Rapidity fluctuations in the initial state

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Motivation/new data

- Old story ...
- New data from the LHC, new methodology (ATLAS notes 2015)
- Longitudinally-extended source model

Goal: understand key elements from an analytic model anatomy of the correlations

3-stage approach



Generation and propagation of e-by-e fluctuations, $\eta_S \to \eta$

New data

$$C(\eta_1, \eta_2) = \frac{\langle N(\eta_1, \eta_2) \rangle}{\langle N(\eta_1) \rangle \langle N(\eta_2) \rangle} = \frac{S(\eta_1, \eta_2)}{B(\eta_1, \eta_2)}$$
$$C_N(\eta_1, \eta_2) = \frac{C(\eta_1, \eta_2)}{C_p(\eta_1) C_p(\eta_2)}, \quad C_p(\eta_1) = \int d\eta_2 C(\eta_1, \eta_2), \ C_p(\eta_2) = \dots$$

 η_1 and η_2 – pseudorapidities of different hadrons



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Rapidity fluctuations



$$\begin{split} \langle N(\eta_1)N(\eta_2) \rangle &= \langle N_A \rangle \langle f_A(\eta_1,\eta_2) \rangle + \langle N_A(N_A-1) \rangle \langle f_A(\eta_1) \rangle \langle f_A(\eta_2) \rangle \\ &+ \langle N_B \rangle \langle f_B(\eta_1,\eta_2) \rangle + \langle N_B(N_B-1) \rangle \langle f_B(\eta_1) \rangle \langle f_B(\eta_2) \rangle \\ &+ \langle N_A N_B \rangle \left[\langle f_A(\eta_1) \rangle \langle f_B(\eta_2) \rangle + \langle f_B(\eta_1) \rangle \langle f_A(\eta_2) \rangle \right] \end{split}$$

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$$\langle N(\eta_1)N(\eta_2)\rangle = \langle N_A\rangle \operatorname{cov}_A(\eta_1,\eta_2) + \langle N_A^2\rangle \langle f_A(\eta_1)\rangle \langle f_A(\eta_2)\rangle + \langle N_B\rangle \operatorname{cov}_B(\eta_1,\eta_2) + \langle N_B^2\rangle \langle f_B(\eta_1)\rangle \langle f_B(\eta_2)\rangle + \langle N_AN_B\rangle [\langle f_A(\eta_1)\rangle \langle f_B(\eta_2)\rangle + \langle f_B(\eta_1)\rangle \langle f_A(\eta_2)\rangle]$$

Production is independent/uncorrelated, unless from the same source. However, even if $cov_{A,B}(\eta_1, \eta_2) = 0$ we have nontrivial $C(\eta_1, \eta_2)$ from the fluctuation of numbers of sources N_A and N_B

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Rapidity fluctuations

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Average number of particles:

$$\langle N(\eta) \rangle = \langle N_A \rangle \langle f_A(\eta) \rangle + \langle N_B \rangle \langle f_B(\eta) \rangle$$

Symmetric and antisymmetric parts (see next slide):

$$\langle f_A(\eta) \rangle = f_s(\eta) + f_a(\eta), \quad \langle f_B(\eta) \rangle = f_s(\eta) - f_a(\eta)$$

After elementary transformations

$$C(\eta_1, \eta_2) = 1 + \frac{1}{\langle N(\eta_1) \rangle \langle N(\eta_2) \rangle} \times \left\{ \langle N_A \rangle \operatorname{cov}_A(\eta_1, \eta_2) + \langle N_B \rangle \operatorname{cov}_B(\eta_1, \eta_2) \right. \\ \left. + \operatorname{var}(N_A + N_B) f_s(\eta_1) f_s(\eta_2) + \operatorname{var}(N_A - N_B) f_a(\eta_1) f_a(\eta_2) \right. \\ \left. + \left[\operatorname{var}(N_A) - \operatorname{var}(N_B) \right] \left[f_s(\eta_1) f_a(\eta_2) + f_a(\eta_1) f_s(\eta_2) \right] \right\}$$

Białas-Czyż triangles

Charged hadron spectra in d+Au @ RHIC [Białas+Czyż 2004]



$$\langle f_A(\eta) \rangle = h(\eta) \frac{y_b + \eta}{2y_b}, \ \langle f_B(\eta) \rangle = h(\eta) \frac{y_b - \eta}{2y_b}$$

Asymmetric profiles used ever since

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Rapidity fluctuations

Observation by Bzdak and Teaney

Vanishing covariance, symmetric system, profiles from the previous slide:

$$\langle N_A \rangle = \langle N_B \rangle, \ \operatorname{var}(N_A) = \operatorname{var}(N_B), \ N_W = N_A + N_B$$

Then

$$C(\eta_1, \eta_2) = 1 + \frac{1}{\langle N_W \rangle^2} \left[\operatorname{var}(N_W) + \operatorname{var}(N_A - N_B) \frac{\eta_1}{y_b} \frac{\eta_2}{y_b} \right]$$
[Bzdak+Teaney 2013]

Fluctuations of N_A vs N_B cause the $\eta_1\eta_2$ structure in $C(\eta_1,\eta_2)$

Fluctuating length

- Idea: entropy deposition from wounded nucleons originates from string-like objects whose other end-point is randomly distributed in η (related to [Brodsky+Gunion+Kuhn 1977])
- String fragmentation
- "Soft particle production in hadronic collisions is dominated by multiple gluon exchanges between partons from the colliding hadrons, followed by radiation of ... partons distributed uniformly in rapidity" [Białas+Jeżabek 2004]
- Torque in p-A collisions (see talk by PB) [PB+WB, arXiv:1506.02817]
- Similar ideas in [Monnai+Schenke, arXiv:1509.04103]



