Parton bremsstrahlung as a mechanism of energy deposition in high-energy hadron-nucleus and nucleus-nucleus collisions

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On the basis of perturbative quantum chromodynamics, we consider the bremsstrahlung energy losses of partons traversing *parton matter* using the well-known quantum electrodynamical formulas. We find the transport equation describing the energy distribution of partons as a function of thickness of a target. The equation is solved numerically and we briefly discuss the relevance of our results for hadron-nucleus and nucleus-nucleus collisions at high energies.

The mechanism of energy deposition in soft hadronic collisions at high energy is not well understood at present. In the most popular stringlike models of hadron interactions, see, e.g., Ref. 1, this mechanism is associated with the nonperturbative sector of quantum chromodynamics (QCD). Namely, the energy is deposited due to the string length increase and the subsequent string decay. The aim of this Brief Report is to discuss the mechanism from the quite different point of view which is very close in spirit to the perturbative QCD. According to our picture the substantial energy loss of the interacting hadron is due to multiple bremsstrahlung in parton-parton collisions. We assume that the time scale of parton-parton collision is much shorter than the hadronization scale, and that the hadronization is soft. In other words, we extrapolate the scenario which is justified in hard hadron collisions to the soft interactions. Such a picture of hadron processes has been advocated in Ref. 2.

To make our consideration definite let us consider an ultrarelativistic parton traversing the layer of *parton matter.* The energy loss of the parton is described by the following equation:

$$\frac{dE}{dl}=-\rho\sigma_r E \ , \tag{1}$$

where E is the parton energy, l is the parton-matter thickness traversed by a parton, ρ is the density of scattering centers, and σ_r is the so-called radiation cross section defined as

$$\sigma_r=\frac{1}{E}\int \omega\frac{d\sigma(E)}{d\omega}d\omega \ , \tag{2}$$

where $d\sigma(E)/d\omega$ is the cross section of emission of a parton with energy ω. It is well known in quantum electrodynamics (QED), see, e.g., Refs. 3 and 4, that the radiation cross section of ultrarelativistic electrons interacting with a screened Coulomb potential is independent of energy of the electron, and that the bremsstrahlung cross section can be approximately written as

$$\frac{d\sigma(E)}{d\omega}=\Theta(E-\omega)\frac{\sigma_r}{\omega} \ . \tag{3}$$

It is also important to notice that in the ultrarelativistic limit the cross sections of bremsstrahlung in electron-electron collisions and in electron interactions with external Coulomb source with unit charge are equal to each other, see, e.g., Ref. 4.

Since the radiation cross section is energy independent Eq. (1) has the trivial exponential solution

$$E(l)=E_0e^{-l/\lambda} \ ,$$

where $1/\lambda=\rho\sigma_r$. Let us estimate the numerical value of radiation length λ for parton interactions. The expression of σ_r found in QED reads[3]

$$\sigma_r=\frac{\alpha^3}{m^2}[4\ln(am)+\tfrac{2}{9}] \ , \tag{4}$$

where α is the fine-structure constant, m is the electron mass, and a is the screening length. Substituting in (4) $\alpha=0.5$, $m=300$ MeV, which corresponds to the constituent quark mass, or more generally, to a typical nonperturbative scale in QCD and $a=1$ fm one finds $\sigma_r=0.17$ fm^2. Identifying the parton density with the number of three valence quarks per volume of a one-Fermi-radius sphere, one finds $\lambda=8$ fm. Therefore, a parton traversing the *parton-matter* layer of one-Fermi thickness loses about 10 percent of its energy. It is a rather conservative estimation since we have not taken into account gluons and sea quarks when the parton density has been considered. Encouraged by the non-negligible value of energy lost due to bremsstrahlung, we propose to analyze the suggested picture of energy deposition in more detail.

Let us start with a little more careful discussion of the formulas (3) and (4) and values of the parameters. At first it should be observed that the bremsstrahlung cross section (3) seems adequate not only in the perturbative regime where it is derived. As known, see, e.g., Ref. 4, the analogous formula is found in the classical radiation theory with no assumption of smallness of the coupling constant. However, the interpretation of the formula is somewhat different.

Because we consider the soft hadronic interactions it seems reasonable to identify the mass parameter from Eq. (4) with the constituent quark mass. In fact, the actual values of the mass, the coupling constant, and the screening length are not very important for us. It is important that the radiation cross section is energy independent and its value should be treated rather as a free parameter. The above number is only a rough estimation of it.

In further analysis we treat quarks and gluons in the same way. The assumption that the *nonperturbative* gluons are massive is supported by phenomenology. It does not alter the formula of the bremsstrahlung cross section (3) as long as the energy of an emitted gluon is much greater than its mass. Since we are interested in partons which are initially ultrarelativistic, this requirement is easily satisfied. The slow partons are, in fact, excluded from our analysis due to the presence of a cutoff parameter (see below).

