Initial conditions for hydro: implications for phenomenology [1]

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[1] Work with Mikołaj Chojnacki, Wojciech Florkowski, and Adam Kisiel

Evolution history

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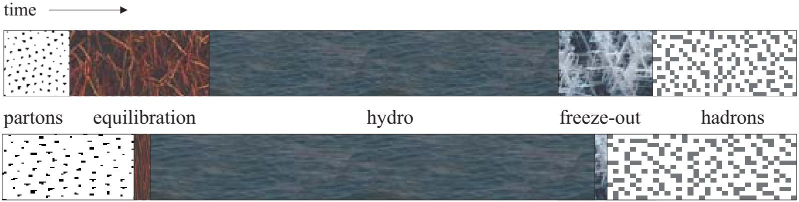

free streaming + sudden equilibration (FS+SE)

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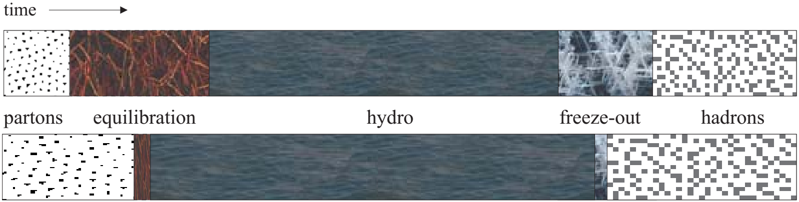

free streaming + sudden equilibration (FS+SE)

- P. F. Kolb, J. Sollfrank, U. Heinz, Phys. Rev. **C62** (2000) 054909

... idealize the initial kinetic equilibration stage of the collision by a stage of collisionless free-streaming followed by hydrodynamic expansion, thereby assuming a sudden, but delayed transition from a non-equilibrium initial state to a fully thermalized fluid ...

- W. Jas, St. Mrówczyński, Phys.Rev.C76 (2007) 044905
- Y. M. Sinyukov, Acta Phys. Polon. **B37** (2006) 3343
- M. Gyulassy, Y. M. Sinyukov, I. Karpenko, A. V. Nazarenko, Braz. J. Phys. **37** (2007) 1031 (similar ideas, azimuthal asymmetry of flow)
- WB, M. Chojnacki, W. Florkowski, A. Kisiel, arXiv:0801.4361v1 [nucl-th]

Geometry and asymmetric flow

Consider a non-central collision. I has been generally thought that FS+SE, which decreases spatial asymmetry, leads automatically to a reduction of the elliptic flow. However ...

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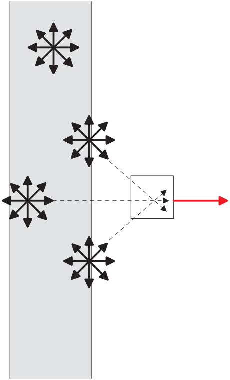

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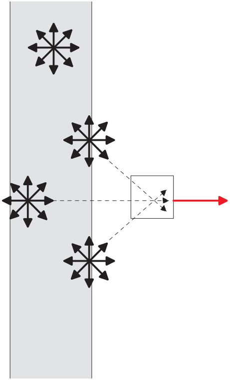

FS+SE from a slab yields to flow velocity $\perp$ to the surface

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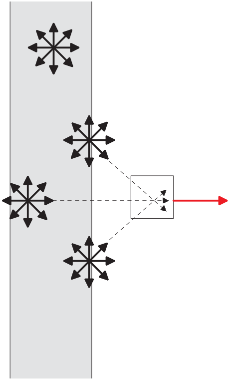

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FS+SE → anisotropic flow velocity of the fluid

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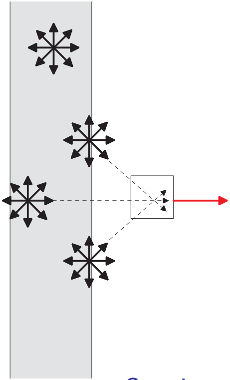

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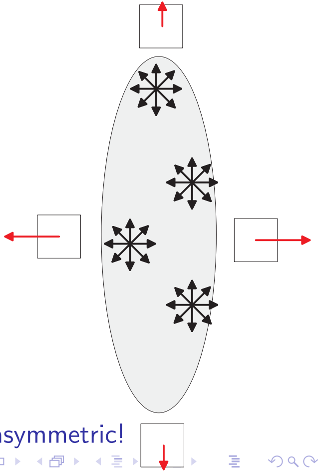
FS+SE $\rightarrow$ anisotropic flow velocity of the fluid



Starting condition of hydro is azimuthally asymmetric!

