}_\mu k_\nu}{(\bar{u}\cdot k)^2}F^{\mu\nu}, \tag{32}$$

$$\delta u^\nu = ig\frac{\bar{n}}{\bar{\epsilon} + \bar{p}}\frac{\bar{u}_\mu}{\bar{u}\cdot k}F^{\mu\nu}. \tag{33}$$

Since $\text{Tr}[F^{\mu\nu}] = 0$, the induced current equals

$$\delta j^\mu = -\frac{g}{2}\sum_\alpha(\bar{n}_\alpha \delta u^\mu_\alpha + \delta n_\alpha \bar{u}^\mu_\alpha), \tag{34}$$

where the index labelling the streams present in the plasma system is restored. Because the linearized strength tensor equals $F^{\mu\nu}(k) = -ik^\mu A^\nu(k) + ik^\nu A^\mu(k)$, one finally finds

$$\delta j^\mu(k) = -\Pi^{\mu\nu}(k)A_\nu(k), \tag{35}$$

with

$$\Pi^{\mu\nu}(k) = -\frac{g^2}{2}\sum_\alpha \frac{\bar{n}^2_\alpha}{\bar{\epsilon}_\alpha + \bar{p}_\alpha}\frac{(\bar{u}_\alpha \cdot k)(k^\mu \bar{u}^\nu_\alpha + k^\nu \bar{u}^\mu_\alpha) - k^2 \bar{u}^\mu_\alpha \bar{u}^\nu_\alpha - (\bar{u}_\alpha \cdot k)^2 g^{\mu\nu}}{(\bar{u}_\alpha \cdot k)^2}. \tag{36}$$

The polarization tensor $\Pi^{\mu\nu}(k)$ is proportional to the unit matrix in color space, it is symmetric and transverse $(k_\mu \Pi^{\mu\nu}(k) = 0)$, and thus it is gauge independent.

The polarization tensor obtained within the kinetic theory in analogous approximation [30] is

$$\Pi^{\mu\nu}(k) = -\frac{g^2}{2}\int_p f(p)\frac{(p\cdot k)(k^\mu p^\nu + k^\nu p^\mu) - k^2 p^\mu p^\nu - (p\cdot k)^2 g^{\mu\nu}}{(p\cdot k)^2}, \tag{37}$$

where $f(p)$ is the distribution function of quarks in the colorless, stationary and homogeneous state. One observes that Eq. (37) transforms into Eq. (36) when

$$f(p) = \sum_\alpha \bar{n}_\alpha \bar{u}^0_\alpha \delta^{(3)}\left(\mathbf{p} - \frac{\bar{\epsilon}_\alpha + \bar{p}_\alpha}{\bar{n}_\alpha}\mathbf{u}_\alpha\right). \tag{38}$$

The deltalike distribution function (38), and consequently the approximation, where the pressure gradients are neglected, hold for systems where the thermal momentum (p_{thermal}) is much smaller than the collective momentum (p_{collec}) of the hydrodynamic flow. In the elec-

tromagnetic plasma of electrons and ions, such a situation occurs for a sufficiently low temperatures when the effects of pressure are indeed expected to be small. For the system of massless partons, the condition $p_{\text{thermal}} \ll p_{\text{collec}}$ is achieved by requiring that p_{collec} is large rather than p_{thermal} is small. For massless particles in local equilibrium $p_{\text{thermal}} \sim T$, where T is the local temperature, while the formula (38) clearly shows that $p_{\text{collec}} \sim T\bar{\gamma}_\alpha \bar{v}_\alpha$ where $\bar{v}_\alpha$ and $\bar{\gamma}_\alpha$ are the velocity and Lorentz factor of the collective flow. Therefore, the condition $p_{\text{collec}} \gg p_{\text{thermal}}$ requires $\bar{\gamma}_\alpha \gg 1$.

B. Effect of pressure gradients included

Equation (17) allows one to close the system of fluid equations, not neglecting the pressure gradients. Using the relation (17), one has to solve three Eqs. (25), (27), and (28).

