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Towards Relativistic Transport Theory of Nuclear Matter

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Starting with the lagrangian density of the Walecka model of nuclear matter, a set of coupled relativistic transport equations for nucleons, antinucleons, scalar and vector mesons is derived in a systematic way by means of the contour Green function technique. The mean field and the collision terms in the equations are discussed in detail. In both cases the spin degrees of freedom are fully taken into account.

1. INTRODUCTION

The success of kinetic models in describing heavy-ion collisions at intermediate energies, see, e.g., the review articles [1], has stimulated systematic studies of the transport equations which form the theoretical basis of these models. The main goal of such studies is to derive transport equations starting from a microscopic model of hadron–hadron interactions. One obvious candidate for such a model is the one proposed by Walecka [2, 3], where nucleon fields interact with scalar and vector meson fields. Since the model is formulated in terms of quantum field theory (QFT), it is fully relativistic and quantum mechanical. However, there are also important disadvantages of this model: It is in practice impossible to go beyond the mean-field approximation because large values of the coupling constants render a perturbative expansion meaningless [4]. While the vector field corresponds to the omega meson, there is no stable or quasistable particle among known mesons corresponding to the scalar field. Finally, it does not explicitly describe the pion degrees of freedom and their coupling to the Δ-resonance (although this can be remedied by adding appropriate terms to the Lagrangian). In spite of these drawbacks, the model is widely discussed in the literature, and it is very instructive, in our opinion, to start the study of hadron–matter transport equations with the Walecka model. The description of nucleons by Dirac spinor fields coupled to

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mesonic fields in this model introduces many of the difficulties encountered with the transport equations for gauge theories (see [5–7] for QED and [8] for QCD), without, however, the essential complications due to gauge invariance in the latter case. In this sense the hadronic transport equations to be discussed in this paper can be also used as a formal testing ground for the more ambitious program of deriving a transport theory of quarks and gluons.

The problem of deriving relativistic nuclear transport equations has been addressed in several papers, see [9–19]. While the papers [9–13] dealt only with the equations in the mean-field (Vlasov) limit, the collision terms of the equations have been considered in [14–19]. However, none of the existing studies contains a fully systematic derivation. In Section 13 we critically discuss the papers [9–19] and compare them with the more rigorous analysis presented here.

Our method is based on the Green function technique [20] and extends previous work [21], where it was used to derive transport equations of self-interacting scalar fields. As will be shown below, the appearance of spin degrees of freedom seriously complicates the whole approach. The form of the transport equations is found directly from the Dyson–Schwinger equations on the basis of very general arguments, but in order to obtain explicit expressions for the self-energies which enter the equations we have to refer to a perturbative expansion. Therefore, we formally treat the Walecka model as *perturbative*; i.e., smallness of the coupling constants is assumed. While collision terms of the Boltzmann–Nordheim–Uehling–Uhlenbeck type have repeatedly been written for this model (see, e.g., [1, 11, 12, 14, 15, 17] and references therein), our analysis shows that it will not be easy to justify this form outside the framework of low-order perturbation theory.

In addition to nucleons, we also derive transport equations for the scalar and vector mesons, treating them as *real* and not only as *effective* particles. We believe that our considerations can be generalized to more realistic models of hadronic matter.

The concrete formulation of the problem discussed in this paper is given at the end of Section 3 after the introduction the formal objects of our considerations, in particular the contour Green functions which play a central role in the method. In Section 2 we briefly present the Walecka model and collect some useful formulae. The exact equations of motion for the Green functions, the Dyson–Schwinger equations, are considered in Section 4. In Section 5 we introduce the essential approximation on the way to bringing them into the form of transport equations, namely that of quasihomogeneity of the system. Section 6 is devoted to a study of the limit of noninteracting fields which will form the basis of the following perturbative expansion in powers of the coupling constant. In Sections 7 and 8 we discuss two alternative approximations (the so-called pairing approximation and the lowest order of the perturbative expansion) which give rise to explicit expressions for the self-energies in the mean-field limit. The perturbative expansion is carried in Section 9 to the next order to derive the lowest order contribution to the collision terms. In Section 10 we extract from the Green functions the distribution functions of (anti-)nucleons, scalar and vector mesons, and in Section 11 the final set of transport equations satisfied by these distribution functions is written. The paper is rounded off with a general discussion of the whole derivation procedure and a critical assessment of the existing literature in Sections 12 and 13, respectively. Several specific questions are considered in the appendices.

Throughout the article we use natural units where $\hbar = c = 1$. The signature of the metric tensor is $(+, -, -, -)$. As far as possible we keep the convention of Bjorken and Drell [22].

2. Preliminaries

Let us begin with writing down the lagrangian density of nucleon (ψ), neutral scalar (ϕ), and vector (V^μ) mesonic fields as proposed by Walecka [2]:

$$\mathscr{L} = \frac{i}{2}\bar{\psi}\,\overleftrightarrow{\partial}^\mu \gamma_\mu \psi - M\bar{\psi}\psi + \frac{1}{2}(\partial^\mu\phi\,\partial_\mu\phi - m_s^2\phi^2) - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \frac{1}{2}m_v^2 V_\mu V^\mu + g_s\bar{\psi}\psi\phi - g_v\bar{\psi}\gamma^\mu\psi V_\mu, \tag{2.1}$$

where

$$F^{\mu\nu} \equiv \partial^\mu V^\nu - \partial^\nu V^\mu,$$

M, m_s, and m_v are the masses of the nucleons, scalar and vector mesons, respectively; g_s and g_v are the respective coupling constants.

The Lagrangian (2.1) leads to the field equations,

$$[i\partial_\mu\gamma^\mu - M]\psi = -g_s\psi\phi + g_v\gamma^\mu\psi V_\mu, \tag{2.2a}$$

$$[\partial^2 + m_v^2]\, V^\nu = g_v\bar{\psi}\gamma^\nu\psi, \tag{2.2b}$$

$$[\partial^2 + m_s^2]\,\phi = g_s\bar{\psi}\psi, \tag{2.2c}$$

where $\partial^2 \equiv \partial^\mu\,\partial_\mu$. In addition to Eqs. (2.2), the lagrangian density (2.1) provides the condition of transversality of the vector field, i.e.,

$$\partial^\mu V_\mu = 0. \tag{2.3}$$

Due to the invariance of the Lagrangian (2.1) under global $U(1)$ transformations of the nucleon field, there is a conserved baryon current which reads

$$j_b^\mu = \bar{\psi}\gamma^\mu\psi. \tag{2.4}$$

Let us also write the energy-momentum tensor of the system:

$$T^{\mu\nu} = \frac{i}{2}\bar{\psi}\gamma^\nu\,\overleftrightarrow{\partial}^\mu\psi + \frac{1}{4}V^\sigma\,\overleftrightarrow{\partial}^\mu\,\overleftrightarrow{\partial}^\nu V_\sigma - \frac{1}{4}\phi\,\overleftrightarrow{\partial}^\mu\,\overleftrightarrow{\partial}^\nu\phi + \frac{1}{2}g^{\mu\nu}(g_s\bar{\psi}\psi\phi - g_v\bar{\psi}\gamma^\sigma\psi V_\sigma). \tag{2.5}$$

As in [21], we have subtracted a full divergence term to obtain the energy-momentum tensor in the most convenient form for our purposes. The fields from Eq. (2.5) are assumed to satisfy Eqs. (2.2).

The system is quantized by postulating the commutation relations [22, 23] for the nucleon field,

$$\{\psi_\alpha(t,\mathbf{x}), \psi^\dagger_\beta(t,\mathbf{y})\} = \delta_{\alpha\beta}\,\delta^{(3)}(\mathbf{x}-\mathbf{y}),$$

$$\{\psi_\alpha(x), \psi_\beta(y)\} = \{\psi^\dagger_\alpha(x), \psi^\dagger_\beta(y)\} = 0;$$

for the vector field,

$$[F_{0i}(t,\mathbf{x}), V_j(t,\mathbf{y})] = -i\delta_{ij}\,\delta^{(3)}(\mathbf{x}-\mathbf{y}),$$

$$[F_{0i}(t,\mathbf{x}), F_{0j}(t,\mathbf{y})] = [V_i(t,\mathbf{x}), V_j(t,\mathbf{y})] = 0;$$

and for the (real) scalar fields

$$[\dot\phi(t,\mathbf{x}), \phi(t,\mathbf{y})] = -i\delta^{(3)}(\mathbf{x}-\mathbf{y}),$$

$$[\phi(t,\mathbf{x}), \phi(t,\mathbf{y})] = [\dot\phi(t,\mathbf{x}), \dot\phi(t,\mathbf{y})] = 0.$$

The dots denote here the time derivatives.

Let us also introduce the singular operators (spectral functions) $\mathscr{G}(x,y)$, $\mathscr{D}(x,y)$, and $\mathscr{S}(x,y)$, defined by

$$\{\psi_\alpha(x), \bar\psi_\beta(y)\} = i\mathscr{G}_{\alpha\beta}(x,y) \tag{2.6a}$$

for the nucleon field,

$$[V^\mu(x), V^\nu(y)] = i\mathscr{D}^{\mu\nu}(x,y) \tag{2.6b}$$

for the vector field, and

$$[\phi(x), \phi(y)] = i\mathscr{S}_{m_s}(x,y) \tag{2.6c}$$

for the scalar field. For non-interacting fields the operators $\mathscr{G}(x,y)$, $\mathscr{D}(x,y)$, and $\mathscr{S}(x,y)$ are c-number functions, and [22]

$$\mathscr{G}_{\alpha\beta}(x,y) = (i\partial^\mu_x\gamma_\mu + M)_{\alpha\beta}\,\mathscr{S}_M(x,y), \tag{2.7a}$$

$$\mathscr{D}^{\mu\nu}(x,y) = \left(g^{\mu\nu} + \frac{1}{m_v^2}\,\partial^\mu_x\,\partial^\nu_x\right)\mathscr{S}_{m_v}(x,y), \tag{2.7b}$$

where

$$i\mathcal{S}_m(x, y) = \int \frac{d^4k}{(2\pi)^4} e^{-ik(x-y)} 2\pi\, \delta(k^2 - m^2)(\Theta(k_0) - \Theta(-k_0))$$

$$= \int \frac{d^3k}{(2\pi)^3\, 2\omega} (e^{-ik(x-y)} - e^{ik(x-y)}), \tag{2.7c}$$

with $k^\mu = (\omega, \mathbf{k})$ and $\omega = (m^2 + \mathbf{k}^2)^{1/2}$.

3. Green Functions

The central role in our considerations is played by the contour Green functions defined as

$$iG_{\alpha\beta}(x, y) \stackrel{\text{def}}{=} \langle \tilde{T}\psi_\alpha(x)\, \bar{\psi}_\beta(y)\rangle \tag{3.1a}$$

for the nucleon field,

$$iD_{\mu\nu}(x, y) \stackrel{\text{def}}{=} \langle \tilde{T} V_\mu(x)\, V_\nu(y)\rangle - \langle V_\mu(x)\rangle\langle V_\nu(y)\rangle \tag{3.1b}$$

for the vector field, and

$$i\,\Delta(x, y) \stackrel{\text{def}}{=} \langle \tilde{T}\phi(x)\, \phi(y)\rangle - \langle\phi(x)\rangle\langle\phi(y)\rangle \tag{3.1c}$$

for the scalar field; $\langle \cdots \rangle$ denotes the ensemble average at time t_0 (usually identified with $-\infty$); $\tilde{T}$ is the time-ordering operation along the directed contour shown in Fig. 1. The parameter $t_{\max}$ is shifted to $+\infty$ in the calculations. In (3.1) the time arguments are complex with an infinitesimal positive or negative imaginary part, which locates them on the upper or on the lower branch of the contour. The ordering operation is defined as

$$\tilde{T}A(x)\, B(y) \stackrel{\text{def}}{=} \Theta(x_0, y_0)\, A(x)\, B(y) \pm \Theta(y_0, x_0)\, B(y)\, A(x),$$

where $\Theta(x_0, y_0)$ equals one if x_0 succeeds y_0 on the contour, and it equals zero when x_0 precedes y_0. The plus sign applies to bosonic operators A and B, while the minus sign applies for fermionic ones. In the case of the bosonic Green functions, the contributions from classical expectation values have been subtracted in order to concentrate on the field fluctuations around the classical values.

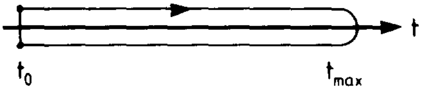

FIG. 1. The contour along the time axis used for the evaluation of operator expectation values.

In addition to the functions (3.1), we use four other types of functions with real time arguments:

$$iG^{>}_{\alpha\beta}(x, y) \stackrel{\text{def}}{=} \langle \psi_\alpha(x)\, \bar{\psi}_\beta(y) \rangle, \tag{3.2a}$$

$$iD^{>}_{\mu\nu}(x, y) \stackrel{\text{def}}{=} \langle V_\mu(x)\, V_\nu(y) \rangle - \langle V_\mu(x) \rangle \langle V_\nu(y) \rangle, \tag{3.2b}$$

$$i\Delta^{>}(x, y) \stackrel{\text{def}}{=} \langle \phi(x)\, \phi(y) \rangle - \langle \phi(x) \rangle \langle \phi(y) \rangle, \tag{3.2c}$$

$$iG^{<}_{\alpha\beta}(x, y) \stackrel{\text{def}}{=} -\langle \bar{\psi}_\beta(y)\, \psi_\alpha(x) \rangle, \tag{3.3a}$$

$$iD^{<}_{\mu\nu}(x, y) \stackrel{\text{def}}{=} \langle V_\nu(y)\, V_\mu(x) \rangle - \langle V_\mu(x) \rangle \langle V_\nu(y) \rangle, \tag{3.3b}$$

$$i\Delta^{<}(x, y) \stackrel{\text{def}}{=} \langle \phi(y)\, \phi(x) \rangle - \langle \phi(x) \rangle \langle \phi(y) \rangle, \tag{3.3c}$$

$$iG^{c}_{\alpha\beta}(x, y) \stackrel{\text{def}}{=} \langle T^c \psi_\alpha(x)\, \bar{\psi}_\beta(y) \rangle, \tag{3.4a}$$

$$iD^{c}_{\mu\nu}(x, y) \stackrel{\text{def}}{=} \langle T^c V_\mu(x)\, V_\nu(y) \rangle - \langle V_\mu(x) \rangle \langle V_\nu(y) \rangle, \tag{3.4b}$$

$$i\Delta^{c}(x, y) \stackrel{\text{def}}{=} \langle T^c \phi(x)\, \phi(y) \rangle - \langle \phi(x) \rangle \langle \phi(y) \rangle, \tag{3.4c}$$

$$iG^{a}_{\alpha\beta}(x, y) \stackrel{\text{def}}{=} \langle T^a \psi_\alpha(x)\, \bar{\psi}_\beta(y) \rangle, \tag{3.5a}$$

$$iD^{a}_{\mu\nu}(x, y) \stackrel{\text{def}}{=} \langle T^a V_\mu(x)\, V_\nu(y) \rangle - \langle V_\mu(x) \rangle \langle V_\nu(y) \rangle, \tag{3.5b}$$

$$i\Delta^{a}(x, y) \stackrel{\text{def}}{=} \langle T^a \phi(x)\, \phi(y) \rangle - \langle \phi(x) \rangle \langle \phi(y) \rangle, \tag{3.5c}$$

where T^c (T^a) prescribes (anti-)chronological time ordering:

$$T^c A(x)\, B(y) \stackrel{\text{def}}{=} \Theta(x_0 - y_0)\, A(x)\, B(y) \pm \Theta(y_0 - x_0)\, B(y)\, A(x),$$
$$T^a A(x)\, B(y) \stackrel{\text{def}}{=} \Theta(y_0 - x_0)\, A(x)\, B(y) \pm \Theta(x_0 - y_0)\, B(y)\, A(x).$$

Again the plus sign is for bosonic operators, and the minus sign for fermionic ones.

The functions (3.2)–(3.5) are related to the one defined by Eqs. (3.1) in the following manner:

$$\Delta^c(x, y) \equiv \Delta(x, y) \quad \text{for } x_0, y_0 \text{ from the upper branch,} \tag{3.6}$$

$$\Delta^a(x, y) \equiv \Delta(x, y) \quad \text{for } x_0, y_0 \text{ from the lower branch,} \tag{3.7}$$

$$\Delta^{>}(x, y) \equiv \Delta(x, y) \quad \text{for } x_0 \text{ from the upper branch and } y_0 \text{ from the lower one,} \tag{3.8}$$

$$\Delta^{<}(x, y) \equiv \Delta(x, y) \quad \text{for } x_0 \text{ from the lower branch and } y_0 \text{ from the upper one.} \tag{3.9}$$

There are analogous relations for the nucleon and vector field Green functions.

The four functions (3.6)–(3.9) are often summarized [24] by a 2×2 matrix representation Δ_{ij}, where $i, j=1$ corresponds to a time argument on the upper branch, and $i, j=2$, to a time argument on the lower branch of the contour:

$$\Delta(x, y)=\begin{pmatrix}\Delta_{11} & \Delta_{12}\\ \Delta_{21} & \Delta_{22}\end{pmatrix}=\begin{pmatrix}\Delta^{c} & \Delta^{>}\\ \Delta^{<} & \Delta^{a}\end{pmatrix}.$$

One further finds the identities, which we write down only for the scalar field Green functions,

$$\Delta^{c}(x, y)=\Theta(x_0-y_0)\,\Delta^{>}(x, y)+\Theta(y_0-x_0)\,\Delta^{<}(x, y), \tag{3.10}$$

$$\Delta^{a}(x, y)=\Theta(y_0-x_0)\,\Delta^{>}(x, y)+\Theta(x_0-y_0)\,\Delta^{<}(x, y). \tag{3.11}$$

One also easily proves that $iG^{\lessgtr}(x, y)\,\gamma^0$, $iD^{\lessgtr}_{\mu\nu}(x, y)$, and $i\Delta^{\lessgtr}(x, y)$ are hermitian (e.g., $(iD^{>}_{\mu\nu}(x, y))^{\dagger}=iD^{>}_{\mu\nu}(x, y)$), and that

$$(iG^{a}(x, y)\,\gamma^0)^{\dagger}=iG^{c}(x, y)\,\gamma^0, \tag{3.12}$$

with similar relations for $iD^{c}_{\mu\nu}(x, y)$ and $i\Delta^{c}(x, y)$. Here $\dagger$ denotes hermitian conjugation, i.e., complex conjugation with an exchange of the Green function arguments and indices.

Because of the relations (2.6), there are also the identities

$$G^{>}(x, y)-G^{<}(x, y)=\langle\mathscr{G}(x, y)\rangle, \tag{3.13a}$$

$$D^{>}(x, y)-D^{<}(x, y)=\langle\mathscr{D}(x, y)\rangle, \tag{3.13b}$$

$$\Delta^{>}(x, y)-\Delta^{<}(x, y)=\langle\mathscr{S}(x, y)\rangle. \tag{3.13c}$$

For the (real) scalar and vector fields the Green functions have the specific property

$$D^{>}_{\mu\nu}(x, y)=D^{<}_{\nu\mu}(y, x); \tag{3.14a}$$

$$\Delta^{>}(x, y)=\Delta^{<}(y, x). \tag{3.14b}$$

Furthermore, (2.3) implies transversality of the vector field Green functions $D^{\gtrless}$:

$$\partial^{\mu}_{x} D^{\gtrless}_{\mu\nu}(x, y)=0=\partial^{\nu}_{y} D^{\gtrless}_{\mu\nu}(x, y). \tag{3.15}$$

Let us now discuss the physical content of these Green functions. The time-ordered Green function $G^{c}(x, y)$ describes the propagation of a disturbance in which a single nucleon is added to the many-particle system at space–time point y and is later removed at space–time point x. An antinucleon (hole) disturbance is propagated backward in time. The meaning of $G^{a}(x, y)$ is analogous but nucleons are propagated backward in time and antinucleons (holes) forward. In the zero density limit $G^{c}(x, y)$ coincides with the nucleon Feynman propagator [22].

