Kwieciński evolution of unitegrated parton distributions

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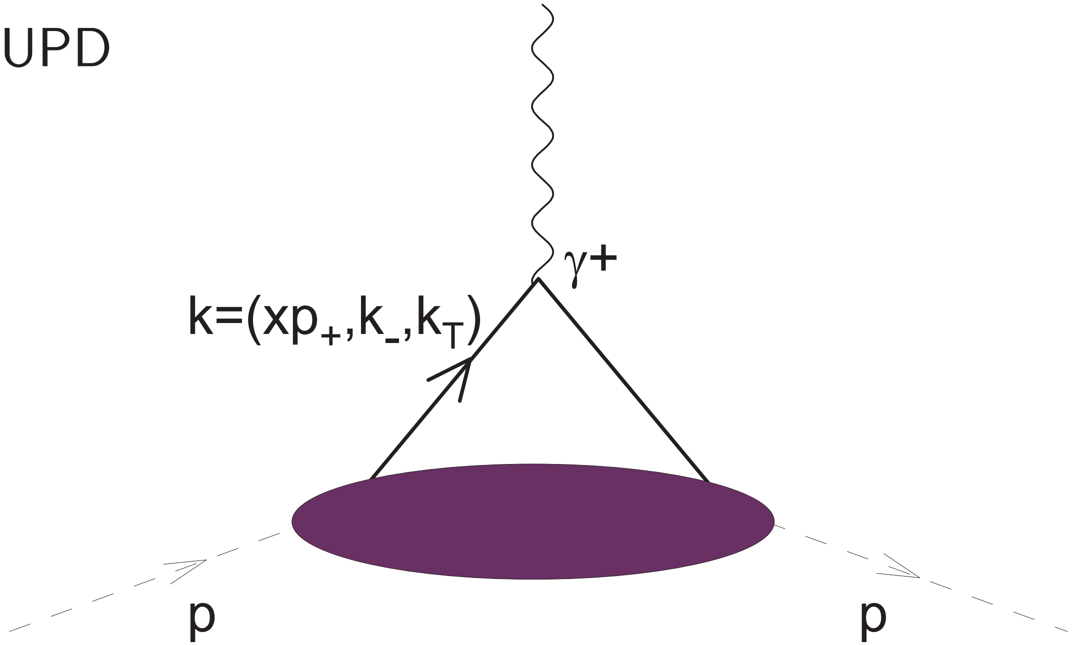
Based on work with J. Kwieciński, E. Ruiz Arriola (Granada) and A. Gawron
AG + JK + WB, Phys. Rev. D **68** (2003) 054001
ERA + WB, hep-ph/0404008

also:
JK, Acta Phys. Polon. B **33** (2002) 1809
AG + JK, Acta Phys. Polon. B **34** (2003) 133 and hep-ph/0309303

Applications/similar approaches: A. Szczurek, H. Jung, L. Lonnblad

Unintegrated parton distributions

Leading-twist UPD



No integration over $k_\perp$!

$k_\perp$ not limited

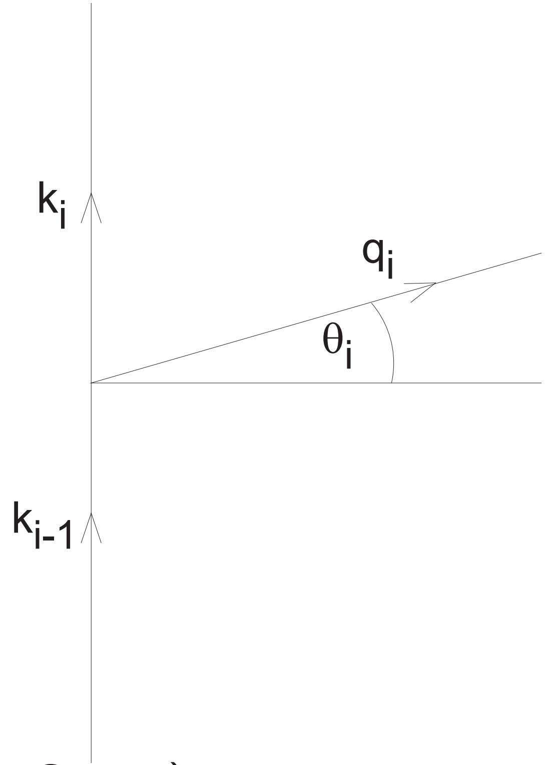
Around since the dawn of QCD, k_T-factorization, CCFM
formal definition (Collins, 2003):

$$p(x, k_\perp) = \int \frac{dy^- d^2 y_\perp}{16\pi^3} e^{-ixp^+ y^- + ik_\perp \cdot y_\perp} \langle p \mid \bar{\psi}(0, y^-, y_\perp) W[y, 0] \gamma^+ \psi(0) \mid p \rangle$$

The Kwieciński equations

Parton cascade, $z_i \equiv \frac{x_i}{x_{i-1}}$, $q'_{\perp,i} \equiv \frac{q_{\perp,i}}{1-z_i}$

$k'_{\perp,i} = k_{\perp,i-1} + (1 - z_i) q'_{\perp,i}$



CCFM angular ordering:

$\theta_i > \theta_{i-1} \Longleftrightarrow q'_{\perp,i} > z_{i-1} q'_{\perp,i-1}$

