Elementary atom interaction with hydrogen

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The interactions of elementary atoms, i.e., Coulomb bound states of elementary particles such as positronium or pionium, with hydrogen atoms are studied. The processes leading to excitation of elementary and/or the hydrogen atom are discussed. The numerical values of cross sections, in particular, the total ones, of seven elementary atoms are given.

I. INTRODUCTION

The elementary atoms are Coulomb bound states of elementary particles as positronium, pionium, or an atom of μ^+ and π^-. The physics of elementary atoms and their history are briefly presented in Ref. 1. The reader is invited to look over this paper for an introduction.

In a series of papers[1–5] the problem of interaction of elementary atoms with matter has been studied. In all these papers, however, heavy matter atoms have been considered and, except Ref. 3, these atoms have been treated as sources of external potential. Therefore the processes leading to the excitations of these atoms have been neglected. As argued in, e.g., Ref. 1 it is approximately correct for atoms with charge Z, which is much greater than unity. The excitations of heavy matter atoms have been discussed in Ref. 3 by means of the Thomas-Fermi approximation.

In this paper we study in detail the case of the hydrogen atom, where its excitations can be treated in an exact way. To our knowledge there is only one paper[6] where the problem has been discussed. Namely, Massey and Mohr have calculated the cross sections of positronium interaction with hydrogen.

In Sec. II we derive the formulas of the cross sections, in particular of the total cross sections. The numerical values of the cross sections are given in Sec. III, and Sec. IV is devoted to the discussion of our results.

II. FORMULAS

The matrix element of the interaction of an elementary atom with hydrogen, in Born approximation, reads

$$\begin{aligned} M_{if} = & -ie2\pi\,\delta(E_i-E_f) \\ & \times\int d^3\mathbf{x}_1 d^3\mathbf{x}_2 d^3\mathbf{y}_1 d^3\mathbf{y}_2 \psi_f^*(\mathbf{x}_1,\mathbf{x}_2)\psi_{\mathrm{H}f}^*(\mathbf{y}_1,\mathbf{y}_2) \\ & \times[U(\mathbf{x}_1-\mathbf{y}_1)-U(\mathbf{x}_1-\mathbf{y}_2) \\ & +U(\mathbf{x}_2-\mathbf{y}_2)-U(\mathbf{x}_2-\mathbf{y}_1)] \\ & \times\psi_i(\mathbf{x}_1,\mathbf{x}_2)\psi_{\mathrm{H}i}(\mathbf{y}_1,\mathbf{y}_2)\ , \end{aligned} \tag{1}$$

where E_i (E_f) is the total energy of initial (final) state; $\mathbf{x}_1,\mathbf{x}_2$ and $\mathbf{y}_1,\mathbf{y}_2$ denote the positions of elementary and hydrogen atom components, respectively; ψ (ψ_{H}) is the two-particle wave function of elementary (hydrogen) atom; $U(\mathbf{x})$ is the Coulomb potential $U(\mathbf{x})=4\pi e/|\mathbf{x}|$.

Introducing the variables of the center of mass and of relative motion for both atoms, and using the plane wave to describe the motion of center of mass of each atom, one gets from Eq. (1) the following amplitude:

$$\begin{aligned} M_{if} = & -4\pi ie^2(2\pi)^4\delta^{(4)}(P_i-P_f) \\ & \times\mathbf{q}^{-2}[F_{if}(\eta\mathbf{q})-F_{if}(\xi\mathbf{q})] \\ & \times[F_{if}^{\mathrm{H}}(\eta_{\mathrm{H}}\mathbf{q})-F_{if}^{\mathrm{H}}(\xi_{\mathrm{H}}\mathbf{q})]\ , \end{aligned} \tag{2}$$

where P_i (P_f) is the total four momentum of the initial (final) state, $\mathbf{q}$ is the momentum transfer to the center of mass of the elementary atom, and F_{if} is the elementary atom form factor defined as

