BALANCE FUNCTIONS IN THE SINGLE-FREEZE-OUT MODEL

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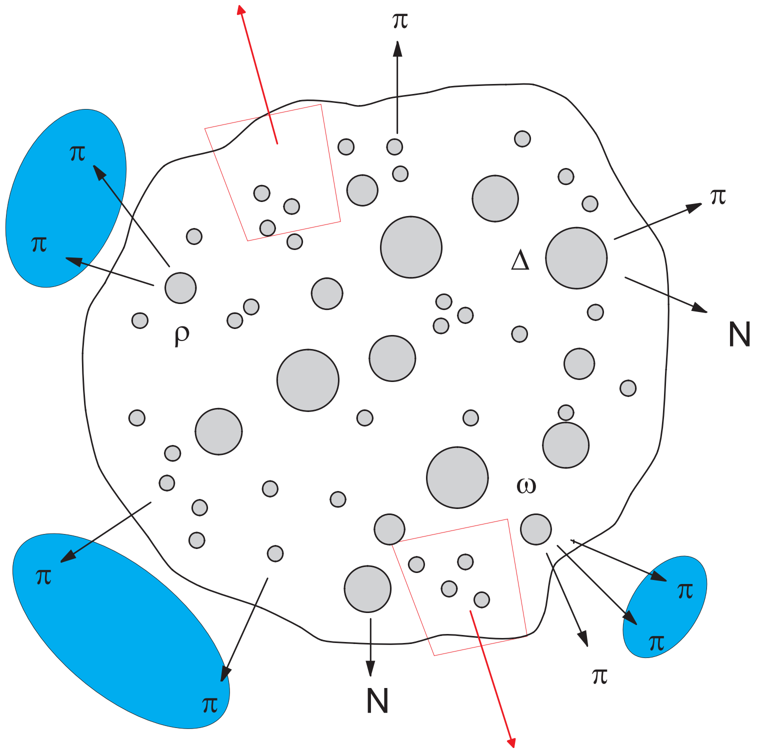


Concept of balance functions

S. Bass, P. Danielewicz, and S. Pratt, PRL 85 (2000) 2689

$$B(\delta, Y) = \frac{1}{2}\left\{\frac{\langle N_{+-}(\delta)\rangle - \langle N_{++}(\delta)\rangle}{\langle N_{+}\rangle} + \frac{\langle N_{-+}(\delta)\rangle - \langle N_{--}(\delta)\rangle}{\langle N_{-}\rangle}\right\}$$

N_{+-} and N_{-+} – number of the unlike-sign pairs
N_{++} and N_{--} – number of the like-sign pairs

The two members of the pair fall into the rapidity window Y, with relative rapidity

$$\delta = \Delta y = |y_2 - y_1|$$

N_{+} (N_{-}) – number of positive (negative) particles in the interval Y

Relation to charge fluctuations

Asakawa, Heinz, and Müller, PRL 85 (2000) 2072, Jeon and Koch, PRL 85 (2000) 2076

After integration over δ

$$\int_0^Y d\delta\, B(\delta, Y) = \frac{1}{2}\left\{\frac{\langle N_+ N_-\rangle - \langle N_+(N_+ - 1)\rangle}{\langle N_+\rangle} + (+ \to -)\right\}$$

charge: $Q = N_+ - N_-$, multiplicity of charged particles: $N_{\rm ch} = N_+ + N_-$

$$\frac{\langle (Q - \langle Q\rangle)^2\rangle}{\langle N_{\rm ch}\rangle} = 1 - \int_0^Y d\delta\, B(\delta, Y)$$