Performing the linearization analogous to that from the previous section and the Fourier transformation, Eqs. (25), (27), and (28) get the form

$$\bar{u}^{\,\mu} k_\mu \delta n + \bar{n} k_\mu \delta u^\mu = 0, \tag{39}$$

$$\bar{u}^{\,\mu} k_\mu \delta \epsilon + \frac{4}{3} \bar{\epsilon} k_\mu \delta u^\mu = 0, \tag{40}$$

$$i \frac{4}{3} \bar{\epsilon} \bar{u}_\mu k^\mu \delta u^\nu + i \frac{1}{3} (\bar{u}^\mu \bar{u}^\nu k_\mu - k^\nu) \delta \epsilon + g \bar{n} \bar{u}_\mu F^{\mu\nu} = 0. \tag{41}$$

Substituting $\delta \epsilon$ obtained from Eq. (40) into the Euler Eq. (41), one gets

$$\left[g^{\nu\mu} + \frac{1}{3(\bar{u} \cdot k)^2} (k^\nu k^\mu - \bar{u}^\nu k^\mu (\bar{u} \cdot k)) \right] \delta u_\mu$$

$$= i \frac{3}{4} g \frac{\bar{n}}{\bar{\epsilon}(\bar{u} \cdot k)} \bar{u}_\mu F^{\mu\nu}. \tag{42}$$

Observing that

$$(k_\sigma k_\nu - \bar{u}_\sigma k_\nu (\bar{u} \cdot k))(k^\nu k_\mu - \bar{u}^\nu k_\mu (\bar{u} \cdot k))$$

$$\propto (k_\sigma k_\mu - \bar{u}_\sigma k_\mu (\bar{u} \cdot k)),$$

the operator in the left-hand-side of Eq. (42) can be inverted as

$$\left[g_{\sigma\nu} - \frac{1}{k^2 + 2(\bar{u} \cdot k)^2} (k_\sigma k_\nu - \bar{u}_\sigma k_\nu (\bar{u} \cdot k)) \right]$$

$$\times \left[g^{\nu\mu} + \frac{1}{3(\bar{u} \cdot k)^2} (k^\nu k^\mu - \bar{u}^\nu k^\mu (\bar{u} \cdot k)) \right] = g_\sigma{}^\mu, \tag{43}$$

and Eq. (42) is exactly solved by

$$\delta u_\sigma = i \frac{3}{4} g \frac{\bar{n}}{\bar{\epsilon}(\bar{u} \cdot k)} \left[g_{\sigma\nu} - \frac{1}{k^2 + 2(\bar{u} \cdot k)^2} \right.$$

$$\left. \times (k_\sigma k_\nu - \bar{u}_\sigma k_\nu (\bar{u} \cdot k)) \right] \bar{u}_\mu F^{\mu\nu}. \tag{44}$$

Substituting δn and δu^μ into the current (34), one finds the polarization tensor as

$$\Pi^{\mu\nu}(k) = -\frac{g^2}{2} \sum_\alpha \frac{3\bar{n}_\alpha^2}{4\bar{\epsilon}_\alpha} \frac{1}{(\bar{u}_\alpha \cdot k)^2} \left[(\bar{u}_\alpha \cdot k)(k^\mu \bar{u}_\alpha^\nu + k^\nu \bar{u}_\alpha^\mu) - (\bar{u}_\alpha \cdot k)^2 g^{\mu\nu} - k^2 \bar{u}_\alpha^\mu \bar{u}_\alpha^\nu \right.$$

$$\left. - \frac{(\bar{u}_\alpha \cdot k)k^2(k^\mu \bar{u}_\alpha^\nu + k^\nu \bar{u}_\alpha^\mu) - (\bar{u}_\alpha \cdot k)^2 k^\mu k^\nu - k^4 \bar{u}_\alpha^\mu \bar{u}_\alpha^\nu}{k^2 + 2(\bar{u}_\alpha \cdot k)^2} \right], \tag{45}$$

where the stream index α has been restored. The first term in Eq. (45) corresponds to the polarization tensor (36) found when the pressure gradients are neglected while the second term gives the effect of the pressure gradients. The first term is, as already mentioned, symmetric and transverse. The second term is symmetric and transverse as well. Thus, the whole polarization tensor (45) is symmetric and transverse.