$$F_{if}(\mathbf{q})=\int d^3\mathbf{r}\, e^{i\mathbf{q}\cdot\mathbf{r}}\phi_f^*(\mathbf{r})\phi_i(\mathbf{r})\ ,$$

where ϕ_i (ϕ_f) is the initial (final) elementary atom wave function of the relative motion. The hydrogen atom form factor F_{if}^{H} is defined in the analogous way. The parameters ξ and η are equal to m_i/M and $-m_2/M$, respectively, where m_1 and m_2 are the elementary atom component masses, and M is the atom mass. Because of the smallness of the mass defect, $\xi-\eta=1$. The parameters ξ_{H} and η_{H} are defined analogously. Because a proton is nearly 2000 times heavier than an electron, $\xi_{\mathrm{H}}\cong 1$ and $\eta_{\mathrm{H}}\cong 0$. Therefore, one usually uses the approximation

$$F_{if}^{\mathrm{H}}(\eta_{\mathrm{H}}\mathbf{q})-F_{if}^{\mathrm{H}}(\xi_{\mathrm{H}}\mathbf{q})\cong\begin{cases}1-F_{ii}^{\mathrm{H}}(\mathbf{q}) & \text{for } i=f\ ,\\ -F_{if}^{\mathrm{H}}(\mathbf{q}) & \text{for } i\neq f\ ,\end{cases}$$

where the initial and final states are assumed orthogonal for the case $i\neq f$.

One finds from of the amplitude (2) that it is zero for the elementary atom composed of particle and antiparticle ($\xi=-\eta=\frac{1}{2}$) if the orbital momentum of the elementary atoms changes in the course of interaction by an even number. The detailed analysis of this selection rule can be found in Ref. 4.

In the standard way one converts the amplitude (2) into the differential cross section of excitation of elemen-

tary and/or hydrogen atom to the discrete state

$$d\sigma_{if}=\frac{8\pi e^4}{\mathbf{v}^2}\,|\,[F_{if}(\eta\mathbf{q})-F_{if}(\xi\mathbf{q})]$$

$$\times[F_{if}^{\mathrm{H}}(\eta_{\mathrm{H}}\mathbf{q})-F_{if}^{\mathrm{H}}(\xi_{\mathrm{H}}\mathbf{q})]\,|\,^2 q^{-3}dq\ , \qquad (3)$$

where $\mathbf{v}$ is the initial relative velocity of the atoms and $q\equiv|\,\mathbf{q}\,|$.

The maximum and minimum values of the momentum transfer q follow from the energy-momentum conservation. If the initial kinetic energy is much greater than the atom binding energy (which is demanded because of the Born approximation used) $q_{\min}$ is much smaller than the characteristic momentum transfer which is of the order of the inverse atom Bohr radius. Then $q_{\min}$ can be shifted to zero when the cross section (3) is integrated over momentum transfer. On the other hand, $q_{\max}$ is much greater than the characteristic momentum transfer and it can be moved to infinity. Therefore the integration limits are independent of the initial and final states of the atoms and the initial energy dependence of the integrated cross section reads

$$\sigma_{if}=\frac{1}{\mathbf{v}^2}\bar{\sigma}_{if}\ , \qquad (4)$$

where the cross section $\bar{\sigma}_{if}$ is independent of initial energy.

As proved in Ref. 1,

$$\sum_f\,|\,F_{if}(\eta\mathbf{q})-F_{if}(\xi\mathbf{q})\,|\,^2=2-2F_{ii}(\mathbf{q})\ , \qquad (5)$$

where the summation is performed over the complete set of final states. Using the sum rule (5) one can find the cross sections summed over the final states of the elementary and/or the hydrogen atom. In particular, one finds the total cross section, i.e., the cross section summed over final states of both atoms

$$\bar{\sigma}^{\mathrm{tot}}=32\pi e^4\int_0^\infty dq\,q^{-3}[1-F_{ii}(\mathbf{q})][1-F_{ii}^{\mathrm{H}}(\mathbf{q})]\ . \qquad (6)$$