What is the energy distribution of partons after traversing the layer of *parton matter?* How thick should the layer be to modify initial parton energy distribution? To address these questions we have formulated the following transportlike equation:

$$\frac{df(E,l)}{dl}=\rho\int dE_1\left[f(E_1,l)\frac{d\sigma(E_1)}{dE}+f(E_1+E,l)\frac{d\sigma(E_1+E)}{dE_1}-f(E,l)\frac{d\sigma(E)}{dE_1}\right], \tag{5}$$

where $f(E,l)dE$ is the number of partons with energy from the interval $(E,E+dE)$ after traversing the layer of a thickness l. The terms from the right-hand side (rhs) of Eq. (5) correspond to three processes which modify the distribution function $f(E,l)$. The first term describes emission of partons with energy E, the second term corresponds to deceleration of partons which initially have energy $E+E_1$ to the final energy E, and the third term is responsible for the decrease of the number of partons with energy E due to bremsstrahlung.

Using the standard methods of transport theory, see, e.g., Ref. 5, one can easily prove that Eq. (5) admits the energy conservation, i.e., $dW/dl=0$, where W is the total energy carried by partons, $W=\int dE\, Ef(E,l)$. Since we work in a reference frame where the target is at rest and the incoming partons are ultrarelativistic the energy carried by partons well approximates the total energy. Therefore, the equation $dW/dl=0$ represents the total energy conservation.

Introducing the dimensionless variables $x=E/E_0$, $z=l/\lambda$ and using the bremsstrahlung cross section in the form (3) we rewrite the transport equation as

$$\frac{df(x,z)}{dz}=\int_0^1 dx_1\left[\left[\frac{1}{x}+\frac{1}{x_1}\right]f(x+x_1,z)-\frac{1}{x_1}\Theta(x-x_1)f(x,z)\right], \tag{6}$$

where $f(x,z)=f(E,l)E_0$. The interesting feature of Eq. (6) is that the second and the third terms of the rhs are infrared logarithmically divergent if they are treated separately from each other. When the terms are combined the integral is finite. Due to infrared catastrophe of perturbative QCD (or QED) the rhs of Eq. (6) is divergent when x goes to zero. However, one should remember that the physically observable quantity $Ef(E,l)$ is finite. As is well known, the difficulty is cured if the energy resolution is finite. A natural infrared cut appears when one uses the numerical methods to solve Eq. (6). Therefore, in further considerations, we will not pay any attention to this problem. However, one should notice that the value of the cutoff parameter could not be too small. The point is that the cross section (3) is valid for ultrarelativistic partons. If one considers partons from a proton of momentum 100 GeV and the cutoff x value equals 0.05, the slowest partons of momentum 2 GeV (and of mass 300 MeV) can be still treated as ultrarelativistic. Note that this cutoff value is substantially greater than the QCD scale parameter.

Let us observe a very nice feature of Eq. (6). It contains no parameters. Therefore, the knowledge of only one parameter (except E_0)—the radiation length—is needed to convert Eq. (6) into Eq. (5).

In Fig. 1(a) we present the distribution function evolution in z variable starting with the deltalike distribution

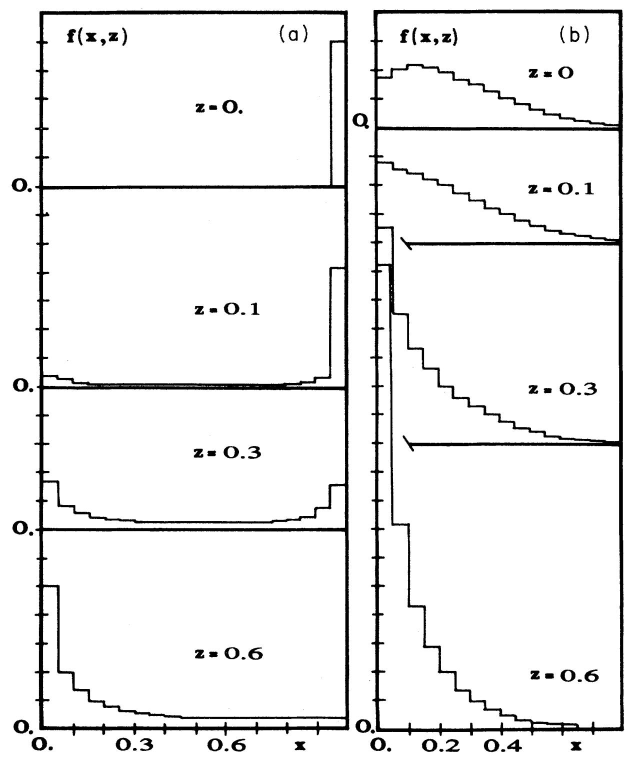

FIG. 1. The evolution of distribution function with the target thickness. The vertical axis units are arbitrary. (a) The initial distribution function is deltalike. (b) The initial distribution function coincides with the valence quark structure function of a nucleon.