As explained below Eq. (38), the pressure gradients can be neglected and the distribution function can be approximated by the deltalike form (38) when $\bar{\gamma}_\alpha \gg 1$. Because the four-velocity is expressed as $\bar{u}_\alpha^\mu = (\bar{\gamma}_\alpha, \bar{\gamma}_\alpha \bar{\mathbf{v}}_\alpha)$, it is easy to check that the second term in Eq. (45) is small, when compared to the first one, for $\bar{\gamma}_\alpha \gg 1$. However, one has to be careful here: when the wave vector $\mathbf{k}$ is exactly parallel to $\bar{\mathbf{v}}_\alpha$ and both are, say, in the z direction, then the two terms of the nonzero components of the polarization tensor (45) ($\Pi^{00}, \Pi^{0z}, \Pi^{zz}$) vanish as $1/\bar{\gamma}_\alpha^2$. For the remaining nonzero components of (45) (Π^{xx}, Π^{yy}), the second term is identically zero. We return to this point when the collective modes are discussed.

V. COLLECTIVE MODES IN THE TWO-STREAM SYSTEM

As an application of the polarization tensors derived in the previous section, we discuss here collective plasma modes in the two-stream system. Within kinetic theory such a system was studied in [31–34]. The two-stream configuration has not much to do with the experimental situation in heavy-ion collisions where the momentum distribution of the produced partons is very different from the two-peak shape characteristic for the two-stream system. Thus, it should be treated only as a toy model. However, the model dynamics is rather nontrivial due to several unstable modes. While the two-stream configuration is rather of academic interest, one can model experimentally relevant situations with several streams. And in the linear regime, such a multistream configuration can be studied along the lines presented below.

Since Fourier transformed chromodynamic field $A^\mu(k)$ satisfies the equation of motion as

$$[k^2 g^{\mu\nu} - k^\mu k^\nu - \Pi^{\mu\nu}(k)]A_\nu(k) = 0, \qquad (46)$$

the general dispersion equation of the collective plasma modes is

$$\det[k^2 g^{\mu\nu} - k^\mu k^\nu - \Pi^{\mu\nu}(k)] = 0. \qquad (47)$$

Because of the transversality of $\Pi^{\mu\nu}(k)$ not all components of $\Pi^{\mu\nu}(k)$ are independent from each other, and consequently the dispersion Eq. (47), which involves a determinant of a 4×4 matrix, can be simplified to the determinant of a 3×3 matrix. For this purpose one usually introduces the color dielectric tensor $\varepsilon^{ij}(k)$, where the indices $i, j = 1$, 2, 3 label three-vector and tensor components, which is expressed through the polarization tensor as

$$\epsilon^{ij}(k) = \delta^{ij} + \frac{1}{\omega^2}\Pi^{ij}(k),$$

where $k \equiv (\omega, \mathbf{k})$. Then, the dispersion equation gets the form

$$\det[\mathbf{k}^2 \delta^{ij} - k^i k^j - \omega^2 \varepsilon^{ij}(k)] = 0. \qquad (48)$$

The relationship between Eq. (47) and (48) is most easily

seen in the Coulomb gauge when $A^0 = 0$ and $\mathbf{k} \cdot \mathbf{A}(k) = 0$. Then, $\mathbf{E} = i\omega\mathbf{A}$ and Eq. (46) is immediately transformed into an equation of motion of $\mathbf{E}(k)$ which further provides the dispersion Eq. (48).