Because of the importance of the total cross section, we have found the analytical formula of it. The initial states of both atoms have been identified with the ground states. Substituting the respective expressions of form factors taken from Ref. 1 into (6) one gets

$$\bar{\sigma}^{\mathrm{tot}}=8\pi e^4\frac{a^2a_{\mathrm{H}}^2}{(a_{\mathrm{H}}^2-a^2)^2}$$

$$\times\left[\frac{a^2+a_{\mathrm{H}}^2}{2}+\frac{a^4+a_{\mathrm{H}}^4-3a^2a_{\mathrm{H}}^2}{a^2-a_{\mathrm{H}}^2}\ln a_{\mathrm{H}}^2/a^2\right]\ ,$$

where a (a_{H}) is the Bohr radius of the elementary (hy-

TABLE I. The cross sections (in cm^2) of A_{2e}.

	$1s$	$2s$	$2p$	$3s$	$3p$	$3d$	$4s$	$4p$	$4d$	$4f$	Σ
$1s$	0	0	0	0	0	0	0	0	0	0	0
$2s$	0	0	0	0	0	0	0	0	0	0	0
$2p$	1.3(−21)	4.7(−22)	4.7(−21)	9.5(−23)	9.5(−22)	6.8(−23)	3.5(−23)	3.5(−22)	3.1(−23)	6.0(−25)	1.5(−20)
$3s$	0	0	0	0	0	0	0	0	0	0	0
$3p$	3.5(−22)	1.1(−22)	9.5(−22)	2.3(−23)	1.9(−22)	1.5(−23)	8.7(−24)	7.4(−23)	7.0(−24)	1.4(−25)	3.6(−21)
$3d$	0	0	0	0	0	0	0	0	0	0	0
$4s$	0	0	0	0	0	0	0	0	0	0	0
$4p$	1.4(−22)	4.4(−23)	3.5(−22)	9.2(−24)	7.4(−23)	5.8(−24)	3.4(−24)	2.8(−23)	2.7(−24)	5.5(−26)	1.3(−21)
$4d$	0	0	0	0	0	0	0	0	0	0	0
$4f$	4.0(−25)	1.2(−25)	6.0(−25)	2.6(−26)	1.4(−25)	1.4(−26)	9.9(−27)	5.5(−26)	6.7(−27)	1.5(−28)	3.2(−24)
Σ	1.7(−20)	1.8(−21)	1.1(−20)	4.1(−22)	2.5(−21)	2.0(−22)	1.6(−22)	9.8(−22)	9.7(−23)	2.0(−24)	8.0(−20)

TABLE II. The cross sections (in cm^2) of $A_{e\mu}$.