For sufficiently large Y we have $\int_0^Y d\delta\, B(\delta, Y) = 1$

Physical significance

The width of B in δ gives info about the hadronization time

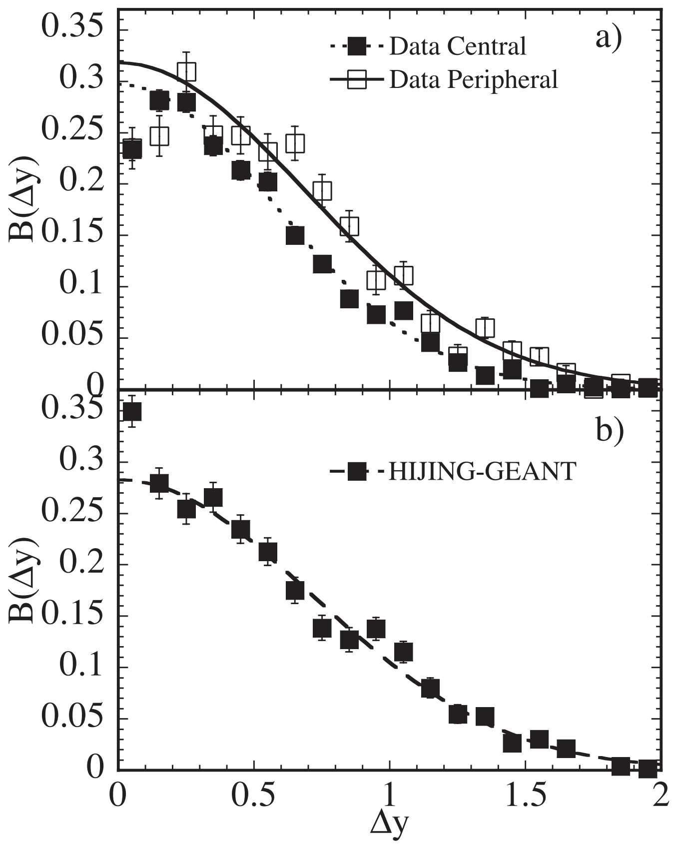
small width $\equiv$ late-stage hadronization
large width $\equiv$ production of hadrons at early stage

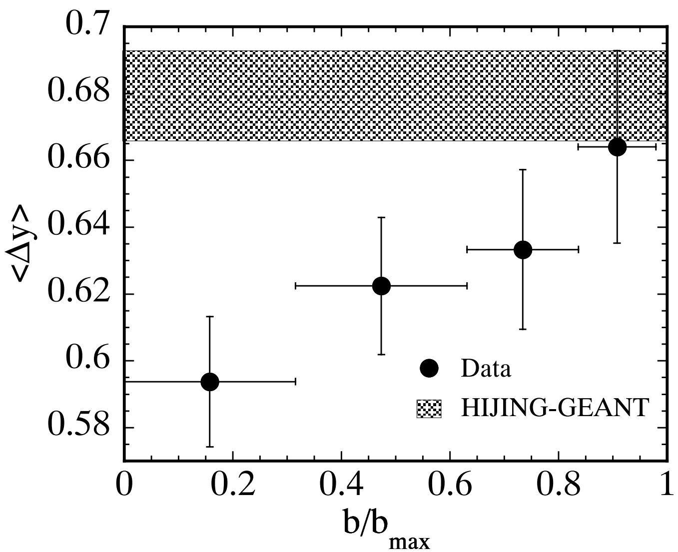
Typical scales: 1 unit of rapidity (for widths in δ), 1 fm/c (for time)

Through the balance functions we acquire insight into dynamics of hadronization

Subtraction of $++$ pairs effectively removes the uncorrelated $+-$ pairs from the distribution

Balance functions measured by STAR





$B(\delta)$ and its width for identified charged pions, $\Delta y \equiv \delta$, b – impact parameter [J. Adams et al., STAR Collaboration, PRL 90 (2003) 172301]

Single-freeze-out model

[WB+WF, PRL 87 (2001) 272302]

I. chemical and thermal freeze-outs occur simultaneously

– TWO thermodynamic parameters, T and μ_B

II. complete treatment of the resonances

III. Hubble-like expansion

– definition of the freeze-out hypersurface (Bjorken, Csörgő-Lörstad, Heinz)