The dielectric tensor given by the polarization tensor (36), which neglects the effect of pressure, is

$$\varepsilon^{ij}(\omega, \mathbf{k}) = \left(1 - \frac{\omega_p^2}{\omega^2}\right)\delta^{ij} - \frac{g^2}{2\omega^2}\sum_\alpha \frac{\bar{n}_\alpha^2}{\bar{\epsilon}_\alpha + \bar{p}_\alpha}$$
$$\times \left(\frac{\bar{v}_\alpha^i k^j + \bar{v}_\alpha^j k^i}{\omega - \mathbf{k} \cdot \bar{\mathbf{v}}_\alpha} - \frac{(\omega^2 - \mathbf{k}^2)\bar{v}_\alpha^i \bar{v}_\alpha^j}{(\omega - \mathbf{k} \cdot \bar{\mathbf{v}}_\alpha)^2}\right), \quad (49)$$

where $\bar{\mathbf{v}}_\alpha$ is the hydrodynamic velocity related to the hydrodynamic four-velocity $\bar{u}_\alpha^\mu$; ω_p is the plasma frequency given as

$$\omega_p^2 \equiv \frac{g^2}{2}\sum_\alpha \frac{\bar{n}_\alpha^2}{\bar{\epsilon}_\alpha + \bar{p}_\alpha}. \qquad (50)$$

The dielectric tensor given by the polarization tensor (45), which includes the effect of pressure, is

$$\varepsilon^{ij}(\omega, \mathbf{k}) = \left(1 - \frac{\omega_p^2}{\omega^2}\right)\delta^{ij} - \frac{3g^2}{8\omega^2}\sum_\alpha \frac{\bar{n}_\alpha^2}{\bar{\epsilon}_\alpha}\left(\frac{\bar{v}_\alpha^i k^j + \bar{v}_\alpha^j k^i}{\omega - \mathbf{k} \cdot \bar{\mathbf{v}}_\alpha} - \frac{(\omega^2 - \mathbf{k}^2)\bar{v}_\alpha^i \bar{v}_\alpha^j}{(\omega - \mathbf{k} \cdot \bar{\mathbf{v}}_\alpha)^2}\right.$$
$$\left. - \frac{(\omega - \mathbf{k} \cdot \bar{\mathbf{v}}_\alpha)(\omega^2 - \mathbf{k}^2)(\bar{v}_\alpha^i k^j + \bar{v}_\alpha^j k^i) - (\omega - \mathbf{k} \cdot \bar{\mathbf{v}}_\alpha)^2 k^i k^j - (\omega^2 - \mathbf{k}^2)^2 \bar{v}_\alpha^i \bar{v}_\alpha^j}{(\omega - \mathbf{k} \cdot \bar{\mathbf{v}}_\alpha)^2(\omega^2 - \mathbf{k}^2 + 2\bar{\gamma}_\alpha^2(\omega - \mathbf{k} \cdot \bar{\mathbf{v}}_\alpha)^2)}\right), \qquad (51)$$

with the plasma frequency given as

$$\omega_p^2 \equiv \frac{3g^2}{8}\sum_\alpha \frac{\bar{n}_\alpha^2}{\bar{\epsilon}_\alpha}. \qquad (52)$$

In the following subsections we consider the collective modes in the two-stream system with the wave vectors perpendicular and parallel to the hydrodynamic velocity. The velocities $\bar{\mathbf{v}}_\alpha$ are chosen to be oriented along the axis z, i.e. $\bar{\mathbf{v}}_\alpha = (0, 0, \bar{v}_\alpha)$. The index α, which labels the streams has two values, $\alpha = \pm$. For simplicity we also assume that

$$\bar{v} \equiv \bar{v}_+ = -\bar{v}_-, \qquad \bar{n} \equiv \bar{n}_+ = \bar{n}_-,$$
$$\bar{\epsilon} \equiv \bar{\epsilon}_+ = \bar{\epsilon}_-, \qquad \bar{p} \equiv \bar{p}_+ = \bar{p}_-. \qquad (53)$$

Then, the plasma frequency equals $\omega_p^2 = g^2 \bar{n}^2/(\bar{\epsilon} + \bar{p})$ or $\omega_p^2 = 3g^2 \bar{n}^2/(4\bar{\epsilon})$.

A. $\mathbf{k} \perp \bar{\mathbf{v}}$

The wave vector is chosen to be parallel to the axis x, $\mathbf{k} = (k, 0, 0)$. Due the conditions (53), the off-diagonal elements of the matrix in Eq. (48) vanish.