function for $z=0$. One sees that the evolution is very fast. After traversing 10 percent of the radiative length λ, the initial distribution function is significantly disturbed. When $z=0.6$ the distribution function is close to the limiting one which locates all partons in the first energy bin. The z evolution of the distribution function, which at $z=0$ is identified with the nucleon structure function of valence quarks

$$f(x,z=0)\sim(x)^{1/2}(1-x)^3 ,$$

is shown in Fig. 1(b). In these calculations the bin width and, consequently, the cutoff parameter equal 0.05. The changes of the cutoff value alter the shape of function $f(x,z)$ in the first bins only.

The question arises how the number of partons increases with the target thickness? Integrating over x Eq. (6) one finds

$$\frac{dn(z)}{dz}=\int_{x_0}^{1} dx \ln(x/x_0)f(x,z) , \tag{7}$$

with

$$n(z)=\int_{x_0}^{1} dx\, f(x,z) ,$$

where x_0 is the infrared cut parameter—the smallest value of x which is taken into account. From Fig. 1 one sees that at sufficiently thick targets, the dominant contribution to the integral from Eq. (7) comes from the x which are close to x_0. Therefore, we expand the logarithm around x_0. Then one gets

$$\frac{dn(z)}{dz}=w/x_0-n(z) , \tag{8}$$

where $w=\int dx\, xf(x,z)$ is the dimensionless analog of the earlier introduced total energy W. One easily finds the solution of Eq. (8) which reads

$$n(z)=[n(z=0)-w/x_0]\exp(-z)+w/x_0 . \tag{9}$$

One sees that the parton multiplicity first linearly increases with the target thickness, then the increase is slower than linear, and finally it approaches the maximal multiplicity equal to w/x_0. Since the thickness of heaviest nuclei is probably not higher than about one radiation length, Eq. (9) predicts an approximately linear particle multiplicity increase with a nucleus thickness in hadron-nucleus collisions at high energies if the number of partons is proportional to the number of hadrons. As is known, the experimental data reveal just such a linear increase.[6]

Experimentally the rapidity distributions of hadrons instead of x distributions of partons are observed in hadron-nucleus collisions. Therefore, to confront the solutions of Eq. (6) with experimental data, let us assume that a parton of x fraction of initial hadron momentum converts in the hadronization process into a pion of the same x. The rapidity distribution of pions coming from the parton x distribution which at $z=0$ coincides with the valence quark structure function is given in Fig. 2. The initial total momentum equals 100 GeV/c. Although some qualitative features of data[6] are seen in Fig. 2, the quantitative agreement is rather poor. Probably nonperturbative effects responsible for hadronization, which are completely neglected here, substantially modify the shape of the rapidity distributions.

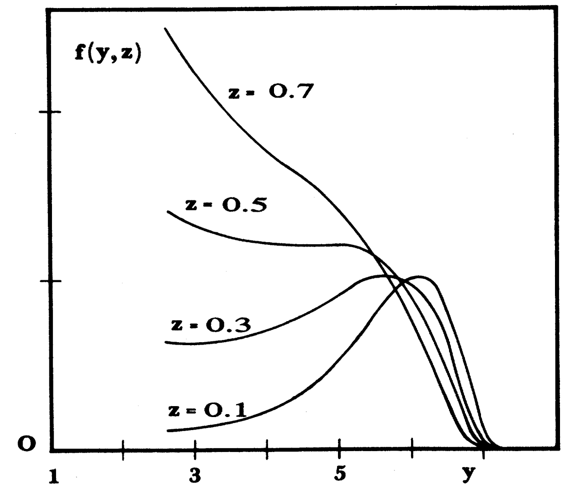

FIG. 2. The rapidity distribution of pions for several values of target thickness. The initial momentum is 100 GeV/c. The vertical axis units are arbitrary.

