The Hagedorn spectra

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Highly Excited Hadrons
Trento, July 2005

Wojciech Florkowski
Leonid Ya. Glozman

WB + W. Florkowski, Phys. Lett. B **490** (2000) 223
WB, in *Few-Quark Problems*, Bled, Slevenia, July 8-15, 2000, [arXiv:hep-ph/0008112]
WB + WF + L. Ya. Glozman, Phys. Rev. **D70** (2004) 117503

The Hagedorn hypothesis

R. Hagedorn, Suppl. Nuovo Cim. **3** (1965) 147
R. Hagedorn and J. Ranft, Suppl. Nuovo Cim. 6 (1968) 169
R. Hagedorn, CERN preprints CERN 71-12 (1971) and CERN-TH.7190/94 (1994)
T. E. O. Ericson and J. Rafelski, CERN Cour. **43N7** (2003) 20

At large masses m the density of hadronic states $\rho(m)$ behaves as

$$\boxed{\rho(m) \sim \exp\left(\frac{m}{T_H}\right)}$$

- The Hagedorn temperature T_H is a scale controlling the asymptotic growth of the spectrum of hadronic resonances, $\rho(m)$
- It is supported by the data
- It follows from certain models
- It has thermodynamic implications
- Different for mesons and baryons

Counting the states

Define N as the number of hadronic states of mass lower than m:

Experimental: $\boxed{N_{\rm exp}(m) = \sum_i g_i \Theta(m - m_i)}$, where

$g_i = (2J_i + 1)(2I_i + 1)$ is the spin-isospin degeneracy of the state.

Theoretical: $\boxed{N_{\rm theor}(m) = \int_0^m \rho_{\rm theor.}(m')dm'}$, where

$$\rho_{\rm theor.}(m) = f(m)\exp(m/T)$$

with $f(m)$ denoting a slowly-varying function. A typical choice is

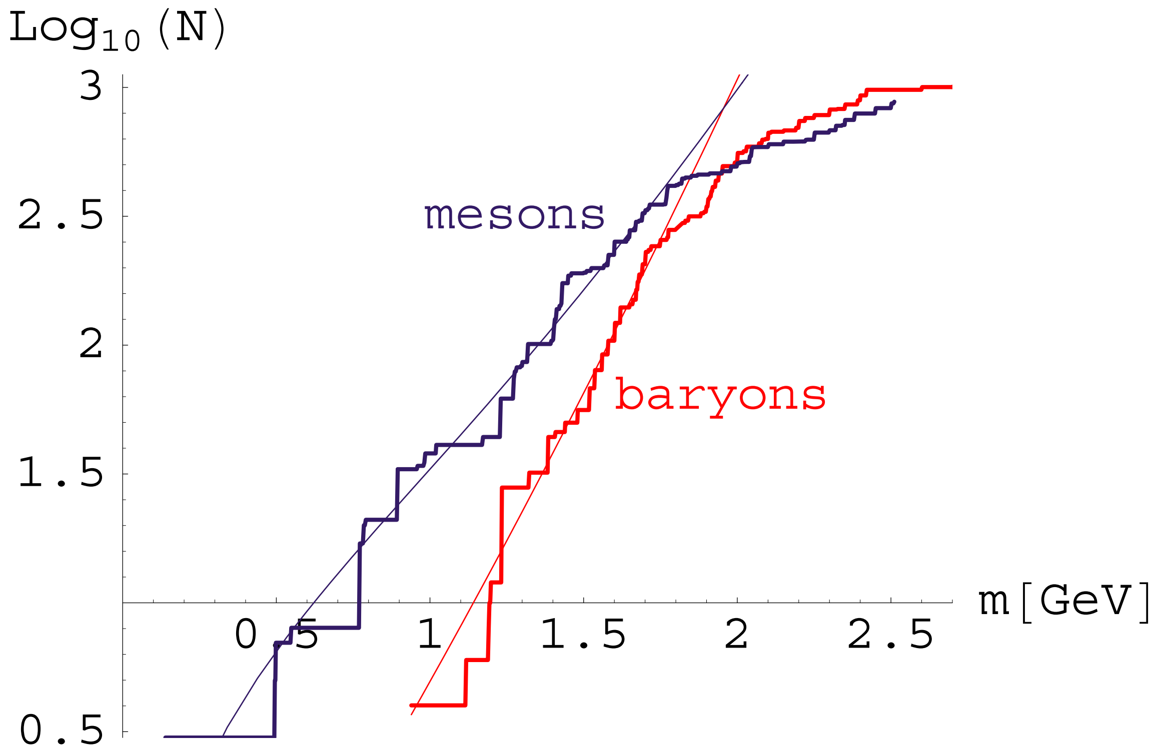
$$f(m) = A/(m^2 + m_0^2)^{5/4} \sim 1/m^{5/2}.$$

- Construct $N_{\rm exp}(m)$ using Particle Data Table
- Fit parameters of $N_{\rm theor}$

...and the results are...