The physical interpretation of the functions $G^{\gtrless}(x, y)$ becomes more transparent when one considers their Wigner transforms:

$$G^{\gtrless}(X, p) \stackrel{\text{def}}{=} \int d^4u \, e^{ip \cdot u} G^{\gtrless}(X + \tfrac{1}{2}u, X - \tfrac{1}{2}u). \tag{3.16}$$

One then finds that the baryon current (2.4) averaged over the ensemble can be expressed as

$$\langle j_b^{\mu}(X) \rangle = -\int \frac{d^4p}{(2\pi)^4} \operatorname{Tr}(\gamma^{\mu} iG^{<}(X, p)), \tag{3.17}$$

where the trace is taken over spinor indices. Further, one can express the nucleon contribution to the ensemble-averaged energy-momentum tensor (2.5) of non-interacting fields as

$$\langle T_N^{\mu\nu}(X) \rangle = -\int \frac{d^4p}{(2\pi)^4} p^{\mu} \operatorname{Tr}(\gamma^{\nu} iG^{<}(X, p)). \tag{3.18a}$$

From Eqs. (3.17) and (3.18a) one sees that $iG^{<}(X, p)$ corresponds to the density of nucleons and antinucleons with four-momentum p in a space–time point X, and consequently, it is the quantum analog of the classical distribution function. This interpretation is supported by the fact that $iG^{<}(X, p)$ is hermitian[1]; however, it is not positive definite, and a probabilistic interpretation is only approximately valid. One should also observe that, in contrast to the classical distribution functions, $iG^{<}(X, p)$ can be nonzero for off-shell four-momenta. The interpretation of $iG^{>}(X, p)$ is similar to that of $iG^{<}(X, p)$, since these functions differ only by the field commutator, cf. Eqs. (3.2) and (3.3). As will be shown in Section 10, the positive energy part of $iG^{<}(X, p)$ corresponds to the nucleon distribution function, while the positive energy part of $iG^{>}(X, p)$ corresponds to the nucleon distribution function minus one. Further, the negative energy part of $iG^{>}(X, p)$ provides the antinucleon distribution function, and the negative energy part of $iG^{<}(X, p)$ provides the antinucleon distribution function minus one.

Let us also write down the vector and scalar field contributions to the energy-momentum tensor (2.5) of noninteracting fields:

$$\langle T_v^{\mu\nu}(X) \rangle = -\int \frac{d^4p}{(2\pi)^4} p^{\mu} p^{\nu} iD_{\sigma}^{<\sigma}(X, p) + \frac{1}{4} \langle V^{\sigma}(X) \rangle \overleftrightarrow{\partial}^{\mu} \overleftrightarrow{\partial}^{\nu} \langle V_{\sigma}(X) \rangle, \tag{3.18b}$$

$$\langle T_s^{\mu\nu}(X) \rangle = \int \frac{d^4p}{(2\pi)^4} p^{\mu} p^{\nu} i\Delta^{<}(X, p) - \frac{1}{4} \langle \phi(X) \rangle \overleftrightarrow{\partial}^{\mu} \overleftrightarrow{\partial}^{\nu} \langle \phi(X) \rangle. \tag{3.18c}$$

[1] The hermitian conjugation of $A_{ab}(x, y)$ is defined as a complex conjugation of A with a transposition of indices (a, b) and arguments (x, y). Consequently, the hermitian conjugation of the Wigner transformed $A_{ab}(X, p)$ demands the complex conjugation and the transposition of indices (a, b) (but no change in the sign of p).

After these preliminaries we are in a position to explain the objective of our considerations. Starting from the Lagrange equations (2.2) we will derive equations of motion for the Green functions $G^{\gtrless}$, $D^{\gtrless}$, and $\Delta^{\gtrless}$. These equations will then be converted into transport equations for the distribution functions. Our goal will be realized in several steps: First we write the exact equations for the contour Green functions (3.1). These equations give us the equations for $G^{\gtrless}$, $D^{\gtrless}$, and $\Delta^{\gtrless}$, which are then subjected to a systematic expansion in gradients, i.e, assuming quasihomogeneity of the system. The self-energies, which enter these equations, are found by means of the so-called pairing approximation and the perturbative expansion. We introduce the distribution functions which are defined only for on-shell momenta. The distribution functions of nucleons and vector mesons are matrices in spin space, and their structure is discussed in detail. Having the self-energies in an explicit form, we finally write down a set of transport equations for the distribution functions.

4. Green Function Equations of Motion

From Eqs. (2.2) and the definitions (3.1) one finds the equations of motion for the contour Green functions:

$$([i\gamma \cdot \partial_x - M]\, G(x, y))_{\alpha\beta} = \delta_{\alpha\beta}\, \delta^{(4)}(x, y) + \int_C d^4x' (\Sigma(x, x')\, G(x', y))_{\alpha\beta}, \tag{4.1a}$$

$$(G(x, y)[-i\overleftarrow{\partial}_y \cdot \gamma - M])_{\alpha\beta} = \delta_{\alpha\beta}\, \delta^{(4)}(x, y) + \int_C d^4x' (G(x, x')\, \Sigma(x', y))_{\alpha\beta}, \tag{4.2a}$$

$$[\partial_x^2 + m_v^2]\, D^{\mu\nu}(x, y) = \left(g^{\mu\nu} + \frac{1}{m_v^2}\, \partial_x^\mu\, \partial_x^\nu\right) \delta^{(4)}(x, y) - \int_C d^4x\, P^\mu{}_\lambda(x, x')\, D^{\lambda\nu}(x', y), \tag{4.1b}$$

$$[\partial_y^2 + m_v^2]\, D^{\mu\nu}(x, y) = \left(g^{\mu\nu} + \frac{1}{m_v^2}\, \partial_y^\mu\, \partial_y^\nu\right) \delta^{(4)}(x, y) - \int_C d^4x'\, D^{\mu\lambda}(x, x')\, P_\lambda{}^\nu(x', y), \tag{4.2b}$$

$$[\partial_x^2 + m_s^2]\, \Delta(x, y) = -\delta^{(4)}(x, y) + \int_C d^4x'\, \Pi(x, x')\, \Delta(x', y), \tag{4.1c}$$

$$[\partial_y^2 + m_s^2]\, \Delta(x, y) = -\delta^{(4)}(x, y) + \int_C d^4x'\, \Delta(x, x')\, \Pi(x', y), \tag{4.2c}$$

where the integration over x_0' is performed along the contour. The function $\delta^{(4)}(x, y)$ is defined on the contour as

$$\delta^{(4)}(x, y) = \begin{cases} \delta^{(4)}(x - y) & \text{for } x_0, y_0 \text{ from the upper branch,} \\ 0 & \text{for } x_0, y_0 \text{ from the different branches,} \\ -\delta^{(4)}(x - y) & \text{for } x_0, y_0 \text{ from the lower branch.} \end{cases}$$

The self-energies $\Sigma(x, x')$, $P(x, x')$, and $\Pi(x, x')$ are defined as

$$\int_C d^4x' \, \Sigma_{\alpha\gamma}(x, x') \, G_{\gamma\beta}(x', y) \stackrel{\text{def}}{=} + ig_s \langle \tilde{T}\psi_\alpha(x) \, \phi(x) \, \bar{\psi}_\beta(y) \rangle$$

$$- ig_v \gamma^\mu_{\alpha\gamma} \langle \tilde{T}\psi_\gamma(x) \, V_\mu(x) \, \bar{\psi}_\beta(y) \rangle, \qquad (4.3a)$$

$$\int_C d^4x' \, P^\mu{}_\lambda(x, x') \, D^{\lambda\nu}(x', y) \stackrel{\text{def}}{=} ig_v (\langle \tilde{T}\bar{\psi}_\alpha(x) \, \gamma^\mu_{\alpha\beta} \psi_\beta(x) \, V^\nu(y) \rangle$$

$$- \langle \bar{\psi}_\alpha(x) \, \gamma^\mu_{\alpha\beta} \psi_\beta(x) \rangle \langle V^\nu(y) \rangle), \qquad (4.3b)$$

$$\int_C d^4x' \, \Pi(x, x') \, \Delta(x', y) \stackrel{\text{def}}{=} -ig_s (\langle \tilde{T}\bar{\psi}_\alpha(x) \, \psi_\alpha(x) \, \phi(y) \rangle$$

$$- \langle \bar{\psi}_\alpha(x) \, \psi_\alpha(x) \rangle \langle \phi(y) \rangle). \qquad (4.3c)$$

Due to conservation of the nucleon current, the vector field self-energy must obey the transversality conditions

$$\partial^x_\mu P^{\mu\nu}(x, y) = \partial^y_\nu P^{\mu\nu}(x, y) = 0.$$

Because the Green functions of the free fields satisfy the equations

$$([i\gamma \cdot \partial_x - M] \, G_0(x, y))_{\alpha\beta} = \delta_{\alpha\beta} \, \delta^{(4)}(x, y),$$

$$[\partial_x^2 + m_v^2] \, D_0^{\mu\nu}(x, y) = \left(g^{\mu\nu} + \frac{1}{m_v^2} \partial^\mu_x \partial^\nu_x \right) \delta^{(4)}(x, y),$$

$$[\partial_x^2 + m_s^2] \, \Delta_0(x, y) = -\delta^{(4)}(x, y),$$

one can rewrite Eqs. (4.1) in symbolic operator notation as

$$G_0^{-1} G = 1 + \Sigma G, \qquad G G_0^{-1} = 1 + G\Sigma,$$

$$D_0^{-1} D = 1 - PD, \qquad D D_0^{-1} = 1 - DP$$

$$\Delta_0^{-1} \Delta = 1 - \Pi\Delta, \qquad \Delta\Delta_0^{-1} = 1 - \Delta\Pi$$

and recover the familiar form of the Dyson–Schwinger equations:

$$G = G_0 + G_0 \Sigma G, \qquad (4.4a)$$

$$D = D_0 - D_0 P D, \qquad (4.4b)$$

$$\Delta = \Delta_0 - \Delta_0 \Pi \Delta. \qquad (4.4c)$$

Let us express the self-energies as

$$\Sigma(x,y)=\Sigma_{\mathrm{MF}}(x)\,\delta^{(4)}(x,y)+\Sigma^{>}(x,y)\,\Theta(x_0,y_0)+\Sigma^{<}(x,y)\,\Theta(y_0,x_0), \tag{4.5a}$$

$$P(x,y)=P_{\mathrm{MF}}(x)\,\delta^{(4)}(x,y)+P^{>}(x,y)\,\Theta(x_0,y_0)+P^{<}(x,y)\,\Theta(y_0,x_0), \tag{4.5b}$$

$$\Pi(x,y)=\Pi_{\mathrm{MF}}(x)\,\delta^{(4)}(x,y)+\Pi^{>}(x,y)\,\Theta(x_0,y_0)+\Pi^{<}(x,y)\,\Theta(y_0,x_0). \tag{4.5c}$$

As we shall see later, Σ_{MF}, P_{MF}, and Π_{MF} describe mean-field effects while $\Sigma^{\gtrless}$, $P^{\gtrless}$, and $\Pi^{\gtrless}$ give rise to the collision terms in the transport equations. Therefore, we call Σ_{MF}, P_{MF}, and Π_{MF} the mean-field self-energies and $\Sigma^{\gtrless}$, $P^{\gtrless}$, and $\Pi^{\gtrless}$, the collisional self-energies.

It is also convenient to introduce the retarded $(+)$ and advanced $(-)$ Green functions,

$$G^{\pm}(x,y)\overset{\mathrm{def}}{=}\pm(G^{>}(x,y)-G^{<}(x,y))\,\Theta(\pm x_0\mp y_0), \tag{4.6a}$$

$$D^{\pm}(x,y)\overset{\mathrm{def}}{=}\pm(D^{>}(x,y)-D^{<}(x,y))\,\Theta(\pm x_0\mp y_0), \tag{4.6b}$$

$$\Delta^{\pm}(x,y)\overset{\mathrm{def}}{=}\pm(\Delta^{>}(x,y)-\Delta^{<}(x,y))\,\Theta(\pm x_0\mp y_0), \tag{4.6c}$$

and the retarded and advanced self-energies $\Sigma^{\pm}$, $P^{\pm}$, and $\Pi^{\pm}$ in an analogous way. With their help Eqs. (4.1) and (4.2) give

$$\begin{aligned}&([i\gamma\cdot\partial_x-M-\Sigma_{\mathrm{MF}}(x)]\,G^{\gtrless}(x,y))_{\alpha\beta}\\&\quad=\int d^4x'(\Sigma^{\gtrless}(x,x')\,G^{-}(x',y)+\Sigma^{+}(x,x')\,G^{\gtrless}(x',y))_{\alpha\beta},\end{aligned} \tag{4.7a}$$

$$\begin{aligned}&(G^{\gtrless}(x,y)[-i\overleftarrow{\partial}_y\cdot\gamma-M-\Sigma_{\mathrm{MF}}(y)])_{\alpha\beta}\\&\quad=\int d^4x'(G^{\gtrless}(x,x')\,\Sigma^{-}(x',y)+G^{+}(x,x')\,\Sigma^{\gtrless}(x',y))_{\alpha\beta},\end{aligned} \tag{4.8a}$$

$$\begin{aligned}&([\partial_x^2+m_v^2+P_{\mathrm{MF}}(x)]\,D^{\gtrless}(x,y))^{\mu\nu}\\&\quad=-\int d^4x'(P^{\gtrless}(x,x')\,D^{-}(x',y)+P^{+}(x,x')\,D^{\gtrless}(x',y))^{\mu\nu},\end{aligned} \tag{4.7b}$$

$$\begin{aligned}&(D^{\gtrless}(x,y)[\overleftarrow{\partial}_y^2+m_v^2+P_{\mathrm{MF}}(y)])^{\mu\nu}\\&\quad=-\int d^4x'(D^{\gtrless}(x,x')\,P^{-}(x',y)+D^{+}(x,x')\,P^{\gtrless}(x',y))^{\mu\nu},\end{aligned} \tag{4.8b}$$

$$\begin{aligned}&[\partial_x^2+m_s^2-\Pi_{\mathrm{MF}}(x)]\,\Delta^{\gtrless}(x,y)\\&\quad=\int d^4x'[\Pi^{\gtrless}(x,x')\,\Delta^{-}(x',y)+\Pi^{+}(x,x')\,\Delta^{\gtrless}(x',y)],\end{aligned} \tag{4.7c}$$

$$\begin{aligned}&[\partial_y^2+m_s^2-\Pi_{\mathrm{MF}}(y)]\,\Delta^{\gtrless}(x,y)\\&\quad=\int d^4x'[\Delta^{\gtrless}(x,x')\,\Pi^{-}(x',y)+\Delta^{+}(x,x')\,\Pi^{\gtrless}(x',y)],\end{aligned} \tag{4.8c}$$

where all time integrations run from $-\infty$ to $+\infty$.

Let us also write the equations satisfied by the functions $G^{\pm}$, $D^{\pm}$, and $\Delta^{\pm}$, which we will need in Section 10,

$$([i\gamma\cdot\partial_x - M - \Sigma_{\rm MF}(x)]\, G^{\pm}(x,y))_{\alpha\beta}$$
$$= \delta_{\alpha\beta}\,\delta^{(4)}(x-y) + \int d^4x'(\Sigma^{\pm}(x,x')\, G^{\pm}(x',y))_{\alpha\beta}, \tag{4.9a}$$

$$(G^{\pm}(x,y)[-i\overleftarrow{\partial}_y\cdot\gamma - M - \Sigma_{\rm MF}(y)])_{\alpha\beta}$$
$$= \delta_{\alpha\beta}\,\delta^{(4)}(x-y) + \int d^4x'(G^{\pm}(x,x')\,\Sigma^{\pm}(x',y))_{\alpha\beta}, \tag{4.10a}$$

$$([\partial_x^2 + m_v^2 + P_{\rm MF}(x)]\, D^{\pm}(x,y))^{\mu\nu}$$
$$= \left(g^{\mu\nu} + \frac{1}{m_v^2}\,\partial_x^{\mu}\,\partial_x^{\nu}\right)\delta^{(4)}(x-y) - \int d^4x'(P^{\pm}(x,x')\, D^{\pm}(x',y))^{\mu\nu}, \tag{4.9b}$$

$$(D^{\pm}(x,y)[\overleftarrow{\partial}_y^2 + m_v^2 + P_{\rm MF}(y)])^{\mu\nu}$$
$$= \left(g^{\mu\nu} + \frac{1}{m_v^2}\,\partial_y^{\mu}\,\partial_y^{\nu}\right)\delta^{(4)}(x-y) - \int d^4x'(D^{\pm}(x,x')\, P^{\pm}(x',y))^{\mu\nu}, \tag{4.10b}$$

$$[\partial_x^2 + m_s^2 - \Pi_{\rm MF}(x)]\, \Delta^{\pm}(x,y)$$
$$= -\delta^{(4)}(x-y) + \int d^4x'\, \Pi^{\pm}(x,x')\, \Delta^{\pm}(x',y), \tag{4.9c}$$

$$[\partial_y^2 + m_s^2 - \Pi_{\rm MF}(y)]\, \Delta^{\pm}(x,y)$$
$$= -\delta^{(4)}(x-y) + \int d^4x'\, \Delta^{\pm}(x,x')\, \Pi^{\pm}(x',y). \tag{4.10c}$$

While in Eqs. (4.7), (4.8) a coupling between the $>$ and $<$ Green functions is obvious, Eqs. (4.9), (4.10) for the retarded and advanced functions appear to close among themselves. Indeed, it was shown in [25] that the retarded self-energies can be expressed in terms of retarded propagators only, so that this superficial appearance is also proved in detail.

It should be stressed that Eqs. (4.7)–(4.10) are exact, and all of them together are equivalent to the field equations of motion.

5. Towards Transport Equations

The transport equations are derived under the assumption that the Green functions and the self-energies depend only weakly on the sum of their arguments and that they are significantly different from zero only when the difference of their arguments is close to zero. To express these properties it is convenient to define a new set of variables as

$$\Delta(X,u) \equiv \Delta(X + \tfrac{1}{2}u,\, X - \tfrac{1}{2}u).$$

For homogeneous systems, translational invariance dictates that the dependence on $X=(x+y)/2$ drops out entirely, and $\Delta(x, y)$ depends only on $u=x-y$. For weakly inhomogeneous, or quasihomogeneous systems, the Green functions and self-energies are assumed to vary slowly with X. We additionally assume that the Green functions and self-energies are strongly *peaked* near $u=0$. With these assumptions, which are discussed in Section 12, where we analyse the whole procedure of deriving kinetic equations, one can, in particular, approximate $\Delta(X+u, u)$ as

$$\Delta(X+u, u) \cong \Delta(X, u) + u^\mu \frac{\partial}{\partial X^\mu} \Delta(X, u). \tag{5.1}$$

We will now convert Eqs. (4.7), (4.8) into transport equations by implementing the above approximation and performing the Wigner transformation (3.16) for all Green functions and self-energies. This is done using the following set of translation rules which can be easily derived:

$$\int d^4x' f(x, x')\, g(x', y) \longrightarrow f(X, p)\, g(X, p) + \frac{i}{2}\left[\frac{\partial f(X, p)}{\partial p_\mu}\frac{\partial g(X, p)}{\partial X^\mu} - \frac{\partial f(X, p)}{\partial X^\mu}\frac{\partial g(X, p)}{\partial p_\mu}\right], \tag{5.2}$$

$$h(x)\, g(x, y) \longrightarrow h(X)\, g(X, p) - \frac{i}{2}\frac{\partial h(X)}{\partial X^\mu}\frac{\partial g(X, p)}{\partial p_\mu}, \tag{5.3}$$

$$h(y)\, g(x, y) \longrightarrow h(X)\, g(X, p) + \frac{i}{2}\frac{\partial h(X)}{\partial X^\mu}\frac{\partial g(X, p)}{\partial p_\mu}, \tag{5.4}$$

$$\partial_x^\mu f(x, y) \longrightarrow (-ip^\mu + \tfrac{1}{2}\partial^\mu) f(X, p), \tag{5.5}$$

$$\partial_y^\mu f(x, y) \longrightarrow (ip^\mu + \tfrac{1}{2}\partial^\mu) f(X, p). \tag{5.6}$$

Here $X=(x+y)/2$, $\partial^\mu \equiv \partial/\partial X_\mu$, and the functions $f(x, y)$ and $g(x, y)$ satisfy the assumptions discussed above.