While the transport equation (5) seems adequate for hadron-nucleus collisions it can be inappropriate for interactions of heavy ions. The point is the following. The partons from the initial beam which are strongly decelerated can interact with incoming partons. In other words, slow partons should contribute to the density of scattering centers, ρ, which is assumed constant in Eq. (5). One observes that implementation of the effect discussed to the transport equation demands introduction of the time dependence. To simplify the problem let us split the incoming beam of partons into bunches corresponding to nucleons which successively approach the target. If we numerate the bunches in such a way that the number one bunch is that one which collides with the target as a first, the bunch number two approaches the target the density of which is increased due to interaction with the first bunch. When the bunch number three interacts with the target its density has been modified by the first and the second bunch. Keeping in mind this picture Eq. (5) changes into the set of equations

$$\begin{aligned}\frac{df_i(E,l)}{dl}=\rho_i(l)\int dE_1 \Bigg[& f(E_1,l)\frac{d\sigma(E_1)}{dE} \\ & +f_i(E_1+E,l)\frac{d\sigma(E_1+E)}{dE_1} \\ & -f_i(E,l)\frac{d\sigma(E)}{dE_1}\Bigg] ,\end{aligned} \tag{10}$$

where f_i is the distribution function of partons originated from ith bunch. The density of partons in the target is defined as

$$\rho_1(l)=\rho ,$$

$$\rho_i(l)=\rho_{i-1}(l)+\frac{1}{s}\int_0^{\epsilon} dE\, df_{i-1}(E,l)/dl ,$$

where ϵ is the maximal energy of partons which contribute to the target density and s is the perpendicular area of the beam.

As we did previously, we rewrite Eq. (10) in the dimensionless form

$$\frac{df_i(x,z)}{dz}=\kappa_i(z)\int_0^1 dx_1\left[\left[\frac{1}{x}+\frac{1}{x_1}\right]f_i(x+x_1,z)-\frac{1}{x_1}\Theta(x-x_1)f_i(x,z)\right],$$

$$\kappa_1(z)=1\,, \tag{11}$$

$$\kappa_i(z)=\kappa_{i-1}(z)+\frac{\sigma}{s}\int_0^{\epsilon/E_0}dx\,df_{i-1}(x,z)/dz\,.$$

To make Eq. (11) definite one has to determine the values of two ratios σ/s and ϵ/E_0. Since Eq. (11) is nonlinear with respect to the distribution function, the normalization of it cannot be chosen arbitrarily.

To get a feeling what kind of effects we can expect, Eq. (11) has been solved numerically with probably somewhat overestimated values of the ratios. Namely we have taken $\sigma/s=0.1$ and $\epsilon/E_0=0.05$. The distribution functions, which are initially deltalike, are normalized as $\int dxf_i(x,0)=3$. The results are presented in Fig. 3, where we show the evolution of the fifth bunch distribution function compared with that of the first one. One sees that for sufficiently thick targets the nonlinear effects can be very important.

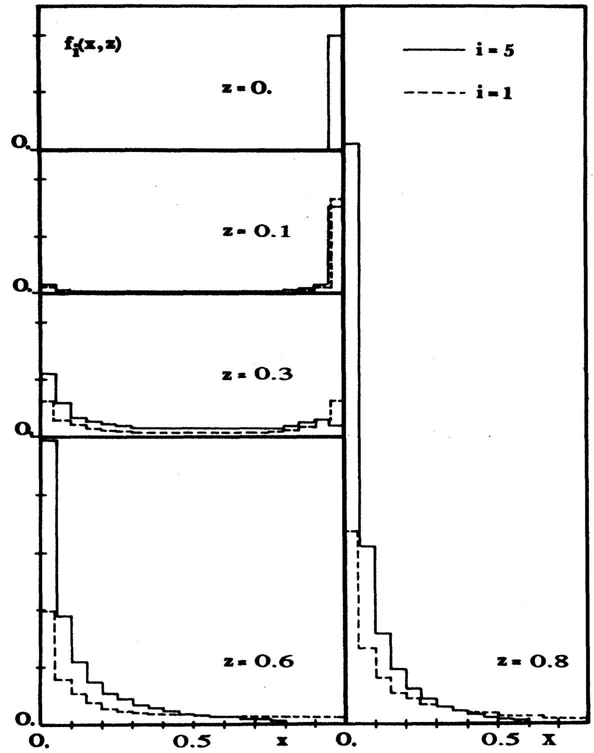

FIG. 3. The evolution of distribution functions of the first and the fifth bunch with the target thickness. The vertical axis units are arbitrary.

We have presented in this paper a first schematic description of parton beam traversing the *parton matter* target. Assuming that the beam evolves due to parton bremsstrahlung we have found the distribution of energy of partons and of their number as a function of target thickness. Although the bremsstrahlung is treated perturbatively, the evolution seems very fast, i.e., the distribution function is substantially modified when the parton beam traverses even a thin layer. Some features of experimental data from hadron-nucleus collisions appear in the results of our approach; however, it is premature to conclude whether or not it provides an adequate description of these data. We have shown how to modify the approach in order to include the case of nucleus-nucleus collisions. First numerical results indicate a rather strong nonlinear response of the target to successive collisions with partons of the projectile nucleus.

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