Kinematics

Initial density profile: $n(x_0, y_0) = \exp\left(-\frac{x_0^2}{2a^2} - \frac{y_0^2}{2b^2}\right)$ (can be any)

Massless partons are formed at the proper time $\tau_0 = \sqrt{t_0^2 - z_0^2}$, move along straight lines at the speed of light, sudden equilibration occurs at $\tau = \sqrt{t^2 - z^2}$

$$\eta_0 = \frac{1}{2}\log\frac{t_0 - z_0}{t_0 + z_0}, \quad \eta = \frac{1}{2}\log\frac{t - z}{t + z}$$

Parton's momentum: $p^\mu = (p_T \mathrm{ch} Y, p_T \cos\phi, p_T \sin\phi, p_T \mathrm{sh} Y)$

Elementary kinematics $\rightarrow$

$$\tau \mathrm{sh}(\eta - Y) = \tau_0 \mathrm{sh}(\eta_0 - Y)$$

$$x = x_0 + \Delta\cos\phi, \quad y = y_0 + \Delta\sin\phi$$

$$\Delta = \frac{t - t_0}{\mathrm{ch} Y} = \tau \mathrm{ch}(\eta - Y) - \sqrt{\tau_0^2 + \tau^2 \mathrm{sh}^2(\eta - Y)}$$

The $\eta \simeq Y$ condition

The phase-space densities of partons at τ_0 and τ are related:

$$\frac{d^6N(\tau)}{dY\,d^2p_T d\eta dx dy} = \int d\eta_0 dx_0 dy_0 \frac{d^6N(\tau_0)}{dY\,d^2p_T d\eta_0 dx_0 dy_0} \times$$

$$\delta(\eta_0 - Y - \mathrm{arcsh}[\frac{\tau}{\tau_0}\mathrm{sh}(\eta - Y)])\delta(x - x_0 - \Delta\cos\phi)\delta(y - y_0 - \Delta\sin\phi)$$

Assume a factorized boost-invariant form (good at midrapidity):

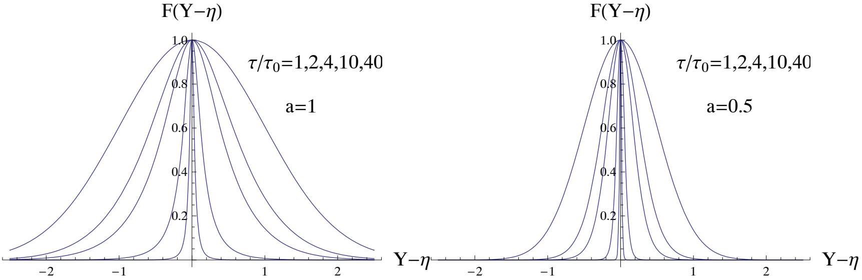
$$\frac{d^6N(\tau_0)}{dY\,d^2p_T d\eta_0 dx_0 dy_0} = n(x_0, y_0)F(Y - \eta_0, p_T)$$

If F is peaked near $Y = \eta_0$, *e.g.* $F \sim \exp[-(Y - \eta_0)^2/(2a^2)]$ with $a \sim 1$, and if $\tau \gg \tau_0$, then
$F \sim \exp\left(-\mathrm{arcsh}^2\left[\frac{\tau}{\tau_0}\mathrm{sh}(Y - \eta)\right]/(2\sigma^2)\right) \sim \delta(Y - \eta)$ and

$$\frac{d^6N(\tau)}{dY\,d^2p_T d\eta dx dy} \sim \delta(Y - \eta)$$

Remark: $\delta(Y-\eta)$ follows from the kinematics at $\tau \gg \tau_0$

It effectively works for $\tau \geq \tau_0$



Transverse flow decreased somewhat by the spread in $Y-\eta$

$T_{\mu\nu}$

Energy-momentum tensor from free streaming from τ_0 to $\tau \gg \tau_0$ at midrapidity:

$$T^{\mu\nu}(x, y, \eta = 0) = \int dY d^2 p_T \frac{d^6 N(\tau)}{dY d^2 p_T d\eta dx dy} p^\mu p^\nu$$

$$= A \int_0^{2\pi} d\phi \, n_0 \left[x - (\tau - \tau_0)\cos\phi, y - (\tau - \tau_0)\sin\phi\right] \times$$

$$\times \begin{pmatrix} 1 & \cos\phi & \sin\phi & 0 \\ \cos\phi & \cos^2\phi & \cos\phi\sin\phi & 0 \\ \sin\phi & \cos\phi\sin\phi & \sin^2\phi & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix},$$