Applying the formulae (5.2)–(5.6) to Eqs. (4.7), (4.8), we obtain

$$\left(\left[\left(p_\mu + \frac{i}{2}\partial_\mu\right)\gamma^\mu - M - \Sigma_{\mathrm{MF}}(X) + \frac{i}{2}\partial_\mu \Sigma_{\mathrm{MF}}(X)\,\partial_p^\mu\right] G^{\gtrless}(X, p)\right)_{\alpha\beta}$$
$$= (\Sigma^{\gtrless}(X, p)\, G^-(X, p) + \Sigma^+(X, p)\, G^{\gtrless}(X, p))_{\alpha\beta}, \tag{5.7a}$$

$$\left(G^{\gtrless}(X, p)\left[\left(p_\mu - \frac{i}{2}\overleftarrow{\partial}_\mu\right)\gamma^\mu - M - \Sigma_{\mathrm{MF}}(X) - \frac{i}{2}\overleftarrow{\partial}_p^\mu\,\partial_\mu \Sigma_{\mathrm{MF}}(X)\right]\right)_{\alpha\beta}$$
$$= (G^{\gtrless}(X, p)\, \Sigma^-(X, p) + G^+(X, p)\, \Sigma^{\gtrless}(X, p))_{\alpha\beta}, \tag{5.8a}$$

$$\left(\left[\frac{1}{4}\partial^2 - ip^\mu\,\partial_\mu - p^2 + m_v^2 + P_{\mathrm{MF}}(X) - \frac{i}{2}\partial_\nu P_{\mathrm{MF}}(X)\,\partial_p^\nu\right] D^{\gtrless}(X, p)\right)^{\rho\sigma}$$
$$= -(P^{\gtrless}(X, p)\, D^-(X, p) + P^+(X, p)\, D^{\gtrless}(X, p))^{\rho\sigma}, \tag{5.7b}$$

$$\left(D^{\gtrless}(X,p)\left[\frac{1}{4}\overleftarrow{\partial}^2 + ip^\mu\,\overleftarrow{\partial}_\mu - p^2 + m_v^2 + P_{\mathrm{MF}}(X) + \frac{i}{2}\overleftarrow{\partial}_p^\nu\,\partial_\nu P_{\mathrm{MF}}(X)\right]\right)^{\rho\sigma}$$

$$= -(D^{\gtrless}(X,p)\,P^{-}(X,p) + D^{+}(X,p)\,P^{\gtrless}(X,p))^{\rho\sigma}, \tag{5.8b}$$

$$\left[\frac{1}{4}\partial^2 - ip^\mu\,\partial_\mu - p^2 + m_s^2 - \Pi_{\mathrm{MF}}(X) + \frac{i}{2}\partial_\mu\Pi_{\mathrm{MF}}(X)\,\partial_p^\mu\right]\Delta^{\gtrless}(X,p)$$

$$= \Pi^{\gtrless}(X,p)\,\Delta^{-}(X,p) + \Pi^{+}(X,p)\,\Delta^{\gtrless}(X,p), \tag{5.7c}$$

$$\left[\frac{1}{4}\partial^2 + ip^\mu\,\partial_\mu - p^2 + m_s^2 - \Pi_{\mathrm{MF}}(X) - \frac{i}{2}\partial_\mu\Pi_{\mathrm{MF}}(X)\,\partial_p^\mu\right]\Delta^{\gtrless}(X,p)$$

$$= \Delta^{\gtrless}(X,p)\,\Pi^{-}(X,p) + \Delta^{+}(X,p)\,\Pi^{\gtrless}(X,p). \tag{5.8c}$$

On the right-hand sides of Eqs. (5.7), (5.8) we have neglected gradient terms like those from Eq. (5.2). This approximation is discussed in Section 12.

Due to Eq. (3.15), the vector field Green functions $D^{\gtrless}(X,p)$ satisfy the conditions

$$(\tfrac{1}{2}\partial^\mu - ip^\mu)\,D^{\gtrless}_{\mu\nu}(X,p) = 0 = (\tfrac{1}{2}\partial^\nu + ip^\nu)\,D^{\gtrless}_{\mu\nu}(X,p). \tag{5.9}$$

There are also analogous transversality conditions for the vector field self-energies.

Let us now take the sum and the difference of Eqs. (5.7) and (5.8). Then, one obtains

$$[p_\mu\gamma^\mu, G^{\gtrless}(X,p)] + \frac{i}{2}\{\gamma^\mu, \partial_\mu G^{\gtrless}(X,p)\} - [\Sigma_{\mathrm{MF}}(X), G^{\gtrless}(X,p)]$$

$$+ \frac{i}{2}\{\partial_\mu\Sigma_{\mathrm{MF}}(X), \partial_p^\mu G^{\gtrless}(X,p)\}$$

$$= \Sigma^{>}(X,p)\,G^{<}(X,p) - \Sigma^{<}(X,p)\,G^{>}(X,p)$$

$$+ [\Sigma^{\gtrless}(X,p), G^{+}(X,p)] + [\Sigma^{-}(X,p), G^{\gtrless}(X,p)], \tag{5.10a}$$

$$\{p_\mu\gamma^\mu - M, G^{\gtrless}(X,p)\} + \frac{i}{2}[\gamma^\mu, \partial_\mu G^{\gtrless}(X,p)] - \{\Sigma_{\mathrm{MF}}(X), G^{\gtrless}(X,p)\}$$

$$+ \frac{i}{2}[\partial_\mu\Sigma_{\mathrm{MF}}(X), \partial_p^\mu G^{\gtrless}(X,p)]$$

$$= \Sigma^{\gtrless}(X,p)\,G^{-}(X,p) + \Sigma^{+}(X,p)\,G^{\gtrless}(X,p)$$

$$+ G^{\gtrless}(X,p)\,\Sigma^{-}(X,p) + G^{+}(X,p)\,\Sigma^{\gtrless}(X,p), \tag{5.11a}$$

$$-2ip^\mu\,\partial_\mu D^{\gtrless}(X,p) + [P_{\mathrm{MF}}(X), D^{\gtrless}(X,p)] - \frac{i}{2}\{\partial_\nu P_{\mathrm{MF}}(X), \partial_p^\nu D^{\gtrless}(X,p)\}$$

$$= -P^{>}(X,p)\,D^{<}(X,p) + P^{<}(X,p)\,D^{>}(X,p)$$

$$-[P^{\gtrless}(X,p), D^{+}(X,p)] - [P^{-}(X,p), D^{\gtrless}(X,p)], \tag{5.10b}$$

$$2\left(\frac{1}{4}\partial^2 - p^2 + m_v^2\right) D^{\gtrless}(X,p) + \{P_{\rm MF}(X), D^{\gtrless}(X,p)\}$$
$$-\frac{i}{2}[\partial_\nu P_{\rm MF}(X), \partial_p^\nu D^{\gtrless}(X,p)]$$
$$= -P^{\gtrless}(X,p)\, D^-(X,p) - P^+(X,p)\, D^{\gtrless}(X,p)$$
$$-D^{\gtrless}(X,p)\, P^-(X,p) - D^+(X,p)\, P^{\gtrless}(X,p), \tag{5.11b}$$

$$-2i(p^\mu \partial_\mu - \tfrac{1}{2}\partial_\mu \Pi_{\rm MF}(X)\, \partial_p^\mu)\, \Delta^{\gtrless}(X,p)$$
$$= -\Pi^>(X,p)\, \Delta^<(X,p) + \Pi^<(X,p)\, \Delta^>(X,p), \tag{5.10c}$$

$$2(\tfrac{1}{4}\partial^2 - p^2 + m_s^2 - \Pi_{\rm MF}(X))\, \Delta^{\gtrless}(X,p)$$
$$= \Pi^{\gtrless}(X,p)(\Delta^+(X,p) + \Delta^-(X,p))$$
$$+ (\Pi^+(X,p) + \Pi^-(X,p))\, \Delta^{\gtrless}(X,p). \tag{5.11c}$$

We have used the fact following from the definition (4.6) that all Green functions and self-energies satisfy identities of the form

$$A^+(X,p) - A^-(X,p) = A^>(X,p) - A^<(X,p). \tag{5.12}$$

For the scalar field, one observes that Eq. (5.10c) has already the form of a transport equation, with Eq. (5.11c) being the associated mass shell constraint (see, e.g., [26]). The equations for the fermion and vector fields demand further analysis.

The retarded and advanced Green functions and self-energies (4.6) can be written as

$$\gamma^0 G^\pm(X,p) = \pm\frac{1}{2}(\gamma^0 G^>(X,p) - \gamma^0 G^<(X,p))$$
$$+ \frac{1}{2\pi i} P \int d\omega' \, \frac{\gamma^0 G^>(X,\omega',\mathbf{p}) - \gamma^0 G^<(X,\omega',\mathbf{p})}{\omega - \omega'}, \tag{5.13}$$

and analogous formulae for $\Sigma^\pm$, $D^\pm$, and $P^\pm$. It will be further shown that the first term on the r.h.s. of Eq. (5.13) corresponds to the on-mass-shell part of the retarded (advanced) Green function (self-energy), while the second one describes the off-shell part.

Using Eq. (5.13), Eqs. (5.10a) and (5.10b) can be manipulated to the form

$$\frac{i}{2}\{\gamma^\mu, \partial_\mu G^{\gtrless}(X,p)\} + \frac{i}{2}\{\partial_\mu \Sigma_{\rm MF}(X), \partial_p^\mu G^{\gtrless}(X,p)\}$$
$$+ [p_\mu \gamma^\mu, G^{\gtrless}(X,p)] - [\Sigma_{\rm MF}(X), G^{\gtrless}(X,p)]$$
$$= \frac{1}{2}\{\Sigma^>(X,p), G^<(X,p)\} - \frac{1}{2}\{\Sigma^<(X,p), G^>(X,p)\}$$
$$+ [\Sigma^{\gtrless}(X,p), G_R^+(X,p)] + [\Sigma_R^-(X,p), G^{\gtrless}(X,p)], \tag{5.14a}$$

$$-2ip^\mu \partial_\mu D^{\gtrless}(X,p) - \frac{i}{2}\{\partial_\nu P_{\rm MF}(X), \partial_p^\nu D^{\gtrless}(X,p)\} + [P_{\rm MF}(X), D^{\gtrless}(X,p)]$$

$$= -\frac{1}{2}\{P^{>}(X,p), D^{<}(X,p)\} + \frac{1}{2}\{P^{<}(X,p), D^{>}(X,p)\}$$

$$- [P^{\gtrless}(X,p), D_R^{+}(X,p)] - [P_R(X,p), D^{\gtrless}(X,p)]. \tag{5.14b}$$

The functions with the index R correspond to the second term on the right-hand side of Eq. (5.13), i.e., to the off-mass-shell parts of the Green functions or self-energies, respectively.

Equations (5.10c), (5.14a), and (5.14b) are the main result of this section. After further preparations in the following section, they will be converted into standard transport equations in Section 11.

6. Noninteracting Fields

It is instructive to first consider non-interacting fields. Then, Eqs. (5.7) and (5.8) reduce to

$$\left[\left(p_\mu + \frac{i}{2}\partial_\mu\right)\gamma^\mu - M\right] G^{\gtrless}(X,p) = 0, \tag{6.1a}$$

$$G^{\gtrless}(X,p)\left[\left(p_\mu - \frac{i}{2}\bar{\partial}_\mu\right)\gamma^\mu - M\right] = 0, \tag{6.2a}$$

$$[\tfrac{1}{4}\partial^2 - ip^\mu\partial_\mu - p^2 + m_{\rm v}^2]\, D^{\gtrless}(X,p) = 0, \tag{6.1b}$$

$$[\tfrac{1}{4}\partial^2 + ip^\mu\partial_\mu - p^2 + m_{\rm v}^2]\, D^{\gtrless}(X,p) = 0, \tag{6.2b}$$

$$[\tfrac{1}{4}\partial^2 - ip^\mu\partial_\mu - p^2 + m_{\rm s}^2]\, \Delta^{\gtrless}(X,p) = 0, \tag{6.1c}$$

$$[\tfrac{1}{4}\partial^2 + ip^\mu\partial_\mu - p^2 + m_{\rm s}^2]\, \Delta^{\gtrless}(X,p) = 0. \tag{6.2c}$$

To simplify the notation we avoid the index zero, which has been previously used to specify Green functions of non-interacting fields. We hope that this will not cause confusion.

To obtain the nucleon transport equation for the free fields one can use a trick, which unfortunately does not work in general. Specifically, Eq. (6.1a) is multiplied by $[(p_\mu + (i/2)\,\partial_\mu)\,\gamma^\mu + M]$ from the left, and Eq. (6.2a) by $[(p_\mu - (i/2)\,\partial_\mu)\,\gamma^\mu + M]$ from the right. Thus, one obtains

$$\left[\left(p_\mu + \frac{i}{2}\partial_\mu\right)^2 - M^2\right] G^{\gtrless}(X,p) = 0, \tag{6.1d}$$

$$\left[\left(p_\mu - \frac{i}{2}\partial_\mu\right)^2 - M^2\right] G^{\gtrless}(X,p) = 0. \tag{6.2d}$$

Subtracting and adding Eqs. (6.2d), (6.2b), (6.2c) and Eqs. (6.1d), (6.1b), (6.1c), respectively, we find

$$p^\mu \partial_\mu G^{\gtrless}(X, p) = 0, \tag{6.3}$$

$$[\tfrac{1}{4}\partial^2 - p^2 + M^2]\, G^{\gtrless}(X, p) = 0, \tag{6.4}$$

plus two identical sets of equations for $D^{\gtrless}$ and $\Delta^{\gtrless}$, with M replaced by the appropriate meson masses.[2]

Equation (6.3) and its analogues for meson fields are identified with the classical relativistic kinetic equations (see, e.g., [26]) in the absence of mean-field and collision terms. However, due to Eq. (6.4), the Green functions $G^{\gtrless}$, $D^{\gtrless}$, and $\Delta^{\gtrless}$ can be different from zero also for off-shell four-momenta. Since kinetic theory deals only with averaged system characteristics which are homogeneous on a scale of the Compton wave length of the individual particles (which is of the order of their inverse mass), we formally impose the condition

$$|G^{\gtrless}(X, p)| \gg \left| \frac{1}{M^2} \partial^2 G^{\gtrless}(X, p) \right|, \tag{6.5}$$

and similar conditions for $D^{\gtrless}$ and $\Delta^{\gtrless}$. Let us observe that the condition of quasihomogeneity of the Green functions, on the one hand, limits the class of systems which can be described by means of transport theory, but, on the other hand, limits the amount of information provided by this theory. This condition is further discussed in Section 12. Upon introducing these requirements into (6.4), we obtain

$$[p^2 - M^2]\, G^{\gtrless}(X, p) = 0, \tag{6.6}$$

plus analogous mass-shell equations for $D^{\gtrless}$ and $\Delta^{\gtrless}$. One sees that the condition (6.5) renders the off-shell contributions to the Green functions $G^{\gtrless}$, $D^{\gtrless}$, and $\Delta^{\gtrless}$ negligible.

Let us also discuss the equations for the (anti-)chronological Green functions $G^{c(a)}$, $D^{c(a)}$, and $\Delta^{c(a)}$. For noninteracting fields, the analogues of (6.3), (6.4) for the time-ordered functions are

$$p^\mu \partial_\mu G^c(X, p) = 0, \tag{6.7a}$$

$$[\tfrac{1}{4}\partial^2 - p^2 + M^2]\, G^c(X, p) = p^\mu \gamma_\mu - M, \tag{6.8a}$$

$$p^\mu \partial_\mu D^c(X, p) = 0, \tag{6.7b}$$

$$\left[\frac{1}{4}\partial^2 - p^2 + m_v^2\right] D^c_{\mu\nu}(X, p) = g_{\mu\nu} - \frac{p_\mu p_\nu}{m_v^2}, \tag{6.8b}$$

$$p^\mu \partial_\mu \Delta^c(X, p) = 0, \tag{6.7c}$$

$$[\tfrac{1}{4}\partial^2 - p^2 + m_s^2]\, \Delta^c(X, p) = -1. \tag{6.8c}$$

[2] Please note that for free fields Eqs. (6.3) and (6.4) can be obtained from the Lagrange equations without making use of the approximations discussed at the beginning of Section 5.

For the antichronological functions G^a, D^a, and Δ^a the right hand sides differ only by a minus sign.

Imposing the conditions of quasihomogeneity, the solutions of Eqs. (6.7), (6.8) can be written as

$$G^c(X,p)=\frac{p^\mu\gamma_\mu - M}{p^2-M^2+i0^+}+\Theta(-p_0)\,G^>(X,p)+\Theta(p_0)\,G^<(X,p), \tag{6.9a}$$

$$G^a(X,p)=\frac{-p^\mu\gamma_\mu + M}{p^2-M^2-i0^+}+\Theta(p_0)\,G^>(X,p)+\Theta(-p_0)\,G^<(X,p), \tag{6.10a}$$

$$D^c_{\mu\nu}(X,p)=\frac{-g_{\mu\nu}+p_\mu p_\nu/m_v^2}{p^2-m_v^2+i0^+}+\Theta(-p_0)\,D^>(X,p)+\Theta(p_0)\,D^<(X,p), \tag{6.9b}$$

$$D^a_{\mu\nu}(X,p)=\frac{g_{\mu\nu}-p_\mu p_\nu/m_v^2}{p^2-m_v^2-i0^+}+\Theta(p_0)\,D^>(X,p)+\Theta(-p_0)\,D^<(X,p), \tag{6.10b}$$

$$\Delta^c(X,p)=\frac{1}{p^2-m_s^2+i0^+}+\Theta(-p_0)\,\Delta^>(X,p)+\Theta(p_0)\,\Delta^<(X,p), \tag{6.9c}$$

$$\Delta^a(X,p)=\frac{-1}{p^2-m_s^2-i0^+}+\Theta(+p_0)\,\Delta^>(X,p)+\Theta(-p_0)\,\Delta^<(X,p), \tag{6.10c}$$

where the free functions $G^{\gtrless}$, $D^{\gtrless}$, and $\Delta^{\gtrless}$ satisfy Eqs. (6.3) and (6.6), i.e., exist only on-shell. The solutions (6.9) possess the initial conditions [22] of the standard Feynman propagator, and (6.9), (6.10) satisfy the general relations (3.10)–(3.12). From the solutions (6.9), (6.10) one also finds

$$G^\pm(X,p)=\frac{p^\mu\gamma_\mu - M}{p^2-M^2\pm ip_0 0^+}, \tag{6.11a}$$

$$D^\pm_{\mu\nu}(X,p)=\frac{-g_{\mu\nu}+p_\mu p_\nu/m_v^2}{p^2-m_v^2\pm ip_0 0^+}, \tag{6.11b}$$

$$\Delta^\pm(X,p)=\frac{1}{p^2-m_s^2\pm ip_0 0^+}, \tag{6.11c}$$

by using the identities

$$\Delta^+(X,p)+\Delta^-(X,p)=\Delta^c(X,p)-\Delta^a(X,p), \tag{6.12}$$

$$\Delta^+(X,p)-\Delta^-(X,p)=\Delta^>(X,p)-\Delta^<(X,p), \tag{6.13}$$

and their analogues for the nucleon and vector field Green functions. These relations follow directly from the definitions (4.6) and are exact also for interacting fields. Let us remember that in the case of noninteracting fields, $G^> - G^<$, $D^> - D^<$, and $\Delta^> - \Delta^<$ are given by the functions (2.7).

7. The Pairing Approximation

In this section we discuss a specific approximation which allows to obtain the nucleon transport equations in the presence of interactions using a method quite similar to that applied to non-interacting fields. Unfortunately, the resulting equations reproduce only the mean-field limit and do not yield any collision terms.