Kwieciński:

1) One-loop CCFM: $q'_{\perp,i} > q'_{\perp,i-1}$

2) include quarks

3) non-Sudakov form factor set to unity

4) $k_\perp$ not limited

$\longrightarrow$ simple equations (validity range: LO DGLAP)

$$\begin{aligned} f_{NS}(x, k_\perp, Q) &= f_{NS}(x, k_\perp, Q_0) + \int_0^1 dz \int_{Q_0^2}^{Q^2} \frac{d^2 Q'}{\pi Q'^2} \frac{\alpha(Q'^2)}{2\pi} P_{qq}(z) \\ &\times \left[\Theta(z - x) f_{NS}\left(\frac{x}{z}, k_\perp + (1 - z) Q', q \right) - f_{NS}(x, k_\perp, q) \right] \end{aligned}$$

SFSC – "similarly for the singlet channel"

Fourier-Bessel transformation

$$f_j(x,b,Q) \equiv \int d^2k_\perp\, e^{-ik_\perp \cdot b} f_j(x,k_\perp,Q) = \int_0^\infty 2\pi dk_\perp\, k_\perp J_0(bk_\perp) f_j(x,k_\perp,Q)$$

diagonalizes the equations in the transverse coordinate b:

$$Q^2 \frac{\partial f_{\rm NS}(x,b,Q)}{\partial Q^2} = \frac{\alpha(Q^2)}{2\pi} \int_0^1 dz\, P_{qq}(z) \left[\Theta(z-x)\, J_0((1-z)Qb)\, f_{\rm NS}\left(\frac{x}{z},b,Q\right) - f_{\rm NS}(x,b,Q) \right]$$

SFSC

Remarks:
$b=0 \rightarrow J_0 = 1 \longrightarrow$ equations identical to DGLAP, with the distributions f_j at $b=0$ becoming the integrated PD's:

$$f_j(x,b=0,Q) = \frac{x}{2} p_j(x,Q)$$

"b-factorization": $f(x,b,Q)$-solution $\longrightarrow F(b) f(x,b,Q)$-solution

Kwieciński: For each b at an initial scale Q_0 the non-perturbative UPD's depending on x and b ($k_\perp$) are perturbatively evolved to a higher scale Q

DGLAP: The non-perturbative PD's depending on x are perturbatively evolved from Q_0 to a higher scale Q

Initial condition assumed, for simplicity, in a factorized form

$$f_j(x, b, Q_0) = F^{\mathrm{NP}}(b)\frac{x}{2}p_j(x, Q_0), \quad F^{\mathrm{NP}}(0) = 1$$

with the (non-perturbative) *initial profile* function $F^{\mathrm{NP}}(b)$ taken to be universal for all species of partons. Certain models do predict a factorized initial condition. The initial profile function factorizes from the evolution equations. Due to evolution, at higher scales Q we have

$$f_j(x, b, Q) = F^{\mathrm{NP}}(b) f_j^{\mathrm{evol}}(x, b, Q)$$

with $f_j^{\mathrm{evol}}(x, b, Q)$ denoting the *the evolution-generated UPD*

Initial profile

1. (Kwieciński + Gawron + WB, '03):

$$p_j(x, Q_0) = \text{GRV/GRS}, \; F(b) = e^{-\frac{b^2}{b_0^2}}$$

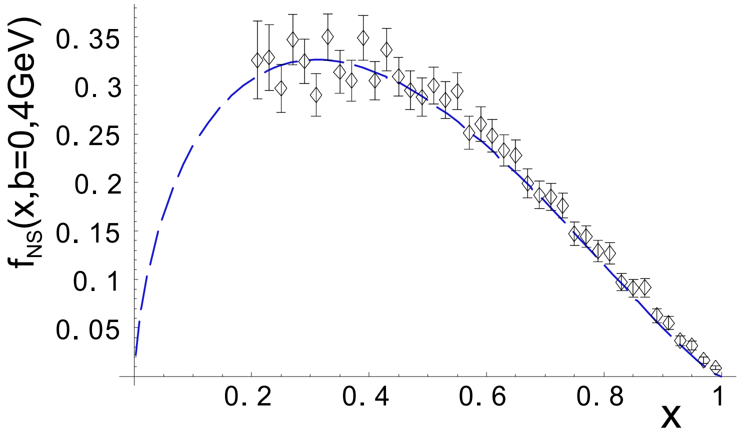
2. (ERA+WB, '04): Chiral quark models give predictions for the pion →

$$\begin{aligned} p_{NS,S}(x, b, Q_0) &= \theta(x)\theta(1-x) \\ p_G(x, b, Q_0) &= 0 \quad (\text{no gluons}) \end{aligned}$$

Momentum sum rule: setting $Q_0 = 313$ MeV leads to the 47% momentum fraction carried by the quarks at Q=2GeV $(\alpha(Q_0^2)/(2\pi) \sim 0.3)$, NLO analysis fine

Davidson+ERA, '95: the NS distribution evolved to 2 GeV agrees very well with the SMRS parameterization of the pion data

