Deciphering azimuthal correlations in relativistic heavy-ion collisions

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(Received 16 November 2010; published 14 February 2011)

We discuss various sources of azimuthal correlations in relativistic heavy-ion collisions. The integral measure Φ is applied to quantify the correlations. We first consider separately the correlations caused by the elliptic flow, resonance decays, jets, and transverse momentum conservation. An effect of randomly lost particles is also discussed. Using the PYTHIA and HIJING event generators we produce a sample of events that mimic experimental data. By means of kinematic cuts and particle selection criteria, the data are analyzed to identify a dominant source of correlations.

DOI: 10.1103/PhysRevC.83.024905 PACS number(s): 25.75.Gz

I. INTRODUCTION

Particles produced in relativistic heavy-ion collisions are correlated in the azimuthal angle from various mechanisms. One mentions here extensively studied jets and minijets resulting from (semi-)hard parton-parton scattering and collective flow from the cylindrically asymmetric pressure gradients (see the review articles [1] and [2], respectively). More exotic sources of correlations are also possible. As argued in Ref. [3], the plasma instabilities, which occur at an early stage of collisions, can generate the azimuthal fluctuations. Except the dynamically interesting mechanisms, there are also rather trivial effects caused by decays of hadronic resonances or by energy-momentum conservation.

There is a variety of methods designed to study fluctuations on an event-by-event basis. In particular, the so-called measure Φ proposed in Ref. [4] was used to measure the transverse momentum [5,6] and electric charge fluctuations [7]. The measure proved to be very sensitive to dynamical correlations and it was suggested to apply it to study azimuthal ones [8]. Such an analysis is underway using experimental data accumulated by the NA49 and NA61 Collaborations and some preliminary results are already published [9]. The aim of this paper is to present model simulations to be used in interpretation of the experimental data. The fact that the measure Φ is sensitive to correlations of various origin is an advantage and disadvantage at the same time, as it is difficult to disentangle different contributions. Therefore, we model the azimuthal correlations driven by several processes and we look how the correlations show up when quantified by the measure Φ. We first consider separately in terms of toy models, the elliptic flow, resonance decays, jets, and transverse momentum conservation. An effect of randomly lost particles is also examined. Then, we analyze the data provided by the PYTHIA and HIJING event generators showing how to identify the main sources of correlations by applying kinematic cuts and particle selection criteria.

A magnitude of Φ of azimuthal correlations measured in Pb-Pb collisions at 158A GeV is of the order of 0.01 radian [9]. Because the effects, which are theoretically studied here, often exceed this order, a realistic model should include *all* the effects, and consequently, the model has to be rather complex. For this reason we do not attempt to compare our model calculations to the preliminary data [9]. There is also an additional reason. The measurement [9] was performed in an acceptance window that is not only limited in particle rapidity and transverse momentum but it is also nonuniform in azimuthal angle. The acceptance, which is characteristic for the NA49 detector, has to be properly included in model calculations for a quantitative comparison with the experimental data. It is beyond the scope of this study. Our aim here is to understand how the collective flow, resonance decays, jets, and transverse momentum conservation contribute to azimuthal correlations and how the contributions can be disentangled.

II. MEASURE Φ

Let us first introduce the correlation measure Φ. One defines the variable $z \overset{\text{def}}{=} x - \overline{x}$, where x is a single particle's characteristics such as the particle transverse momentum, electric charge, or azimuthal angle. The overline denotes averaging over a single-particle inclusive distribution. In the subsequent sections, x will be identified with the particle azimuthal angle ϕ and the fluctuation measure will be denoted as Φ_ϕ. The event variable Z, which is a multiparticle analog of z, is defined as $Z \overset{\text{def}}{=} \sum_{i=1}^{N}(x_i - \overline{x})$, where the summation runs over particles from a given event. By construction, $\langle Z\rangle = 0$, where $\langle\dots\rangle$ represents averaging over events (collisions). The measure Φ is finally defined as

$$\Phi \overset{\text{def}}{=} \sqrt{\frac{\langle Z^2\rangle}{\langle N\rangle}} - \sqrt{\overline{z^2}}. \quad (1)$$

It is evident that $\Phi = 0$, when no interparticle correlations are present. The measure also possesses a less trivial property—it is *independent* of the distribution of the number of particle sources if the sources are identical and independent from each other. Thus, the measure Φ is "blind" to the impact parameter variation as long as the "physics" does not change with the collision centrality. In particular, Φ is independent of the impact parameter if the nucleus-nucleus collision is a simple superposition of nucleon-nucleon interactions. In the following sections we discuss how various mechanisms responsible for azimuthal correlations contribute to Φ_ϕ. Then, using the event generators we show how the dominant contributions can be identified.

III. COLLECTIVE FLOW

Particles produced in relativistic heavy-ion collisions reveal a collective behavior that is naturally described in terms of hydrodynamics where the collective flow is caused by pressure gradients. The collective flow as quantified by the measure Φ_ϕ was studied in Ref. [8]. Here we only recapitulate the results derived in Ref. [8].

Because the inclusive azimuthal distribution is flat, $\overline{\phi} = \pi$ and $\overline{\phi^2} = \frac{4}{3}\pi^2$ for $\phi \in [0, 2\pi]$, and thus $\overline{z^2} = \frac{1}{3}\pi^2$. A single-particle azimuthal distribution in a given event is

$$P(\phi) = \frac{1}{2\pi}\left[1 + 2\sum_{n=1}^{\infty} v_n \cos(n(\phi - \phi_R))\right], \tag{2}$$

where $0 \leqslant \phi \leqslant 2\pi$; ϕ_R is the reaction plane angle and v_n denotes an amplitude of the nth Fourier harmonics. The N-particle distribution is assumed to be a product on N distributions (2) multiplied by a multiplicity distribution. Consequently, the collective flow is the only source of azimuthal correlations in the system. Averaging over particles is performed by integrating over ϕ_i with $i = 1, 2, \ldots N$ and averaging over events is achieved by integrating over ϕ_R and summing over N. The distribution of the reaction plane angle is obviously flat. Thus, one finds the measure Φ of azimuthal correlations caused by the flow as

$$\Phi_\phi = \sqrt{\frac{\pi^2}{3} + \left(\frac{\langle N^2\rangle - \langle N\rangle}{\langle N\rangle}\right) S} - \frac{\pi}{\sqrt{3}}. \tag{3}$$

where $\langle N^m\rangle$ is the mth moment of multiplicity distribution and

$$S \equiv 2\left\langle \sum_{n=1}^{\infty}\left(\frac{v_n}{n}\right)^2\right\rangle. \tag{4}$$