What it yields?



$$f_A(\eta; y) = \theta(y < \eta < y_b), \quad f_B(\eta; y) = \theta(-y_b < \eta < -y)$$

(uniform string fragmentation function) Random end y is uniformly selected from $[-y_b, y_b]$, where y_b is the beam rapidity Averaging over $y \rightarrow$ "triangles":

$$\langle f_{A,B}(\eta) \rangle = \int_{-y_b}^{y_b} \frac{dy}{2y_b} f_{A,B}(\eta;y) = \frac{y_b \pm \eta}{2y_b}$$

Correlations from length fluctuations:

$$\langle f_{A,B}(\eta_1,\eta_2) \rangle = \int_{-y_b}^{y_b} dy f_{A,B}(\eta_1;y) f_{A,B}(\eta_2;y) = \frac{y_b \pm \min(\eta_1,\eta_2)}{2y_b} \\ \operatorname{cov}_{A,B}(\eta_1,\eta_2) = \frac{y_b^2 - \eta_1 \eta_2 - y_b |\eta_1 - \eta_2|}{4y_b^2}$$

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Fluctuating strength

An additional ingredient:

$$f_A(\eta; y) = \omega \theta(y < \eta < y_b), \quad f_B(\eta; y) = \omega \theta(-y_b < \eta < -y)$$

 ω – random strength of the source (included in models to increase multiplicity fluctuations)

Also known as overlaid distribution

(we use $\sigma(\omega)/\langle\omega
angle=1$ on top of fluctuating length)



Generation of the ridge (structure from $-|\eta_1 - \eta_2|$) Fluctuating length essential

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Generation of the saddle in the ridge (seen in experiment) is trivial Fluctuating length essential

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$$a_{nm} = \int_{-Y}^{Y} \frac{d\eta_1}{Y} \int_{-Y}^{Y} \frac{d\eta_2}{Y} C(\eta_1, \eta_2) T_n\left(\frac{\eta_1}{Y}\right) T_m\left(\frac{\eta_1}{Y}\right)$$
$$T_n(x) = \sqrt{\frac{2n+1}{2}} P_n(x) \qquad [\text{Bzdak+Teaney 2013, Jia 2015}]$$
$$(\text{play analogous role to flow coefficients in harmonic flow})$$

$$a_{nn} = \frac{\frac{\operatorname{var}(N_A - N_B)}{\langle N_W \rangle} + \frac{\operatorname{var}(\omega)}{\langle \omega \rangle^2}}{12 \langle N_W \rangle} \frac{Y^2}{y_p^2} \delta_{n1} - \frac{\frac{\operatorname{var}(\omega)}{\langle \omega \rangle^2} + 1}{6 \langle N_W \rangle} \frac{Y^2}{y_p^2} \delta_{n1} + \frac{\frac{\operatorname{var}(\omega)}{\langle \omega \rangle^2} + 1}{(2n-1)(2n+3) \langle N_W \rangle} \frac{Y}{y_p}}{y_p}$$
$$a_{n,n+2} = -\frac{\frac{\operatorname{var}(\omega)}{\langle \omega \rangle^2} + 1}{(2n+3)\sqrt{(2n+1)(2n+5)} \langle N_W \rangle} \frac{Y}{y_p} \quad \text{(length fluct.)}$$

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Results for a_{nm} (from C_N)



Results for a_{nm} (from C_N)



Results for a_{nm} (from C_N)



Need linear response from N_{sources} to $N_{ch}!$

From initial state to final hadrons

- $\eta_S
 ightarrow \eta$, some extra hydro push ightarrow reduction of a_{nm}
- resonance decays relevant effect [PB+WB+A. Olszewski 2015] \rightarrow increase
- charge conservation, other correlations not included
- removal of the short-range effects (discussed in a while)

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Hydro (no length fluctuations included here):

 $C_{N}(\eta_{1},\eta_{2})$ c=30-40%



a_{nm} from hydro



 $\langle a_n a_m \rangle = a_{nm}$, "torque"=our model without length fluct., "oversampled"=no resonance decays

About 50% of $\sqrt{a_{11}}$ from resonances!

a_{nm} from hydro



 $\langle a_n a_m \rangle = a_{nm}$, "torque"=our model without length fluct., "oversampled"=no resonance decays

Removing the short-range defects

Rather involved experimental procedure, which gets rid of correlations shorter that ~ 1 unit in $\eta_1-\eta_2$



Removes resonances, jets, ..., some of the source fluctuations After the procedure only a_{11} survives in exp., other =0 (do not throw out the baby with the bath water!)

It should be carried out in a model simulation

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Conclusions

- Data: need for extra longitudinal fluctuations in the initial state (see also the talk by Piotr Bożek, torque in p+Pb)
- No collectivity, no flow needed robust analysis
- Anatomy:
 - Early stage: N_A vs N_B fluctuations, strength fluctuations, length fluctuations
 - Late stage: resonance decays (large effect), conservation laws
- Many sources required, 1/N effect
- $1/\sqrt{N_{ch}}$ scaling of $\sqrt{a_{11}} \rightarrow$ linear relation between $N_{sources}$ and N_{ch}
- Intricate procedures: $C \to C_N$, normalization, removing short-range correlations necessary in modeling for 1-1 comparisons