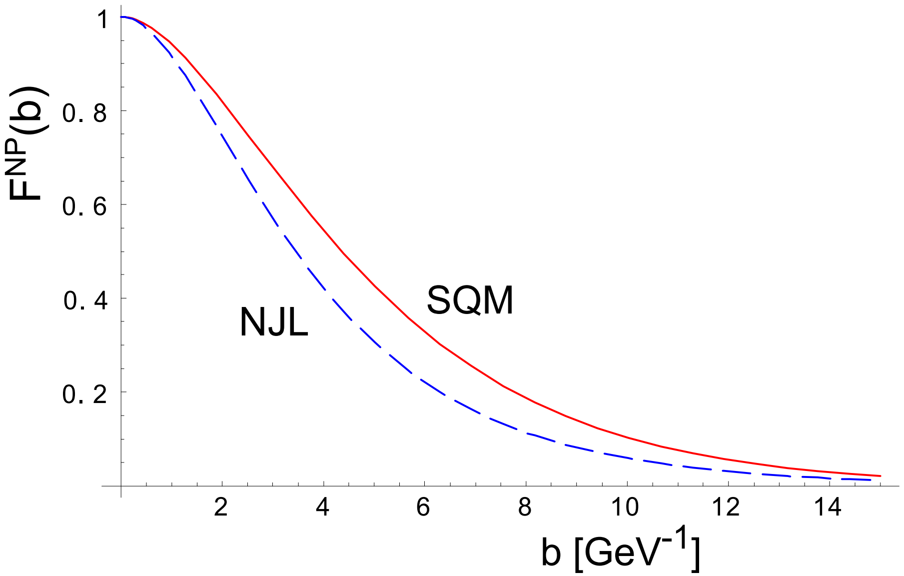
WB+ERA, '03: compares favorably to the E615 data at 4 GeV



NJL with PV regulator:

$$q(x, k_\perp, Q_0) = \frac{\Lambda^4 M^2 N_c}{4 f_\pi^2 \pi^3 \left(k_\perp^2 + M^2\right) \left(k_\perp^2 + \Lambda^2 + M^2\right)^2} \theta(x)\theta(1-x)$$

$$\langle k_\perp^2 \rangle_{\rm NP}^{\rm NJL} = (626 \text{ MeV})^2 \qquad (M = 280 \text{ MeV}, \Lambda = 871 \text{ MeV})$$



At large b fall off exponentially, at large $k_\perp$ fall off as a power law

. . . now we run the evolution

Kwieciński equations in the Mellin space

(ERA+WB, 2004) The Mellin moments are

$$f_j(n, b, Q) = \int_0^1 dx\, x^{n-1} f_j(x, b, Q)$$

Evolution involves the b-dependent anomalous dimensions

$b = 0 \rightarrow$ DGLAP

$$\gamma_{n,ab}(Qb) = 4 \int_0^1 dz\, \left[z^n J_0\left((1-z)Qb\right) - 1\right] P_{ab}(z)$$

Explicitly,

$$\gamma_{n,NS}(Qb) = \gamma_{n,NS}^{(0)} + \frac{4C_F}{(1+n)\,(2+n)} \left[-3 - 2\,n + 2\,(2+n)\; {}_1F_2\left(\frac{1}{2}; \frac{2+n}{2}, \frac{3+n}{2}; -\frac{Q^2 b^2}{4}\right)\right.$$
$$\left. - \; {}_1F_2\left(\frac{3}{2}; \frac{3+n}{2}, \frac{4+n}{2}; -\frac{Q^2 b^2}{4}\right) + \frac{Q^2 b^2}{2}\; {}_3F_4\left((1, 1, \frac{3}{2}; 2, 2, \frac{3+n}{2}, \frac{4+n}{2}; -\frac{Q^2 b^2}{4}\right)\right]$$

where ${}_pF_q$ are the generalized hypergeometric functions and

SFSC

$$\gamma_{n,NS}^{(0)} = 2C_F \left(-3 + \frac{2}{1+n} + \frac{2}{2+n} + 4 \sum_{k=1}^{n} \frac{1}{k}\right)$$

We find

$$Q^2 \frac{df_{\rm NS}(n, b, Q)}{dQ^2} = -\frac{\alpha(Q^2)}{8\pi} \gamma_{n,NS}(Qb) f_{\rm NS}(n, b, Q)$$

with the formal solution

$$\frac{f_{\rm NS}(n, b, Q)}{f_{\rm NS}(n, b, Q_0)} = \exp\left[-\int_{Q_0^2}^{Q^2} \frac{dQ'^2 \alpha(Q'^2)}{8\pi Q'^2} \gamma_{\rm NS}(n, b, Q') \right]$$