Let us return to Eqs. (4.3) defining the self-energies and assume that the expectation values of the field operator products can be factorized into expectation values of products of at most two operators. Then

$$\langle \tilde{T}\psi_\alpha(x)\,\phi(x)\,\bar{\psi}_\beta(y)\rangle = \langle\phi(x)\rangle\langle \tilde{T}\psi_\alpha(x)\,\bar{\psi}_\beta(y)\rangle \tag{7.1a}$$

and

$$\langle \tilde{T}\psi_\alpha(x)\,V_\mu(x)\,\bar{\psi}_\beta(y)\rangle = \langle V_\mu(x)\rangle\langle \tilde{T}\psi_\alpha(x)\,\bar{\psi}_\beta(y)\rangle\,. \tag{7.1b}$$

Consequently (cf. Eq. (4.3)),

$$\int_C d^4x'\,\Sigma(x,x')\,G(x',y) = (-g_s\langle\phi(x)\rangle + g_v\langle V_\mu(x)\rangle\,\gamma^\mu)\,G(x,y),$$

$$\int_C d^4x'\,P(x,x')\,D(x',y) = 0,$$

$$\int_C d^4x'\,\Pi(x,x')\,\Delta(x',y) = 0.$$

Therefore,

$$\Sigma(x,y) = (-g_s\langle\phi(x)\rangle + g_v\langle V_\mu(x)\rangle\,\gamma^\mu)\,\delta^{(4)}(x,y),$$

$$P(x,y) = 0,$$

$$\Pi(x,y) = 0.$$

Comparing with Eqs. (4.5), we see that the approximation (7.1) provides a nonzero mean-field part only for the nucleon self-energy,

$$\Sigma_{\mathrm{MF}}(x) = -g_s\langle\phi(x)\rangle + g_v\langle V_\mu(x)\rangle\,\gamma^\mu, \tag{7.2}$$

with all the other self-energies vanishing. One sees that in the framework of the pairing approximation, the vector and scalar mesons behave as free particles. So, let us concentrate on nucleons. Substituting the self-energy (7.2) into Eqs. (5.7a) and (5.8a), we obtain

$$\Big[\Big(p_\mu + \frac{i}{2}\partial_\mu - g_v\langle V_\mu(X)\rangle + \frac{i}{2}g_v\,\partial_\nu\langle V_\mu(X)\rangle\,\partial_p^\nu\Big)\gamma^\mu$$
$$- M + g_s\langle\phi(X)\rangle - \frac{i}{2}g_s\,\partial_\mu\langle\phi(X)\rangle\,\partial_p^\mu\Big]\,G^{\gtrless}(X,p) = 0, \tag{7.3}$$

$$G^{\gtrless}(X,p)\left[\left(p_\mu-\frac{i}{2}\overleftarrow{\partial}_\mu-g_v\langle V_\mu(X)\rangle-\frac{i}{2}g_v\overleftarrow{\partial}_p^\nu\partial_\nu\langle V_\mu(X)\rangle\right)\gamma^\mu\right.$$
$$\left.-M+g_s\langle\phi(X)\rangle+\frac{i}{2}g_s\overleftarrow{\partial}_p^\mu\partial_\mu\langle\phi(X)\rangle\right]=0. \tag{7.4}$$

As always in the context of interactions with an (electromagnetic) vector field [27], it is more convenient to work with the kinetic rather than the canonical momentum. For the Walecka model it is achieved *via* the replacement [9]

$$p^\mu\to p^\mu-g_v\langle V^\mu(X)\rangle.$$

Then,

$$\partial^\mu\to\partial^\mu-g_v\,\partial^\mu\langle V_\nu(X)\rangle\,\partial_p^\nu.$$

After these changes, Eqs. (7.3) and (7.4) are

$$\left[\left(p_\mu+\frac{i}{2}\partial_\mu-\frac{i}{2}g_v\langle F_{\mu\nu}(X)\rangle\,\partial_p^\nu\right)\gamma^\mu-M^*(X)\right.$$
$$\left.-i\frac{1}{2}g_s\,\partial_\mu\langle\phi(X)\rangle\,\partial_p^\mu\right]G^{\gtrless}(X,p)=0, \tag{7.5}$$

$$G^{\gtrless}(X,p)\left[\left(p_\mu-\frac{i}{2}\overleftarrow{\partial}_\mu+\frac{i}{2}g_v\overleftarrow{\partial}_p^\nu\langle F_{\mu\nu}(X)\rangle\right)\gamma^\mu\right.$$
$$\left.-M^*(X)+\frac{1}{2}g_s\overleftarrow{\partial}_p^\mu\partial_\mu\langle\phi(X)\rangle\right]=0, \tag{7.6}$$

ignoring terms of third and fourth order in the gradients. We have introduced the effective mass

$$M^*(X)=M-g_s\langle\phi(X)\rangle.$$

Now we apply a similar trick as in Eqs. (6.1a), (6.2a): we multiply Eq. (7.5) from the left by

$$\left[\left(p_\mu+\frac{i}{2}\partial_\mu-\frac{i}{2}g_v\langle F_{\mu\nu}(X)\rangle\,\partial_p^\nu\right)\gamma^\mu+M^*(X)+\frac{i}{2}g_s\,\partial_\mu\langle\phi(X)\rangle\,\partial_p^\mu\right],$$

and Eq. (7.6) from the right by the complex conjugate of this expression. As a result we obtain

$$[-\tfrac{1}{4}\partial^2+ip^\mu\,\partial_\mu+p^2-ig_vp^\mu\langle F_{\mu\nu}(X)\rangle\,\partial_p^\nu-M^{*2}(X)-ig_sM^*(X)\,\partial_\mu\langle\phi(X)\rangle\,\partial_p^\mu$$
$$-ig_s\,\partial_\mu\langle\phi(X)\rangle\,\gamma^\mu-ig_v\langle F_{\mu\nu}(X)\rangle\,\gamma^\mu\gamma^\nu]\,G^{\gtrless}(X,p)=0, \tag{7.7}$$

$$G^{\gtrless}(X,p)[-\tfrac{1}{4}\overleftarrow{\partial}^2-ip^\mu\,\overleftarrow{\partial}_\mu+p^2+ig_vp^\mu\langle F_{\mu\nu}(X)\rangle\,\overleftarrow{\partial}_p^\nu-M^{*2}(X)$$
$$-ig_sM^*(X)\,\partial_\mu\langle\phi(X)\rangle\,\overleftarrow{\partial}_p^\mu+ig_s\,\partial_\mu\langle\phi(X)\rangle\,\gamma^\mu+ig_v\langle F_{\mu\nu}(X)\rangle\,\gamma^\mu\gamma^\nu]=0, \tag{7.8}$$

again ignoring terms of third and fourth order in gradients and using the identity

$$(A_\mu \gamma^\mu + B)(A_\mu \gamma^\mu - B) = A_\mu A^\mu - B^2 - [A_\mu, B]\,\gamma^\mu + [A_\mu, A_\nu]\,\gamma^\mu \gamma^\nu.$$

Subtracting and adding Eqs. (7.7) and (7.8), one finds

$$\begin{aligned}&[p^\mu \partial_\mu - g_v p^\mu \langle F_{\mu\nu}(X)\rangle\, \partial_p^\nu - g_s M^*(X)\, \partial_\mu \langle \phi(X)\rangle\, \partial_p^\mu]\, G^{\gtrless}(X, p)\\ &\quad = -\tfrac{1}{2} g_s\, \partial_\mu \langle \phi(X)\rangle \{\gamma^\mu, G^{\gtrless}(X, p)\} - \tfrac{1}{2} g_v \langle F_{\mu\nu}(X)\rangle \{\gamma^\mu \gamma^\nu, G^{\gtrless}(X, p)\}, \qquad (7.9)\\ &[-\tfrac{1}{4}\partial^2 + p^2 - M^{*2}(X)]\, G^{\gtrless}(X, p)\\ &\quad = i g_s \partial_\mu \langle \phi(X)\rangle [\gamma^\mu, G^{\gtrless}(X, p)] + i g_v \langle F_{\mu\nu}(X)\rangle [\gamma^\mu \gamma^\nu, G^{\gtrless}(X, p)]. \qquad (7.10)\end{aligned}$$

To close the set of Eqs. (7.9), (7.10), the classical fields $\langle V_\mu(x)\rangle$ and $\langle \phi(X)\rangle$ should be self-consistently generated from the nucleon currents. Using Eqs. (2.2b), (2.2c), and (3.3a) one finds

$$[\partial^2 + m_v^2]\langle V_\mu(x)\rangle = -i g_v\, \mathrm{Tr}(\gamma_\mu G^<(x, x)), \qquad (7.11a)$$

$$[\partial^2 + m_s^2]\langle \phi(x)\rangle = -i g_s\, \mathrm{Tr}\, G^<(x, x). \qquad (7.11b)$$

Assuming that in the case of vanishing nucleon currents the mean fields vanish as well, these equations are solved by

$$\langle V_\mu(x)\rangle = -i g_v \int d^4x'\, D^+_{\mu\nu}(x, x')\, \mathrm{Tr}(\gamma^\nu G^<(x', x')), \qquad (7.12a)$$

$$\langle \phi(x)\rangle = i g_s \int d^4x'\, \Delta^+(x, x')\, \mathrm{Tr}\, G^<(x', x'), \qquad (7.12b)$$

where the retarded Green functions D^+ and Δ^+ are given by Eqs. (6.11b), (6.11c). Equations (7.9), (7.10), together with Eqs. (7.12), form the set of kinetic equations for the nucleon Green function in the mean field limit.

8. Perturbative Approach to the Mean Field

As discussed in, e.g., [28, 29] (see also [30]) the contour Green functions admit a perturbative expansion very similar to that known from vacuum QFT [22] with essentially the same Feynman rules. However, the time integrations do not run from $-\infty$ to $+\infty$, but along the contour shown in Fig. 1. The right turning point of the contour ($t_{\max}$) must be above the largest time argument of the evaluated Green function. In practice, t_0 is shifted to $-\infty$ and $t_{\max}$ to $+\infty$. The second difference to vacuum QFT is the appearance of tadpoles, i.e., loops formed by single lines. These give zero contribution in vacuum QFT due to the normal ordering of operators in the lagrangian. A tadpole corresponds to a Green function with two equal space–time arguments. However, the Green function $G(x, y)$ is not well defined for $x=y$, and a prescription is needed for how to perform this limit. We ascribe the function $-iG^{<}(x,x)$ to each tadpole.

It follows from Eqs. (6.6) that for noninteracting fields the functions $G^{\gtrless}(X,p)$, $D^{\gtrless}(X,p)$, and $\Delta^{\gtrless}(X,p)$ are nonzero only for on-shell four-momenta. In the case of interacting fields this is no longer true, in general. However, one expects that it is approximately true as long as the perturbative expansion is justified. Then, calculating the self-energies we are interested in their values only for on-shell momenta. As we will see below this circumstance essentially simplifies the calculations. Again, this approximation and the complications which would arise from giving it up will be further discussed in Section 12.

In this section we consider the lowest-order contributions to the self-energies, performing a perturbative expansion in the coupling constant. As we will see, these contributions correspond to the mean field effects discussed in Section 7. The Green functions which enter into the Feynman diagrams correspond, as usual, to the non-interacting fields. To simplify the notation we omit, as previously, the index "0."

In the case of the nucleon field the lowest-order contributions to self-energy, represented by the graphs shown in Fig. 2a and Fig. 2b, respectively, are

$$-i\Sigma_a(x,y) = -(-ig_v)^2\,\delta^{(4)}(x-y)\,\gamma^\mu \int_C d^4x'\, iD_{\mu\nu}(x,x')\,\mathrm{Tr}(\gamma^\nu iG^{<}(x',x')), \tag{8.1a}$$

$$-i\Sigma_b(x,y) = -(-ig_s)^2\,\delta^{(4)}(x-y) \int_C d^4x'\, i\Delta(x,x')\,\mathrm{Tr}\, iG^{<}(x',x'). \tag{8.1b}$$

The exchange or Fock graphs from Fig. 2c and Fig. 2d correspond to, respectively,

$$-i\Sigma_c(x,y) = (-ig_v)^2\,\gamma^\mu i\, D_{\mu\nu}(x,y)\,\gamma^\nu iG(x,y), \tag{8.1c}$$

$$-i\Sigma_d(x,y) = (-ig_s)^2\, i\Delta(x,y)\, iG(x,y). \tag{8.1d}$$

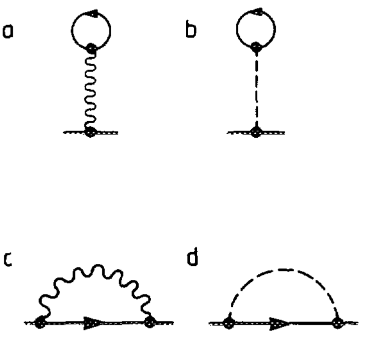

FIG. 2. The lowest-order diagrams for the nucleon self-energy. Solid lines correspond to nucleons, wavy lines to vector mesons, and dashed lines to scalar mesons.

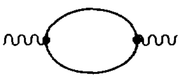

FIG. 3. The lowest-order diagram for the self-energy of the vector field.

Comparing Eqs. (8.1) with Eq. (4.5a), one finds that the contributions (8.1a) and (8.1b) give the mean-field self-energy, while those of (8.1c) and (8.1d) give the collisional self-energy as

$$\Sigma_{\mathrm{MF}}(x) = -ig_{\mathrm{v}}^2 \gamma^\mu \int_C d^4x'\, D_{\mu\nu}(x, x')\, \mathrm{Tr}(\gamma^\nu G^<(x', x'))$$

$$- ig_{\mathrm{s}}^2 \int_C d^4x'\, \Delta(x, x')\, \mathrm{Tr}\, G^<(x', x'), \tag{8.2}$$

$$\Sigma^{\gtrless}(x, y) = ig_{\mathrm{v}}^2 \gamma^\mu D_{\mu\nu}^{\gtrless}(x, y)\, \gamma^\nu G^{\gtrless}(x, y) + ig_{\mathrm{s}}^2\, \Delta^{\gtrless}(x, y)\, G^{\gtrless}(x, y). \tag{8.3}$$

One notes that the self-energy (8.3) vanishes if the functions $G^{\gtrless}$, $D^{\gtrless}$, and $\Delta^{\gtrless}$ are nonzero only on the mass shell, because this contribution corresponds to the nucleon decay into a nucleon and a meson, i.e., $N \to N + M$, which is forbidden due to energy-momentum conservation.[3] We will return to this point in Section 12. An elaborate discussion of the Fock diagram is given in [31].

Let us further discuss the mean-field self-energy (8.2). Locating the argument x on the upper branch of the contour, one finds

$$\Sigma_{\mathrm{MF}}(x) = -ig_{\mathrm{v}}^2 \gamma^\mu \int d^4x' (D_{\mu\nu}^c(x, x') - D_{\mu\nu}^>(x, x'))\, \mathrm{Tr}(\gamma^\nu G^<(x', x'))$$

$$- ig_{\mathrm{s}}^2 \int d^4x' (\Delta^c(x, x') - \Delta^>(x, x'))\, \mathrm{Tr}\, G^<(x', x'), \tag{8.4}$$

where the time integration runs from $-\infty$ to $+\infty$. One proves that an equivalent result is obtained if the argument x is located on the lower branch.

Using Eqs. (6.9), (6.11), (3.13), and (2.7) one shows that $\Delta^c - \Delta^> = \Delta^+$ and, consequently, Eq. (8.4) reduces to

$$\Sigma_{\mathrm{MF}}(x) = -ig_{\mathrm{v}}^2 \gamma^\mu \int d^4x'\, D_{\mu\nu}^+(x, x')\, \mathrm{Tr}(\gamma^\nu G^<(x', x'))$$

$$- ig_{\mathrm{s}}^2 \int d^4x'\, \Delta^+(x, x')\, \mathrm{Tr}\, G^<(x', x'). \tag{8.5}$$

[3] The decay $N \to N + M$ is allowed if the four-momentum of the meson is space-like. If in the medium mesons with space-like momenta can propagate, this process can give rise to Cherenkov radiation. For massive mesons this requires a very strong modification of the dispersion relation ($|m^* - m| > m$) which would be hard to reconcile with the perturbative approach taken here.

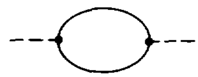

FIG. 4. The lowest-order diagram for the self-energy of the scalar field.

In the case of the meson field there are no self-energy graphs analogous to those in Figs. 2a, b, which are proportional to $\delta^{(4)}(x-y)$. Therefore, the mean-field contributions to the meson self-energies vanish. The lowest-order contributions to the collisional self-energies $P^{\gtrless}$ and $\Pi^{\gtrless}$ correspond to the graphs shown in Figs. 3 and 4. Arguments analogous to those used in the discussion of Eq. (8.3) lead us to the conclusion that these contributions vanish due to the mass-shell constraints.

Comparing Eq. (8.5) to Eqs. (7.2) and (7.12), we conclude that the lowest-order perturbative calculations exactly reproduce the results found in the pairing approximation in Section 7.

9. Higher Order Self Energies

In this section we go to the next order in the perturbative expansion. This will produce the lowest-order nonvanishing contributions to the collisional self energies.

The fourth-order contributions to the self-energies are represented by the diagrams shown in Figs. 5 and 6. (One should remember that the self-energies

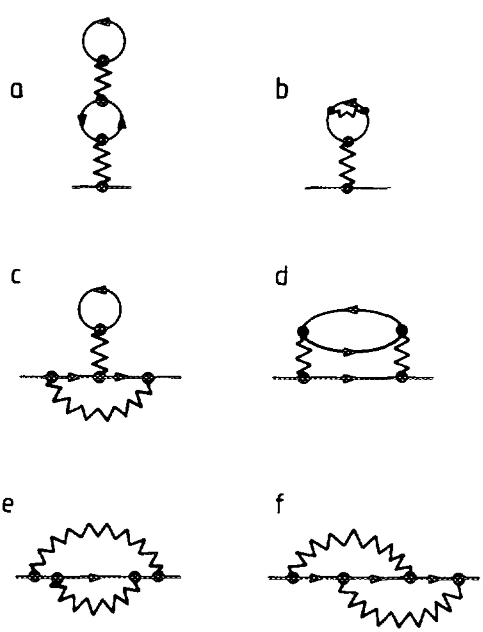

FIG. 5. The fourth-order diagrams for the nucleon self-energy. The zig-zag lines correspond to the sum of vector and scalar field contributions.

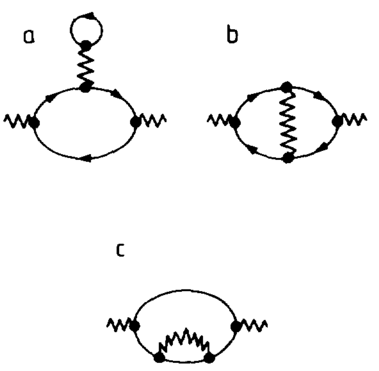

FIG. 6. The fourth-order diagrams for the self-energies of the meson fields.

defined as Eqs. (4.4) relate only to one-particle-irreducible diagrams.) Zig-zag lines correspond to the sum of scalar and vector Green functions. In nonhomogeneous systems there can occur a scalar–vector mixing due to the longitudinal component of the vector field [32]. The point is that, in general, $p^{\mu}D_{\mu\nu}(X, p) \neq 0$ (cf. Eq. (5.9)). The graph from Fig. 5a yields a contribution which only *renormalizes* [33] the mean-field self-energy given by Eq. (8.5), and we will not discuss it here. The remaining diagrams from Figs. 5 and 6, contributing to the collisional self-energy, demand a more careful analysis.

Up to now we have always calculated the contour self-energy, and then extracted the mean-field and collisional self-energies from the definition (4.5). In the case of more complicated diagrams the latter step is algebraically quite complicated. There-

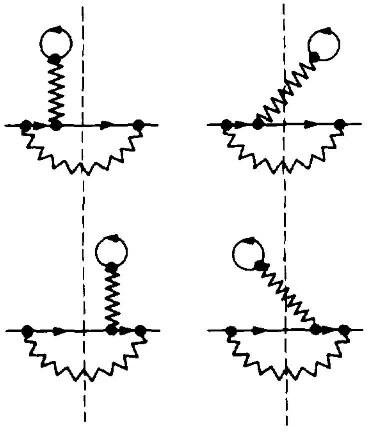

FIG. 7. The contributions to the nucleon collisional self energies corresponding to the graph in Fig. 5c.

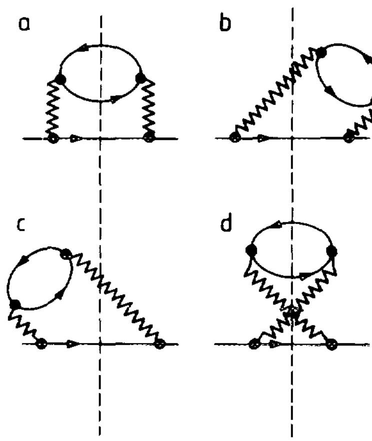

FIG. 8. The contributions to the nucleon collisional self-energies corresponding to the graph in Fig. 5d.

fore it is better to calculate the collisional self-energies directly by means of the following graphical method [28].

We draw a line dividing the plane into two parts, left and right, which will correspond to the two time branches, the left part to the chronological (upper) branch and the right part to the antichronological (lower) branch. Then we draw all topologically distinct diagrams locating the interaction vertices on both half-planes in all possible ways. For example, calculating the self-energy $\Sigma^{>}(x, y)$ related to the diagram from Fig. 5c we place the x point in the left half-plane and the y point in the right one. The remaining two vertices can be placed in four possible ways as shown in Fig. 7. By virtue of the relations (3.6)–(3.9) the lines in the diagrams are identified with the functions having the indices c, a, $>$, and $<$, according to the following rules:

1. When both end points are on the left (right) side of the plane, the line corresponds to the function with the index c (a).

2. When the start point is in the left (right) side of the plane and the end point is in the right (left) part, the line relates to the function with the index $>$ ($<$).[4]

The vertex positions are integrated over with the time integration running from $-\infty$ to $+\infty$. However, each integration corresponding to a vertex placed on the antichronological (right) half-plane is associated with a factor -1.

[4] For meson lines which do not have an orientation, the two possible assignments of start and end points yield equivalent results, due to the symmetry $D^{>}_{\mu\nu}(X, p) = D^{<}_{\nu\mu}(X, -p)$ (cf. (3.14)).