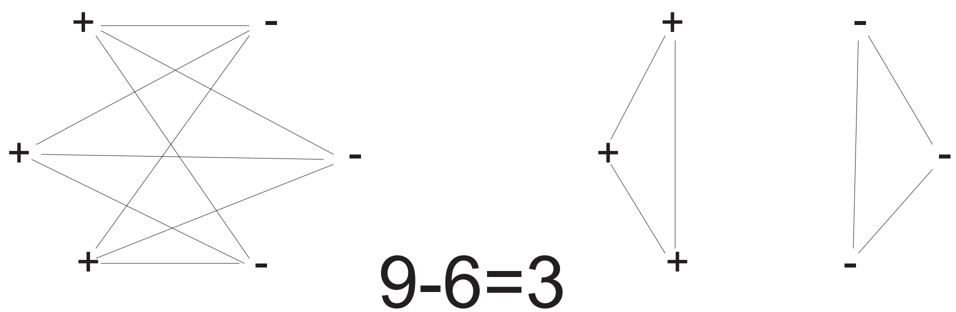
$$\tau = \sqrt{t^2 - r_z^2 - r_x^2 - r_y^2} = \text{const}, \quad u^\mu = \partial^\mu \tau = \frac{x^\mu}{\tau} = \frac{t}{\tau}\left(1, \frac{r_z}{t}, \frac{r_x}{t}, \frac{r_y}{t}\right)$$

– TWO expansion parameters: τ fixes overall normalization, ρ_{max}/τ determines the shape of the spectra $\sqrt{r_x^2 + r_y^2} < \rho_{\text{max}}$

The Białas clusters

Because of the charge conservation, in the late-hadronization scenario, the opposite-charge particles may be treated as created from neutral clusters. In the calculation of the two-particle distributions one has to take into account that particles of the same charge must originate from different clusters, whereas the particles of opposite charge may come either from different cluster or from the same cluster (A. Białas, hep-ph/0308245)

In this case the difference of the two-particle distributions, $\rho_{+-}(p_1, p_2) - \rho_{++}(p_1, p_2)$, may be reduced to a two-particle distribution in a single cluster



QUESTION: What are the clusters in a thermal model?

Two contributions for the $\pi^+\pi^-$ balance function

1) RESONANCE CONTRIBUTION (R) is determined by the decays of neutral hadronic resonances which have a $\pi^+\pi^-$ pair in the final state

$$K_S,\ \eta,\ \eta',\ \rho^0,\ \omega,\ \sigma,\ f_0$$

2) NON-RESONANCE CONTRIBUTION (NR) other possible correlations among the charged pions

The pion balance function is constructed as a sum of the two terms

$$B(\delta, Y) = B_{\rm R}(\delta, Y) + B_{\rm NR}(\delta, Y)$$

Resonance contribution

$$\frac{dN_R^{+-}}{dy_1 dy_2} = \int dy d^2 p_\perp \int d^2 p_1^\perp d^2 p_2^\perp C_\pi \frac{dN_R}{dy d^2 p_\perp} \rho_{R\to\pi^+\pi^-}(p, p_1, p_2)$$

C_π indicates the kinematic cuts for the pions $(|\eta| < 1.3,\ p_\perp > 100\mathrm{MeV})$ The momentum distribution of the resonance R is obtained from the Cooper-Frye formula

$$\frac{dN_R}{dy d^2 p_\perp} = \int d\Sigma(x) \cdot p\, f_R(p \cdot u(x))$$

where f_R is the phase-space distribution function of the resonance

The two-particle pion momentum distribution in a two-body $(\pi^+\pi^-)$ resonance decay is

$$\rho_{R\to\pi^+\pi^-} = \frac{b_{\pi\pi}}{N_2} \delta^{(4)}(p - p_1 - p_2)$$

$b_{\pi\pi}$ – the branching ratio, $N_2 = \int \frac{d^3 p_1}{E_1} \frac{d^3 p_2}{E_2} \delta^{(4)}(p - p_1 - p_2)$ – normalization