	1s	2s	2p	3s	3p	3d	4s	4p	4d	4f	Σ
1s	4.4(−21)	1.3(−22)	3.3(−22)	3.4(−23)	8.9(−23)	8.4(−24)	1.4(−23)	3.6(−23)	4.3(−24)	1.0(−25)	1.2(−20)
2s	1.3(−22)	2.6(−23)	1.1(−22)	5.9(−24)	2.8(−23)	2.6(−24)	2.3(−24)	1.1(−23)	1.3(−24)	3.0(−26)	8.0(−22)
2p	3.4(−22)	1.2(−22)	1.1(−21)	2.4(−23)	2.4(−22)	1.7(−23)	9.0(−24)	9.0(−23)	7.9(−24)	1.5(−25)	4.0(−21)
3s	3.4(−23)	5.9(−24)	2.3(−23)	1.3(−24)	5.8(−24)	5.6(−25)	5.4(−25)	2.3(−24)	2.8(−25)	6.5(−27)	1.8(−22)
3p	9.2(−23)	2.8(−23)	2.4(−22)	5.9(−24)	5.0(−23)	3.8(−24)	2.2(−24)	1.8(−23)	1.7(−24)	3.6(−26)	9.1(−22)
3d	8.5(−24)	2.6(−24)	1.7(−23)	5.6(−25)	3.7(−24)	3.4(−25)	2.1(−25)	1.4(−24)	1.6(−25)	3.5(−27)	7.5(−23)
4s	1.4(−23)	2.3(−24)	8.8(−24)	5.4(−25)	2.1(−24)	2.1(−25)	2.1(−25)	8.7(−25)	1.0(−25)	2.4(−27)	7.4(−23)
4p	3.7(−23)	1.1(−23)	8.9(−23)	2.3(−24)	1.8(−23)	1.4(−24)	8.8(−25)	7.1(−24)	6.8(−25)	1.4(−26)	3.5(−22)
4d	4.3(−24)	1.3(−24)	7.8(−24)	2.8(−25)	1.7(−24)	1.6(−25)	1.0(−25)	6.7(−25)	7.7(−26)	1.6(−27)	3.6(−23)
4f	1.0(−25)	3.0(−26)	1.5(−25)	6.5(−27)	3.5(−26)	3.5(−27)	2.4(−27)	1.3(−26)	1.6(−27)	3.8(−29)	8.0(−25)
Σ	1.2(−20)	8.1(−22)	4.0(−21)	1.9(−22)	9.1(−22)	7.6(−23)	7.4(−23)	3.5(−22)	3.7(−23)	8.0(−25)	4.3(−20)

TABLE III. The cross sections (in cm^2) of $A_{e\pi}$.

	1s	2s	2p	3s	3p	3d	4s	4p	4d	4f	Σ
1s	4.4(−21)	1.3(−22)	3.3(−22)	3.4(−23)	8.9(−23)	8.4(−24)	1.4(−23)	3.6(−23)	4.3(−24)	1.0(−25)	1.2(−20)
2s	1.3(−22)	2.6(−23)	1.1(−22)	5.9(−24)	2.8(−23)	2.6(−24)	2.3(−24)	1.1(−23)	1.3(−24)	3.0(−26)	8.0(−22)
2p	3.4(−22)	1.1(−22)	1.1(−21)	2.4(−23)	2.4(−22)	1.7(−23)	8.9(−24)	8.9(−23)	7.9(−24)	1.5(−25)	4.0(−21)
3s	3.4(−23)	5.9(−24)	2.3(−23)	1.3(−24)	5.8(−24)	5.6(−25)	5.4(−25)	2.3(−24)	2.8(−25)	6.5(−27)	1.8(−22)
3p	9.1(−23)	2.8(−23)	2.4(−22)	5.9(−24)	5.0(−23)	3.8(−24)	2.2(−24)	1.8(−23)	1.7(−24)	3.6(−26)	9.1(−22)
3d	8.5(−24)	2.6(−24)	1.7(−23)	5.6(−25)	3.7(−24)	3.4(−25)	2.1(−25)	1.4(−24)	1.6(−25)	3.5(−27)	7.5(−23)
4s	1.4(−23)	2.3(−24)	8.8(−24)	5.4(−25)	2.1(−24)	2.1(−25)	2.1(−25)	8.7(−25)	1.0(−25)	2.4(−27)	7.4(−23)
4p	3.7(−23)	1.1(−23)	8.9(−23)	2.3(−24)	1.8(−23)	1.4(−24)	8.8(−25)	7.1(−24)	6.8(−25)	1.4(−26)	3.5(−22)
4d	4.3(−24)	1.3(−24)	7.8(−24)	2.8(−25)	1.7(−24)	1.6(−25)	1.0(−25)	6.7(−25)	7.7(−26)	1.6(−27)	3.6(−23)
4f	1.0(−25)	3.0(−26)	1.5(−25)	6.5(−27)	3.5(−26)	3.5(−27)	2.4(−27)	1.3(−26)	1.6(−27)	3.8(−29)	8.0(−25)
Σ	1.2(−20)	8.0(−22)	3.9(−21)	1.9(−22)	9.0(−22)	7.6(−23)	7.4(−23)	3.5(−22)	3.7(−23)	8.0(−25)	4.3(−20)

TABLE IV. The cross sections (in cm^2) of $A_{2\mu}$.