We have first verified the effect of second Fourier coefficient v_2 on Φ_ϕ by Monte Carlo simulations. For this purpose we have generated events of particle multiplicity given by either Poisson or negative binomial (NB) distribution. The latter is defined as

$$P_N = \frac{\Gamma(N+k)}{\Gamma(N+1)\Gamma(k)} \frac{\langle N\rangle^N k^k}{(\langle N\rangle + k)^{N+k}}, \tag{5}$$

where $\Gamma(k)$ is the Gamma function, which for positive integer arguments equals $\Gamma(k) = (k-1)!$; the parameter k can be expressed through the variance of the distribution $\mathrm{Var}(N) \equiv \langle N^2\rangle - \langle N\rangle^2$ and the average value $\langle N\rangle$ as

$$k = \frac{\langle N\rangle^2}{\mathrm{Var}(N) - \langle N\rangle}. \tag{6}$$

The parameter k is chosen in such a way in all our simulations that $\sqrt{\mathrm{Var}(N)} = \langle N\rangle/2$. Then, the multiplicity distribution approximately obeys the Wróblewski's formula [10], which is known to hold for proton-proton interactions in a wide collision energy range. The simulations are performed for both the negative binomial and Poisson distributions as the former distribution is much broader than the latter one for $\langle N\rangle \gg 1$. We note here that the width of multiplicity distributions in relativistic heavy-ion collisions strongly depends on centrality selection criteria. Thus, it is important to see how the correlation signal changes with the width of multiplicity distribution.

The azimuthal angle of each particle was generated from the distribution,

$$P(\phi) = \frac{1}{2\pi}(1 + 2v_2\cos(2(\phi - \phi_R))), \tag{7}$$

where $0 \leqslant \phi \leqslant 2\pi$; the reaction plane angle ϕ_R of a given event was generated from the flat distribution. The results of our simulations are shown in Fig. 1 for both the Poisson [Fig. 1(a)] and NB [Fig. 1(b)] multiplicity distributions. As seen, the analytical formula (3) works perfectly well.

There are large (~40%) event-by-event fluctuations of v_2 observed [2] at BNL RHIC. The v_2 fluctuations are dominated by the fluctuations of eccentricity of the overlap region of colliding nuclei (see, e.g., [11]). We have introduced the v_2 fluctuations in our simulations in the following way. For each event the value of v_2 was generated from the Gaussian distribution of the dispersion σ_{v_2}. The fluctuations have been restricted to vary within $2\sigma_{v_2}$ around the mean $\langle v_2\rangle$ that is $\langle v_2\rangle - 2\sigma_{v_2} \leqslant v_2 \leqslant \langle v_2\rangle + 2\sigma_{v_2}$. Then, v_2 remains positive unless $\sigma_{v_2}/\langle v_2\rangle$ exceeds 0.5.

In Fig. 2 we demonstrate the effect of flow fluctuations relative to the effect of flow. Specifically, we show the difference of the correlation measures computed for the fluctuating v_2 and fixed $v_2 = \langle v_2\rangle$. The particle multiplicity was generated from the NB distribution with $\langle N\rangle = 400$. As seen, the flow fluctuations of a relative magnitude of ~40% noticeably increase the value of Φ_ϕ if $\langle v_2\rangle$ is not too small.

IV. RESONANCE DECAYS

Let us start the discussion of effects of resonance decays with the toy model where *all* produced particles come from heavy resonances that have vanishing transverse velocity and decay back to back into pairs of particles. The particle multiplicity is an arbitrary but fixed even number. Then, as shown in the appendix, we have

$$\Phi_\phi = \frac{1-\sqrt{2}}{\sqrt{6}}\pi \approx -0.531. \tag{8}$$

When only a fraction f of all produced particles comes from the back-to-back decays of resonances whereas the remaining particles are produced independently from each other, the calculations presented in the appendix lead to