In the singlet channel

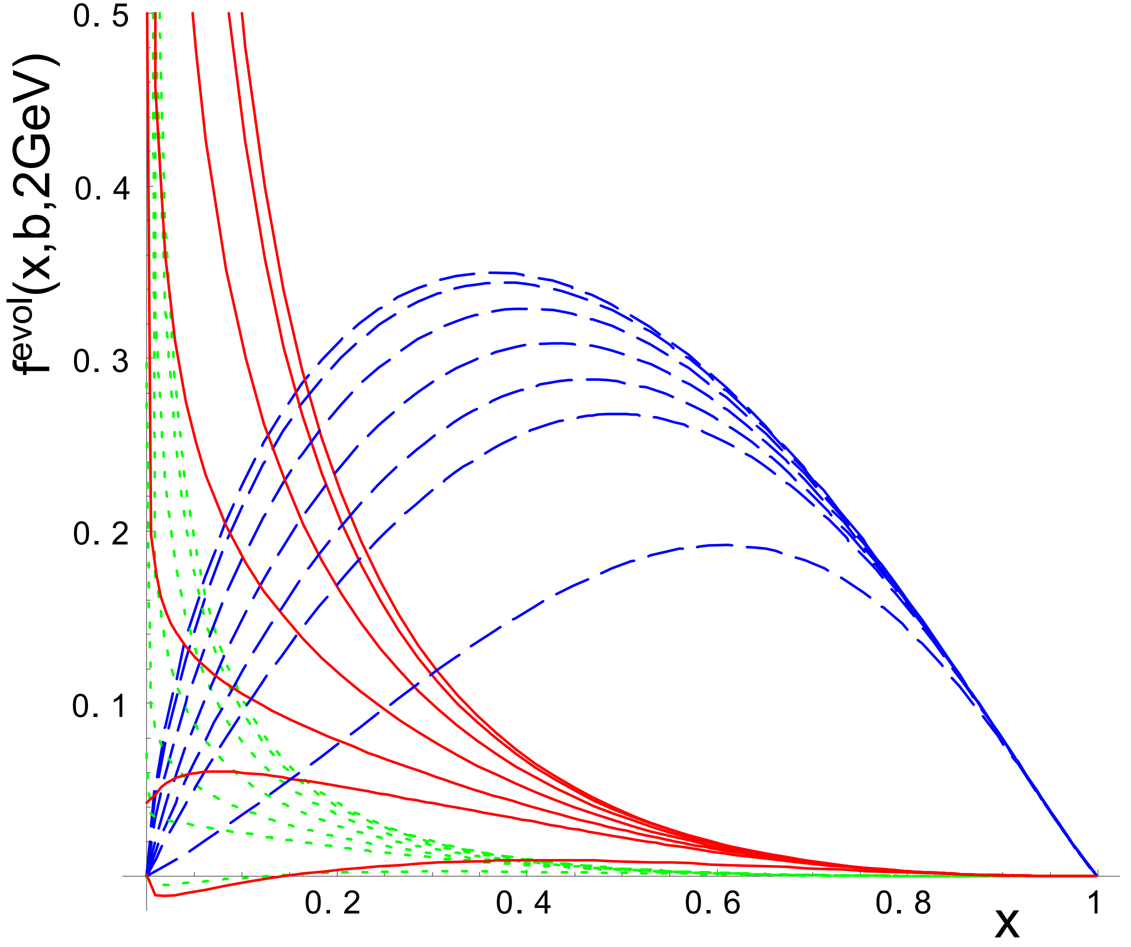
$$\begin{pmatrix} f_S(n, b, Q) \\ f_G(n, b, Q) \end{pmatrix} = \mathcal{P} \exp\left[-\int_{Q_0^2}^{Q^2} \frac{dQ'^2 \alpha(Q'^2)}{8\pi Q'^2} \Gamma_n(Qb) \right] \begin{pmatrix} f_S(n, b, Q_0) \\ f_G(n, b, Q_0) \end{pmatrix},$$

$$\Gamma_n(Qb) = \begin{pmatrix} \gamma_{n,qq}(Qb) & \gamma_{n,qG}(Qb) \\ \gamma_{n,Gq}(Qb) & \gamma_{n,GG}(Qb) \end{pmatrix}$$

$\mathcal{P}$ indicates ordering along the integration path. The above equations are solved numerically for any value of n and b. Then the inverse Mellin transform is carried out,

$$f_j(x, b, Q) = \int_C \frac{dn}{2\pi i} x^{-n} f_j(n, b, Q)$$

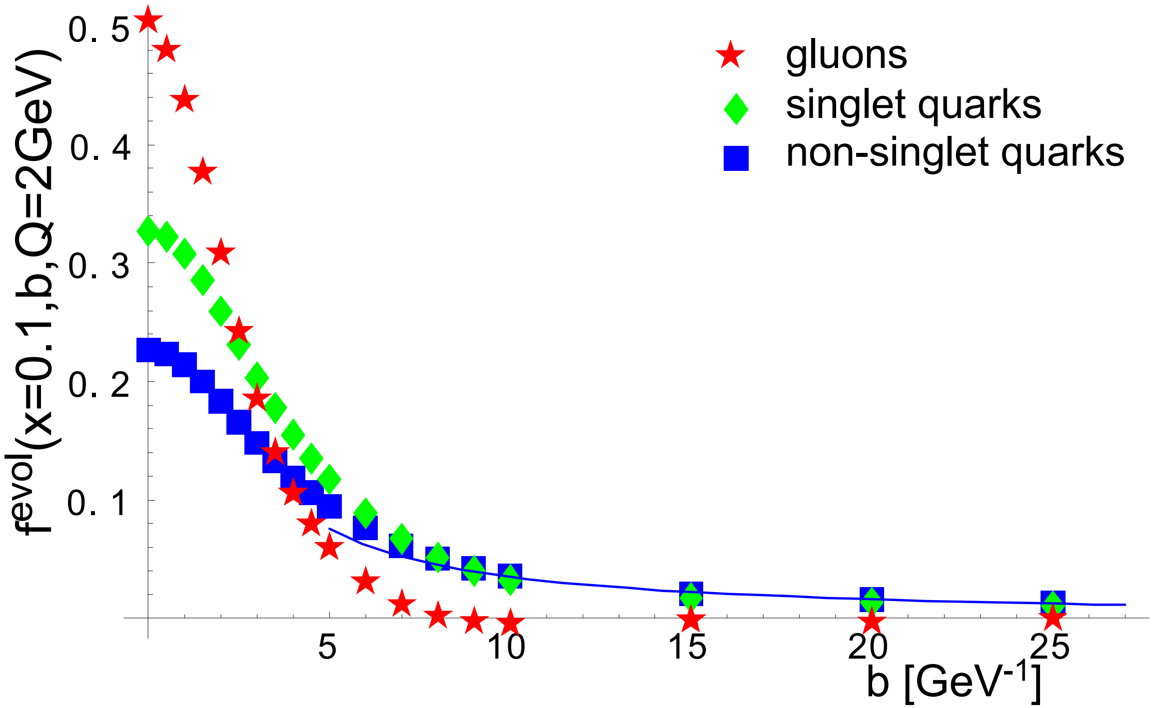
Numerical solution, $Q^2 = 4\ \mathrm{GeV}^2$