Three-body decays $(\pi^+\pi^-\pi^0)$ are treated in an analogous way, with the assumption of a constant transition matrix element

Finally,

$$B_{\rm R}(\delta) = \frac{1}{N_\pi} \sum_R \int dy_1 dy_2 C_\pi \frac{dN_R^{+-}}{dy_1 dy_2} \delta(|y_2 - y_1| - \delta)$$

Model parameters (fixed earlier by the ratios and spectra)
$T = 165 \mathrm{MeV}, \quad \langle \beta_\perp \rangle = 0.5$

Non-resonance contribution

$$\frac{dN^{+-}_{NR}}{dy_1 dy_2} = A \int d^2p_1^{\perp} d^2p_2^{\perp} C_\pi \int d\Sigma(x) p_1 \cdot u(x) f^{\pi}_{NR}(p_1 \cdot u(x))\, p_2 \cdot u(x) f^{\pi}_{NR}(p_2 \cdot u(x))$$

f^{π}_{NR} – phase-space distribution function of non-resonance pions

normalization constant A obtained from the condition $\int dy_2 \left(\frac{dN^{+-}_{NR}}{dy_1 dy_2}\right) = \frac{dN^{\pi}_{NR}}{dy_1}$

$$\tilde{B}_{\rm NR}(\delta) = \frac{1}{N_\pi} \int dy_1 dy_2 C_\pi \frac{dN^{+-}_{NR}}{dy_1 dy_2} \delta(|y_2 - y_1| - \delta)$$

R + NR contributions

$\int_0^Y d\delta B_R(\delta) = N_{\rm R}^{\pi}/N_{\pi}, \quad \int_0^Y d\delta \tilde{B}_{NR}(\delta) = N_{NR}^{\pi}/N_{\pi}$

Since some of the non-resonance pions are balanced by other charged hadrons, the final expression for the pion balance function is

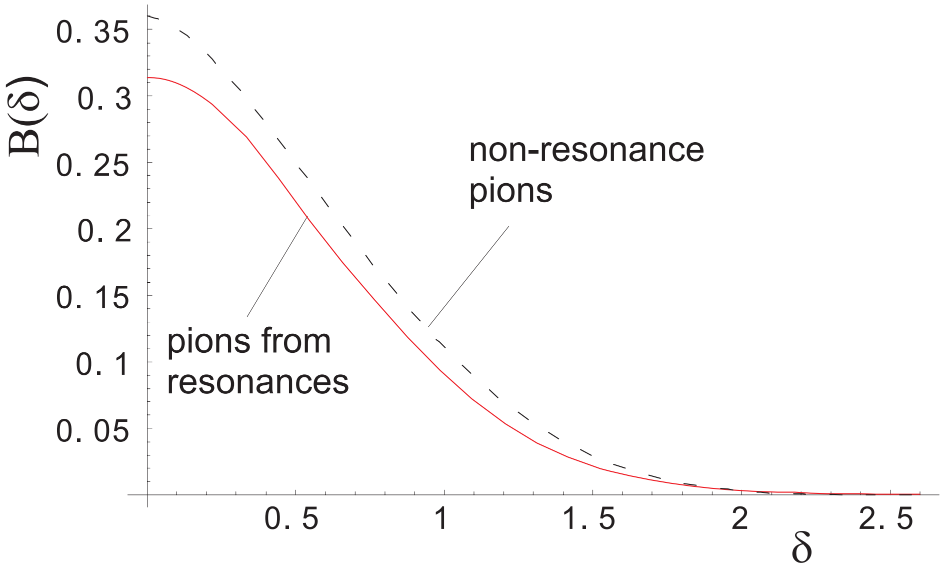
$$B(\delta) = B_{\rm R}(\delta) + \frac{N_{\rm NR}^{\pi}}{N_{\rm charged} - N_{\rm R}^{\pi}} \tilde{B}_{\rm NR}(\delta)$$