	1s	2s	2p	3s	3p	3d	4s	4p	4d	4f	Σ
1s	0	0	0	0	0	0	0	0	0	0	0
2s	0	0	0	0	0	0	0	0	0	0	0
2p	7.7(−24)	6.0(−26)	2.4(−25)	1.4(−26)	5.7(−26)	4.8(−27)	5.9(−27)	2.2(−26)	2.4(−27)	5.3(−29)	1.7(−23)
3s	0	0	0	0	0	0	0	0	0	0	0
3p	1.3(−24)	9.7(−27)	3.8(−26)	2.3(−27)	9.1(−27)	7.8(−28)	9.4(−28)	3.5(−27)	3.8(−28)	8.4(−30)	2.9(−24)
3d	0	0	0	0	0	0	0	0	0	0	0
4s	0	0	0	0	0	0	0	0	0	0	0
4p	4.6(−25)	3.3(−27)	1.3(−26)	8.2(−28)	3.1(−27)	2.7(−28)	3.2(−28)	1.2(−27)	1.3(−28)	2.9(−30)	1.0(−24)
4d	0	0	0	0	0	0	0	0	0	0	0
4f	2.1(−28)	1.0(−36)	7.2(−37)	3.1(−37)	2.3(−37)	1.6(−38)	1.3(−37)	1.0(−37)	9.1(−39)	1.8(−40)	4.2(−28)
Σ	1.5(−23)	1.0(−25)	4.3(−25)	2.6(−26)	1.0(−25)	8.7(−27)	1.0(−26)	4.0(−26)	4.3(−27)	9.5(−29)	3.4(−23)

TABLE V. The cross sections (in cm^2) of $A_{\mu\pi}$.

	1s	2s	2p	3s	3p	3d	4s	4p	4d	4f	Σ
1s	2.0(−26)	4.3(−32)	6.5(−32)	1.2(−32)	1.9(−32)	1.6(−33)	4.9(−33)	7.9(−33)	8.8(−34)	1.9(−35)	4.1(−26)
2s	4.8(−27)	4.3(−32)	6.4(−32)	1.1(−32)	1.8(−32)	1.6(−33)	4.9(−33)	7.8(−33)	8.7(−34)	1.9(−35)	9.7(−27)
2p	6.1(−24)	4.6(−26)	1.8(−25)	1.1(−26)	4.4(−26)	3.7(−27)	4.5(−27)	1.7(−26)	1.8(−27)	4.0(−29)	1.3(−23)
3s	1.0(−27)	5.8(−33)	8.7(−33)	1.6(−33)	2.5(−33)	2.2(−34)	6.6(−34)	1.0(−33)	1.1(−34)	2.6(−36)	2.0(−27)
3p	1.0(−24)	7.5(−27)	3.0(−26)	1.8(−27)	7.0(−27)	6.0(−28)	7.3(−28)	2.7(−27)	2.9(−28)	6.5(−30)	2.2(−24)
3d	7.6(−28)	1.1(−32)	1.7(−32)	3.2(−33)	5.0(−33)	4.4(−34)	1.3(−33)	2.1(−33)	2.3(−34)	5.3(−36)	1.5(−27)
4s	3.8(−28)	1.9(−33)	2.8(−33)	5.3(−34)	8.3(−34)	7.3(−35)	2.1(−34)	3.5(−34)	3.8(−35)	8.7(−37)	7.7(−28)
4p	3.7(−25)	2.6(−27)	1.0(−26)	6.4(−28)	2.4(−27)	2.0(−28)	2.5(−28)	9.6(−28)	1.0(−28)	2.2(−30)	8.1(−25)
4d	3.4(−28)	4.5(−33)	6.7(−33)	1.2(−33)	1.9(−33)	1.7(−34)	5.1(−34)	8.2(−34)	9.1(−35)	2.0(−36)	6.9(−28)
4f	1.6(−28)	5.5(−37)	3.7(−37)	1.6(−37)	1.2(−37)	8.4(−39)	6.9(−38)	5.3(−38)	4.7(−39)	9.4(−41)	3.2(−28)
Σ	1.2(−23)	8.4(−26)	3.3(−25)	2.0(−26)	7.9(−26)	6.7(−27)	8.2(−27)	3.1(−26)	3.3(−27)	7.3(−29)	2.7(−23)