$$\Phi_\phi = \frac{\sqrt{2-f} - \sqrt{2}}{\sqrt{6}}\pi. \tag{9}$$

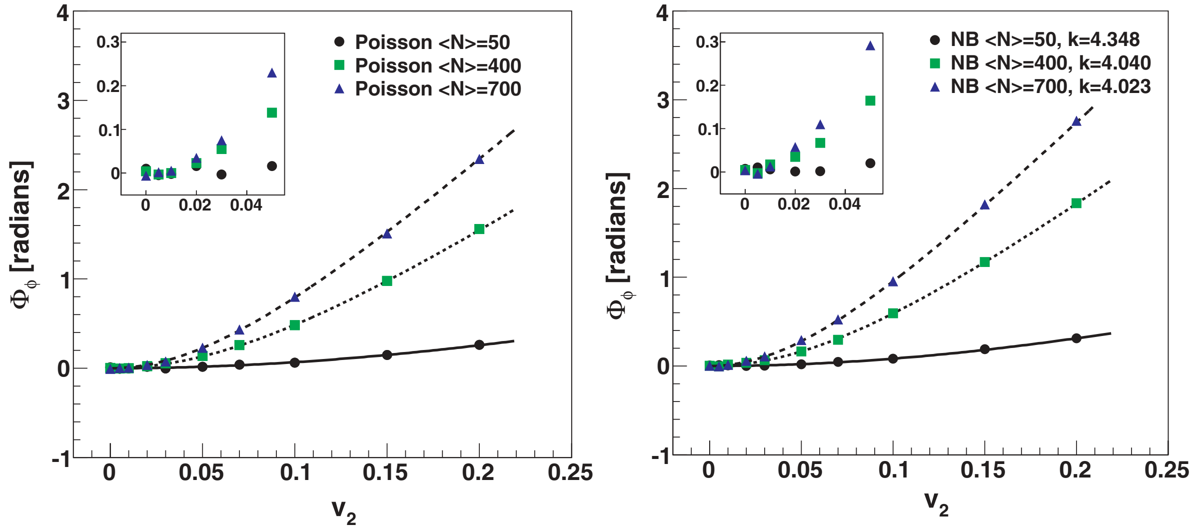

FIG. 1. (Color online) Φ_ϕ as a function of the second Fourier coefficient v_2 for the Poisson (a) and NB (b) multiplicity distributions. The lines represent the analytical formula (3). The insets show Φ_ϕ for small values of v_2.

As seen, for $f = 0$ the formula (9) gives, as expected, $\Phi = 0$ and for $f = 1$ we get the value (8).

We have checked the formula (9) by Monte Carlo simulations and then we have considered the model where the particle multiplicity is not fixed but it is given by either Poisson or NB distribution with the average multiplicity equal 50, 400, or 700 particles. For a given fraction f of particles coming from the back-to-back decays of resonances, the number of particles coming from the decays in the event of multiplicity N was the even number that is the nearest to fN. The measure Φ_ϕ as a function of f is shown in Fig. 3. As seen, the formula (9) still works very well.

When a resonance that is at rest decays back to back, the difference of azimuthal angles of the decay products is $\Delta\phi = \pi$. When the resonance has a finite velocity, the difference of azimuthal angles of the decay products is smaller than π. When the resonance's kinetic energy is much larger than the energy released in its decay, $\Delta\phi$ is zero. Therefore, we consider a model where a fraction of particles comes from the resonance decays and the particles are emitted in pairs with the difference of their azimuthal angles $\Delta\phi$ varying from 0 to π. We first assume that *all* particles are emitted in pairs and the relative azimuthal angle of two correlated particles equals $\Delta\phi$. As shown in the appendix, we then have

$$\Phi_\phi = \sqrt{\frac{2}{3}\pi^2 - \Delta\phi\,\pi + \frac{1}{2}(\Delta\phi)^2} - \frac{\pi}{\sqrt{3}}. \qquad (10)$$

As seen that Φ_ϕ changes its sign from positive to negative with growing $\Delta\phi$; Φ_ϕ vanishes when

$$\Delta\phi = \pi\left(1 - \frac{1}{\sqrt{3}}\right) \approx 1.328, \qquad (11)$$

and for $\Delta\phi = \pi$ we deal with the model described by the formula (8). Further on, we have considered a model where only a fraction f of particles is emitted in correlated pairs. Then, as explained in the appendix, Eq. (10) gets the form,

$$\Phi_\phi = \sqrt{\frac{\pi^2}{3} + f\left(\frac{\pi^2}{3} - \Delta\phi\,\pi + \frac{1}{2}(\Delta\phi)^2\right)} - \frac{\pi}{\sqrt{3}}. \qquad (12)$$

As previously, Φ_ϕ changes its sign and $\Phi_\phi = 0$ for $\Delta\phi$ given by Eq. (11).

In Fig. 4 we compare the formula (12) with the results of Monte Carlo simulation of Φ_ϕ as a function of $\Delta\phi$. The fraction of particles emitted in pairs equals 0.3, 0.5, 0.7, or 1.0. The remaining particles, which are not emitted in pairs, carry no correlations. The particle multiplicity is generated from the NB distribution with $\sqrt{\mathrm{Var}(N)} = \langle N\rangle/2 = 50$. As seen, the formula (12) works perfectly well.

We next discuss the combined effect of resonance decays and elliptic flow. The multiplicity of events is generated from the Poisson or NB binomial distribution. For each event 30% of particles is assumed to originate from heavy resonances that

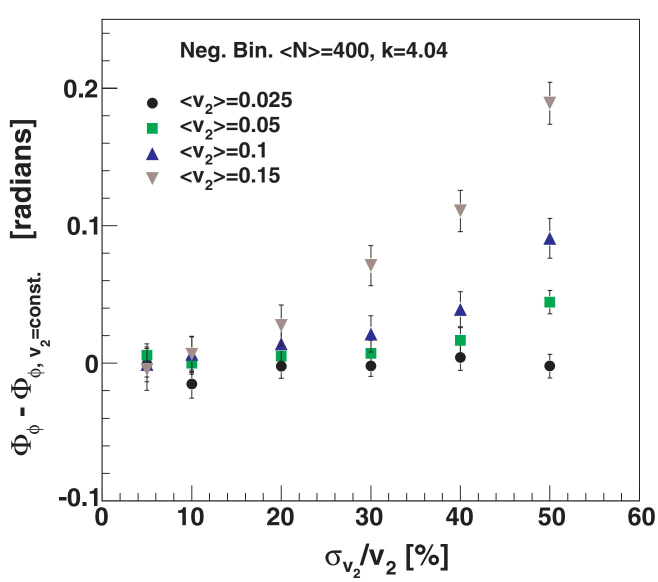

FIG. 2. (Color online) The difference of Φ_ϕ computed for the fluctuating and fixed v_2.