Landau matching

$T_{\mu\nu}u^{\nu} = \epsilon g_{\mu\nu}u^{\nu}$ - Landau matching (LM) condition

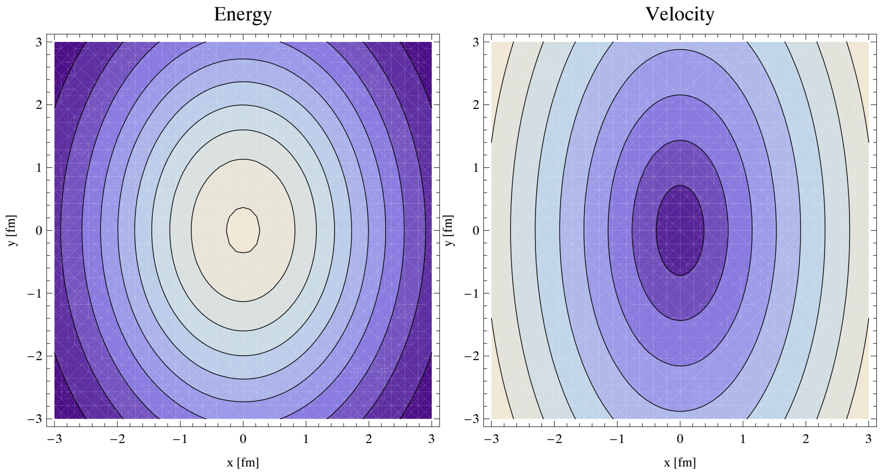
$u = \gamma(1, \vec{v})$ - flow velocity

$u^{\mu}T_{\mu\nu}u^{\nu} = \epsilon$

In the rest frame (RF) of the fluid element $u = (1, 0, 0, 0)$ and $\epsilon = T_{00}^{\mathrm{RF}}$

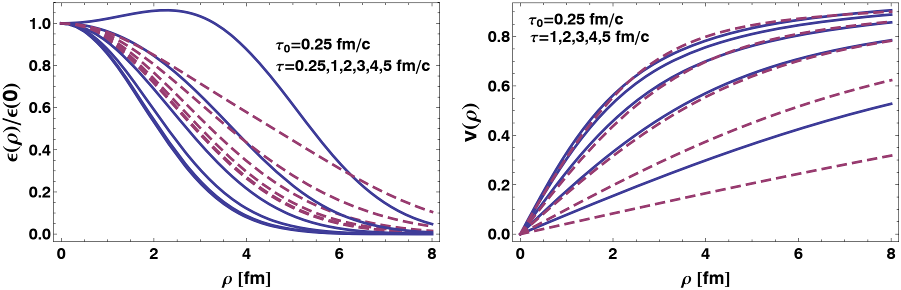
- LM is solved numerically

FS from $\tau_0 = 0.25$ fm to $\tau = 1$ fm



Flow velocity steeper along the x axis (in-plane)

