On the Phase Space of Tachyons.

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The aim of this paper is to present the phase-space properties of the systems that contain bradyons, luxons and tachyons (1). It is shown that particulary at low energy, these properties are quite different from the well-known properties of bradyons.

Although the progress in understanding tachyon-tachyon and tachyon-bradyon interactions has been obtained (2) the suitable dynamics still needs to be formulated, so the knowledge of the phase-space could be valuable, since it is expected that in the threshold region (low energy) phase-space effects are dominant.

The volume of the N-particle phase-space in the centre-of-mass system is defined by

$$L^N(S, m_1, \ldots, m_N) = \int \prod_{i=1}^{N} \frac{\mathrm{d}^3 p_i}{2E_i}\, \delta\left(\sqrt{S} - \sum_{i=1}^{N} E_i\right) \delta^{(3)}\left(\sum_{i=1}^{N} \overline{p}_i\right), \tag{1}$$

where $\sqrt{S}$ is the energy of the N-particle system in the centre-of-mass frame ($S \geqslant 0$), $E_i = \sqrt{p_i^2 \pm m_i^2}$ is the energy of the i-th particle, the upper sign is for the bradyon and the lower one for the tachyon.

The integration is taken over all values of the momentum when the particle is the bradyon, but for tachyon we have the condition $|\overline{p}_i| \geqslant m_i$.

In the case of two particles, L^2 can be found directly from formula (1) and the result is as follows:

$$L^2(S, m_1, m_2) = \begin{cases} 0\,, & \text{for } \sqrt{S} < E_0\,, \\ \dfrac{\pi}{2}\sqrt{\dfrac{(S - \varepsilon_1 m_1^2 - \varepsilon_2 m_2^2)^2 - 4\varepsilon_1\varepsilon_2 m_1^2 m_2^2}{S^2}}\,, & \text{for } \sqrt{S} \geqslant E_0\,, \end{cases}$$

(1) It is assumed that the reader is familiar with the terminology and main features of Special Theory of Relativity extended to faster-than-light objects. If not, see E. RECAMI and R. MIGNANI: *Riv. Nuovo Cimento*, **4**, 209, 398 (1974).

(2) See *e.g. Proceedings of Session « Tachyons, Monopoles and Related Topics » Erice 1976*, edited by E. RECAMI (Amsterdam, 1978); G. D. MACCARRONE and E. RECAMI: *Nuovo Cimento A*, **57**, 85 (1980).

ε_i is the two-value function: $\varepsilon_i = +1$, if the i-th particle is a bradyon and $\varepsilon_i = -1$, if the i-th particle is a tachyon. E_0 is the threshold energy and

$$E_0 = \begin{cases} m_1 + m_2\,, & \text{for } \varepsilon_1 = \varepsilon_2 = +1\,, \\ \sqrt{m_1^2 + m_2^2}\,, & \text{for } \varepsilon_1 = -\varepsilon_2\,, \\ \sqrt{|m_1^2 - m_2^2|}\,, & \text{for } \varepsilon_1 = \varepsilon_2 = -1\,. \end{cases}$$

The existence of thresholds in the systems containing tachyons is connected with the fact that the momentum of tachyons cannot be smaller than its mass; this corresponds to the property of bradyon that its energy is always bigger than its mass. The volume of the phase-space of bradyons is continuous as a function of energy, but when the system contains at least one tachyon, this function « jumps » at threshold from zero to a finite value.

In fig. 1 it is shown the volume of the two-particle phase-space when the masses of particles are the same and equal to m. Let us notice that in this case there is no threshold in the two tachyon systems and L^2 goes to infinity when $\sqrt{S}$ goes to zero. Such a property can lead to the peculiar instability of the vacuum, since the vacuum could decay into real (not virtual) tachyons, let us say tachyon-antitachyon pairs. These tachyons carry nearly zero energy, but momenta higher than their masses. The possibility and some consequences of such a vacuum instability was discussed previously ([3]), but this phase-space aspect of the quoted phenomenon have not been examined.

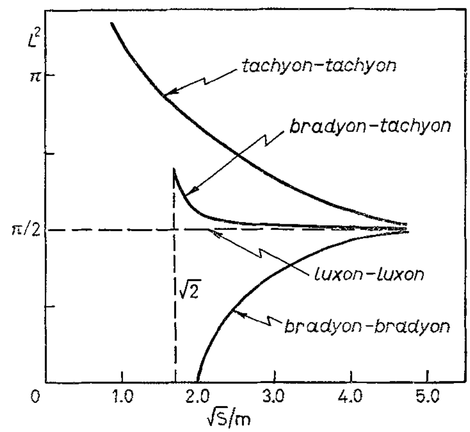

Fig. 1.

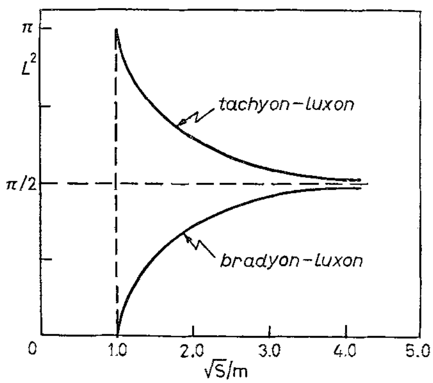

Fig. 2.

In fig. 2 the volume of the phase space of bradyon-luxon and tachyon-luxon systems is presented. When we consider the reaction

$$B \to T + L\,,$$

where B is the bradyon, T the tachyon, L the luxon, we find that the most favourable

([3]) R. MIGNANI and E. RECAMI: *Phys. Lett. B*, **65**, 148 (1976) and reference therein.

configuration from the point of view of kinematics arises when the rest masses of B and T are the same and the velocity of the tachyon is infinite. If the « tachyon electron » e_t is an analog of « bradyon electron » e_b, *i.e.* ordinary electron exists, the above observation means that the following transition or even oscillation could occur:

$$e_b \leftrightarrow e_t + \gamma .$$

However, this possibility (noticed by LEMKE [4]) seems to be less probable than the previous one which concerns the vacuum instability, since in this case the volume of the phase space is finite.

Summarizing the kinematic properties of the two-particle system, let us notice that when at least one particle is a tachyon L^2 decreases with energy.

The much more complicated problem of a phase-space with more than two particles will be discussed in the future. Here we only mention that this property of decreasing L^2 with energy in the case of tachyons does not hold for systems containing more than two particles, since at high energy the L^N of massive particles (bradyons as well as tachyons) goes to the volume of the phase-space of massless particles (luxons) [*] but

$$L^N(S, m_1 = 0, \dots, m_N = 0) = \left(\frac{\pi}{2}\right)^{N-1} \frac{S^{N-2}}{(N-2)!\,(N-1)!}$$

and increases with energy.

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(4) H. LEMKE: *Phys. Rev. D*, **22**, 1342 (1980).

(*) The L^N for luxons we found in the following way: the recurrence formula for $(N+1)$-particle phase-space was introduced which in the case of luxons takes the form

$$(2)\qquad L^{N+1}(S, m_1 = 0, \dots, m_N = 0, m_{N+1} = 0) = 2\pi \int_0^{\sqrt{S}/2} \mathrm{d}E\, E L^N(S - 2\sqrt{S}E, m_1 = 0, \dots, m_N = 0).$$

Then it was inductively proved that

$$(3)\qquad L^N = \alpha_N S^{N-2},$$

where α_N is constant.

α_N was determined from the ratio α_{N+1}/α_N that can be found by putting (3) into (2).