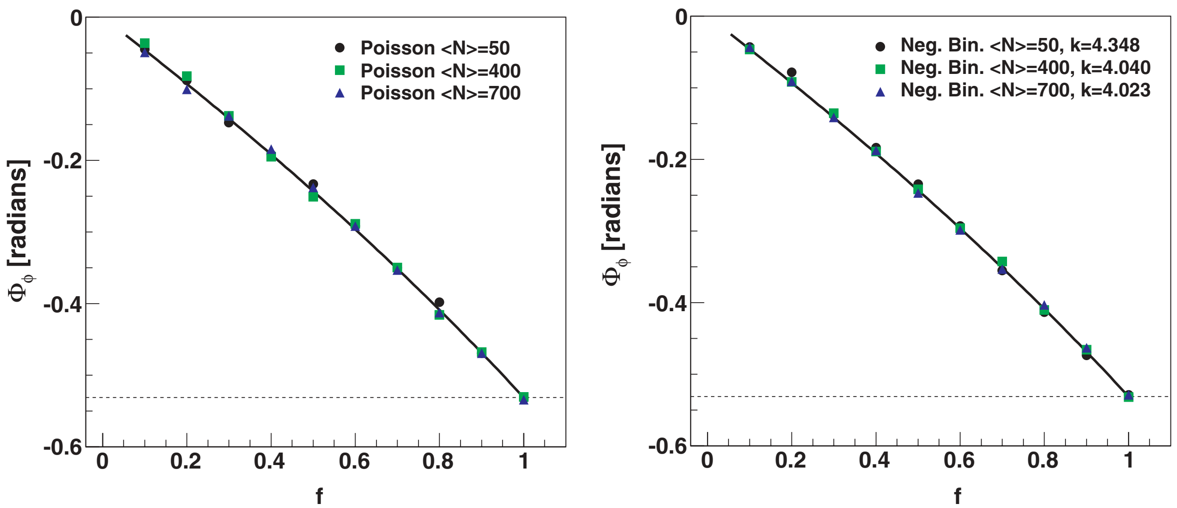

FIG. 3. (Color online) Φ_ϕ as a function of fraction of particles coming from the back-to-back resonance decays. The particle multiplicity is distributed according to the Poisson (a) or NB (b) distribution. The dashed and solid lines represent the analytical formulas (8) and (9), respectively.

decay back to back into pairs of particles. Neither resonances nor their decay products experience any flow, but the remaining 70% of particles manifest the collective elliptic flow according to Eq. (7). The results of the simulation are shown in Fig. 5. When the particle multiplicity or v_2 are sufficiently small, the effects of resonance decays dominates and Φ_ϕ is negative. It becomes positive when the effect of elliptic flow takes over.

V. "DIJETS"

We call a dijet the two groups (jets) of particles flying in exactly opposite directions. Particles from each jet are distributed within a cone of the azimutal angle $\sigma\phi$. We have considered the dijets of $2+2$, $5+5$, and $10+10$ particles. The total particle multiplicity is correspondingly 4, 10, 20 as there is exactly one dijet per event and there are no other particles. The results of dijet simulation are shown in Fig. 6.

We have here two sources of azimuthal correlations that counteract each other. As we already know, the back-to-back emission of particles generates negative correlations whereas the collinear emission leads to positive ones. When the particle's multiplicity of dijets is sufficiently high and $\sigma\phi$ is sufficiently small, the effect of collinear emission wins and Φ_ϕ is positive.

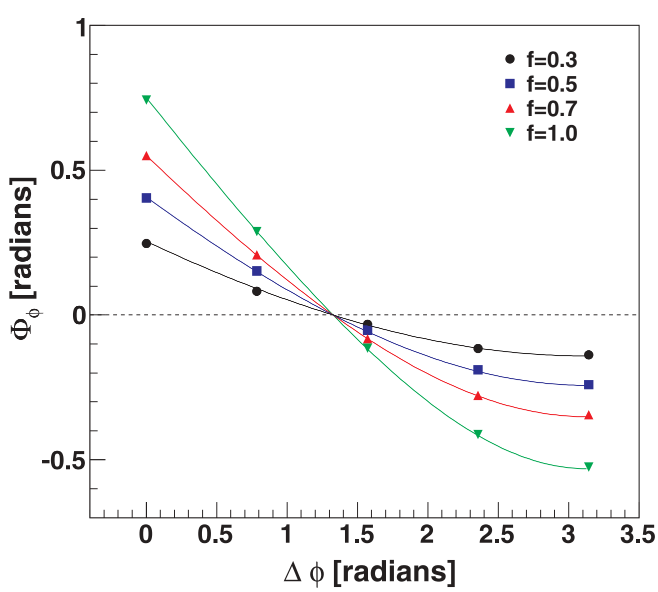

FIG. 4. (Color online) Φ_ϕ as a function of the correlation angle $\Delta\phi$ for varying fraction f of particles emitted in pairs. The particle multiplicity is given by the NB distribution.

VI. MOMENTUM CONSERVATION

The momentum conservation obviously leads to interparticle correlations. We have studied the effect on Φ_ϕ, generating the sets of particles of multiplicity N. The azimuthal angle distribution of a single particle is assumed to be flat whereas the transverse momentum distribution is chosen in the form,

$$P(p_T) = \beta^2 p_T e^{-\beta p_T}, \tag{13}$$

with the slope parameter $\beta^{-1} = 200$ MeV. For each particle the x and y components of its momentum have been computed as $p_x = p_T \cos\phi$ and $p_y = p_T \sin\phi$. To make the total transverse momentum of N particles vanish, the x and y component of momentum of each particle was shifted as

$$p_x \to p_x - \frac{1}{N}\sum_{i=1}^{N} p_x^i, \qquad p_y \to p_y - \frac{1}{N}\sum_{i=1}^{N} p_y^i. \tag{14}$$

The simulation showing the effect of transverse momentum conservation is illustrated in Fig. 7. The particle multiplicity was generated according to NB distribution with $\sqrt{\mathrm{Var}(N)} = \langle N\rangle/2$. As seen, the effect of momentum conservation is sizable and it survives to large multiplicities.