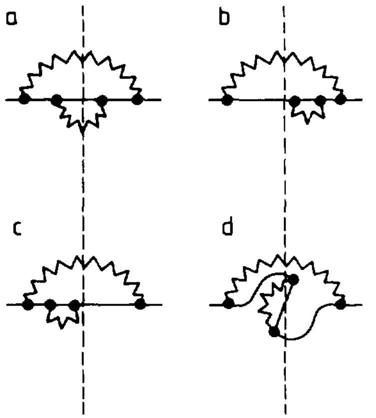

FIG. 9. The contributions to the nucleon collisional self-energies corresponding to the graph in Fig. 5e.

Because the self-energies are calculated for on-shell momenta and the Green functions with indices > and < are assumed to satisfy the mass-shell constraints (6.6), the method allows immediately to exclude those diagrams which give zero contribution due to energy-momentum conservation.

Now we can return to the analysis of the diagrams from Figs. 5 and 6. The contributions from Fig. 5c to the self-energy $\Sigma^{\gtrless}$ are shown in Fig. 7. One finds that each of them vanishes since the four-momenta of the lines crossing the plane division must always stay off-shell by kinematical reasons, but the Green functions assigned to them, according to the second rule given above, contribute only on-shell.

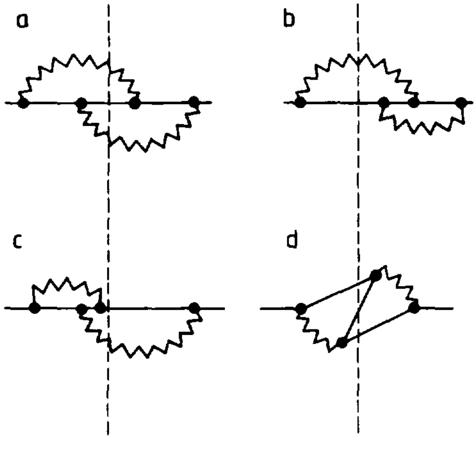

FIG. 10. The contributions to the nucleon collisional self-energies corresponding to the graph in Fig. 5f.

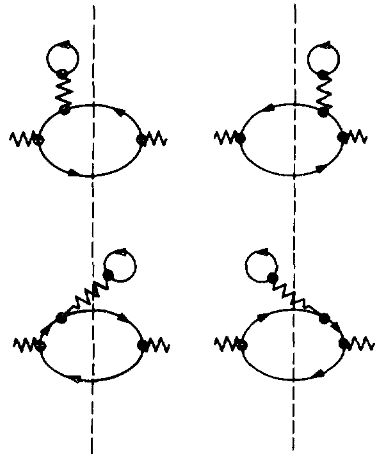

FIG. 11. The contributions to the collisional self-energies of the meson fields originating from the graph in Fig. 6a.

The contributions to the self-energy $\Sigma^{\gtrless}$ related to the graphs from Figs. 5d–f are shown in Figs. 8–10, respectively. We immediately find that among the 12 graphs only four (Figs. 8a, 9a, 10a, and 10d) give a nonzero contribution. The contributions from Fig. 6 to the self-energies $P^{\gtrless}$ and $\Pi^{\gtrless}$ are shown in Figs. 11–13. Since in Fig. 11 all lines crossing the plane carry again off-shell four-momenta, none of these diagrams contributes. Nonzero contributions come from the graphs 12a, 12b, and 13a. We do not write down the very lengthy explicit expressions corresponding to these diagrams, since this is easily done using the standard Feynman rules [22] supplemented by those presented above.

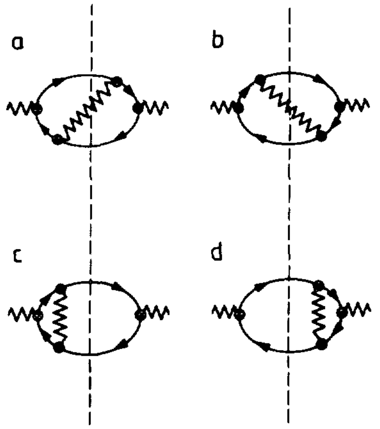

FIG. 12. The contributions to the collisional self-energies of the meson fields originating from the graph in Fig. 6b.

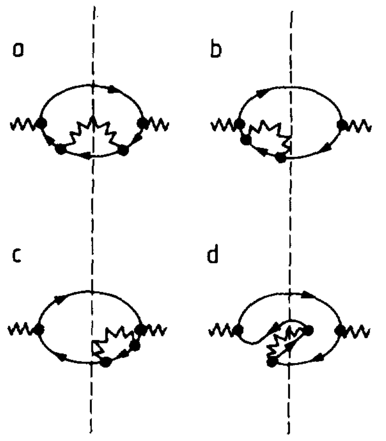

FIG. 13. The contributions to the collisional self-energies of the meson fields originating from to the graph in Fig. 6c.

It is interesting to note that if the meson exchange is replaced by an instantaneous interaction (as in non-relativistic nuclear physics), of all the diagrams in Figs. 7–13 only those in Figs. 8a and 10d survive. They correspond to nucleon–nucleon and nucleon–antinucleon scattering as shown in Figs. 14a, b. The other non-vanishing contributions from Figs. 9a, 10a, 10d, 12a, b, and 13a disappear in this limit because they correspond to the Compton-scattering diagrams, Fig. 14c, and require propagating meson fields (real mesons) which do not exist in the non-relativistic limit. This shows that a simple extrapolation from the non-relativistic case [20] yields the correct result as long as the meson fields of the Walecka model

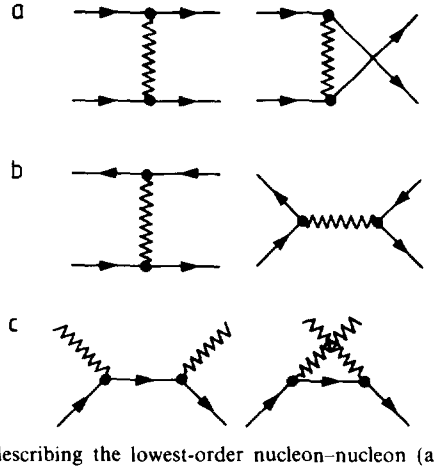

FIG. 14. The diagrams describing the lowest-order nucleon–nucleon (a), nucleon–antinucleon (b), and nucleon–meson (c) scattering processes.

are only taken into account as classical fields. However, as seen in our derivation, all this holds true only if the propagators crossing the dashed line are assumed to vanish off-shell. Any modification at this point will immediately cause many more diagrams which contribute to the collisional self-energies.

The perturbative expansion of the contour Green functions, as most perturbative expansions in field theory, suffers from the appearance of infinities. The specific divergences are due to the tadpole diagrams. We briefly discuss them in Section 10, where the distribution functions are introduced. Their renormalization is based on the *physical* argument that the tadpole contributions should vanish in the vacuum limit; i.e., in a vacuum these contributions should be compensated by counterterms. Other than tadpole divergences have not appeared explicitly in our considerations due to our *practical* attitude to the problem: The mean-field divergent diagram from Fig. 5a, which (as discussed in [33]) only *renormalizes* the lower order contribution, has not been studied here. The contributions from the remaining graphs in Figs. 5 and 6 are finite because we have imposed the mass-shell constraints. They will be discussed more explicitly in Section 12.

10. Distribution Functions

Our first goal in this section is to determine the particle dispersion relations in g^2 order of the perturbative expansion and zeroth order of the gradient one. Thus, all terms proportional to g^4 or gradients are neglected. In this limit the collisional self-energies (those with indices $<$ and $>$) are, as discussed in Section 9, zero at the mass-shell of non-interacting or weakly interacting fields. Thus, Eqs. (5.7) and (5.8) read

$$[p_\mu \gamma^\mu - M - \Sigma_{\mathrm{MF}}(X) - \Sigma^+(X,p)]\, G^{\gtrless}(X,p) = 0, \tag{10.1a}$$

$$G^{\gtrless}(X,p)[p_\mu \gamma^\mu - M - \Sigma_{\mathrm{MF}}(X) - \Sigma^-(X,p)] = 0, \tag{10.2a}$$

$$[-p^2 + m_v^2 + P_{\mathrm{MF}}(X) + P^+(X,p)]\, D^{\gtrless}(X,p) = 0, \tag{10.1b}$$

$$D^{\gtrless}(X,p)[-p^2 + m_v^2 + P_{\mathrm{MF}}(X) + P^-(X,p)] = 0, \tag{10.2b}$$

$$[-p^2 + m_s^2 - \Pi_{\mathrm{MF}}(X) - \Pi^+(X,p)]\, \Delta^{\gtrless}(X,p) = 0, \tag{10.1c}$$

$$[-p^2 + m_s^2 - \Pi_{\mathrm{MF}}(X) - \Pi^-(X,p)]\, \Delta^{\gtrless}(X,p) = 0. \tag{10.2c}$$

For the collisional self-energies, which vanish at the mass-shell, Eq. (5.12) tells us that the retarded (+) and advanced (−) self-energies are equal to each other at the mass-shell. Furthermore, Eq. (5.13) implies

$$A^+(X,p) + A^-(X,p) = \frac{1}{\pi i} P \int d\omega' \, \frac{A^>(X,\omega',\mathbf{p}) - A^<(X,\omega',\mathbf{p})}{\omega - \omega'}, \tag{10.3}$$

for any of the self-energies ($A = \Sigma, P, \Pi$). We see that the vanishing of the collisional self-energies at the mass-shell causes $A^+(X, p) = A^-(X, p)$ to be *small* there since the function under the integral (10.3) is zero for ω' which is close to ω. Thus, the retarded or advanced self-energies are neglected in (10.1), (10.2).

Equations (10.1), (10.2) provide the dispersion relations for quasi-particles and quasi-antiparticles, including mean-field effects. From (7.2) we know that the nucleon self-energy in the mean-field limit has the spinor structure

$$\Sigma_{\mathrm{MF}}(X) = \Sigma_s(X) + \gamma_\mu \Sigma_v^\mu(X). \tag{10.4}$$

The analogous formula for the vector field self-energy is

$$P_{\mathrm{MF}}^{\mu\nu}(X) = \left(g^{\mu\nu} - \frac{p^\mu p^\nu}{m_{\mathrm{v}}^2}\right) P(X). \tag{10.5}$$

Actually, the self-energy (10.5) vanishes exactly in the mean-field limit, and we keep it only for methodological reasons.

The dispersion relations thus read

$$\det[\gamma_\mu(p^\mu - \Sigma_v^\mu(X)) - (M + \Sigma_s(X))] = 0, \tag{10.6a}$$

$$\det\left[(-p^2 + m_{\mathrm{v}}^2 + P(X))\, g^{\mu\nu} - \frac{p^\mu p^\nu}{m_{\mathrm{v}}^2} P(X)\right] = 0, \tag{10.6b}$$

$$-p^2 + m_{\mathrm{s}}^2 - \Pi_{\mathrm{MF}}(X, p) = 0. \tag{10.6c}$$

For the nucleons, it is again convenient to introduce the kinetic momentum

$$p^{*\mu} = p^\mu - \Sigma_v^\mu(X), \tag{10.7}$$

in terms of which the nucleon dispersion relation reads

$$\det[\gamma_\mu p^{*\mu} - M^*(X)] = 0, \tag{10.8}$$

where the effective mass is given by

$$M^*(X) = M + \Sigma_s(X). \tag{10.9a}$$

Defining also effective vector and scalar meson masses by

$$m_{\mathrm{v}}^{*2}(X) = m_{\mathrm{v}}^2 + P(X), \tag{10.9b}$$

$$m_{\mathrm{s}}^{*2}(X) = m_{\mathrm{s}}^2 - \Pi_{\mathrm{MF}}(X), \tag{10.9c}$$

and evaluating the determinant of the matrix $(\gamma_\mu p^{*\mu} - M^*(X))$ according to Appendix 1, we obtain

$$(p^{*2} - M^{*2})^2 = 0, \tag{10.10a}$$

$$(p^2 - m_{\mathrm{v}}^{*2})^3\, (p^2 - m_{\mathrm{v}}^2) = 0, \tag{10.10b}$$

$$p^2 - m_{\mathrm{s}}^{*2} = 0. \tag{10.10c}$$

In each case we find pairs of solutions with positive and negative frequencies, which we assign to particles and antiparticles, respectively,

$$p_0^* = \pm \omega_{p^*} \equiv \pm \sqrt{\mathbf{p}^{*2} + M^{*2}}. \tag{10.11a}$$

We distinguish between the *frequencies* ω_{p^*}, which satisfy "usual" mass-shell constraints $\omega_{p^*}^2 = \mathbf{p}^{*2} + M^{*2}$, and *energies* $E_p = \sqrt{\mathbf{p}^{*2} + M^{*2}} + \Sigma_v^0$, $-\bar{E}_p = -\sqrt{\mathbf{p}^{*2} + M^{*2}} + \Sigma_v^0$. While the frequencies are much more convenient to work with, and in particular their sign serves to uniquely separate particles from antiparticles, the physical energy of the total system is more directly given in terms of energies, which contain the influence of the vector repulsion. This will be discussed in connection with the energy-momentum tensor at the end of this section. For the meson fields such a distinction is not necessary, and we find as particle and antiparticle solutions,

$$p_0 = \pm E_p = \pm \sqrt{\mathbf{p}^2 + m_i^{*2}}, \qquad i = \text{v, s}. \tag{10.11b}$$

The duplication of both positive and negative frequency solutions of (10.10a) is due to the two possible helicity states in each case, and we see that the dispersion relation is identical for both polarizations within the zeroth order of the gradient expansion. There are two types of solutions of Eq. (10.10b). However, it can be easily shown that the solution $p^2 - m_\text{v}^2 = 0$ corresponds to the *unphysical* time-like vector mesons. Let us observe that in the zeroth order of the gradient expansion the *magnetic* part of the vector-meson field, which is responsible for the interaction with particle spins, vanishes. Thus it is not surprising that we have obtained helicity-independent dispersion relations.

We can now introduce on-shell distribution functions of particles and antiparticles through the positive and negative frequency components of the Green functions, respectively, by writing (repeated spin indices r, s are always summed over)

$$\begin{aligned} \Theta(p_0^*)\, iG_{\alpha\beta}^<(X, p^*) &= -\Theta(p_0^*)\, 2\pi\delta(p^{*2} - M^{*2}(X))\, 2M^*(X) \\ &\quad \times u_\alpha(r, p^*)\, \bar{u}_\beta(s, p^*)\, f_N^{rs}(X, p^*) \\ &= -\frac{\pi}{\omega_{p^*}}\, \delta(\omega_{p^*} - p_0^*)\, 2M^*(X)\, u_\alpha(r, p^*)\, \bar{u}_\beta(s, p^*)\, f_N^{rs}(X, p^*), \end{aligned} \tag{10.12a}$$

$$\begin{aligned} \Theta(p_0)\, iD_{\mu\nu}^<(X, p) &= -\Theta(p_0)\, 2\pi\delta(p^2 - m_\text{v}^{*2}(X))\, \varepsilon_\mu(r, p)\, \varepsilon_\nu(s, p)\, f_\text{v}^{rs}(X, p) \\ &= -\frac{\pi}{E_p}\, \delta(E_p - p_0)\, \varepsilon_\mu(r, p)\, \varepsilon_\nu(s, p)\, f_\text{v}^{rs}(X, p), \end{aligned} \tag{10.12b}$$

$$\begin{aligned} \Theta(p_0)\, i\Delta^<(X, p) &= \Theta(p_0)\, 2\pi\delta(p^2 - m_\text{s}^{*2}(X))\, f_\text{s}(X, p) \\ &= \frac{\pi}{E_p}\, \delta(E_p - p_0)\, f_\text{s}(X, p), \end{aligned} \tag{10.12c}$$

and (please note the order of the spin indices in the nucleon case)

$$\begin{aligned}\Theta(-p_0^*)\, iG^>_{\alpha\beta}(X, p^*) &= \Theta(-p_0^*)\, 2\pi\delta(p^{*2} - M^{*2}(X))\, 2M^*(X) \\ &\quad \times v_\alpha(s, -p^*)\, \bar{v}_\beta(r, -p^*)\, \bar{f}^{rs}_N(X, -p^*) \\ &= \frac{\pi}{\omega_{p^*}}\, \delta(\omega_{p^*} + p_0^*)\, 2M^*(X)\, v_\alpha(s, -p^*)\, \bar{v}_\beta(r, -p^*)\, \bar{f}^{rs}_N(X, -p^*),\end{aligned} \tag{10.13a}$$

$$\begin{aligned}\Theta(-p_0)\, iD^>_{\mu\nu}(X, p) &= -\Theta(p_0)\, 2\pi\delta(p^2 - m_v^{*2}(X))\, \varepsilon_\mu(s, -p)\, \varepsilon_\nu(r, -p)\, \bar{f}^{rs}_v(X, -p) \\ &= -\frac{\pi}{E_p}\, \delta(E_p + p_0)\, \varepsilon_\mu(s, -p)\, \varepsilon_\nu(r, -p)\, \bar{f}^{rs}_v(X, -p),\end{aligned} \tag{10.13b}$$

$$\begin{aligned}\Theta(-p_0)\, i\Delta^>(X, p) &= \Theta(-p_0)\, 2\pi\delta(p^2 - m_s^{*2}(X))\, \bar{f}_s(X, -p) \\ &= \frac{\pi}{E_p}\, \delta(E_p + p_0)\, \bar{f}_s(X, -p).\end{aligned} \tag{10.13c}$$

Here $f^{rs}_N(X, p^*)$ $(\bar{f}^{rs}_N(X, p^*))$ is the (anti-)nucleon distribution function, with the indices r, s labelling the spin states; $u(s, p)$ and $v(s, p)$ are the Dirac spinors which satisfy

$$(\gamma^\mu p^*_\mu - M^*(X))\, u(s, p^*) = 0,$$

$$(\gamma^\mu p^*_\mu + M^*(X))\, v(s, p^*) = 0,$$

and they are normalized by the condition

$$\bar{u}_\alpha(s, p)\, u_\alpha(r, p) = -\bar{v}_\alpha(s, p)\, v_\alpha(r, p) = \delta^r_s;$$

$f^{rs}_v(X, p)$ $(\bar{f}^{rs}_v(X, p))$ is the distribution function of vector (anti-)mesons; $\varepsilon_\mu(s, p)$ is the polarization vector, which satisfies the transversality condition

$$p^\mu \varepsilon_\mu(s, p) = 0$$

and is normalized as

$$\varepsilon_\mu(s, p)\, \varepsilon^\mu(r, p) = -\delta^{rs};$$

$f_s(X, p)$ $(\bar{f}_s(X, p))$ is the distribution function of scalar (anti-)mesons. Obviously the distribution functions of nucleons and vector mesons are matrices in spin space. Since these functions are defined only for on-shell momenta (the four-momenta satisfy the dispersion equations (10.11)), it is essential that the dispersion relations are independent of the particle polarization. Otherwise, the off-diagonal elements of the distribution functions would be meaningless because they correspond to mixed polarization states which would not have a well-defined dispersion relation.

Please note that in the distribution functions f, spinors u, v, and polarization vectors ε, the four-momentum always satisfies the mass-shell constraint (10.11) and that these functions thus depend only on $\mathbf{p}$. Consequently, derivatives with respect to p_0 in the kinetic equations act only on the mass-shell δ-functions.

Due to the definitions (10.12), (10.13) the positive frequency ($p_0 > 0$) parts of $G^<$, $D^<$, and $\Delta^<$ can be expressed through the particle distribution functions, while the negative frequency parts of $G^>$, $D^>$, and $\Delta^>$ are expressed in terms of the antiparticle distribution functions. We will now derive relations which allow us to express both types of Green functions for positive and negative energies through the distribution functions for particles and antiparticles.

As discussed in Section 6, for noninteracting fields $G^> - G^<$, $D^> - D^<$, and $\Delta^> - \Delta^<$ are given by the functions (2.7). Our task is to derive analogous relations for interacting fields. Since these will be based on the relations (6.13), let us write the equations for the retarded and advanced Green functions. To lowest order in the gradient expansion Eqs. (4.9) yield

$$[p_\mu \gamma^\mu - M - \Sigma_{\rm MF}(X)]\, G^\pm(X, p) = 1, \tag{10.14a}$$

$$([-p^2 + m_{\rm v}^2 + P_{\rm MF}(X)]\, D^\pm(X, p))^{\mu\nu} = g^{\mu\nu} - \frac{p^\mu p^\nu}{m_{\rm v}^2}, \tag{10.14b}$$

$$[-p^2 + m_{\rm s}^2 - \Pi_{\rm MF}(X)]\, \Delta^\pm(X, p) = 1. \tag{10.14c}$$

This immediately provides

$$G^\pm(X, p^*) = \frac{p^{*\mu}\gamma_\mu - M^*}{p^{*2} - M^{*2} \pm ip_0 0^+}, \tag{10.15a}$$

$$D^\pm_{\mu\nu}(X, p) = \frac{-g_{\mu\nu} + p_\mu p_\nu / m_{\rm v}^{*2}}{p^2 - m_{\rm v}^{*2} \pm ip_0 0^+}, \tag{10.15b}$$

$$\Delta^\pm(X, p) = \frac{1}{p^2 - m_{\rm s}^{*2} \pm ip_0 0^+}, \tag{10.15c}$$

where $m_i^* \equiv m_i^*(X)$, $i = N, {\rm v}, {\rm s}$, and the infinitesimal imaginary terms are the remnants of ${\rm Im}(\Sigma^\pm \gamma^0)$, ${\rm Im}\, P^\pm$, and ${\rm Im}\, \Pi^\pm$, respectively. We have used Eqs. (10.4) to find Eqs. (10.14a), (10.14b). The final form of Eq. (10.15b) has been obtained by means of the transversality condition, which in zeroth order of the gradient expansion is $p^\mu D_{\mu\nu} = D_{\mu\nu} p^\nu = 0$.