TABLE VI. The cross sections (in cm^2) of $A_{2\pi}$.

	1s	2s	2p	3s	3p	3d	4s	4p	4d	4f	Σ
1s	0	0	0	0	0	0	0	0	0	0	0
2s	0	0	0	0	0	0	0	0	0	0	0
2p	4.7(−24)	3.4(−26)	1.3(−25)	8.5(−27)	3.2(−26)	2.7(−27)	3.3(−27)	1.2(−26)	1.3(−27)	3.0(−29)	1.0(−23)
3s	0	0	0	0	0	0	0	0	0	0	0
3p	8.0(−25)	5.5(−27)	2.2(−26)	1.3(−27)	5.2(−27)	4.4(−28)	5.4(−28)	2.0(−27)	2.2(−28)	4.8(−30)	1.7(−24)
3d	0	0	0	0	0	0	0	0	0	0	0
4s	0	0	0	0	0	0	0	0	0	0	0
4p	2.8(−25)	1.9(−27)	7.7(−27)	4.7(−28)	1.8(−27)	1.5(−28)	1.8(−28)	7.1(−28)	7.6(−29)	1.6(−30)	6.2(−25)
4d	0	0	0	0	0	0	0	0	0	0	0
4f	1.2(−28)	2.0(−37)	1.3(−37)	6.0(−38)	4.4(−38)	3.0(−39)	2.5(−38)	1.9(−38)	1.7(−39)	3.4(−41)	2.4(−28)
Σ	9.5(−24)	6.2(−26)	2.5(−25)	1.5(−26)	5.8(−26)	5.0(−27)	6.1(−27)	2.3(−26)	2.4(−27)	5.4(−29)	2.0(−23)

TABLE VII. The cross sections (in cm^2) of $A_{\pi K}$.

	1s	2s	2p	3s	3p	3d	4s	4p	4d	4f	Σ
1s	9.5(−26)	6.6(−32)	9.9(−32)	1.8(−32)	2.9(−32)	2.5(−33)	7.5(−33)	1.2(−32)	1.3(−33)	3.0(−35)	1.9(−25)
2s	2.0(−26)	6.5(−32)	9.8(−32)	1.8(−32)	2.8(−32)	2.5(−33)	7.4(−33)	1.1(−32)	1.3(−33)	2.9(−35)	4.1(−26)
2p	2.0(−24)	1.4(−26)	5.7(−26)	3.5(−27)	1.3(−26)	1.1(−27)	1.3(−27)	5.2(−27)	5.6(−28)	1.2(−29)	4.4(−24)
3s	4.1(−27)	8.8(−33)	1.3(−32)	2.4(−33)	3.8(−33)	3.3(−34)	1.0(−33)	1.6(−33)	1.7(−34)	4.0(−36)	8.3(−27)
3p	3.4(−25)	2.2(−27)	9.1(−27)	5.6(−28)	2.1(−27)	1.8(−28)	2.2(−28)	8.4(−28)	9.0(−29)	2.0(−30)	7.5(−25)
3d	3.2(−27)	1.7(−32)	2.6(−32)	4.9(−33)	7.7(−33)	6.7(−34)	2.0(−33)	3.2(−33)	3.5(−34)	8.0(−36)	6.4(−27)
4s	1.5(−27)	2.9(−33)	4.3(−33)	8.0(−34)	1.2(−33)	1.1(−34)	3.3(−34)	5.3(−34)	5.9(−35)	1.3(−36)	3.1(−27)
4p	1.2(−25)	7.9(−28)	3.1(−27)	1.9(−28)	7.5(−28)	6.4(−29)	7.7(−29)	2.9(−28)	3.1(−29)	6.9(−31)	2.6(−25)
4d	1.4(−27)	6.9(−33)	1.0(−32)	1.9(−33)	3.0(−33)	2.6(−34)	7.8(−34)	1.2(−33)	1.3(−34)	3.1(−36)	2.8(−27)
4f	3.9(−29)	5.3(−38)	3.5(−38)	1.5(−38)	1.1(−38)	8.0(−40)	6.6(−39)	5.0(−39)	4.5(−40)	9.0(−42)	7.8(−29)
Σ	4.2(−24)	2.5(−26)	1.0(−25)	6.3(−27)	2.4(−26)	2.0(−27)	2.5(−27)	9.5(−27)	1.0(−27)	2.2(−29)	9.2(−24)