Hubble flow

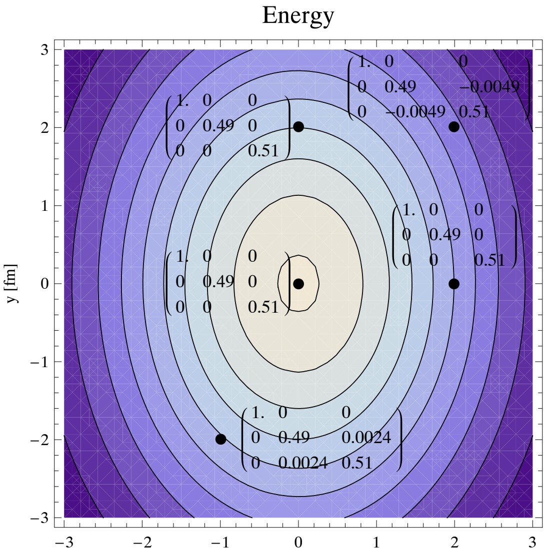


ρ - transverse radius, solid: in-plane, dashed: out-of-plane

Hubble flow follows from the Gaussian profile at low $\rho(\tau - \tau_0)$, as

$$\mathbf{v} \equiv (v_x, v_y, v_z) = -\frac{\tau - \tau_0}{3}\frac{\nabla n(x,y)}{n(x,y)} = \frac{\tau - \tau_0}{3}\left(\frac{x}{a^2}, \frac{y}{b^2}, 0\right)$$

$T_{\mu\nu}$ in the local rest frame



Pass to the rest frame of the fluid element

$T_{\mu\nu}^{\mathrm{RF}}/\epsilon$,
$\mu, \nu = 0, 1, 2$

Almost the "transverse hydro" form

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1/2 & 0 \\ 0 & 0 & 1/2 \end{pmatrix}$$

[see talks by M. Chojnacki and P. Bożek]

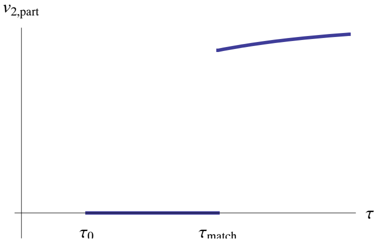
$T_{\mu\nu} \rightarrow \epsilon \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1/2 & 0 \\ 0 & 0 & 1/2 \end{pmatrix}$ - matching to transverse (ideal boost-invariant) hydro (very close)

Larger deviation in T_{xx} and T_{yy} than in T_{xy}, as in viscous hydro

$T_{\mu\nu} \rightarrow \begin{pmatrix} \epsilon & 0 & 0 & 0 \\ 0 & p & 0 & 0 \\ 0 & 0 & p & 0 \\ 0 & 0 & 0 & p \end{pmatrix}$ - matching to 3-dim. isotropic hydro (carried out here)

Eccentricity and "v_2"

$$\epsilon_{\text{part}} = \frac{\langle y\rangle^2 - \langle x\rangle^2}{\langle y\rangle^2 + \langle x\rangle^2} \qquad v_{2,\text{part}} = \frac{\langle T_{xx}\rangle - \langle T_{yy}\rangle}{\langle T_{xx}\rangle + \langle T_{yy}\rangle}$$



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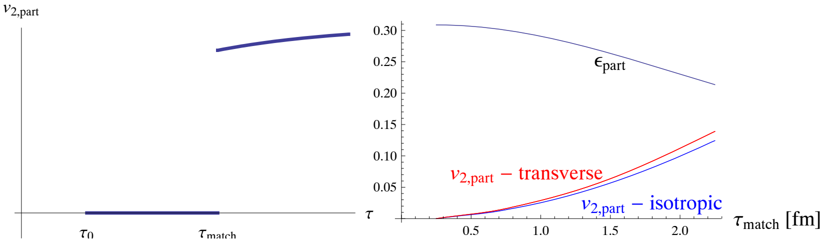


Early v_2 is generated by FS+SE

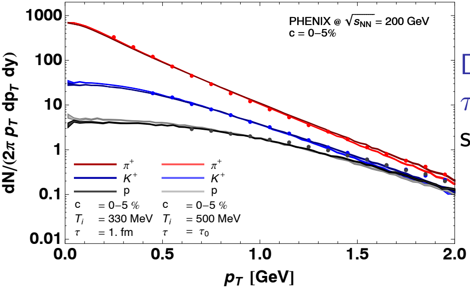
Decrease of spatial asymmetry compensated by asymmetric flow

Delayed hydro

Results with FS from $\tau_0 = 0.25$ fm up to $\tau \sim 1$ fm/c followed by SE and hydro are virtually indistinguishable from those without FS, *i.e.* with hydro started at τ_0