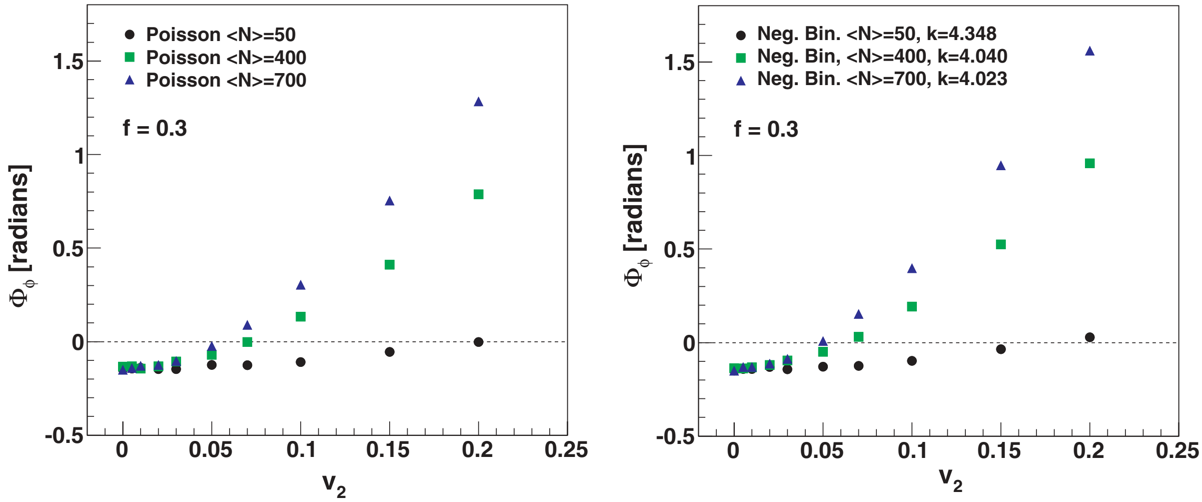

FIG. 5. (Color online) Φ_ϕ resulting from the combined effect of elliptic flow and resonance decays. The particle multiplicity is distributed according to the Poisson (a) or NB (b) distribution.

In real experiments only a fraction of all produced particles is observed because of a finite detector efficiency and acceptance. We model the effect of detector efficiency by randomly loosing particles independently of their azimuthal angle. In Fig. 8 we show how the effect of detector efficiency modifies the correlations caused by the transverse momentum conservation. As seen, the random losses of particles lead to the dilution of the correlation that is Φ_ϕ monotonically goes to zero as the fraction of registered particles $f_{\text{reg}} \to 0$.

VII. PROTON-PROTON COLLISIONS IN PYTHIA

After the discussion of various mechanisms responsible for azimuthal correlations, let us now consider a more realistic situation where several mechanisms of azimuthal correlations are present at the same time. We used the PYTHIA generator [12] to simulate *p-p* collisions at several collision energies accessible at SPS ($\sqrt{s_{NN}} = 6.27$, 7.62, 8.73, 12.3, 17.3 GeV) and RHIC ($\sqrt{s_{NN}} = 19.6$, 62.4, 130, 200 GeV). For every energy a set of minimum bias events was collected. We treated as stable the following particles: μ^-, π^0, π^+, K^0, K^+, K_L^0, K_S^0, Λ, Σ^+, Σ^-, Ξ^0, Ξ^-, Ω^-, and their antiparticles. No acceptance cuts were applied.

For every energy we computed Φ_ϕ separately for positive, negative, and all charged particles. The results are shown in Fig. 9. As seen, Φ_ϕ is negative and weakly depends on collision energy. To understand why Φ_ϕ is so different for negative and for positive particles, we excluded protons from all charged and from positive particles. The corresponding values of Φ_ϕ are

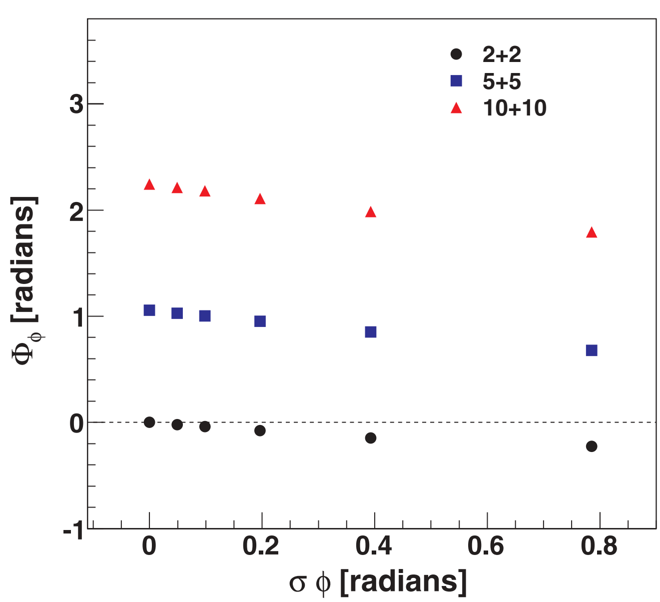

FIG. 6. (Color online) Φ_ϕ as a function of the jet opening angle $\sigma\phi$ for several numbers of particles in a jet.

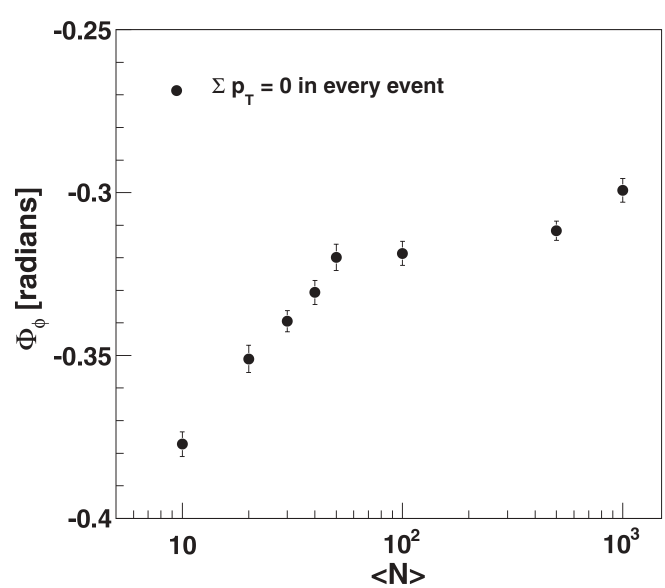

FIG. 7. Φ_ϕ as a function of average multiplicity in events where total transverse momentum exactly vanishes.