b=0
1
2
3
4
5
10
[GeV^{-1}]

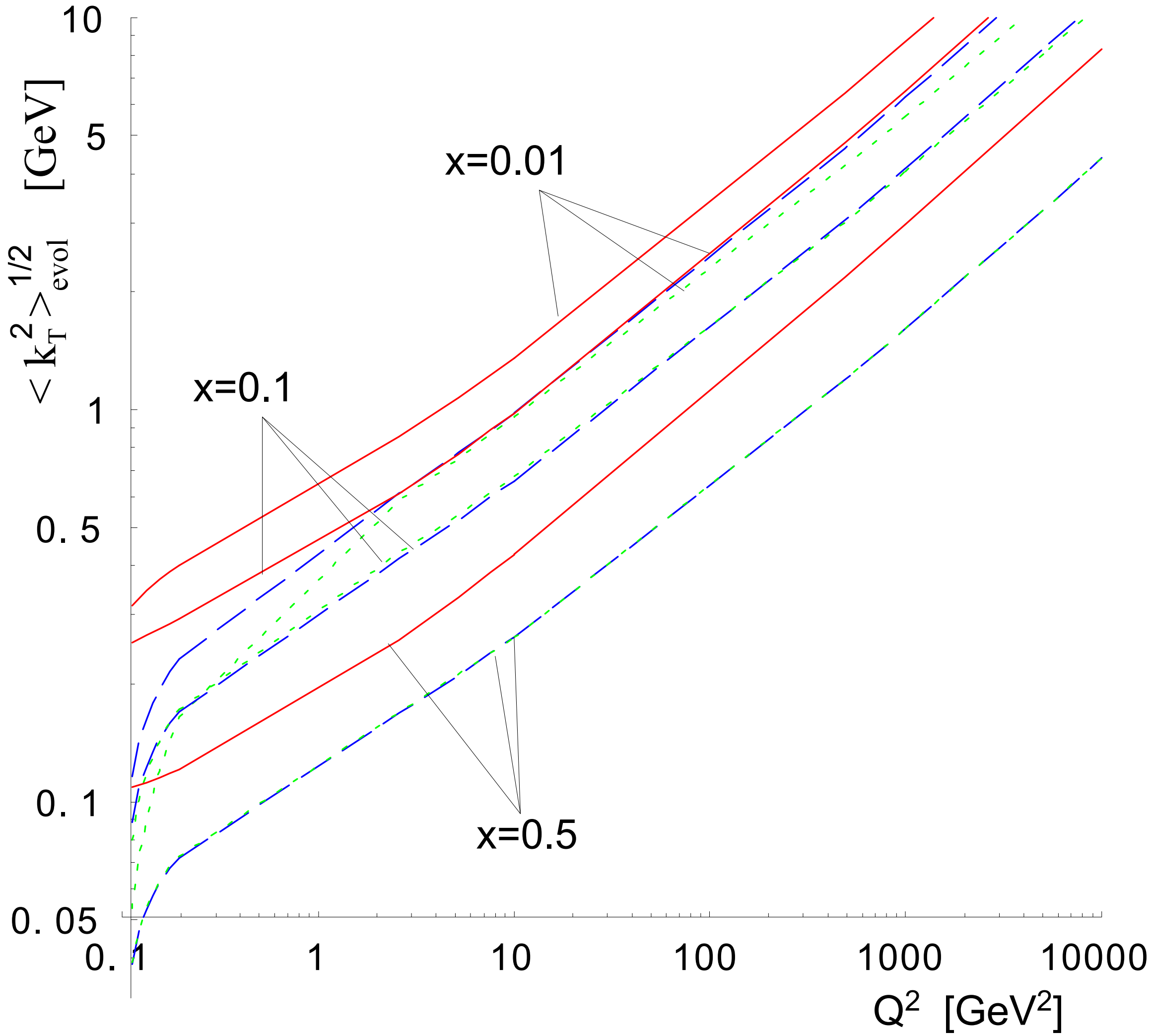
(non-singlet (valence) quarks, sea quarks $(S - NS)$, and gluons)