drogen) atom. When $a = a_H$, the total cross section reads

$$\tilde{\sigma}^{\text{tot}} = \frac{28\pi}{3} e^4 a^2 .$$

III. NUMERICAL RESULTS

In our further discussion we assume that the initial states of the elementary and hydrogen atoms coincide with the ground states. Substituting in Eq. (3) the form factors from Ref. 1 and performing the numerical integration, one gets the cross sections related to the excitations of the elementary and/or hydrogen atom. In the tables we give the numerical values of the initial-energy-independent cross sections from Eq. (4) expressed in cm^2. Tables I–VII relate to the elementary atoms A_{2e}, $A_{\mu e}$, $A_{\pi e}$, $A_{2\mu}$, $A_{\mu\pi}$, $A_{2\pi}$, and $A_{\pi K}$, respectively. (A_{ab} denotes the atom composed of particle a and of particle b, while the atom made of particle a and antiparticle $\tilde{a}$ is labeled by A_{2a}). The final state of the hydrogen atom is denoted in the first line by means of the standard spectroscopical symbols. The elementary atom final state is given in the first column. The sign $\sum$ labels the cross sections summed over the final states of the elementary or the hydrogen atom. The cross section from the lower right-hand corner is, of course, the total one. The zeros in the tables mean that the respective cross sections are zero in Born approximation. The numbers from the tables should be understood as in the example $1.3(-21) = 1.3 \times 10^{-21}\ \text{cm}^2$.

IV. DISCUSSION

One sees from the Tables that the sum of cross sections related to the excitations of the lowest energy levels does not give the cross section summed over the final states by means of the sum rule (5). It is not surprising since the summation in Eq. (5), and consequently the total cross section, includes continuum excited states. The calculation of ionization cross section, however, is faced with difficulties. One has to use the exact Coulomb wave function of continuous energy (Sommerfeld function). If the plane wave is used, the integration over plane-wave momentum is equivalent to the summation from Eq. (5), and one gets the total cross section instead of the ionization one.

Tables II and III are approximately symmetric with respect to the diagonal. It is not surprising since the Bohr radii of $A_{\pi e}$ and $A_{\mu e}$ are nearly equal to the radius of the hydrogen atom. On the other hand, Tables VI and VII are essentially asymmetric, which reflects the difference of the radii of the respective atoms. It is also seen that the cross sections of hydrogen atom excitations in collisions with $A_{2\pi}$ or $A_{\pi K}$ are much smaller than the cross sections of processes where the hydrogen atom remains in the ground state.

The cross sections calculated in this paper have been found in the nonrelativistic approximation. However, as discussed in detail in Refs. 4 and 5, the results are approximately valid for collisions of relativistic elementary atoms.

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