Unstable deltas in nuclear matter ☆

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Nuclear matter with delta isobars is considered in the framework of the mean-field Walecka model and a phenomenological extension of it. Particular attention is paid to the role of the finite delta decay width. It is found that even at normal nuclear density up to 0.05 of the baryon charge may be carried by deltas.

Delta isobars play an important role in nuclear matter, in particular, at densities higher than the normal one. The problem has been studied in numerous papers [1–3], for a review see refs. [4–6]. Since the delta is a meson–nucleon resonance, it usually appears in calculations as an intermediate state of nucleon–nucleon interaction driven by a meson exchange [1,2,4–6]. The delta decay width is not much smaller than the mass difference between deltas and nucleons, and consequently one should take into account an imaginary part of the delta mass, or ore precisely, of the delta self-energy. This energy can be self-consistently calculated within the boson-exchange model, as has been done in ref. [4]; however, the whole approach becomes very complex.

There are models of nuclear matter [2], where delta isobars are treated in the same way as nucleons, i.e., the deltas are real particles and not only intermediate states of nucleon interactions. In ref. [2] deltas have been assumed to be stable particles, but as mentioned above, this is not quite justified. Deltas with finite decay width have been considered to some extent in ref. [3] in such a model.

The aim of this paper is to propose a simple Walecka-type model [7] of nuclear matter, where particular attention is paid to the finite delta decay width. We use an approach initially discussed in the context of classical (nonquantum) transport theory [8], where resonances are treated as stable particles with a mass distribution described by a profile function. This function is uniquely expressed through the resonance formation cross section and the resonance width due to an unitarity condition. It has been later proved [9] that in the case of equilibrium systems, the approach [8] provides results equivalent to those obtained in the Dashen–Ma–Berstein formulation of statistical mechanics, where a partition function is expressed through S-matrix elements. In fact, this equivalence holds for classical systems. Since nuclear matter at low temperature is essentially a quantum system, we modify here the approach of ref. [8] tak-

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ing into account the resonance mass and width dependence on nuclear matter density and temperature.

The model discussed in this paper can be very useful for the description of heavy-ion collisions, where one deals with nuclear matter in a wide temperature range, and an application of the sophisticated models mentioned above [4,5] is cumbersome. However, a reasonable treatment of deltas is very important here because a big fraction of experimentally observed pions from these collisions come from delta decays.

Our model is formulated in two steps. At first we treat the deltas as stable particles, which we incorporate in the Walecka [7] model following ref. [3]. One should remember that the deltas are spin 3/2 particles and consequently are described by Rarita–Schwinger spinors. As the analysis performed in ref. [3] shows, the delta dispersion relation in nuclear matter is analogous to that of nucleons as long as we work in the mean-field approximation. Then, one also observes that the deltas contribute in the same way as nucleons to the equations describing the generation of vector and scalar fields.

In the second step, we take into account the finite decay width of the deltas. We average thermodynamical characteristics found in the mean-field approximation over the delta mass spectrum as has been proposed in refs. [8,9]. In order not to limit our discussion to a specific model, we follow ref. [10] and construct a class of thermodynamically consistent models which is a phenomenological extension of the mean-field Walecka model and contains it as a limiting case.

The basic formulae of the model are written for any temperature. The thermodynamic consistency is also discussed for this general case. The numerical calculations, however, have been performed only for zero-temperature symmetric nuclear matter. We intend to present predictions of our model for the best known case.

The nuclear matter energy density and pressure are expressed as

$$\epsilon(\mu, T) = \gamma_N \int \frac{d^3k}{(2\pi)^3} (\boldsymbol{k}^2 + M_N^{*2})^{1/2} [f_N(k) + f_{\bar N}(k)] + \gamma_\Delta \int \frac{d^3k\, dM_\Delta^*}{(2\pi)^3} \mathscr{P}(M_\Delta^*) (\boldsymbol{k}^2 + M_\Delta^{*2})^{1/2} \times [f_\Delta(k) + f_{\bar\Delta}(k)] + \tfrac{1}{2} C_s^2 \rho_s^2 + \int_0^{\rho_B} d\rho_{B'}\, U(\rho_{B'}) , \quad (1)$$