Numerical solution, $Q^2 = 4\ \mathrm{GeV}^2$, $x = 0.1$



Shrinking in b (spreading in $k_\perp$) as Q grows!
effect increases with increasing Q and dropping x, largest for gluons
Long, power-law tail in b

Spreading in $k_\perp$

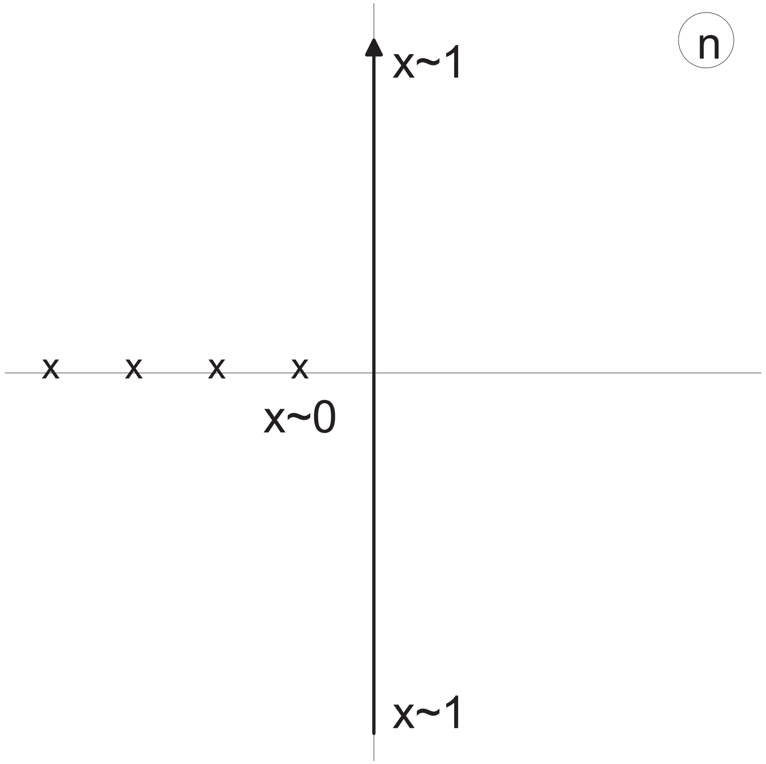


(non-singlet (valence) quarks, singlet quarks, and gluons)

Asymptotically $\langle k_\perp^2 \rangle_{\rm evol} \sim Q^2 \alpha(Q^2)$ Full width: $\langle k_\perp^2 \rangle = \langle k_\perp^2 \rangle_{\rm NP} + \langle k_\perp^2 \rangle_{\rm evol}$

Mathematical properties

. . . follow from the properties of the Mellin transform



for a generalized initial condition of the form $x^\alpha (1-x)^\beta \times F(b)$

$x \sim 0$

$$f^{\rm evol}_{\rm NS,S}(x,b,Q^2) \sim x \exp\left(2\sqrt{C_F A \log\frac{1}{x}}\right)$$

$$f^{\rm evol}_{G}(n,b,Q) \sim \exp\left(2\sqrt{2N_c A \log\frac{1}{x}}\right), \quad A \geq 0$$

$$A = \int_{Q_0^2}^{Q^2} \frac{dQ^2}{2\pi Q^2}\alpha(Q^2) J_0(Qb)$$

Generalization of DLLA, since for $b=0$ $A \sim \log(Q^2)$

For $b>0$ we may have $A<0$ and then $f_j^{\rm evol}(n,b,Q)$ oscillate

$x \sim 1$ The integrated non-singlet distribution behaves as

$$f_{\rm NS}(x,0,Q^2) \sim \frac{e^{2C_F(3-4\gamma)r_0}}{\Gamma(1+8C_F r_0)}(1-x)^{\beta+8C_F r_0}$$

$$r_k = r_k(Q_0^2,Q^2) = \int_{Q_0^2}^{Q^2} \frac{dQ'^2 \alpha(Q'^2)}{8\pi Q'^2} Q'^{2k}$$

For UPD's

$$\frac{f^{\rm evol}_{\rm NS}(x,b,Q^2)}{f^{\rm evol}_{\rm NS}(x,0,Q^2)} = 1 - \frac{2C_F b^2 r_1 (1-x)^2}{(1+8C_F r_0)(2+8C_F r_0)} + \mathcal{O}((1-x)^3)$$

Large bQ From asymptotic forms of $\gamma_n(bQ)$

$$f_{\rm NS,S}(x,b,Q) \sim b^{-8C_F r_0(Q_0^2,Q^2)},$$
$$f_{\rm G}(x,b,Q) \sim b^{-8N_c r_0(Q_0^2,Q^2)}$$

Low b At $x \longrightarrow 0$

$$\langle k_\perp^2 \rangle_{\rm NS}^{\rm evol} \sim \sqrt{-\frac{C_F \log x}{r_0} r_1} \sim \sqrt{\frac{2\beta_0 C_F \log \frac{1}{x}}{\log \frac{\alpha(\Lambda^2)}{\alpha(Q^2)}} \frac{1}{8\pi}} \alpha(Q^2) Q^2$$