1. No pressure gradient effect

With the dielectric tensor given by Eq. (49), the dispersion equation is

$$(\omega^2 - \omega_p^2)(\omega^2 - \omega_p^2 - k^2)\left(\omega^2 - \omega_p^2 - k^2 - \lambda^2 \frac{k^2 - \omega^2}{\omega^2}\right)$$
$$= 0, \quad (54)$$

where $\lambda^2 \equiv \omega_p^2 \bar{v}^2$. As solutions of the equation, one finds the stable longitudinal mode with $\omega^2 = \omega_p^2$ and the stable transverse mode with $\omega^2 = \omega_p^2 + k^2$. There are also transverse modes with

$$\omega_\pm^2 = \frac{1}{2}(\omega_p^2 - \lambda^2 + k^2 \pm \sqrt{(\omega_p^2 - \lambda^2 + k^2)^2 + 4\lambda^2 k^2}). \qquad (55)$$

As seen, $\omega_+^2 > 0$ but $\omega_-^2 < 0$. Thus, the mode ω_+ is stable and there are two modes with pure imaginary frequency corresponding to $\omega_-^2 < 0$. The first mode is overdamped while the second one is the well-known unstable Weibel mode leading to the filamentation instability.

As discussed below Eq. (38), the pressure gradients can be neglected when $\bar{\gamma} \gg 1$, and thus, the solutions (55) are physically relevant in this limit. Therefore, we write down the solutions (55) for $\bar{\gamma} \gg 1$ which are

$$\omega_\pm^2 = \frac{1}{2}(k^2 \pm \sqrt{k^4 + 4\omega_p^2 k^2}). \qquad (56)$$

As seen in Eq. (56), the modes are independent of $\bar{\gamma}$, if it is sufficiently large.

2. Effect of pressure gradients included

As in the "pressureless" case, there are two stable modes $\omega^2 = \omega_p^2$ and the stable transverse mode with $\omega^2 = \omega_p^2 + k^2$. The transverse modes corresponding to those given by Eq. (55) are obtained by solving the equation

$$\omega^4 - \omega^2(k^2 + \omega_p^2) + \lambda^2(\omega^2 - k^2) - \lambda^2 \frac{(\omega^2 - k^2)^2}{\omega^2 - k^2 + 2\bar{\gamma}^2\omega^2}$$
$$= 0. \quad (57)$$

Defining the dimensionless variables $a \equiv \omega/\omega_p$ and $b \equiv k/\omega_p$, the above equation can then be rewritten as follows

$$a^2[a^4 - F(b, \bar{v})a^2 - G(b, \bar{v})] = 0, \qquad (58)$$

where

$$F(b, \bar{v}) \equiv \frac{(3 + 2b^2)(1 - \bar{v}^2) + 2b^2}{3 - \bar{v}^2},$$
$$G(b, \bar{v}) \equiv b^2 \frac{3\bar{v}^2 - 1 - b^2(1 - \bar{v}^2)}{3 - \bar{v}^2}. \qquad (59)$$

Solutions of Eq. (58) are given by $a^2 = 0$ and

$$a_\pm^2 = \frac{1}{2}(F \pm \sqrt{F^2 + 4G}) \qquad (60)$$

The solution with a_-^2 describes the unstable mode ($a_-^2 < 0$) provided G is positive. The condition $G > 0$ gives

$$\bar{v}^2 > \frac{1}{3} \quad \text{and} \quad k^2 < \frac{3\bar{v}^2 - 1}{1 - \bar{v}^2}\omega_p^2. \qquad (61)$$

As seen, the instability appears when the stream velocity is larger than the speed of sound in the ideal gas of massless partons and when the wavelength is sufficiently long. We also note that in the limit $\bar{\gamma} \gg 1$ the solutions (60) reproduce, as expected, those given by Eq. (56).