From the thermal model

$N_{\rm charged} = N_{\rm R}^{\pi} + N_{\rm NR}^{\pi} + \Delta N \ \rightarrow \ N_{\rm NR}^{\pi} + \Delta N = N_{\rm charged} - N_{\rm R}^{\pi}$

$N_{NR}^{\pi}/(N_{\rm charged} - N_{\rm R}^{\pi}) = 0.68$

Results

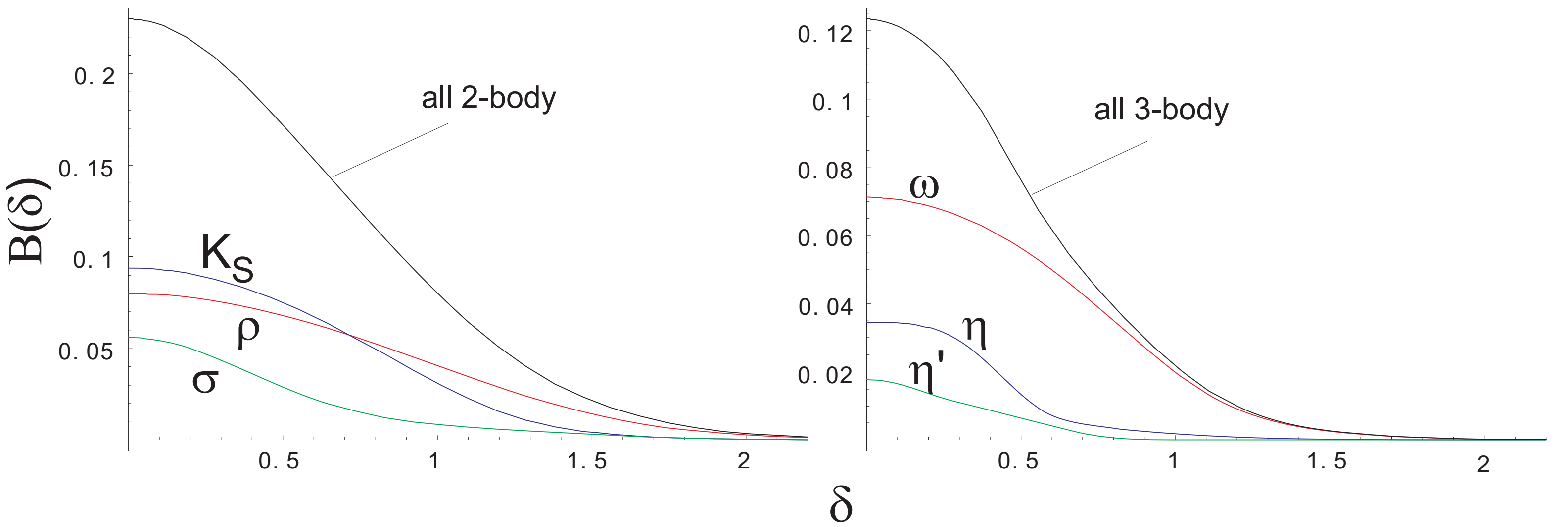


$$\rho_{\max}/\tau = 0.9 \rightarrow \langle \beta_\perp \rangle = 0.5$$

$$\langle \delta \rangle \equiv \int_{0.2}^{2.4} \delta\, B(\delta)\, d\delta / \int_{0.2}^{2.4} B(\delta)\, d\delta$$

$\langle \delta \rangle_{NR} = 0.67$, $\langle \delta \rangle_{R} = 0.65$, $\langle \delta \rangle_{R+NR} = 0.66$, (exp :0.59 – 0.66)