$$p(\mu, T) = -T\gamma_N \int \frac{d^3k}{(2\pi)^3} \{\ln[1 - f_N(k)] + \ln[1 - f_{\bar N}(k)]\} - T\gamma_\Delta \int \frac{d^3k\, dM_\Delta^*}{(2\pi)^3} \mathscr{P}(M_\Delta^*) \times \{\ln[1 - f_\Delta(k)] + \ln[1 - f_{\bar\Delta}(k)]\} - \tfrac{1}{2} C_s^2 \rho_s^2 + \rho_B U(\rho_B) - \int_0^{\rho_B} d\rho_{B'}\, U(\rho_{B'}) , \quad (2)$$

where μ is the chemical potential associated with the baryon density ρ_B; $T = \beta^{-1}$ is the temperature; γ_N and γ_Δ are the numbers of nucleon and delta internal degrees of freedom; $f_{N(\bar N)}(k)$ and $f_{\Delta(\bar\Delta)}(k)$ are the distribution functions of (anti-) nucleons and of (anti-)nucleons and of (anti-)deltas which are

$$f_{N(\bar N)}(k) = \frac{1}{\exp\{\beta[(\boldsymbol{k}^2 + M_N^{*2})^{1/2} \pm U(\rho_B) \mp \mu]\} + 1} ,$$

$$f_{\Delta(\bar\Delta)}(k) = \frac{1}{\exp\{\beta[(\boldsymbol{k}^2 + M_\Delta^{*2})^{1/2} \pm U(\rho_B) \mp \mu]\} + 1} ;$$

$\mathscr{P}(M_\Delta^*)$ is the delta isobar profile function defined as

$$\mathscr{P}(M_\Delta^*) = \xi \frac{\Gamma^*}{(M_\Delta^* - \bar M_\Delta^*)^2 + \Gamma^{*2}/4} \theta(M_\Delta^* - M_N^*) , \quad (3)$$

where Γ^* and $\bar M_\Delta^*$ are the delta width and average mass in nuclear matter; the parameter ξ is determined by the normalization condition

$$\int_0^\infty dM_\Delta^*\, \mathscr{P}(M_\Delta^*) = 1 ;$$

the effective nucleon mass M_N^* and the average effective delta mass $\bar M_\Delta^*$ are

$$M_{\rm N}^* = M_{\rm N} - C_{\rm s}^2 \rho_{\rm s}\,, \quad \bar{M}_\Delta^* = \bar{M}_\Delta - C_{\rm s}^2 \rho_{\rm s}\,,$$

where $M_{\rm N}$ and $\bar{M}_\Delta$ are the respective vacuum values; $\rho_{\rm s}$ is expressed as

$$\rho_{\rm s} = \gamma_{\rm N} \int \frac{{\rm d}^3 k}{(2\pi)^3} \frac{M_{\rm N}^*}{(k^2 + M_{\rm N}^{*2})^{1/2}} [f_{\rm N}(k) + f_{\bar{\rm N}}(k)]$$
$$+ \gamma_\Delta \int \frac{{\rm d}^3 k\, {\rm d}M_\Delta^*}{(2\pi)^3} \mathscr{P}(M_\Delta^*) \frac{M_\Delta^*}{(k^2 + M_\Delta^{*2})^{1/2}}$$
$$\times [f_\Delta(k) + f_{\bar{\Delta}}(k)]\,; \tag{4}$$

and the baryon density $\rho_{\rm B}$ equals

$$\rho_{\rm B} = \gamma_{\rm N} \int \frac{{\rm d}^3 k}{(2\pi)^3} [f_{\rm N}(k) - f_{\bar{\rm N}}(k)]$$
$$+ \gamma_\Delta \int \frac{{\rm d}^3 k\, {\rm d}M_\Delta^*}{(2\pi)^3} \mathscr{P}(M_\Delta^*) [f_\Delta(k) - f_{\bar{\Delta}}(k)]\,; \tag{5}$$

for the Walecka model $U(\rho_{\rm B}) = C_{\rm v}^2 \rho_{\rm B}$, and $C_{\rm s}$, $C_{\rm v}$ are parameters related to the coupling constant of the baryon fields with the scalar and vector fields, respectively. In our considerations deltas and nucleons are coupled to the meson fields with the same constants ($C_{\rm s}^{\rm N} = C_{\rm s}^\Delta$ and $C_{\rm v}^{\rm N} = C_{\rm v}^\Delta$).