B. k ∥ v̄

The velocities and the wave vector are chosen to be oriented along the $z-$ axis i.e. $\bar{\mathbf{v}}_\pm = (0, 0, \pm\bar{v})$ and $\mathbf{k} =$

$(0, 0, k)$. Then, the matrix in Eq. (48) is diagonal. As in the case $\mathbf{k} \perp \bar{\mathbf{v}}$, we discuss separately the collective modes given by the dielectric tensors (49) and (51). However, it is physically rather unjustified because, as explained at the very end of Sec. IV, the tensor (49) cannot be treated as an approximation of (51) even for $\gamma \gg 1$. The effects of the mean-field and of the pressure gradients appear to be equally important. Therefore, only the results of subsection V B 2 are physically reliable.

1. No pressure gradient effect

With the dielectric tensor given by Eq. (49), the dispersion equation reads

$$(\omega^2 - \omega_p^2 - k^2)^2\left(\omega^2 - \omega_p^2 - \omega_p^2\left(\frac{k\bar{v}}{\omega - k\bar{v}} + \frac{(k^2 - \omega^2)\bar{v}^2}{2(\omega - k\bar{v})^2}\right.\right.$$
$$\left.\left. - \frac{k\bar{v}}{\omega + k\bar{v}} + \frac{(k^2 - \omega^2)\bar{v}^2}{2(\omega + k\bar{v})^2}\right)\right) = 0. \quad (62)$$

There are two transverse stable modes with $\omega^2 = \omega_p^2 + k^2$. The longitudinal modes are solutions of the equation which can be rewritten as

$$1 - \omega_0^2\left(\frac{1}{(\omega - k\bar{v})^2} + \frac{1}{(\omega + k\bar{v})^2}\right) = 0, \qquad (63)$$

where $\omega_0^2 \equiv \omega_p^2/2\bar{\gamma}^2$. With the dimensionless quantities $x \equiv \omega/\omega_0$, $y \equiv k\bar{v}/\omega_0$, Eq. (63) is

$$(x^2 - y^2)^2 - 2x^2 - 2y^2 = 0, \qquad (64)$$

and it is solved by

$$x_\pm^2 = y^2 + 1 \pm \sqrt{4y^2 + 1}. \qquad (65)$$

As seen, x_+^2 is always positive and thus, it gives two real (stable) modes; x_-^2 is negative for $0 < y < \sqrt{2}$ and then, there are two pure imaginary modes. The unstable one corresponds to the two-stream electrostatic instability.

2. Effect of pressure gradients included

The transverse modes are the same as in the "pressureless" case while the longitudinal modes are solutions of the equation

$$1 - \omega_0^2\left(\frac{1}{(\omega - k\bar{v})^2} + \frac{1}{(\omega + k\bar{v})^2}\right) + \frac{\omega_p^2}{2}\left(\frac{(\omega\bar{v} - k)^2}{(\omega - k\bar{v})^2(2\bar{\gamma}^2(\omega - k\bar{v})^2 + \omega^2 - k^2)}\right.$$
$$\left. + \frac{(\omega\bar{v} + k)^2}{(\omega + k\bar{v})^2(2\bar{\gamma}^2(\omega + k\bar{v})^2 + \omega^2 - k^2)}\right) = 0, \quad (66)$$

which, in terms of the dimensionless quantities introduced in the previous section, is

$$1 - \frac{1}{(x-y)^2}\left(1 + \frac{(\bar{v}x - y/\bar{v})^2}{2(x-y)^2 + \frac{1}{\bar{\gamma}^2}(x^2 - y^2/\bar{v}^2)}\right) - \frac{1}{(x+y)^2}\left(1 + \frac{(\bar{v}x + y/\bar{v})^2}{2(x+y)^2 + \frac{1}{\bar{\gamma}^2}(x^2 - y^2/\bar{v}^2)}\right) = 0. \qquad (67)$$