At $x \longrightarrow 1$

$$\langle k_\perp^2 \rangle_{\rm NS}^{\rm evol} \longrightarrow \frac{2C_F(1-x)^2 r_1}{(1+8C_F r_0)(2+8C_F r_0)} \sim \frac{\beta_0^2 (1-x)^2}{64\pi C_F \left[\log \frac{\alpha(\Lambda^2)}{\alpha(Q^2)}\right]^2} \alpha(Q^2) Q^2$$

$\langle k_\perp^2 \rangle_{\rm NS}^{\rm evol} \longrightarrow \infty$ at $x \longrightarrow 0$ and $\langle k_\perp^2 \rangle_{\rm NS}^{\rm evol} \longrightarrow 0$ at $x \longrightarrow 1$

For the gluons and singlet quarks a similar asymptotic behavior of $\langle k_\perp^2 \rangle^{\rm evol}$ is found.
Thus, all UPD's spread in $k_\perp$ at large Q as $Q^2 \alpha(Q^2)$

Conclusions

The Kwieciński evolution is diagonal in b. It relates the UPD's at one scale to UPD's at another scale in a well-determined way. Non-perturbative and perturbative physics factorized

Equations are "semi-analytic"

UPD's spread in $k_\perp$ as the probing scale Q grows. Asymptotically, $\langle k_\perp^2 \rangle^{\rm evol}_{\rm NS,S,G} \sim Q^2 \alpha(Q^2)$. Spreading largest for gluons, and at low x

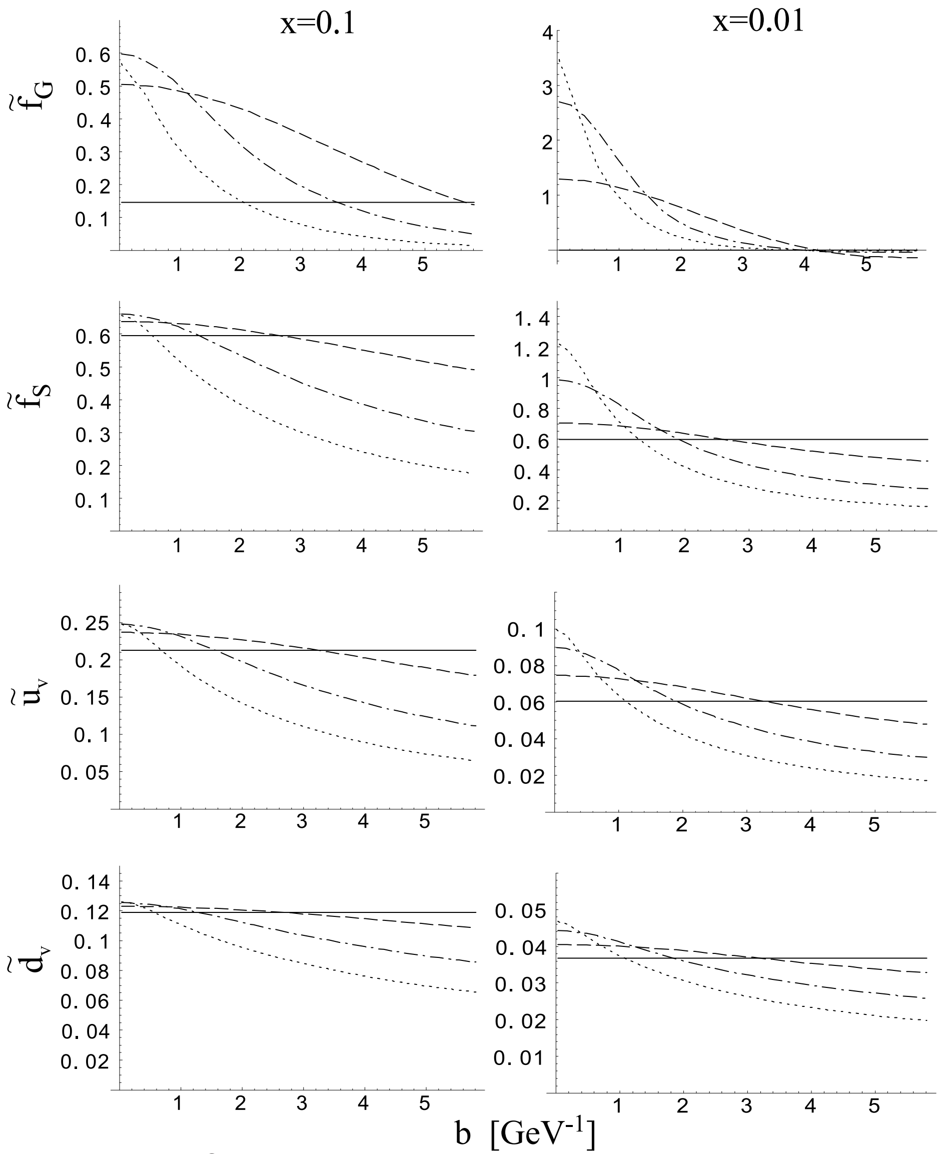
Long tails of the evolution-generated UPD's at large b