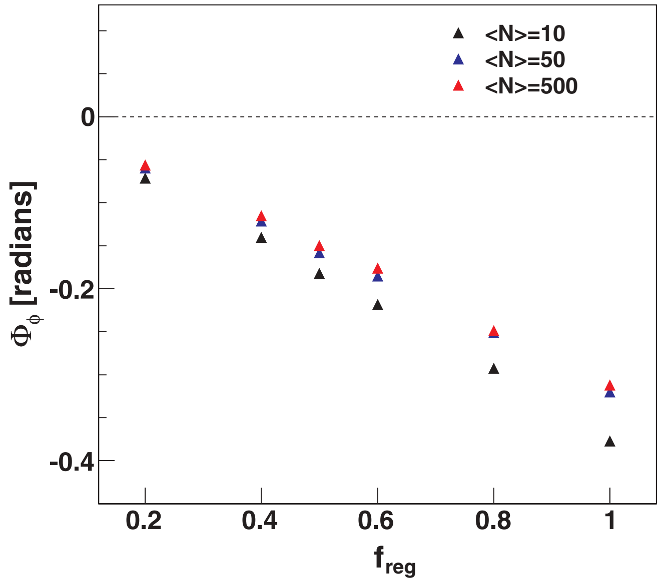

FIG. 8. (Color online) Φ_ϕ as a function of fraction of registered particles $f_{\rm reg}$. The correlation results from the transverse momentum conservation.

also shown in Fig. 9. After excluding protons, the correlations among negative particles and among positive are very similar to each other.

What is the mechanism responsible for negative values of Φ_ϕ? We first checked that high p_T particles play no role here, as Φ_ϕ does not significantly change when particles with $p_T > 1.5$ GeV are excluded. The correlations among charged particles can be caused by the effect resonance decays but the effect is certainly very minor for same-sign particles, as there are very few resonances decaying into two positive or two negative particles.

The transverse momentum conservation, which is discussed in Sec. VI, is another possible source of negative values of Φ_ϕ. We checked that the PHYTHIA events indeed obey the transverse momentum conservation. Specifically, we proved vanishing of the total momentum in x and in y directions of all particles (charged and neutral) from every event. To quantitatively study the effect of transverse momentum conservation we proceeded as follows. For every collision energy we determined the average multiplicity of positive, negative, and neutral particles. Then, we performed the simple simulations described in Sec. VI, generating events of a given total multiplicity that satisfy the transverse momentum conservation. Then, a fraction of particles was randomly eliminated to get the multiplicity of charged, positive, or negative particles. The values of Φ_ϕ computed for such events are shown in Fig. 10. As seen, the values of Φ_ϕ for the PYTHIA events agree quite well with the results of our toy-model simulations, which take into account only the effect of transverse momentum conservation. It is somewhat surprising that the agreement for same-sign particles is not much better that that for all charged particles. It means that the resonance decays do not generate strong correlations in the PYTHIA events. We note, however, that the effect of transverse momentum conservation overshoots the correlations of the like-sign particles and it undershoots the correlations of all charged particles. The latter results presumably signal the presence of resonances decaying into pairs of one positive and one negative particle.

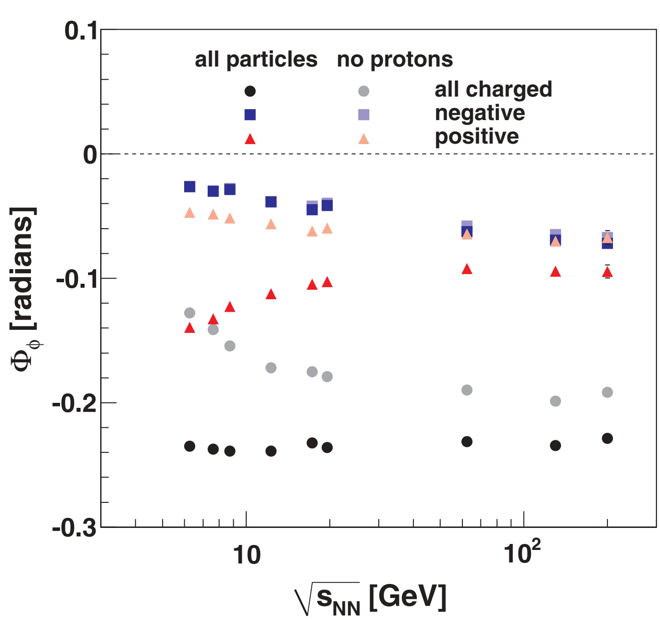

FIG. 9. (Color online) The energy dependence of Φ_ϕ for positive, negative, and all charged particles in the PYTHIA simulations of p-p collisions. The pale symbols correspond to the results with protons excluded from the positive and all charged particles.