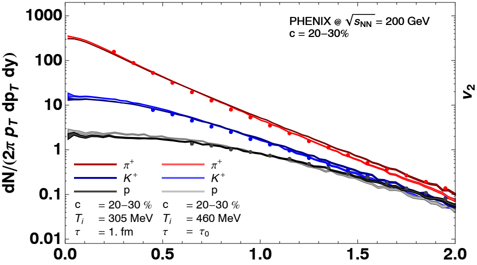
$\longrightarrow$

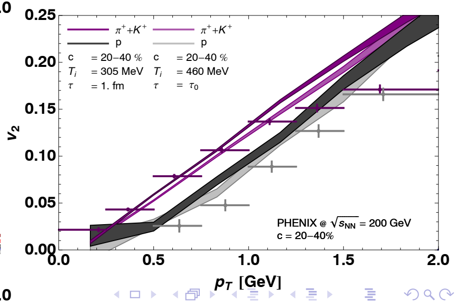
p_T-spectra and v_2 with and without FS



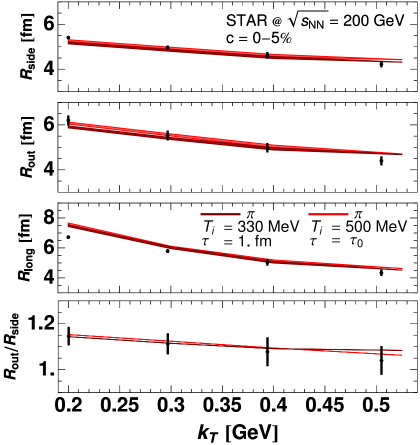
Darker lines/bands – with FS to $\tau = 1$ fm

slightly larger flow





Pionic HBT radii with and without FS



Darker lines/bands – with FS

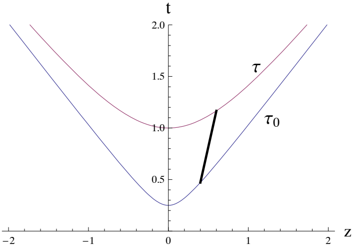
[see the talk by W. Florkowski]

Conclusions

- FS+SE mimics viscous hydro
- allows to delay the start of hydrodynamics to realistic times
- helps with the "early thermalization puzzle"
- For non-central collisions the mechanism generates the initial azimuthally asymmetric flow
- For non-boost-invariant systems the longitudinal flow also develops from FS+SE
- Early transverse flow helps tremendously with the phenomenology → a uniform description of the RHIC data [WB, MCh, WF, AK, PRL **101** (2008) 022301, see talks by W. Florkowski and S. Pratt - SP, J. Vredevoogd, arXiv:0809.0516]
- Landau matching condition very smoothly joins FS to transverse hydrodynamics
- Possible scenario:
  FS → SE → transverse hydro → isotropic hydro

Backup slides

Kinematics of FS



$$z = z_0 + (t - t_0)\mathrm{th}Y$$
$$\tau\mathrm{sh}\eta = \tau_0\mathrm{sh}\eta_0 + (\tau\mathrm{ch}\eta - \tau_0\mathrm{ch}\eta_0)\mathrm{th}Y$$
$$\tau\mathrm{sh}(\eta - Y) = \tau_0\mathrm{sh}(\eta_0 - Y)$$

above $\rightarrow$

$$\frac{t-t_0}{\mathrm{ch}Y} = \tau\mathrm{ch}(\eta_0 - Y) - \sqrt{\tau_0^2 + \tau^2\mathrm{sh}^2(\eta - Y)}$$

$T_{\mu\nu}$ in the boost-invariant model with $\delta(Y-\eta)$

$$T^{\mu\nu}(x,y,\eta) = \int dY d^2p_T \frac{d^6N(\tau)}{dY d^2p_T d\eta dx dy} p^\mu p^\nu$$

$$= A \int_0^{2\pi} d\phi \, n\left(x-(\tau-\tau_0)\cos\phi, y-(\tau-\tau_0)\sin\phi\right) \times$$

$$\begin{pmatrix} \cosh^2\eta & \cosh\eta\cos\phi & \cosh\eta\sin\phi & \cosh\eta\sinh\eta \\ \cosh\eta\cos\phi & \cos^2\phi & \cos\phi\sin\phi & \cos\phi\sinh\eta \\ \cosh\eta\sin\phi & \cos\phi\sin\phi & \sin^2\phi & \sin\phi\sinh\eta \\ \cosh\eta\sinh\eta & \cos\phi\sinh\eta & \sin\phi\sinh\eta & \sinh^2\eta \end{pmatrix}$$