Generalized DLLA at low x

Method simple to implement

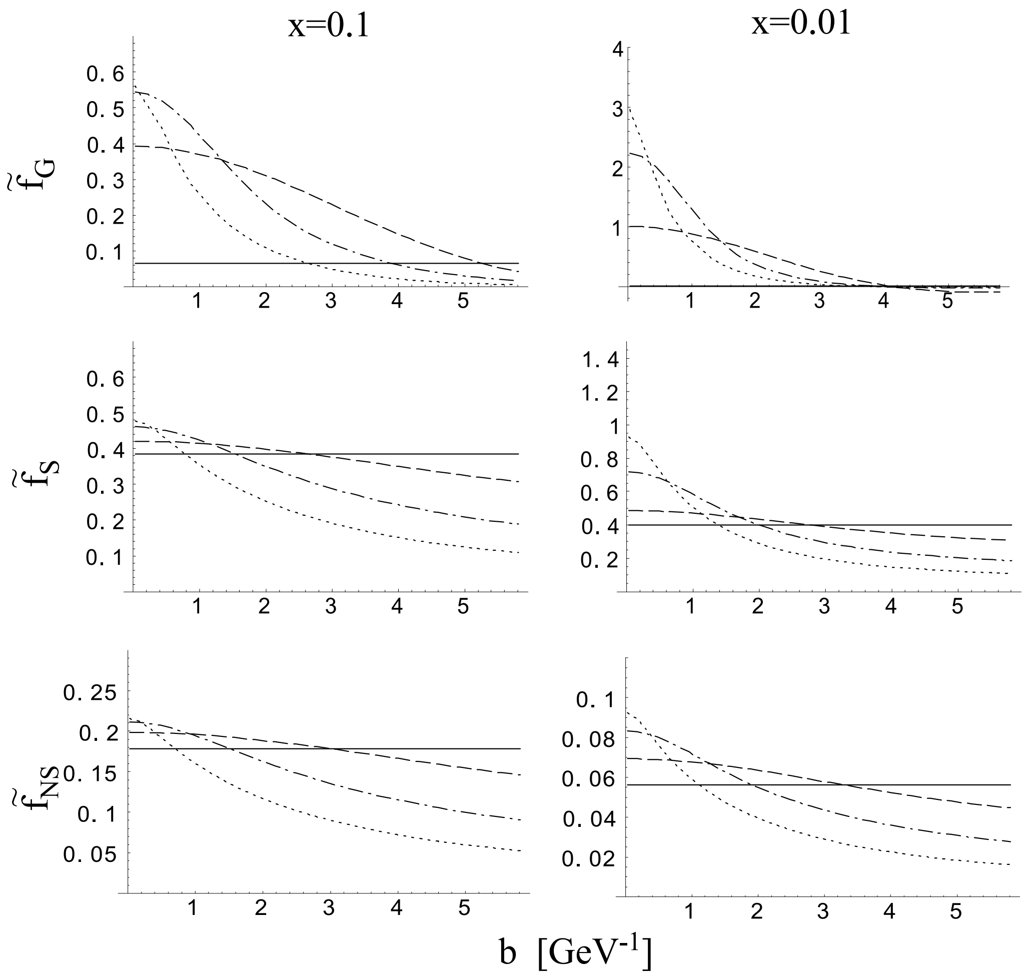
Back-up slides

Nucleon, GRV



(Q^2=0.26, 1, 10, and 100 GeV^2)

Pion, GRS



(Q^2=0.26, 1, 10, and 100 GeV^2)

Kimber + Martin + Ryskin

$$f_g(x, k_\perp) = \left. \frac{d(xg(x, Q^2))}{dQ^2} \right|_{Q^2 = k_\perp^2}$$

Generalized hypergeometric function

$$ {}_pF_q(a_1,\ldots,a_p;b_1,\ldots,b_q;z) = \sum_{k=0}^{\infty} \frac{(a_1)_k \ldots (a_p)_k}{k!\,(b_1)_k \ldots (b_q)_k} z^k $$

where

$$ (a)_k \equiv a(a+1)(a+2)\ldots(a+k-1) = \frac{\Gamma(a+k)}{\Gamma(a)} $$

Initial coditions

SQM:

$$q(x, k_\perp, Q_0) = \frac{6m_\rho^3}{\pi(k_\perp^2 + m_\rho^2/4)^{5/2}}\theta(x)\theta(1-x),$$

$$F_{\rm SQM}^{\rm NP}(b) = \left(1 + \frac{bm_\rho}{2}\right) \exp\left(-\frac{m_\rho b}{2}\right)$$

$$\langle k_\perp^2 \rangle_{\rm NP}^{\rm SQM} = \frac{m_\rho^2}{2} = (544 \text{ MeV})^2$$

NJL (with PV regularization):

$$q(x, k_\perp, Q_0) = \frac{\Lambda^4 M^2 N_c}{4f_\pi^2\pi^3 \left(k_\perp^2 + M^2\right)\left(k_\perp^2 + \Lambda^2 + M^2\right)^2}\theta(x)\theta(1-x)$$

$$F_{\rm NJL}^{\rm NP}(b) = \frac{M^2 N_c}{4f_\pi^2\pi^2}\left(2K_0(bM) - 2K_0(b\sqrt{\Lambda^2 + M^2}) - \frac{b\Lambda^2 K_1(b\sqrt{\Lambda^2 + M^2})}{\sqrt{\Lambda^2 + M^2}}\right)$$

$$\langle k_\perp^2 \rangle_{\rm NP}^{\rm NJL} = (626 \text{ MeV})^2 \qquad (M = 280 \text{ MeV}, \Lambda = 871 \text{ MeV})$$