The explicit expressions (10.15) can now be used with Eq. (6.13) to write the required relations (again $m_i^* \equiv m_i^*(X)$):

$$iG^>(X, p^*) - iG^<(X, p^*)$$
$$= (\gamma^\mu p_\mu^* + M^*)\, 2\pi\delta(p^{*2} - M^{*2})(\Theta(p_0^*) - \Theta(-p_0^*)), \tag{10.16a}$$

$$iD^>_{\mu\nu}(X, p) - iD^<_{\mu\nu}(X, p)$$
$$= \left(g_{\mu\nu} - \frac{p_\mu p_\nu}{m_{\rm v}^{*2}}\right) 2\pi\delta(p^2 - m_{\rm v}^{*2})(\Theta(p_0) - \Theta(-p_0)), \tag{10.16b}$$

$$i\Delta^>(X, p) - i\Delta^<(X, p)$$
$$= 2\pi\delta(p^2 - m_{\rm s}^{*2})(\Theta(p_0) - \Theta(-p_0)), \tag{10.16c}$$

valid in zeroth order in the gradient expansion (cf. Eqs. (3.13)). Since the left-hand sides denote the spectral functions for the respective particles, Eqs. (10.16) simply mean that at this order the system consists of quasiparticles with dispersion relations $p^{*2}=m^{*2}$, showing the consistency of the definition of on-shell distribution functions.

By virtue of the relation (10.16), one finds

$$\Theta(p_0^*)\, iG^{>}_{\alpha\beta}(X,p^*) = -\frac{\pi}{\omega_{p^*}}\,\delta(\omega_{p^*}-p_0^*)\,2M^*(X) \times u_\alpha(r,p^*)\,\bar{u}_\beta(s,p^*)[f^{rs}_N(X,p^*)-\delta^{rs}], \tag{10.17a}$$

$$\Theta(p_0)\, iD^{>}_{\mu\nu}(X,p) = -\frac{\pi}{E_p}\,\delta(E_p-p_0)\,\varepsilon_\mu(r,p)\,\varepsilon_\nu(s,p)[f^{rs}_v(X,p)+\delta^{rs}], \tag{10.17b}$$

$$\Theta(p_0)\, i\Delta^{>}(X,p) = \frac{\pi}{E_p}\,\delta(E_p-p_0)[f_s(X,p)+1], \tag{10.17c}$$

and

$$\Theta(-p_0^*)\, iG^{<}_{\alpha\beta}(X,p^*) = \frac{\pi}{\omega_{p^*}}\,\delta(\omega_{p^*}+p_0^*)\,2M^*(X)\,v_\alpha(s,-p^*) \times \bar{v}_\beta(r,-p^*)[\bar{f}^{rs}_N(X,-p^*)-\delta^{rs}], \tag{10.18a}$$

$$\Theta(-p_0)\, iD^{<}_{\mu\nu}(X,p) = -\frac{\pi}{E_p}\,\delta(E_p+p_0)\,\varepsilon_\mu(s,-p) \times \varepsilon_\nu(r,-p)[\bar{f}^{rs}_v(X,-p)+\delta^{rs}], \tag{10.18b}$$

$$\Theta(-p_0)\, i\Delta^{<}(X,p) = \frac{\pi}{E_p}\,\delta(E_p+p_0)[\bar{f}_s(X,-p)+1], \tag{10.18c}$$

where we have used the identities [22]

$$u_\alpha(s,p^*)\,\bar{u}_\beta(s,p^*) = \left(\frac{p^{*\mu}\gamma_\mu + M^*(X)}{2M^*(X)}\right)_{\alpha\beta},$$

$$v_\alpha(s,p^*)\,\bar{v}_\beta(s,p^*) = \left(\frac{p^{*\mu}\gamma_\mu - M^*(X)}{2M^*(X)}\right)_{\alpha\beta},$$

$$\varepsilon_\mu(s,p)\,\varepsilon_\nu(s,p) = g_{\mu\nu} - \frac{p_\mu p_\nu}{m_v^{*2}}.$$

In the case of (real) vector and scalar fields, particles and antiparticles are indistinguishable, and thus, the distribution functions of particles and antiparticles must coincide. We will prove now that our definitions of the distribution functions

(10.12), (10.13) indeed satisfy this requirement. This property is expressed by the relations (3.14), which for the Wigner transformed functions are

$$D^{>}_{\mu\nu}(X, p) = D^{<}_{\nu\mu}(X, -p), \tag{10.19a}$$

$$\Delta^{>}(X, p) = \Delta^{<}(X, -p). \tag{10.19b}$$

With the help of (10.18), we obtain from Eqs. (10.12), (10.13), (10.17), (10.18) the expected result

$$f^{rs}_{\rm v}(X, p) = \bar{f}^{rs}_{\rm v}(X, p),$$
$$f_{\rm s}(X, p) = \bar{f}_{\rm s}(X, p).$$

In fact, the definitions (10.12), and (10.13) have been constructed in a way to satisfy the above relations.

Equations (10.12), (10.13), together with (10.17), (10.18), can now be used to express the Green functions $G^{\gtrless}$, $D^{\gtrless}$, and $\Delta^{\gtrless}$ for both positive and negative energies in terms of the particle and antiparticle distribution functions. Combining (10.18) with (10.12) we find the important relations:

$$\begin{aligned} iG^{<}_{\alpha\beta}(X, p^*) = &-\frac{\pi}{\omega_{p^*}}\,\delta(\omega_{p^*} - p_0^*)\,2M^*(X)\,u_\alpha(r, p^*)\,\bar{u}_\beta(s, p^*)\,f^{rs}_N(X, p^*) \\ &+\frac{\pi}{\omega_{p^*}}\,\delta(\omega_{p^*} + p_0^*)\,2M^*(X)\,v_\alpha(s, -p^*) \\ &\times \bar{v}_\beta(r, -p^*)[\bar{f}^{rs}_N(X, -p^*) - \delta^{rs}], \end{aligned} \tag{10.20a}$$

$$\begin{aligned} iD^{<}_{\mu\nu}(X, p) = &-\frac{\pi}{E_p}\,\delta(E_p - p_0)\,\varepsilon_\mu(r, p)\,\varepsilon_\nu(s, p)\,f^{rs}_{\rm v}(X, p) \\ &-\frac{\pi}{E_p}\,\delta(E_p + p_0)\,\varepsilon_\mu(s, -p)\,\varepsilon_\nu(r, -p)[f^{rs}_{\rm v}(X, -p) + \delta^{rs}], \end{aligned} \tag{10.20b}$$

$$i\Delta^{<}(X, p) = \frac{\pi}{E_p}\,\delta(E_p - p_0)\,f_s(X, p) + \frac{\pi}{E_p}\,\delta(E_p + p_0)[f_s(X, -p) + 1]. \tag{10.20c}$$

Similarly, Eqs. (10.17) and (10.13) combine to give

$$\begin{aligned} iG^{>}_{\alpha\beta}(X, p^*) = &-\frac{\pi}{\omega_{p^*}}\,\delta(\omega_{p^*} - p_0^*)\,2M^*(X)\,u_\alpha(r, p^*)\,\bar{u}_\beta(s, p^*)[f^{rs}_N(X, p^*) - \delta^{rs}] \\ &+\frac{\pi}{\omega_{p^*}}\,\delta(\omega_{p^*} + p_0^*)\,2M^*(X)\,v_\alpha(s, -p^*)\,\bar{v}_\beta(r, -p^*)\,\bar{f}^{rs}_N(X, -p^*). \end{aligned} \tag{10.21a}$$

$$iD^{>}_{\mu\nu}(X,p) = -\frac{\pi}{E_p}\,\delta(E_p - p_0)\,\varepsilon_\mu(r,p)\,\varepsilon_\nu(s,p)[f^{rs}_{\rm v}(X,p) + \delta^{rs}]$$

$$-\frac{\pi}{E_p}\,\delta(E_p + p_0)\,\varepsilon_\mu(s,-p)\,\varepsilon_\nu(r,-p)\,f^{rs}_{\rm v}(X,-p), \tag{10.21b}$$

$$i\Delta^{>}(X,p) = \frac{\pi}{E_p}\,\delta(E_p - p_0)[f_s(X,p) + 1] + \frac{\pi}{E_p}\,\delta(E_p + p_0)\,f_s(X,-p). \tag{10.21c}$$

We can now insert this into the definition (3.17) for the current and find

$$\langle j^\mu_b(X)\rangle = \int \frac{d^3p^*}{(2\pi)^3}\,\frac{p^{*\mu}}{\omega_{p^*}}\,[f^{ss}_N(X,p^*) - \bar{f}^{ss}_N(X,p) + 2], \tag{10.22}$$

using the equality [22]

$$\bar{u}(s,p^*)\,\gamma^\mu u(r,p^*) = \bar{v}(s,p^*)\,\gamma^\mu v(r,p^*) = \frac{p^{*\mu}}{M^*}\,\delta^{rs}.$$

The integral in (10.22) is divergent and in the vacuum limit $(f_N(X,p), \bar{f}_N(X,p) \to 0)$ gives

$$2\int \frac{d^3p}{(2\pi)^3}\,\frac{p^\mu}{\omega_p} = 2\int \frac{d^3p^*}{(2\pi)^3}\,\frac{p^{*\mu}}{\omega_{p^*}}.$$

This type of divergence, which also appears in the tadpole contributions, is well known in field theory. In the case of vacuum QFT, it is regularized away by the normal ordering prescription for the field operators defining the lagrangian density [22]. Subtracting the vacuum value from the right-hand side of Eq. (10.22), the current equals

$$\langle j^\mu_b(X)\rangle = \int \frac{d^3p^*}{(2\pi)^3}\,\frac{p^{*\mu}}{\omega_{p^*}}\,[f^{ss}_N(X,p^*) - \bar{f}^{ss}_N(X,p^*)]. \tag{10.23}$$

A similar subtraction is needed for the energy-momentum tensor (2.5). The finite result reads

$$\begin{aligned}\langle T^{\mu\nu}(X)\rangle = &\int \frac{d^3p^*}{(2\pi)^3}\,\frac{p^{*\mu}p^{*\nu}}{\omega_{p^*}}\,[f^{ss}_N(X,p^*) + \bar{f}^{ss}_N(X,p^*)]\\ &+ \Sigma^\mu_v(X)\langle j^\nu_b(X)\rangle - \frac{1}{2}\,g^{\mu\nu}(\Sigma_s(X)\,\rho_s(X) + \Sigma^\sigma_v(X)\langle j_{b\,\sigma}(X)\rangle)\\ &+ \int \frac{d^3p}{(2\pi)^3\,E_p}\,p^\mu p^\nu f^{ss}_{\rm v}(X,p) + \int \frac{d^3p}{(2\pi)^3\,E_p}\,p^\mu p^\nu f_s(X,p)\\ &- \frac{1}{2}\,\langle V^\sigma(X)\rangle\,\overleftrightarrow{\partial}^\mu\,\overleftrightarrow{\partial}^\nu\langle V_\sigma(X)\rangle - \frac{1}{4}\,\langle\phi(X)\rangle\,\overleftrightarrow{\partial}^\mu\,\overleftrightarrow{\partial}^\nu\langle\phi(X)\rangle,\end{aligned} \tag{10.24}$$

where ρ_s is the so-called scalar density defined as

$$\rho_s(X) = \int \frac{d^3p^*}{(2\pi)^3} \frac{M^*(X)}{\omega_{p^*}} [f_N^{ss}(X, p^*) + \bar{f}_N^{ss}(X, p)].$$

Since the distribution functions are defined within the mean field limit, the interaction terms from Eq. (2.5) have been calculated also in this limit (cf. Sections 7, 8). The last two terms in (10.24) describe the energy-momentum tensor of the *classical* fields, i.e., the fields which appear due to nonvanishing expectation values of the mesonic fields.

Let us also write the energy density of the system,

$$\begin{aligned}\langle T^{00}(X)\rangle &= \int \frac{d^3p^*}{(2\pi)^3} [E_{p^*} f_N^{ss}(X, p^*) + \bar{E}_{p^*} \bar{f}_N^{ss}(X, p^*)] \\ &\quad - \frac{1}{2} (\Sigma_s(X)\, \rho_s(X) + \Sigma_v^\sigma(X) \langle j_{b\,\sigma}(X)\rangle) \\ &\quad + \int \frac{d^3p}{(2\pi)^3} E_p f_v^{ss}(X, p) + \int \frac{d^3p}{(2\pi)^3} E_p f_s(X, p) \\ &\quad - \frac{1}{2} \langle V^\sigma(X)\rangle \overleftarrow{\partial}^0 \vec{\partial}^0 \langle V_\sigma(X)\rangle - \frac{1}{4} \langle \phi(X)\rangle \overleftarrow{\partial}^0 \vec{\partial}^0 \langle \phi(X)\rangle, \end{aligned} \qquad (10.25)$$

where the energies of the nucleon and the antinucleon are $E_{p^*} = \sqrt{M^{*2} + \mathbf{p}^{*2}} + \Sigma_v^0$ and $\bar{E}_p = \sqrt{M^{*2} + \mathbf{p}^{*2}} - \Sigma_v^0$, respectively. One sees that the energy of the system is expressed more naturally in terms of (anti-)nucleon energies E_{p^*} ($\bar{E}_{p^*}$) than frequencies ω_{p^*}. Equations (10.22), (10.25) also show that our normalization of the distribution functions defined in (10.12), (10.13) coincides with the conventional one.

11. Transport Equations

In this section we will make the final step in the derivation of the transport equations to be satisfied by the distribution functions. Specifically, we will substitute the Green functions in the form (10.20), (10.21) into the Green function transport equations (5.10c) and (5.14) found in Section 5. Before this, however, we still have to manipulate the Green function transport equation (5.14a).

Changing the momentum variable $p^\mu \to p^\mu - \Sigma_v^\mu(X)$ ($\partial^\mu \to \partial^\mu - \partial^\mu \Sigma_{v\,\nu}(X)\, \partial_p^\nu$), the Green function transport equation (5.14a) is rewritten as[5]

[5] To simplify the notation in this section we skip the index $*$ labeling the kinetic momentum equal $p^\mu - \Sigma_v^\mu(X)$ and identify E with ω_{p^*}.

$$\frac{i}{2}\{\gamma^\mu, \partial_\mu G^{\gtrless}(X,p)\} + \frac{i}{2}\{U_\mu(X), \partial_p^\mu G^{\gtrless}(X,p)\}$$
$$+ [p_\mu \gamma^\mu, G^{\gtrless}(X,p)] - [\Sigma_{\mathrm{MF}}(X), G^{\gtrless}(X,p)]$$
$$= \frac{1}{2}\{\Sigma^>(X,p), G^<(X,p)\} - \frac{1}{2}\{\Sigma^<(X,p), G^>(X,p)\}, \tag{11.1}$$

where the force $U_\mu(X)$ is

$$U^\mu(X) \equiv \partial^\mu \Sigma_s(X) - \gamma_\nu \mathscr{F}^{\nu\mu}(X) = \partial^\mu M^*(X) - \gamma_\nu \mathscr{F}^{\nu\mu}(X),$$

with

$$\mathscr{F}^{\mu\nu}(X) \equiv \partial^\mu \Sigma_v^\nu(X) - \partial^\nu \Sigma_v^\mu(X).$$

Σ_s and Σ_v^μ are the scalar and vector parts, respectively, of the mean-field self-energy (cf. Eqs. (10.4) and (10.9)). Anticipating the substitution of on-shell distribution functions (10.20a), (10.21a) below, we have neglected in Eq. (11.1) the terms $[\Sigma^{\gtrless}(X,p), G_R^+(X,p)]$, $[\Sigma_R^-(X,p), G^{\gtrless}(X,p)]$ which are present in Eq. (5.14a). The point is that $G_R^+(X,p)$ $(\Sigma_R^-(X,p))$, which is defined as the second term of the r.h.s. of Eq. (5.13), represents only the off-mass-shell part of $G^+(X,p)$ $(\Sigma_R^-(X,p))$, while their on-shell part is given by the first term in (5.13) (cf. Eq. (10.15)). This separation into on-shell and off-shell contributions is equivalent to the separation of the imaginary and real parts of Eqs. (10.15).

The collisional self-energies "mix" the positive and negative energy parts of the Green functions $G^{\gtrless}$ due to particle–antiparticle collisions. However, the transport equations split into separate equations for particles and antiparticles, which are coupled to each other via the collision terms.

We now substitute the Green functions in the forms (10.20a), (10.21a) into Eq. (11.1) and "sandwich" this equation between spinors u and $\bar{u}$ and v and $\bar{v}$ for the positive and negative energy parts, respectively. Due to the δ-functions present in (10.20) and (10.21) one must carefully distinguish between on- and off-shell four-momenta. In particular, the four-momentum which appears as $p^\mu = 2M^*\bar{u}(s,p)\,\gamma^\mu u(s,p)$ is on-mass-shell, but the four-momentum which is the argument of the δ-function is off-shell, and its four components are independent from each other. To remove the δ-functions and their derivatives from the final transport equations one integrates them over p_0, or equivalently over p^2.

In order to avoid these rather cumbersome manipulations of the δ-functions and the distinction between on- and off-shell momenta, one can alternatively introduce Green functions obtained from (10.20) and (10.21) by integration over p_0. These functions satisfy an equation that is nearly identical to (11.1), but with ∂_p^μ replaced by $(0, \nabla_p)$. Choosing one or the other procedure one arrives after rather lengthy calculations at the transport equations

$$p^\mu \partial_\mu f_N + (M^* \partial_\mu M^* - p^\nu \mathscr{F}_{\nu\mu})\, \partial_p^\mu f_N + \tfrac{1}{2}\,[\mathscr{A}, f_N]$$
$$= \tfrac{1}{2}\,\{\mathscr{T}^{>}, f_N\} - \tfrac{1}{2}\,\{\mathscr{T}^{<}, f_N - 1\}, \tag{11.2}$$
$$p^\mu \partial_\mu \bar{f}_N + (M^* \partial_\mu M^* + p^\nu \mathscr{F}_{\nu\mu})\, \partial_p^\mu \bar{f}_N + \tfrac{1}{2}\,[\bar{\mathscr{A}}, \bar{f}_N]$$
$$= \tfrac{1}{2}\,\{\bar{\mathscr{T}}^{<}, \bar{f}_N\} - \tfrac{1}{2}\,\{\bar{\mathscr{T}}^{>}, \bar{f}_N - 1\}, \tag{11.3}$$

where

$$\mathscr{T}_{rs}^{\gtrless}(X, p) \equiv -iM^*(X)\, \bar{u}(r, p)\, \Sigma^{\gtrless}(X, p)\, u(s, p), \tag{11.4a}$$
$$\bar{\mathscr{T}}_{rs}^{\gtrless}(X, p) \equiv -iM^*(X)\, \bar{v}(r, p)\, \Sigma^{\gtrless}(X, -p)\, v(s, p). \tag{11.5a}$$

The matrices $\mathscr{A}_{rs}$ and $\bar{\mathscr{A}}_{rs}$, which are responsible for the evolution of the spin degrees of freedom due to the mean field, are the antihermitian parts of $\mathscr{A}'_{rs}$ and $\bar{\mathscr{A}}'_{rs}$, respectively, $(\mathscr{A} = \frac{1}{2}(\mathscr{A}' - \mathscr{A}'^{\dagger}))$ with

$$\mathscr{A}'_{rs}(X, p) \equiv M^*(X)\, \bar{u}(r, p) \left(\gamma^\mu \partial_\mu + \left(2\partial_\mu M^* - \left(\gamma^\nu + \frac{p^\nu}{M^*} \right) \mathscr{F}_{\nu\mu} \right) \partial_p^\mu \right) u(s, p)$$
$$+ \frac{1}{M^*}\, p^\mu \partial_\mu M^* \delta^{rs}, \tag{11.4b}$$

$$\bar{\mathscr{A}}'_{rs}(X, p) \equiv M^*(X)\, \bar{v}(r, p) \left(\gamma^\mu \partial_\mu - \left(2\partial_\mu M^* + \left(\gamma^\nu + \frac{p^\nu}{M^*} \right) \mathscr{F}_{\nu\mu} \right) \partial_p^\mu \right) v(s, p)$$
$$+ \frac{1}{M^*}\, p^\mu \partial_\mu M^* \delta^{rs}. \tag{11.5b}$$

To derive Eqs. (11.4b) and (11.5b) one needs to observe that the spinors $u(p)$ and $v(p)$ depend on X only through $M^*(X)$ and that

$$\bar{u}(r, p) \frac{\partial}{\partial M^*} u(s, p) = \bar{v}(r, p) \frac{\partial}{\partial M^*} v(s, p) = 0,$$

which is proved in Appendix 2. In fact, the direct calculation, which leads from Eq. (11.1) to Eqs. (11.2) and (11.3), provides the terms $(\mathscr{A}' f_N + f_N \mathscr{A}'^{\dagger})$ and $(\bar{\mathscr{A}}' \bar{f}_N + \bar{f}_N \bar{\mathscr{A}}'^{\dagger})$ instead of $[\mathscr{A}, f_N]$ and $[\bar{\mathscr{A}}, \bar{f}_N]$, respectively, which are written in Eqs. (11.2) and (11.3). The reasons why the hermitian parts of matrices $\mathscr{A}'$ and $\bar{\mathscr{A}}'$ have to be neglected are carefully discussed in Appendix 3.