The equation can be converted to

$$(x^2 - y^2)^2[x^4 - B(y, \bar{v})x^2 - C(y, \bar{v})] = 0 \qquad (68)$$

with

$$B(y, \bar{v}) \equiv \frac{18 + 6\bar{v}^2 y^2 + 6y^2/\bar{v}^2 - 4y^2 - 6\bar{v}^2}{(3 - \bar{v}^2)^2},$$

$$C(y, \bar{v}) \equiv y^2 \frac{(3 - \frac{1}{\bar{v}^2})[6 - y^2(3 - \frac{1}{\bar{v}^2})]}{(3 - \bar{v}^2)^2}, \qquad (69)$$

which is solved by $x^2 = y^2$ and

$$x_{\pm}^2 = \frac{1}{2}(B \pm \sqrt{B^2 + 4C}). \qquad (70)$$

As for the case when the wave vector is perpendicular to the velocity, there are only instabilities when the stream velocity is bigger than the speed of sound in the ideal gas of massless partons, as $C > 0$ is satisfied when

$$\bar{v}^2 > \frac{1}{3} \quad \text{and} \quad k^2 < 3\frac{1 - \bar{v}^2}{3\bar{v}^2 - 1}\omega_p^2. \qquad (71)$$

It is also interesting to note that for the particular case $\bar{v}^2 = 1/3$, $C = 0$, and the solutions of the dispersion relations get a very simple form. Namely, we have $\omega_-^2 = 0$ and

$$\omega_+^2 = \frac{3}{4}(\omega_p^2 + k^2). \qquad (72)$$

VI. CONCLUDING REMARKS

The fluid Eqs. (10) and (12) with n_α^μ and $T_\alpha^{\mu\nu}$ given by the formulas (14) and (15) form a closed set of equations when supplemented by the relation (17). If the chromodynamic field is generated self-consistently by the color fluid, the Yang-Mills Eq. (9) with the color current of the form (16) must be added. The derivation of these equations is the main result of our paper. Although some information is lost when going from kinetic theory to hydrodynamics, the dynamical content of the fluid equations is still very rich. The results of Sec. V, where the two-stream system is studied, demonstrate it convincingly. The system is shown to be unstable with respect to the magnetic and electric modes, if the hydrodynamic velocity is sufficiently large.

As noted in the Introduction, the hydrodynamic approach is frequently used in studies of electromagnetic plasmas. Numerical simulations of the fluid equations are much simpler than those of kinetic theory because the distribution function depends on four-position and momentum while in the fluid approach one deals with a few functions of four-position only. For this reason we expect that the equations derived here will be useful in numerical studies of the unstable quark-gluon plasma. Hopefully, the late stage of instability development, when the dynamics is strongly nonlinear, can be attacked within the fluid approach. In the case of the two-stream system, our analytic results from Sec. V can be used to gauge numerical calculations. While the two-stream configuration is rather academic, one can model situations, which are relevant for quark-gluon plasma from relativistic heavy-ion collisions, with several streams. And then, our fluid equations can be applied.

We also note that the hydrodynamic approach presented here can be extended to include the effect of collisions which can be modeled with transport coefficients (viscosities, conductivities) and additional terms in the fluid equations. Then, the approach would be applicable to longer time scales.

Our derivation of the chromohydrodynamic equations is based on the parton kinetic theory valid in the regime of very weak couplings or, equivalently, of very high temperatures $T \gg T_c$, where T_c is the temperature of the deconfinement phase transition. One may wonder what happens in the regime of not so weak couplings. Because the hydrodynamic equations are the expressions of the conservation laws, one can hope that the structure of the equations survives when the plasma is no longer weakly coupled. Lattice studies indicate that an equilibrium quark-gluon plasma at temperatures close to T_c is very different from the ideal gas of massless quasiparticles [35]. The deviations can be, at least partially, accounted for in a quasiparticle model when the gluon dispersion relation is modified, see e.g. [36–38]. In such a case, our approach can be easily adapted for by changing the relation (17). We hope to report about those results in the near future.

[1] N. A. Krall and A. W. Trivelpiece, *Principles of Plasma Physics* (Mc-Graw-Hill, New York, 1973).

[2] K. Kajantie and C. Montonen, Phys. Scr. **22**, 555 (1980).

[3] D. D. Holm and B. A. Kupershmidt, Phys. Lett. A **105**, 225 (1984).

[4] D. D. Holm and B. A. Kupershmidt, Phys. Rev. D **30**, 2557 (1984).