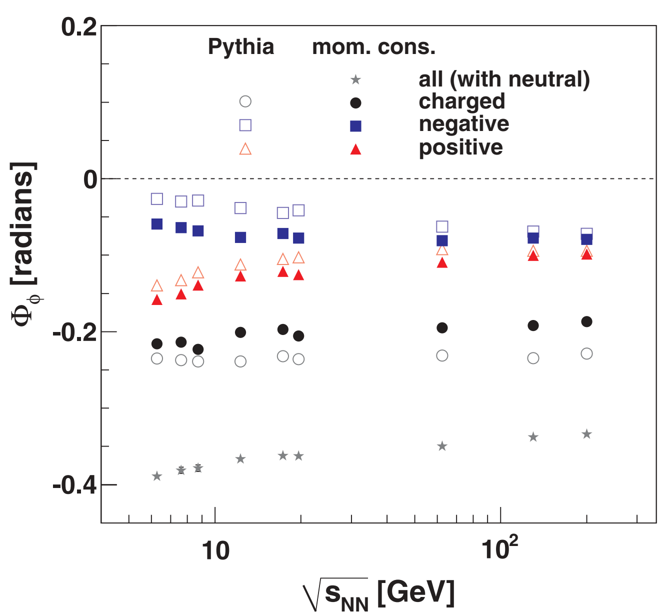

FIG. 10. (Color online) Φ_ϕ for the PYTHIA events (open symbols) compared to the results of toy-model simulations (solid symbols), which take into account only the effect of transverse momentum conservation. The asterisks show the toy-model results for all (neutral and charged) particles.

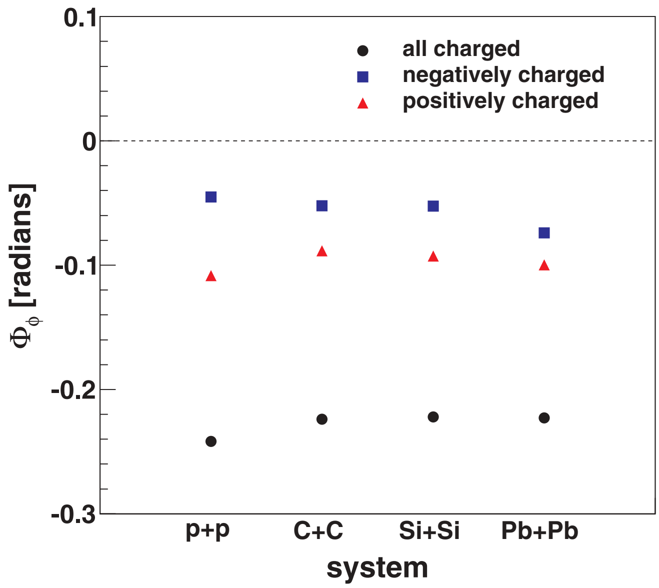

FIG. 11. (Color online) The Hijing simulation of the system size dependence of Φ_ϕ for nucleus-nucleus collisions at $\sqrt{s_{NN}} = 17.3$ GeV with no acceptance cuts.

VIII. NUCLEUS-NUCLEUS COLLISIONS IN HIJING

We have also performed simulations of nucleus-nucleus collisions using the HIJING [13] event generator. We have simulated the collisions of *p-p*, C-C, Si-Si, and Pb-Pb at $\sqrt{s_{NN}} = 17.3$ GeV and Φ_ϕ was computed separately for positive, negative, and all charged particles coming from minimum bias events. The results are shown in Fig. 11. As seen, Φ_ϕ is almost independent of the mass number of colliding nuclei and the values of Φ_ϕ are very close to those found using PYTHIA. It is by no means accidental. When a nucleus-nucleus collision is a simple superposition of nucleon-nucleon interactions, the value of Φ is exactly the same for *p-p* interactions and nucleus-nucleus collisions at any centrality. In the HIJING model a nucleus-nucleus collision is not exactly a superposition of nucleon-nucleon collisions but it is almost so. And the treatment of proton-proton interactions is essentially the same in PYTHIA and HIJING. For these reasons our analysis of PYTHIA events presented in the previous section fully applies here.

IX. SUMMARY AND OUTLOOK

Azimuthal correlations of final state particles from high-energy collisions carry valuable information on the collision dynamics. It motivates the analysis of experimental data collected by the NA49 and NA61 Collaborations, which is in progress with some preliminary results already published [9]. The integral measure Φ_ϕ, which proved to be very sensitive to various dynamical correlations, is used in the analysis. To interpret the experimental results it should be understood how different sources of correlations manifest themselves when measured by means of Φ_ϕ. This was the aim of our study. We performed several simulations to analyze separately the azimuthal correlations caused by the elliptic flow, resonance decays, jets, and transverse momentum conservation. We also discussed how the correlations are diluted from randomly lost particles. Finally we used the PYTHIA and HIJING event generators to produce a big sample of events that mimic experimental data from *p-p* and nucleus-nucleus collisions at the SPS and RHIC collision energies. Φ_ϕ appeared to be surprisingly independent of the collision energy and of the size of colliding systems. Applying some kinematic cuts and selection criteria of particles, we showed that the azimuthal correlations are dominated by a rather trivial effect of transverse momentum conservation that appeared to be almost independent of the particle's multiplicity, which changes dramatically for collision energies and system sizes under consideration.

The experience gathered in the course of this theoretical study will be used to better understand experimental data. The quantitative analysis of several simple mechanisms of azimuthal correlations we discussed will facilitate an observation of possible phenomena, such as critical fluctuations at phase boundaries of strongly interacting matter or plasma instabilities from the early stage of relativistic heavy-ion collisions.

ACKNOWLEDGMENTS

We are grateful to Maciej Rybczyński for providing us with the sample of nucleus-nucleus collisions at $\sqrt{s_{NN}} = 17.3$ GeV simulated with the HIJING event generator. We thank to Marek Gaździcki for critical reading of the manuscript and his numerous comments. This work was partially supported by Polish Ministry of Science and Higher Education under Grants No. N N202 204638 and No. 667/N-CERN/2010/0.

APPENDIX: TOY MODEL OF RESONANCE DECAYS

The inclusive distribution of the azimuthal angle is assumed to be flat, that is,

$$P_{\rm inc}(\phi) = \frac{1}{2\pi}\,\Theta(\phi)\,\Theta(2\pi - \phi), \tag{A1}$$

which gives $\bar{\phi} = \pi$ and $\bar{\phi^2} = 4\pi^2/3$. Consequently, $\bar{z}^2 = \pi^2/3$.