Anatomy of the resonance contribution



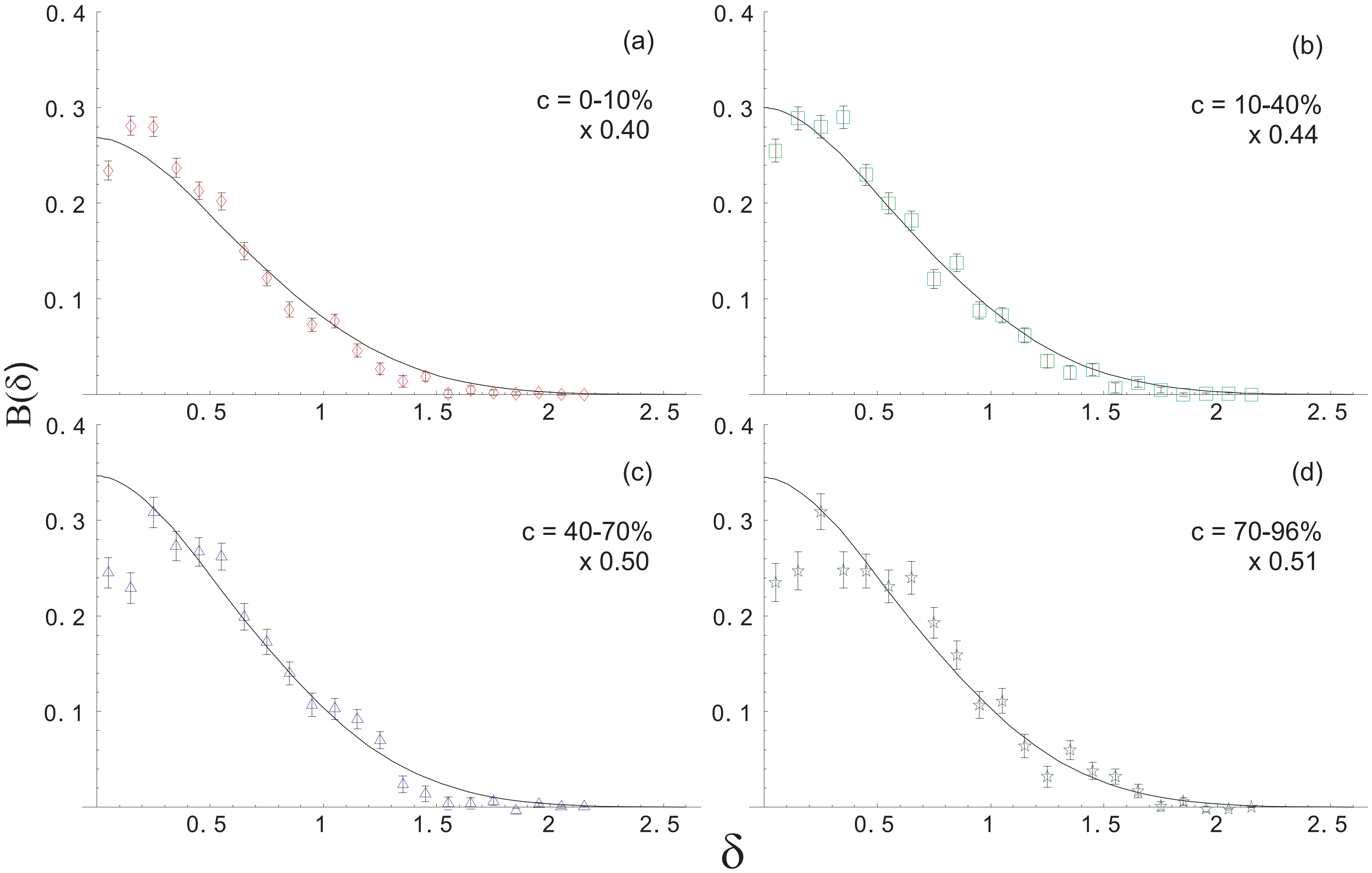
heavier resonance – wider (more phase space)

two-body wider than three-body

higher T – wider

higher flow – narrower

Fit to the STAR data



Rescaling factors (from χ^2 fits) are poor man's way of taking into account the detector acceptance and efficiency