The quantities $\mathscr{F}$, $\mathscr{A}$, and M^* are determined solely by the mean field $\Sigma_{\rm MF}$, which according to Eq. (8.5) is

$$\Sigma_{\rm MF}(x) = -ig_{\rm v}^2 \gamma^\mu \int d^4x'\, D_{\mu\nu}^{+}(x, x') \int \frac{d^3p}{(2\pi)^3} \frac{p^\nu}{E}\, [f_N^{ss}(x', p) - \bar{f}_N^{ss}(x', p)]$$
$$- ig_{\rm s}^2 \int d^4x'\, \Delta^{+}(x, x') \int \frac{d^3p}{(2\pi)^3}\, [f_N^{ss}(x', p) + \bar{f}_N^{ss}(x', p)]. \tag{11.6}$$

The first term gives the vector part of the mean-field self-energy, while the second one gives the scalar part. In principle, the mean-field self-energy enters also on the right-hand side of Eq. (11.6) (through the energies of nucleons and antinucleons). However, this equation has been obtained in second order of the perturbative expansion, and consequently it is sufficient to use the vacuum dispersion relations on the right-hand side of it without spoiling its exactness. Knowing the effective mass, one finds the spinors u and v, and the matrix $\mathscr{A}$, the explicit form of which is given in Appendix 4.

Let us briefly discuss the transport equations (11.2) and (11.3). The left-hand sides of these equations contain terms corresponding to free streaming and to an evolution due to the mean field. The terms proportional to $(M^* \partial_\mu M^* \pm p^\nu \mathscr{F}_{\nu\mu})$ coincide with those given in [9, 11–13, 15, 17]. The third term on the left-hand side of (11.2) and the respective term in (11.3) have been derived, to our best knowledge, for the first time. These terms provide an evolution in spin space due to the mean field. We show in Appendix 5 that Eqs. (11.2) and (11.3) conserve baryon number, and that entropy is, as expected, produced only due to collisions. These proofs are somewhat more complicated than those usually found in textbooks, see, e.g., [26], because here the (effective) particle mass is not constant but position-dependent.

The mean-field self-energies of the vector and scalar fields are zero in the pairing approximation discussed in Section 7 and in the perturbative approach presented in Section 8. Therefore, the dispersion relations of mesons are as in a vacuum, and the masses are, obviously, x-independent. Thus, one immediately finds from Eqs. (5.14b) and (5.10c) the form of the transport equations of f_v and f_s as

$$p^\mu \partial_\mu f_v = \tfrac{1}{2}\{\mathscr{P}^>, f_v\} - \tfrac{1}{2}\{\mathscr{P}^<, f_v + 1\}, \tag{11.7}$$

$$p^\mu \partial_\mu f_s = \frac{1}{2i} \Pi^> f_s - \frac{1}{2i} \Pi^< (f_s + 1), \tag{11.8}$$

where

$$\mathscr{P}^{\gtrless}_{rs}(X, p) \equiv \frac{1}{2i}\, \varepsilon^\mu(r, p)\, \varepsilon^\nu(s, p)\, P^{\gtrless}_{\mu\nu}(X, p). \tag{11.9}$$

To complete the derivation of the transport equations one has to express the collisional self-energies found in Section 9 through the distribution functions and interaction matrix elements. After a very tedious evaluation of Eq. (11.4a), according to the rules given in Section 9, one finds for the collisional self-energies

$$\begin{aligned} \mathscr{T}^>(X, p) = \sum_{ii'} \int & \frac{d^3p_i}{(2\pi)^3\, 2E_i} \frac{d^3p'}{(2\pi)^3\, 2E'} \frac{d^3p'_i}{(2\pi)^3\, 2E'_i} (2\pi)^4\, \delta^{(4)}(p + p_i - p' - p'_i) \\ & \times \mathrm{Tr}_i(f_i \mathscr{M}_{ii'} (f'_N - 1)(f'_i \pm 1)\, \mathscr{M}^\dagger_{i'i}), \end{aligned} \tag{11.10a}$$

$$\begin{aligned} \mathscr{T}^<(X, p) = \sum_{ii'} \int & \frac{d^3p_i}{(2\pi)^3\, 2E_i} \frac{d^3p'}{(2\pi)^3\, 2E'} \frac{d^3p'_i}{(2\pi)^3\, 2E'_i} (2\pi)^4\, \delta^{(4)}(p + p_i - p' - p'_i) \\ & \times \mathrm{Tr}_i((f_i \pm 1)\, \mathscr{M}_{ii'} f'_N f'_i \mathscr{M}^\dagger_{i'i}). \end{aligned} \tag{11.10b}$$

Here the summation runs over nucleons, antinucleons, vector mesons, and scalar mesons, and $\mathcal{M}_{ii'} \equiv \mathcal{M}(X; ps, p_i s_i \mid p's', p_i' s_i')$ is the matrix element of nucleon–nucleon, nucleon–antinucleon, nucleon–meson scatterings, represented by the Feynman diagrams shown in Fig. 14. The trace Tr_i should be taken over spin indices of the i-particle. The minus sign is for fermions, and the plus sign for bosons. In spite of the scalar–vector mixing, the collisional self-energies (11.10) do not include inelastic meson–nucleon interactions, which convert vector mesons into scalar ones or *vice versa*. The point is that the Green functions (10.20b) and (10.21b), which are found in zeroth order of gradient expansion, are purely transversal, i.e., $p^\mu D^{\gtrless}_{\mu\nu}(X, p) = p^\nu D^{\gtrless}_{\mu\nu}(X, p) = 0$.

The collisional self-energies of antinucleons, vector and scalar mesons are analogous to those given by Eqs. (11.10); only the nucleon distribution function should be substituted by antinucleon or meson distribution functions, and the matrix elements should be respectively modified. The self-energies (11.10) thus yield collision terms of the form which appears in the so-called relativistic Waldmann–Snider equation studied in [26] (see also [14]). Inserting the collisional self-energies (11.10) into Eqs. (11.2), (11.3), (11.6), (11.7), and (11.8), we obtain the final set of transport equations, which is the main result of this study.

At the end, let us consider the transport equations for an unpolarized system, where the distribution functions of (anti-)nucleons and vector mesons can be expressed as

$$f_N^{rs}(X, p) = \tfrac{1}{2}\,\delta^{rs} f_N^0(X, p), \qquad \bar{f}_N^{rs}(X, p) = \tfrac{1}{2}\,\delta^{rs} \bar{f}_N^0(X, p), \tag{11.11a}$$

$$f_V^{rs} = \tfrac{1}{3}\,\delta^{rs} f_V^0. \tag{11.11b}$$

Substituting the distribution functions (11.11) into Eqs. (11.2), (11.3), and (11.7) and taking the trace of the resulting equations over spin indices, one finds

$$p^\mu \partial_\mu f_N^0 + (M^* \partial_\mu M^* - p^\nu \mathcal{F}_{\nu\mu})\, \partial_p^\mu f_N^0 = \mathcal{T}_0^> f_N^0 - \mathcal{T}_0^< (f_N^0 - 1), \tag{11.12}$$

$$p^\mu \partial_\mu \bar{f}_N^0 + (M^* \partial_\mu M^* + p^\nu \mathcal{F}_{\nu\mu})\, \partial_p^\mu \bar{f}_N^0 = \bar{\mathcal{T}}_0^> \bar{f}_N^0 - \bar{\mathcal{T}}_0^< (\bar{f}_N^0 - 1), \tag{11.13}$$

$$p^\mu \partial_\mu f_V^0 = \mathcal{P}_0^> f_V^0 - \mathcal{P}_0^< (f_V^0 + 1), \tag{11.14}$$

where

$$\mathcal{T}_0^>(X, p) = \sum_{ii'} \int \frac{d^3 p_i}{(2\pi)^3\, 2E_i} \frac{d^3 p'}{(2\pi)^3\, 2E'} \frac{d^3 p_i'}{(2\pi)^3\, 2E_i'} (2\pi)^4\, \delta^{(4)}(p + p_i - p' - p_i')$$
$$\times \frac{1}{2\lambda_i} \mathrm{Tr}_{iN}(\mathcal{M}_{ii'} \mathcal{M}^\dagger_{i'i})\, f_i^0 (f_N^{0'} - 1)(f_i^{0'} \pm 1), \tag{11.15a}$$

$$\mathcal{T}_0^<(X, p) = \sum_{ii'} \int \frac{d^3 p_i}{(2\pi)^3\, 2E_i} \frac{d^3 p'}{(2\pi)^3\, 2E'} \frac{d^3 p_i'}{(2\pi)^3\, 2E_i'} (2\pi)^4\, \delta^{(4)}(p + p_i - p' - p_i')$$
$$\times \frac{1}{2\lambda_i} \mathrm{Tr}_{iN}(\mathcal{M}_{ii'} \mathcal{M}^\dagger_{i'i})(f_i^0 \pm 1)\, f_N^{0'} f_i^{0'}. \tag{11.15b}$$

Tr_{iN} means that the trace should be taken over the spin indices of the ith particle and the nucleon; λ_i equals 1, 2, or 3 depending on whether the ith particle is a scalar meson, an (anti-)nucleon, or a vector meson. The antinucleon and vector meson collisional self-energies are of the same form as the nucleon ones but the nucleon distribution functions are replaced by the antinucleon or meson functions and the matrix elements are respectively modified.

12. Discussion of the Method

Our derivation of the transport equations is based on several restrictive assumptions and approximations. Let us now discuss the most important of them, keeping in mind that these assumptions and approximations, on one hand, impose some restrictions on the physical systems which can be described in the framework of transport theory, but which, on the other hand, limit the amount of information about the system which can be obtained from this theory.

The essential simplifications have been made in Section 5, where we have assumed that $\Delta(X, u)$ and other Green functions are slowly varying functions of X and are strongly *peaked* for $u \cong 0$. This assumption can be written as

$$|\Delta(X,p)| \gg \left|\frac{\partial}{\partial X^\mu}\frac{\partial}{\partial p_\mu}\Delta(X,p)\right| \gg \left|\left(\frac{\partial}{\partial X^\mu}\frac{\partial}{\partial p_\mu}\right)^2 \Delta(X,p)\right| \gg \cdots, \tag{12.1}$$

which is equivalent to the requirement that

$$\Delta X^\mu \, \Delta p_\mu \gg 1, \tag{12.2}$$

where ΔX^μ and Δp^μ are the characteristic lengths over which the function $\Delta(X, p)$ varies in position and momentum space. In physical units the right-hand side of the inequality (12.2) equals $\hbar$.

If $\Delta(X, p)$ provides the *exact* description of a single-particle system, the relation (12.2) cannot be satisfied since $\Delta X^\mu \sim 1/\Delta p^\mu$ in this case. For a single-particle system, Eq. (12.2) is the condition for validity of a classical description of the system, i.e., the description with *poor* position and/or momentum resolutions. Therefore, to satisfy the condition (12.2), the single-particle function $\Delta(X, p)$ must be averaged over space–time cells that are much bigger than the single-particle de Broglie wavelength. In the case of a many-particle system, the function $\Delta(X, p)$, being defined in terms of an ensemble average, anyhow, carries only averaged information from all particles. Thus, the size of the averaging cell can be, in principle, smaller than a single-particle de Broglie wave length. In any case, the fact that the distribution functions obey quantum statistics is not affected by the condition (12.2) which is often referred to as the semiclassical approximation.

An obvious consequence of (12.1) is that on a macroscopic level the on-shell distribution function must also satisfy

$$f(X,p) \gg \left|\frac{\partial}{\partial X^\mu}\frac{\partial}{\partial p_\mu} f(X,p)\right| \gg \left|\left(\frac{\partial}{\partial X^\mu}\frac{\partial}{\partial p_\mu}\right)^2 f(X,p)\right| \gg \cdots. \tag{12.3}$$

Because the distribution of a many particle system is never momentum independent (Δp_μ never approaches infinity), the requirement (12.3) limits the kinetic description to systems where the rate of temporal changes and the inverse gradients are much smaller than the particle energies and momenta, respectively.

In Section 5 we have also assumed that the self-energies satisfy a condition analogous to (12.1). The characteristic length Δp^μ, at which the self-energy $\Pi(X, p)$ varies in four-momentum space, corresponds to the inverse space–time interaction range. Therefore, the requirement $\Delta X^\mu \, \Delta p_\mu \gg 1$, applied to the self-energies, demands shortness of the space–time interaction range when compared with the scale of space–time inhomogeneities in the system.

The condition (12.1) justifies the expansion *in gradients* and, in particular, the formulae (5.2)–(5.6). Deriving the transport equations we have kept only quantities which are at most of first-order in the gradients. However, while keeping the first-order gradients to the mean-field self-energy, we have neglected such terms in the collisional self-energies. Such a procedure is justified when the interaction in the system is *weak* and a perturbative expansion is allowed, since the mean-field contribution appears at a lower order in the coupling constants than the collisional one.

It proved crucial for the calculation of the collisional self-energies that the incoming four-momentum p could be taken on-shell. In fact in Section 9 we used free-field dispersion relations, but everything would formally have gone through in the same way had we used the mean-field modified quasiparticle dispersion relations derived in Section 10. The on-shell conditions allowed us to neglect several diagrams because of energy-momentum conservation which, in particular, forbids the decay $A \to A + B$. Our analysis remains valid as long the mass shift due to interactions is much smaller than the mass itself ($|m^* - m| \ll m$). This is the case for *perturbative* interactions between massive fields. For massless fields our arguments do not hold since already a small (*perturbative*) modification of the particle dispersion relation can open channels for processes which are forbidden in a vacuum, as it happens with Cherenkov radiation.

When introducing in Section 10 the dispersion relations of quasiparticles, we neglected the possible contribution of the collisional self-energies to the effective mass. It was argued that the mean-field contribution is leading here. As discussed in [17], a contribution of collisional self-energies would cause the quasiparticles to acquire a finite width, i.e., go off mass shell. This would, on the one hand, be a desired effect because it can lead to a momentum dependence of the effective mass, but on the other hand, it would cause serious complications in the further derivation of the collision terms of the transport equations, and we do not know how to deal with them at present.

Due to mass-shell constraints we have also avoided the problem of renormalization of loop diagrams, which usually appears in perturbative expansions beyond the Born approximation. Our collisional self-energies calculated in fourth order of the coupling constant are finite. This is not surprising since they correspond to the imaginary parts of the self-energies in a vacuum QFT, which are also finite

[22]. The real parts which usually cause troubles are exactly zero in the approach presented in this paper as long as the mass-shell restrictions are assumed.

It has been stressed in several publications [34–36] that the mean-field force, which appears in the transport equations of nuclear matter, should be momentum-dependent. Indeed, the force $(M^*(X)\,\partial_\mu M^*(X) - p^\nu \mathscr{F}_{\nu\mu}(X))$ in Eq. (11.2) has this feature. Unfortunately, the momentum dependence is not exactly as required by the real part of the optical potential of proton-nucleus scatterings [36]. One can try to remove this defect of the transport equation (11.2) by taking into account contributions of fourth order in the coupling constant and/or second-order contributions in the gradient expansion. In the present study these contributions have been neglected.

13. Comparison with Previous Work and Conclusions

The problem of systematically deriving kinetic equations for relativistic nuclear matter has attracted a lot of attention in the last few years. In this section we discuss the relation of our work to the papers [9–19], where transport equations of relativistic nuclear matter have been studied before.

The papers [9–13] present the derivation of transport equations in the mean-field limit only. The meson fields have been treated as classical in [9–12], while a lowest-order perturbative approximation has been performed in [13]. These two equivalent approaches correspond to our considerations from Section 7 and 8, respectively. In contrast to our study, the spin degrees of freedom have been eliminated in [9–13] due to the (explicit or implicit) assumption that the system is spin saturated, i.e., unpolarized. Therefore, the equations derived in [9–13] do not contain terms describing the evolution in spin space.

Attempts to derive collision terms have been made in [14–17], where, as in our work, the contour Green function technique has been used. The considerations presented in [15] are, to a large extent, based on intuitive arguments and lack rigor. For example, it appears that in converting Eq. (9) from [15] into Eq. (12), the authors of [15] have assumed $\mathrm{Tr}(\gamma^\mu G^>(x,p)) = (p^\mu/m)\,\mathrm{Tr}(G^>(x,p))$, which in general is not true. As explained in our Section 9, it is a fairly complicated problem to extract the collisional self-energies from the contour self-energies. For its resolution we have applied the graphical method described in Section 9. While in [15] the collision term is written down by essentially copying the nonrelativistic expressions from Kadanoff and Baym [19], we point out at the end of Section 9 that the structure of the relativistic collisional self-energies reduces to the nonrelativistic form only after imposing the mass-shell constraints which are very restrictive. Furthermore, it is not clear how the spin degrees of freedom have been treated in [15] going from their Eq. (18) to (25).

The study [14] is more complete than [15], but the part dealing with relativistic nuclear matter is also not quite satisfactory in our opinion. For example, the spinor structure of the self-energies described by Eq. (6.50) from [14] is exact only for equilibrium nuclear matter, and consequently Eq. (6.51), which is sufficient for a near-equilibrium expansion, does not really represent "the most general expression." The transformation from spinor to spin basis performed in [14] is not quite correct. In particular, the authors of [14] appear to have missed that, through the effective mass, the spinors u and v depend on X. Therefore, the term in our Eq. (11.2) containing the matrix $\mathscr{A}$ is missing in their final transport equation (6.109). It is claimed in [14] that the relativistic equation of motion of $G^>$ is "formally identical" to the nonrelativistic one, and by means of this observation the collisional self-energies are found. As already mentioned and clarified by the present analysis, this observation is correct only under specific approximations. Similar to [15] the mesons have been treated in [14] only as effective fields mediating the interaction, but not as real particles.

In the paper [16] a system of nucleons interacting with scalar mesons is considered, eliminating spin degrees of freedom by the explicit assumption of spin equilibrium. The transport equation is given in the one-loop approximation in a treatment very similar to that of [5], where the relativistic electrodynamic plasma has been studied. However, as the authors of [16] admit, the collision terms in their transport equations (3.90), (3.91), which correspond to the Cherenkov radiation or Landau damping terms from [5], are negligibly small within their approximations. There is a very important difference between the electrodynamic plasma with massless photons and a system of nucleons interacting via massive mesons. When one deals with massive particles, the gradient expansion leads (through condition (6.5)) to the very restrictive mass-shell constraints resulting in vanishing contributions to the meson self-energies (Figs. 3, 4) in the one-loop approximation. Therefore, the meson dispersion relations coincide with the vacuum ones, and processes similar to the plasma Cherenkov radiation or Landau damping are kinematically forbidden. In the case of massless photons the condition (6.5) cannot be imposed, and thus, one obtains nontrivial collision terms for the electron transport equations even in the one-loop approximation. At the very end of the paper [16], the nucleon transport equations at the two-loop level are also written, without, however, giving any details of the derivation.