Let us briefly discuss the form of the profile function given by eq. (3). We have used the standard Breit–Wigner parametrization, which is cut off for masses smaller than the effective nucleon mass. In the case of a delta decaying in the vacuum, it seems reasonable to cut the mass spectrum at a mass equal the sum of the nucleon and the pion mass. When a delta is produced in nuclear matter, then it can disappear due to the process $\Delta{\rm N} \to {\rm NN}$. The existence of this process suggests a cut-off mass equal to $M_{\rm N}^*$ [#1].

The delta decay width in nuclei differs from the value observed in vacuum. In fact, there are several competing mechanisms responsible for this difference [6]. However, the process $\Delta{\rm N} \to {\rm NN}$ seems to dominate [6]. Therefore, we parametrize the effective decay width as function of the baryon density in the following way:

[#1] This value is obviously incorrect in the limit of zero baryon density. However, our results are sensitive to the value of the cut-off mass only when the delta decay width substantially exceeds the vacuum value. This only happens at high densities.

$$\Gamma^*(\rho_{\rm B}) = \Gamma + \delta\Gamma \frac{\rho_{\rm B}}{\rho_0}\,, \tag{6}$$

where $\Gamma = 115$ MeV is the delta decay width in vacuum and ρ_0 is the normal nuclear density. The value of Γ^* at ρ_0 we identify with that one measured in pion–nucleus scattering [11]. The largest value of about 320 MeV has been observed in the interaction with an iron nucleus [11]. For heavier nuclei the decay width seems to saturate, or even to decease. We take $\Gamma^*(\rho_0) = 320$ MeV, which gives $\delta\Gamma = 205$ MeV.

The delta width observed in pion–nucleus interactions should be treated rather as an upper limit (leading to maximal effects discussed below) of the intrinsic decay width of a delta in nuclear matter. The point is that elastic pion–nucleon scatterings provide a significant contribution to the delta width broadening in pion–nucleus collisions [6]. The delta width found in photon–nucleus interactions, where these elastic scatterings are absent, is smaller than that in pion–nucleus collisions [3,6].

As long as we deal with a nuclear-matter model directly derived from a field-theory lagrangian, the model is thermodynamically self-consistent, i.e., thermodynamical identities are satisfied. The phenomenological potential $U(\rho_{\rm B})$ is also introduced in eqs. (1), (2) in a self-consistent way, see ref. [10]. However, if one introduces other modifications, in particular, if one takes into account a finite delta decay width, the consistency of the model might be broken. Let us briefly discuss this problem.

The thermodynamic quantities given by eqs. (1), (2) and (5) must be related to each other by the equations, see e.g. ref. [12],

$$\epsilon = T\left(\frac{\partial p}{\partial T}\right)_\mu + \mu\rho_{\rm B} - p\,, \tag{7}$$

$$\rho_{\rm B} = \left(\frac{\partial p}{\partial \mu}\right)_T. \tag{8}$$

Because the effective nucleon and delta masses are density and temperature dependent dynamical quantities, they must maximise the pressure, i.e.,

$$\left(\frac{{\rm d}p}{{\rm d}M_{\rm N}^*}\right)_{T,\mu} = 0\,, \quad \left(\frac{{\rm d}p}{{\rm d}\bar{M}_\Delta^*}\right)_{T,\mu} = 0\,. \tag{9}$$

If the delta decay width is also a dynamical quantity then

$$\left(\frac{dp}{d\Gamma^*}\right)_{T,\mu}=0 .$$

A direct calculation shows that the quantities given by eqs. (1), (2), (4) and (5) indeed satisfy eqs. (7)-(9), if Γ^* is a fixed parameter. When one uses the parametrization (6) the consistency requirements are violated, and to restore them one needs to modify the energy density and pressure. Such modifications are not considered in this paper. It is interesting to observe that the thermodynaical consistency of eqs. (1)-(5) requires the equality of the constants C_s^N and C_s^Δ. In this case the difference between the effective masses of deltas and nucleons is density and temperature independent, and it equals $\bar{M}_\Delta - M_N$. If this difference is not a constant, eqs. (1)-(5) are not thermodynamically consistent because, formally speaking, the normalization coefficient ξ depends on T and μ.