Let us first assume that *all* produced particles come from heavy resonances that are at rest and decay back to back into two particles. When one particle is emitted at the azimuthal angle ϕ_1 and $0 \leqslant \phi_1 < \pi$, the second particle is emitted at $\phi_2 = \phi_1 + \pi$. When $\pi \leqslant \phi_1 < 2\pi$, then $\phi_2 = \phi_1 - \pi$. Therefore, the two-particle distribution of azimuthal angles reads

$$P_2(\phi_1, \phi_2) = \frac{1}{2\pi}\,\Theta(\pi - \phi_1)\,\delta(\phi_1 - \phi_2 + \pi) + \frac{1}{2\pi}\,\Theta(\phi_1 - \pi)\,\delta(\phi_1 - \phi_2 - \pi). \tag{A2}$$

One observes that

$$\int d\phi_1\, P_2(\phi_1, \phi) = \int d\phi_2\, P_2(\phi, \phi_2) = P_{\rm inc}(\phi), \tag{A3}$$

and computes

$$\int d\phi_1 d\phi_2 \phi_1 \phi_2 P_2(\phi_1, \phi_2) = \frac{5}{6}\pi^2. \tag{A4}$$

We further assume that the particle multiplicity is an arbitrary but fixed even number N. Then, the N-particle distribution of azimuthal angles is

$$P_N(\phi_1, \phi_2, \ldots, \phi_N) = P_2(\phi_1, \phi_2)\, P_2(\phi_3, \phi_4) \cdots \times P_2(\phi_{N-1}, \phi_N). \tag{A5}$$

The variable Z is defined as $Z = \phi_1 + \phi_2 + \ldots + \phi_N - N\pi$ and one computes $\langle Z^2 \rangle$ in the following way:

$$\begin{aligned}\langle Z^2 \rangle &= \int d\phi_1 d\phi_2 \ldots d\phi_N (\phi_1 + \phi_2 + \ldots + \phi_N - N\pi)^2 P_N(\phi_1, \phi_2, \ldots, \phi_N) \\ &= N\bar{\phi^2} + N \int d\phi_1 d\phi_2 \phi_1 \phi_2 P_2(\phi_1, \phi_2) + N(N-2)\bar{\phi}^2 - 2N^2\bar{\phi}\pi + N^2\pi^2 = \frac{N}{6}\pi^2.\end{aligned} \tag{A6}$$

Using the result (A6) and keeping in mind that $\bar{z}^2 = \pi^2/3$, we find the formula (8) from the definition (1).

Let us now assume that N_1 particles come from resonances and additional N_2 particles are produced independently from each other and from resonances. Then, we still have $\bar{z}^2 = \pi^2/3$ and $\langle Z^2 \rangle$ is computed as

$$\begin{aligned}\langle Z^2 \rangle = N\bar{\phi^2} + N_1 \int d\phi_1 d\phi_2 \phi_1 \phi_2 P_2(\phi_1, \phi_2) + N_1(N_1 - 2)\bar{\phi}^2 \\ + 2N_1 N_2 \bar{\phi}^2 + N_2(N_2 - 1)\bar{\phi}^2 - 2N^2\bar{\phi}\pi \\ + N^2\pi^2 = \frac{\pi^2}{6}(2N - N_1),\end{aligned} \tag{A7}$$

where $N \equiv N_1 + N_2$. The result (A7) with $f \equiv N_1/N$ gives the formula (9).

The model can be easily generalized to the situation when the particles from a correlated pair are not emitted back to back but their relative azimuthal angle is fixed and equal $\Delta\phi$. Then, the two-particle distribution of azimuthal angles is

$$\begin{aligned}P_2(\phi_1, \phi_2) = \frac{1}{2\pi}\, \Theta(2\pi - \Delta\phi - \phi_1)\, \delta(\phi_1 - \phi_2 + \Delta\phi) \\ + \frac{1}{2\pi}\, \Theta(\phi_1 - 2\pi + \Delta\phi)\, \delta(\phi_1 - \phi_2 + \Delta\phi - 2\pi),\end{aligned} \tag{A8}$$

and instead of Eq. (A4) we have

$$\int d\phi_1 d\phi_2 \phi_1 \phi_2 P_2(\phi_1, \phi_2) = \frac{4}{3}\pi^2 - \Delta\phi\, \pi + \frac{1}{2}(\Delta\phi)^2. \tag{A9}$$

Substituting the result (A9) to Eq. (A6) or Eq. (A7), one finds $\langle Z^2 \rangle$, which finally gives the formula (10) or (12), respectively.

[1] J. Casalderrey-Solana and C. A. Salgado, Acta Phys. Polon. B **38**, 3731 (2007).
[2] S. A. Voloshin, A. M. Poskanzer, and R. Snellings, arXiv:0809.2949 [nucl-ex].
[3] St. Mrówczyński, J. Phys. Conf. Ser. **27**, 204 (2005).
[4] M. Gaździcki and St. Mrówczyński, Z. Phys. C **54**, 127 (1992).
[5] T. Anticic *et al.* (NA49 Collaboration), Phys. Rev. C **70**, 034902 (2004).
[6] T. Anticic *et al.* (NA49 Collaboration), Phys. Rev. C **79**, 044904 (2009).
[7] C. Alt *et al.* (NA49 Collaboration), Phys. Rev. C **70**, 064903 (2004).
[8] St. Mrówczyński, Acta Phys. Polon. B **31**, 2065 (2000).
[9] T. Cetner and K. Grebieszkow (for the NA49 Collaboration), arXiv:1008.3412 [nucl-ex].
[10] A. Wróblewski, Acta Phys. Polon. B **4**, 857 (1973).
[11] W. Broniowski, P. Bożek, and M. Rybczyński, Phys. Rev. C **76**, 054905 (2007).
[12] T. Sjöstrand, S. Mrenna, and P. Skands, Comput. Phys. Commun. **178**, 852 (2008).
[13] M. Gyulassy and X. N. Wang, Comput. Phys. Commun. **83**, 307 (1994).