[5] St. Mrówczyński, Phys. Lett. B **202**, 568 (1988).

[6] St. Mrówczyński, in *Quark-Gluon Plasma* edited by R. Hwa, Adv. Ser. Direct. High Energy Phys. (World Scientific, Singapore, 1990), Vol. 6, p. 185.

[7] J. Bhatt, P. Kaw, and J. Parikh, Phys. Rev. D **39**, 646 (1989).

[8] R. Jackiw, V. P. Nair, and S. Y. Pi, Phys. Rev. D **62**, 085018 (2000).

[9] B. Bistrovic, R. Jackiw, H. Li, V. P. Nair, and S. Y. Pi, Phys. Rev. D **67**, 025013 (2003).

[10] C. Manuel and St. Mrówczyński, Phys. Rev. D **68**, 094010 (2003).

[11] J. Dai and V. P. Nair, Phys. Rev. D **74**, 085014 (2006).

[12] B. A. Bambah, S. M. Mahajan, and C. Mukku, Phys. Rev. Lett. **97**, 072301 (2006).

[13] J. P. Blaizot and E. Iancu, Phys. Rep. **359**, 355 (2002).

[14] H. T. Elze and U. W. Heinz, Phys. Rep. **183**, 81 (1989).

[15] St. Mrówczyński, Phys. Rev. D **39**, 1940 (1989).

[16] D. F. Litim and C. Manuel, Phys. Rep. **364**, 451 (2002).

[17] St. Mrówczyński, Acta Phys. Pol. B **37**, 427 (2006).

[18] A. Rebhan, P. Romatschke, and M. Strickland, Phys. Rev. Lett. **94**, 102303 (2005).

[19] A. Dumitru and Y. Nara, Phys. Lett. B **621**, 89 (2005).

[20] P. Arnold, G. D. Moore, and L. G. Yaffe, Phys. Rev. D **72**, 054003 (2005).

[21] A. Rebhan, P. Romatschke, and M. Strickland, J. High Energy Phys. 09 (2005) 041.

[22] P. Arnold and G. D. Moore, Phys. Rev. D **73**, 025006 (2006).

[23] A. Dumitru, Y. Nara, and M. Strickland, hep-ph/0604149.

[24] P. Romatschke and A. Rebhan, hep-ph/0605064.

[25] E. Iancu and R. Venugopalan, in *Quark-Gluon Plasma 3*, edited by R. C. Hwa and X. N. Wang (World Scientific, Singapore, 2004).

[26] P. Romatschke and R. Venugopalan, Phys. Rev. Lett. **96**, 062302 (2006).

[27] T. Lappi and L. McLerran, Nucl. Phys. A **772**, 200 (2006).

[28] C. Manuel and St. Mrówczyński, Phys. Rev. D **70**, 094019 (2004).

[29] L. D. Landau and E. M. Lifshitz, *Fluid Mechanics* (Pergamon, Oxford, 1963).

[30] St. Mrówczyński and M. H. Thoma, Phys. Rev. D **62**, 036011 (2000).

[31] Y. E. Pokrovsky and A. V. Selikhov, Pis'ma Zh. Eksp. Teor. Fiz. **47**, 11 (1988); [JETP Lett. **47**, 12 (1988)].

[32] Y. E. Pokrovsky and A. V. Selikhov, Yad. Fiz. **52**, 229 (1990); [Sov. J. Nucl. Phys. **52**, 146 (1990)].

[33] Y. E. Pokrovsky and A. V. Selikhov, Yad. Fiz. **52**, 605 (1990); [Sov. J. Nucl. Phys. **52**, 385 (1990)].

[34] St. Mrówczyński, Phys. Lett. B **214**, 587 (1988).

[35] F. Karsch, Nucl. Phys. A **698**, 199 (2002).

[36] M. I. Gorenstein and S. N. Yang, Phys. Rev. D **52**, 5206 (1995).

[37] D. Zwanziger, Phys. Rev. Lett. **94**, 182301 (2005).

[38] M. Bluhm, B. Kampfer, and G. Soff, Phys. Lett. B **620**, 131 (2005).