We have performed numerical calculations for zero temperature symmetric ($\gamma_N=4$, $\gamma_\Delta=16$) nuclear matter with the potential $U(\rho_B)$ in the form

$$U(\rho_B)=C_v^2\rho_B - C_d^2\rho_B^{1/3} ,$$

which is discussed in detail in ref. [13], see also ref. [14]. When $C_d=0$ one deals with the Walecka model. The parameters C_s, C_v and C_d have been chosen to reproduce the binding energy $W\equiv\epsilon/\rho_B - M_N = -16$ MeV at the saturation density $\rho_B=\rho_0=0.16$ fm^{-3}. However, this condition does not fix the parameters completely and we have considered several sets of them, presented in table 1 together with the corresponding values of the incompressibility

Table 1
The effective nucleon mass, the incompressibility and the fraction of the baryon charge carried by deltas at normal nuclear density for several sets of the model parameters C_v^2, C_s^2, C_d^2 and four values of the effective delta decay width Γ^*.

C_v^2 (GeV^{-2})	C_s^2 (GeV^{-2})	C_d^2	Γ^* (MeV)	M_N^*/M_N	K_0 (MeV)	ρ_Δ/ρ_B
286.0	377.6	0	0	0.543	553	0
286.0	377.6	0	1	0.543	553	2.2×10^{-4}
290.9	382.5	0	115	0.537	552	2.5×10^{-2}
297.3	388.8	0	320	0.529	552	5.8×10^{-2}
235.2	291.8	0.191	0	0.640	291	0
235.2	291.8	0.191	1	0.640	291	1.9×10^{-4}
236.5	291.5	0.199	115	0.640	280	2.1×10^{-2}
238.2	291.1	0.210	320	0.640	267	4.8×10^{-2}
227.1	279.8	0.211	0	0.654	265	0
227.1	279.8	0.211	1	0.654	265	1.8×10^{-4}
228.3	279.5	0.218	115	0.654	255	2.0×10^{-2}
230.0	279.2	0.228	320	0.654	243	4.7×10^{-2}
220.0	269.6	0.227	0	0.666	244	0
220.0	269.6	0.227	1	0.666	244	1.8×10^{-4}
221.2	269.4	0.234	115	0.666	235	2.0×10^{-2}
222.7	269.1	0.242	320	0.666	224	4.7×10^{-2}
203.7	247.0	0.260	0	0.693	203	0
203.7	247.0	0.260	1	0.693	203	1.7×10^{-4}
204.8	246.8	0.266	115	0.693	196	1.9×10^{-2}
206.1	246.5	0.273	320	0.693	187	4.4×10^{-2}
170.4	203.1	0.312	0	0.746	141	0
170.4	203.1	0.312	1	0.746	141	1.6×10^{-4}
171.2	203.0	0.317	115	0.746	137	1.8×10^{-2}
172.3	202.8	0.322	320	0.746	131	4.2×10^{-2}

$$K_0 \equiv 9 \left(\frac{\partial p}{\partial \rho_B}\right)_{T=0, \rho_B=\rho_0},$$

the effective nucleon mass, and the fraction of baryon charge carried by deltas in the ground state ($T=0$, $\rho_B=\rho_0$). We have chosen four values of the delta decay width: $\Gamma^*=0$, $\Gamma^*=1$ MeV, $\Gamma^*=115$ MeV corresponding to decays in vacuum and $\Gamma^*=320$ MeV. The analysis of the experimental data suggests a value of the incompressibility coefficient of about 300 MeV [15] and of the effective mass of about $0.7M_N$ [16], both with considerable errors.

In fig. 1 we present the energy per baryon $W(\rho_B)$ as a function of the baryon density for several sets of the parameters from table 1. In fig. 1a the Walecka model ($C_d=0$) predictions are shown, while in fig. 1b the model with $C_d \neq 0$ is considered. In both cases we present the models with no deltas and the predictions corresponding to deltas with $\Gamma^*=320$ MeV. One observes that the equation of state is rather insensitive to the presence of deltas. In fact, this is no longer true if the coupling constants of nucleons and deltas are not equal to each other [2]. However, the case of equal delta and nucleon coupling constants is privileged in our approach, since it automatically leads to a thermodynamically self-consistent model.