Our study has probably the largest overlap with the analysis of Schönhofen, Cubero *et al.* [17]. In this work a thorough discussion of the derivation of transport equations for the Walecka model is given. The gradient expansion, quasiparticle approximation, and perturbative nature of the approach are critically discussed in a spirit similar to our own presentation. Although not stated explicitly, even the graphical method of Section 9 to select the allowed diagrams for the collision terms appears to have been used by these authors. Furthermore, their work additionally includes pions and Δ's. However, the pions appear as real particles only in the context of the reaction $\pi N \leftrightarrow \Delta$. ($\pi$-production outside the Δ-resonance would require to take into account sixth-order diagrams.) Pion Compton scattering (the process corresponding to our Figs. 9a, 10a) is neglected. In contrast to our work, in [17] the scalar and vector mesons are treated everywhere only as effective interaction-mediating particles. The transport equations are derived there only for unpolarized systems and thus lack the spin dynamics contained in ours.

The considerations presented in [18] are very different from those in [14–17] and from our study. Instead of deriving transport equations, the authors of [18] look for a numerically tractable form of the Dyson–Schwinger equations. Their approach is fully quantum mechanical but the interactions in the system are introduced in a rather simplified way by means of Skyrme-like forces. The paper [18] deals only with equations of motion of time-ordered Green functions (propagators) and consequently does not provide, in our opinion, a complete description of a many-body system out of equilibrium. For such a description one additionally needs equations for the Green functions $G^{\gtrless}$, which correspond to distribution functions.

At the end of this comparison let us mention the very recent paper [19]. The authors of it start with the relativistic Lagrangian of a Walecka-like model but then use the approximation of instantaneous meson exchange, which makes the whole approach rather nonrelativistic than relativistic.

We conclude our study as follows. Starting with the quantum field theory Lagrangian we have derived the set of relativistic transport equations of nucleons interacting with mesons. The derivation assumes quasi-homogeneity of the system, which justifies the gradient expansion, and *weakness* of the interaction, which in turn permits a perturbative expansion. The equations have been obtained in the lowest nontrivial order[6] of the two expansions. The procedure enables one to include higher order corrections but serious technical difficulties have to be resolved. It remains a real challenge to clarify whether the kinetic description is valid when one of the basic assumptions made here is completely relaxed.

Appendix 1

In this appendix the determinant of a matrix $(p\cdot\gamma \pm M)$ is calculated.[7] Using the well-known identity $\det A = \exp \operatorname{Tr} \ln A$, we find by expanding the logarithm in the exponent

$$\det(p\cdot\gamma \pm M) = \exp\left(4\ln M - \sum_{n=1}^{\infty} \frac{(\mp 1)^n}{n} \operatorname{Tr}\left(\frac{p\cdot\gamma}{M}\right)^n\right). \tag{A1.1}$$

Then one calculates

$$\begin{aligned}\sum_{n=1}^{\infty} \frac{(\mp 1)^n}{n} \operatorname{Tr}\left(\frac{p\cdot\gamma}{M}\right)^n &= \sum_{n=1}^{\infty} \frac{1}{2n} \operatorname{Tr}\left(\frac{p\cdot\gamma}{M}\right)^{2n} \\ &= 2\sum_{n=1}^{\infty} \frac{1}{n}\left(\frac{p^2}{M^2}\right)^n = -2\ln\left(1-\frac{p^2}{M^2}\right),\end{aligned} \tag{A1.2}$$

[6] The lowest order in coupling constant, i.e., the mean-field limit of the transport equations, is rather trivial from the point of view of kinetic theory, since the system described by such equations never reaches equilibrium.

[7] We are grateful to Teiji Kunihiro for suggesting this method.

where we have proven by induction that $\mathrm{Tr}(p\cdot\gamma)^{2n}=4p^{2n}$. Substituting Eq. (A1.2) into Eq. (A1.1) one finally finds

$$\det(p\cdot\gamma\pm M)=(p^2-M^2)^2.$$

Appendix 2

We prove that

$$\bar{u}(r,p)\frac{\partial}{\partial M}u(s,p)=\bar{v}(r,p)\frac{\partial}{\partial M}v(s,p)=0. \tag{A2.1}$$

The spinors $u(r,p)$ and $\bar{u}(s,p)$ can be expressed as

$$u(r,p)=\frac{p\cdot\gamma+M}{\sqrt{2M(E+M)}}u(r,0), \qquad \bar{u}(r,p)=\bar{u}(r,0)\frac{p\cdot\gamma+M}{\sqrt{2M(E+M)}}, \tag{A2.2}$$

where $E=\sqrt{\mathbf{p}^2+M^2}$ and $u(r,0)\equiv u(r,E=M,\mathbf{p}=0)$. Keeping in mind that the spinor $u(r,0)$ satisfies the equation $(\gamma^0-1)\,u(r,0)=0$, one finds

$$\frac{\partial}{\partial M}u(r,p)=\frac{1}{E}\sqrt{\frac{E+M}{2M}}u(r,0)-\frac{E+M}{2ME}u(r,p).$$

Observing that $(p\cdot\gamma+M)^2=2M(p\cdot\gamma+M)$, and using Eq. (A2.2) we obtain

$$\bar{u}(r,p)\,u(s,0)=\delta^{rs}\sqrt{(E+M)/2M},$$

and finally prove Eq. (A2.1) for the spinor u. The proof for the spinor v is obviously very similar.

Appendix 3

The aim of this appendix is to discuss those terms of the transport equations (11.2) and (11.3) which are responsible for the mean-field evolution of spin degrees of freedom. The direct calculation, which leads from Eq. (11.1) to Eqs. (11.2) and (11.3), yields the terms $(\mathscr{A}'f_N+f_N\mathscr{A}'^{\dagger})$ and $(\bar{\mathscr{A}}'\bar{f}_N+\bar{f}_N\mathscr{A}'^{\dagger})$ instead of $[\mathscr{A},f_N]$ and $[\bar{\mathscr{A}},\bar{f}_N]$, respectively, which were written down in Eqs. (11.2), (11.3). With these terms the transport equations are found to violate baryon number conservation and to lead to entropy production even by the mean-field spin dynamics. Specifically, following the derivation in Appendix 5 we obtain

$$\partial_\mu\langle j_b^\mu(X)\rangle=\frac{1}{2}\mathrm{Tr}\int\frac{d^3p}{(2\pi)^3\,E}\left[(\mathscr{A}'f_N+f_N\mathscr{A}'^{\dagger})-(\bar{\mathscr{A}}'\bar{f}_N+\bar{f}_N\bar{\mathscr{A}}'^{\dagger})\right]. \tag{A3.1}$$

$$\partial_\mu S_b^\mu(X) = \text{"collisions"} - \frac{1}{2}\,\mathrm{Tr}\int \frac{d^3p}{(2\pi)^3 E}\,\mathrm{Tr}\Big[(\mathscr{A}' f_N + f_N \mathscr{A}'^\dagger)\ln\Big(\frac{f_N}{1-f_N}\Big)$$
$$+ (\bar{\mathscr{A}}' \bar{f}_N + \bar{f}_N \bar{\mathscr{A}}'^\dagger)\ln\Big(\frac{\bar{f}_N}{1-\bar{f}_N}\Big)\Big]. \tag{A3.2}$$

As seen in Eqs. (A3.1) and (A3.2), these two problems would be absent if the matrices $\mathscr{A}'$ and $\bar{\mathscr{A}}'$ were purely antihermitian. Indeed, the hermitian parts seem to be physically meaningless. To clarify this statement let us consider, for simplicity, the unpolarized system. Then, the distribution functions are as in Eqs. (11.11), and the positive energy part of the Green function (10.12a) reads

$$i\Theta(p_0)\,G^<_{\alpha\beta}(X,p) = -\frac{\pi}{2E}\,\delta(E-p_0)(\gamma_\mu p^\mu + M^*)\,f^0_N(X,p). \tag{A3.3}$$

Let us observe that

$$\bar{u}_\alpha(r,p)\,i\Theta(p_0)\,G^<_{\alpha\beta}(X,p)\,u_\beta(s,p) = -\frac{\pi M^*}{E}\,\delta^{rs}\,\delta(E-p_0)\,f^0_N(X,p), \tag{A3.4a}$$

$$\bar{v}_\alpha(r,p)\,i\Theta(p_0)\,G^<_{\alpha\beta}(X,p)\,v_\beta(s,p) = 0. \tag{A3.4b}$$

Now we will derive the transport equation for $f^0_N(X,p)$. We substitute the Green function (A3.3) into Eq. (11.1) and integrate the resulting equation over p_0. In this way we obtain

$$(p^\mu + M^*\gamma^\mu)\,\partial_\mu f^0_N + ((p^\mu\gamma_\mu + M^*)\,\partial_i M^* - (p^\nu + M^*\gamma^\nu)\,\mathscr{F}_{\nu i})\,\partial^i_p f^0_N$$
$$+\Big(\frac{M^*\dot{M}^*}{E} + \gamma^\mu\partial_\mu M^* - \frac{M^*\partial_\mu M^*}{E^2}(p^\mu + M^*\gamma^\mu) + \frac{p^i\partial_i M^*}{E^2}(p^\mu\gamma_\mu + M^*)$$
$$-\frac{p^i\partial_i M^*}{E}\gamma^0 + \gamma^i\partial_i M^* - \frac{p^i}{E^2}(p^\nu + M^*\gamma^\nu)\,\mathscr{F}_{\nu i} + \frac{p^i}{E}\mathscr{F}_{0i}\Big) f^0_N = \text{"collisions,"} \tag{A3.5}$$

where the dot denotes time derivative.

Projecting Eq. (A3.5) on particle ($\bar{u}$, u) and antiparticle ($\bar{v}$, v) states, one finds, respectively,

$$p^\mu\partial_\mu f^0_N + (M^*\partial_i M^* - p^\nu\mathscr{F}_{\nu i})\,\partial^i_p f^0_N$$
$$+\frac{1}{2}\Big(\frac{p^\mu\partial_\mu M^*}{M^*} - \frac{M^*\dot{M}^*}{E} - \frac{p^i}{E}\mathscr{F}_{0i}\Big) f^0_N = \text{"collisions,"} \tag{A3.6a}$$

$$\Big(\frac{p^\mu\partial_\mu M^*}{M^*} - \frac{M^*\dot{M}^*}{E} - \frac{p^i}{E}\mathscr{F}_{0i}\Big) f^0_N = 0. \tag{A3.6b}$$

The result expressed by Eq. (A3.6b) is surprising: The kinetic operator from the left-hand side of Eq. (11.1) acting on the Green function (A3.3) which represents only particle states (no antiparticles, cf. Eq. (A3.4)), produces a nonvanishing antiparticle component (found by projecting onto $\bar{v}, v$). One should keep in mind here that the mean field is assumed to be *weak* and the system *quasihomogenous*, thus no particle–antiparticle mixing should occur. The collision terms in Eq. (11.1) for particles depend lineary on $\Theta(p_0)\, G^{\lessgtr}_{\alpha\beta}(X, p)$, and consequently: $\bar{v}$ "collisions"$v = 0$. Therefore, according to Eq. (A3.6b), the antiparticle component must be zero. Unfortunately, we do not have a simple interpretation of this constraint. Inserting Eq. (A3.6b) back into Eq. (A3.6a), we find

$$p^\mu \partial_\mu f^0_N + (M^* \partial_i M^* - p^\nu \mathscr{F}_{\nu i})\, \partial^i_p f^0_N = \text{"collisions,"} \tag{A3.7}$$

which is the standard kinetic equations of unpolarized nucleons in the Walecka model. In contrast to Eq. (A3.6a), the transport equation (A3.7) does conserve baryon number and leads to the entropy production only *via* collisions, see Appendix 5.

A more complicated but similar analysis of a polarized system shows that the false antiparticle (particle) component is contained in the hermitian part of the matrix $\mathscr{A}'$ ($\bar{\mathscr{A}}'$). The constraints analogous to Eq. (A3.6b) are $\mathscr{A}' - \mathscr{A}'^\dagger = 0$ and $\bar{\mathscr{A}}' - \bar{\mathscr{A}}'^\dagger = 0$, respectively. In the transport equations (11.2), (11.3) and (11.12), (11.13) we have already implemented these constraints.

Appendix 4

Her we find the explicit form of the matrix $\mathscr{A}$, defined as the antihermitian part of the matrix $\mathscr{A}'$ given by Eq. (11.4b). Expressing the spinors as in Eq. (A2.2) and using the formulae given in Appendix J of [37], one finds after rather lengthy calculations the following results, valid for the Dirac representation of the γ-matrices,

$$2\bar{u}(r, p)\, \gamma^\mu\, \partial_\mu u(s, p) = \partial_\mu \left(\frac{p^\mu}{M^*}\right) \delta^{rs} + \frac{i}{M^* E} \nabla M^* \cdot (\boldsymbol{\sigma}_{rs} \times \mathbf{p}), \tag{A4.1a}$$

$$2\bar{u}(r, p)\, \partial^i_p u(s, p) = -\frac{i}{M^*(E + M^*)} (\boldsymbol{\sigma}_{rs} \times \mathbf{p})^i, \tag{A4.1b}$$

$$2\bar{u}(r, p)\, \gamma^0\, \partial^i_p u(s, p) = \frac{1}{M^* E} p^i \delta^{rs} + \frac{i}{M^*(E + M^*)} (\boldsymbol{\sigma}_{rs} \times \mathbf{p})^i, \tag{A4.1c}$$

$$2\bar{u}(r, p)\, \gamma^j\, \partial^i_p u(s, p) = \frac{1}{M^*} \delta^{ij} \delta^{rs} + \frac{i}{M^* E(E + M^*)} p^i (\boldsymbol{\sigma}_{rs} \times \mathbf{p})^j + \frac{i}{M^*} \varepsilon_{ijk} \sigma^k_{rs}, \tag{A4.1d}$$

where $\boldsymbol{\sigma} \equiv (\sigma_1, \sigma_2, \sigma_3)$ is the vector of Pauli matrices and ε_{ijk} is the completely asymmetric tensor. Substituting Eqs. (A4.1) in Eq. (11.4b) and removing the hermitian part one obtains

$$\mathscr{A}_{rs}(X,p) = \frac{i}{2}\left[\frac{3E+M^*}{E(E+M^*)}\nabla M^* - \frac{1}{M^*}\mathbf{E}\right]\cdot(\boldsymbol{\sigma}_{rs}\times\mathbf{p}) + \frac{i}{2M^*E}\mathbf{B}\cdot(\mathbf{p}\times(\boldsymbol{\sigma}_{rs}\times\mathbf{p})) + \frac{i}{2}\mathbf{B}\cdot\boldsymbol{\sigma}_{rs},$$

where $\mathbf{E}$ and $\mathbf{B}$ are the electric and magnetic vectors related to the stress tensor $\mathscr{F}$ as

$$E_i = \mathscr{F}_{0i}, \qquad B_i = \tfrac{1}{2}\varepsilon_{ijk}\mathscr{F}_{jk}.$$

Appendix 5

In this appendix we show that the transport equations (11.2) and (11.3) satisfy baryon number conservation and that entropy is produced only by collisions. Let us start with the current expressed by Eq. (10.23). The divergence of the current reads[8]

$$\partial_\mu \langle j^\mu_b(X)\rangle = \int \frac{d^3p}{(2\pi)^3 E}\left[\frac{E\partial_\mu p^\mu - p^\mu\partial_\mu E}{E}\left[f^{ss}_N(X,p) - \bar{f}^{ss}_N(X,p)\right] + p^\mu\left[\partial_\mu f^{ss}_N(X,p) - \partial_\mu \bar{f}^{ss}_N(X,p)\right]\right]. \tag{A5.1}$$

The first term on the left-hand side of Eq. (A5.1), which is usually absent in discussion of current conservation (see, e.g., [26]), appears because the (effective) particle mass is not constant but position dependent. Using the transport equations (11.2) and (11.3) one expresses $p^\mu\partial_\mu f^{ss}_N$ and $p^\mu\partial_\mu \bar{f}^{ss}_N$ as

$$p^\mu\partial_\mu f^{ss}_N = -(M^*\partial_\mu M^* - p^\nu\mathscr{F}_{\nu\mu})\,\partial^\mu_p f^{ss}_N + C^{ss}, \tag{A5.2a}$$

$$p^\mu\partial_\mu \bar{f}^{ss}_N = -(M^*\partial_\mu M^* + p^\nu\mathscr{F}_{\nu\mu})\,\partial^\mu_p \bar{f}^{ss}_N + \bar{C}^{ss}, \tag{A5.2b}$$

where C and $\bar{C}$ represent the collision terms,

$$C \equiv \tfrac{1}{2}\{\mathscr{T}^>, f_N\} - \tfrac{1}{2}\{\mathscr{T}^<, f_N - 1\},$$

$$\bar{C} \equiv \tfrac{1}{2}\{\bar{\mathscr{T}}^<, \bar{f}_N\} - \tfrac{1}{2}\{\bar{\mathscr{T}}^>, \bar{f}_N - 1\},$$

[8] To simplify the notation, we drop, as in Section 11, the index $*$ and identify E with ω_p.

and the trace is taken over spin indices. Let us observe that these terms of the transport equations which are responsible for the mean-field spin dynamics are absent in Eqs. (A5.2), because the traces of the anticommutators $[\mathscr{A}, f_N]$ and $[\bar{\mathscr{A}}, \bar{f}_N]$ vanish.

One easily proves using symmetry arguments (see, e.g., [26]) and the respective properties of the collisional self-energies $\mathscr{T}^{\gtrless}$ and $\bar{\mathscr{T}}^{\gtrless}$ that

$$\int \frac{d^3p}{(2\pi)^3 E} \operatorname{Tr} C = \int \frac{d^3p}{(2\pi)^3 E} \operatorname{Tr} \bar{C} = 0, \tag{A5.3}$$

which expresses baryon number conservation in particle collisions.

Substituting Eqs. (A5.2) into Eq. (A5.1) and using the relation (A5.3), one finds

$$\begin{aligned} \partial_\mu \langle j_b^\mu(X) \rangle = \int \frac{d^3p}{(2\pi)^3 E} \bigg[& \frac{E \partial_\mu p^\mu - p^\mu \partial_\mu E}{E} [f_N^{ss}(X,p) - \bar{f}_N^{ss}(X,p)] \\ & - [(M^* \partial_\mu M^* - p^\nu \mathscr{F}_{\nu\mu}) \partial_p^\mu f_N^{ss} - (M^* \partial_\mu M^* + p^\nu \mathscr{F}_{\nu\mu}) \partial_p^\mu \bar{f}_N^{ss}]\bigg]. \end{aligned} \tag{A5.4}$$

Performing a partial integration of the second term on the right-hand side of Eq. (A5.4) and assuming that the distribution functions vanish for infinite momenta, one finds that the two terms in Eq. (A5.4) cancel each other and

$$\partial_\mu \langle j_b^\mu(X) \rangle = 0.$$

Let us now discuss the entropy. The baryon contribution to the entropy flow is defined as

$$\begin{aligned} S_b^\mu(X) = -\operatorname{Tr} \int \frac{d^3p}{(2\pi)^3} \frac{p^\mu}{E} & [f_N \ln f_N - (1 - f_N) \ln(1 - f_N) \\ & + \bar{f}_N \ln \bar{f}_N - (1 - \bar{f}_N) \ln(1 - \bar{f}_N)], \end{aligned} \tag{A5.5}$$

and the entropy production is

$$\begin{aligned} \partial_\mu S_b^\mu(X) = -\operatorname{Tr} \int \frac{d^3p}{(2\pi)^3 E} \bigg[& \frac{E \partial_\mu p^\mu - p^\mu \partial_\mu E}{E} [f_N \ln f_N - (1 - f_N) \ln(1 - f_N) \\ & + \bar{f}_N \ln \bar{f}_N - (1 - \bar{f}_N) \ln(1 - \bar{f}_N)] \\ & + p^\mu \left[\partial_\mu f_N \ln\left(\frac{f_N}{1 - f_N}\right) + \partial_\mu \bar{f}_N \ln\left(\frac{\bar{f}_N}{1 - \bar{f}_N}\right)\right]\bigg]. \end{aligned} \tag{A5.6}$$

In a way very similar to the case of baryon number conservation above we arrive at the formula

$$\partial_\mu S_b^\mu(X) = -\operatorname{Tr} \int \frac{d^3p}{(2\pi)^3 E} \left[C \ln\left(\frac{f_N}{1 - f_N}\right) + \bar{C} \ln\left(\frac{\bar{f}_N}{1 - \bar{f}_N}\right)\right]. \tag{A5.7}$$

Thus, entropy is produced, as expected, only by collisions. The terms of the transport equations corresponding to the mean-field spin dynamics do not contribute to the entropy production because of the property

$$\mathrm{Tr}([A, B] \ln B) = 0, \tag{A5.8}$$

which is valid for any matrices A and B.

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