Conclusions

1. Balance functions can be calculated in the thermal model: the resonance contribution (R) is determined in the unique way, the form of the non-resonance contribution (NR) involves additional assumptions (in our approach the non-resonance two-particle distribution is determined by the local relative thermal momenta of particles)
2. The two calculated contributions (R+NR) have similar δ-dependence, the width of the sum is somewhat larger than the width measured by STAR (0.66 vs. 0.59-0.66), while the shape is right except for the very small values of δ where the Bose-Einstein correlations are important (see also the recent nucl-th/0401008 by S. Cheng *et al.*)
3. The overall normalization must be fitted in order to take into account the effect of the efficiency of the detector, this brings a relatively large factor $\sim 1/2$
4. The detector efficiency and acceptance should be incorporated in a more detailed analysis. It may affect the normalization, as well as the shape in δ
5. No significant dependence on centrality can be produced in the thermal approach
6. Our analysis brings a further indication that the single-freeze-out model is a very reasonable first-order approximation of the final state in ultra-relativistic heavy-ion collisions. The model reproduces the ratios, spectra, $R_{\rm out}/R_{\rm side} \sim 1$, $\pi^+\pi^-$ invariant mass distributions, and the balance functions)

BACKUP SLIDES

Dependence on temperature

T [MeV]	$\langle \delta \rangle_{\mathrm{NR}}$
100	0.629
125	0.653
150	0.670
200	0.695

Relation to inclusive distributions

A. Białas and V. Koch, Phys. Lett. B456 (1999) 1

Event-by-event fluctuations are related to inclusive distributions, balance functions may be expressed in terms of one-particle and two-particle observables

$$\langle N_\pm \rangle = \int d^3p \rho_\pm(p)$$

pairs of identical (positive or negative) particles – $\langle N_\pm (N_\pm - 1) \rangle = \int d^3p_1 d^3p_2 \rho_{\pm\pm}(p_1, p_2)$
pairs of non-identical (with opposite charge) particles – $\langle N_\pm N_\mp \rangle = \int d^3p_1 d^3p_2 \rho_{\pm\mp}(p_1, p_2)$

The balance function essentially measures the difference

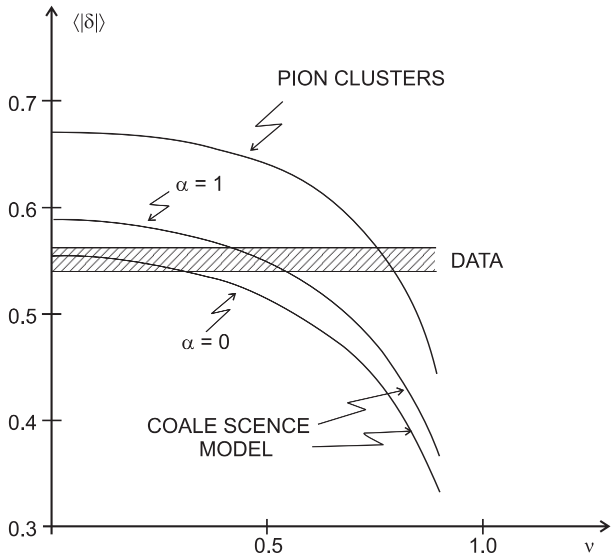
$$\rho_{+-}(p_1, p_2) - \rho_{++}(p_1, p_2)$$

Balance functions in coalescence model

A. Białas, hep-ph/0308245

results of the pion-cluster model are confronted with the quark-antiquark coalescence mechanism for pion production

conclusion: the coalescence mechanism implies a substantial reduction of the width of the balance functions



coalescence model: T. S. Biro, P. Levai, and J. Zimanyi, Phys. Lett. B347 (1995)