In fig. 2 we present the ratio of delta density [the second term in eq. (5)] to baryon density as a function of baryon density for several values of the delta

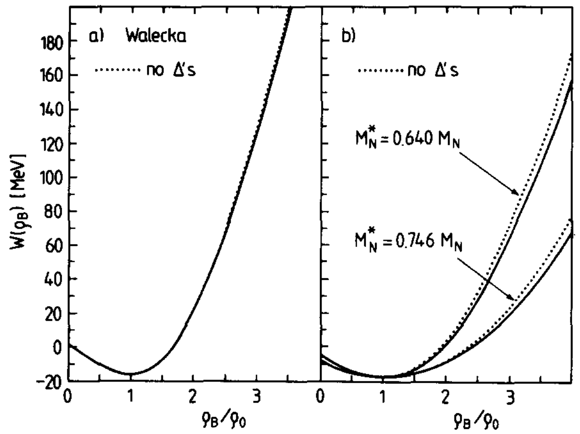

Fig. 1. The binding energy per baryon as a function of baryon density in the Walecka model (a) and in its phenomenological extension (b) with no deltas, and with deltas of $\Gamma^*=320$ MeV.

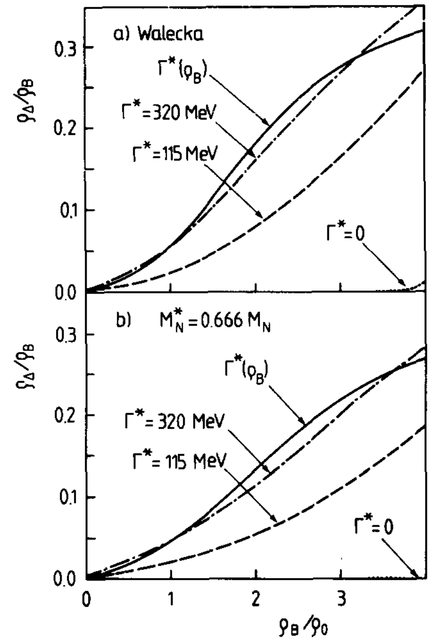

Fig. 2. The fraction of baryon charge carried by deltas as a function of baryon density in the Walecka model (a) and in its phenomenological extension (b) for several values of the delta decay width and for the decay-width parametrization (6).

decay width. The calculations with the parametrization (6) are also presented. The values of the coupling constants are then taken as for the $\Gamma^*=320$ MeV case. Since we do not consider any other thermodynamical functions, we can forget for a moment about the thermodynamical consistency. Fig. 2a corresponds to the Walecka model ($C_d=0$), while fig. 2b to the model with $C_d \neq 0$ and $M_N^*=0.666M_N$ at normal nuclear density. one sees that, in contrast to the equation of state, the fraction of baryon charge carried by deltas strongly depends on Γ^*. Let us observe that the delta fractions calculated with the density dependent width (6) and with a fixed $\Gamma^*=320$ MeV, which corresponds to the delta width observed in pion interactions with heavy nuclei [11], are very similar in the baryon density interval from 0 to $3\rho_0$. At normal nuclear density this fraction equals about 0.05. When $\Gamma^*=115$ MeV, which corresponds to the vacuum value, this admixture is reduced to about 0.02.

A comment is in order. One may expect that the delta admixture depends on the effective mass of the delta, and consequently a large delta admixture found in our model is the result of an underestimation of this mass. This is, however, not true. In fact, the delta admixture depends crucially on the difference between delta and nucleon mass, and not on the delta mass by itself. As mentioned above, this difference is a constant in our calculations because we use equal nucleon and delta coupling constants. Therefore, as the results presented in table 1 show, the delta admixture depends very weakly on the effective mass.

Within the approach discussed here we find a strong increase of the number of deltas due to the finite decay width also at nonzero temperature. This point is important for the hydrodynamic or thermodynamic description of nuclear collisions, where the number of secondary pions equals the number of pions plus the number of deltas present in the system at the freeze-out density, when the system decouples into noninteracting particles. The respective calculations will be presented elsewhere.

Let us summarize our considerations. A model to study unstable deltas in nuclear matter has been proposed. After discussion of its thermodynamical consistency the model has been applied to zero temperature symmetric nuclear matter. The fraction of baryon charge carried by deltas strongly depends on their decay width while the equation of state appears rather insensitive to the presence of deltas in the system. We have shown that there are situations, where the zero-width approximation for the delta resonances seems completely inadequate.

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