Experiment

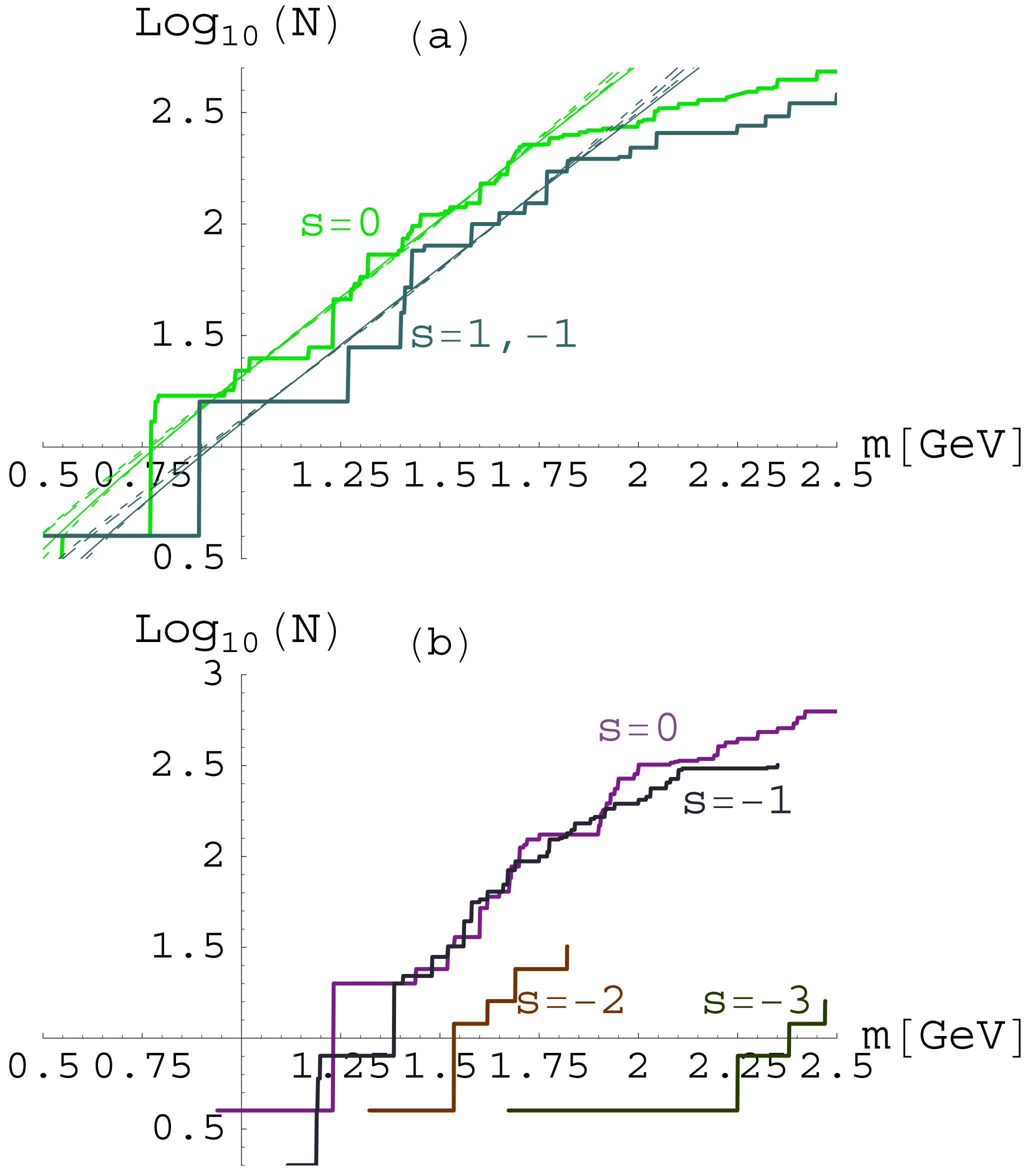
[data from PDG, Eur. Phys. J. C**3** (1998) 1, all ****, ***, **, * states]



Meson and baryon spectra (step lines) and the Hagedorn-like fits (thin lines)

- Different behavior for mesons and baryons!

Flavor universality



Strange vs. non-strange mesons (a) and baryons (b)

Hagedorn-like fits

fit range up to 1800MeV

	m_0 MeV	$T_{\rm mes}$ MeV	$T_{\rm bar}$ MeV
$\frac{A}{(m^2+m_0^2)^{5/4}}\exp(\frac{m}{T})$	500	195	141
- - -	1000	228	152
- - -	250	177	136
$\frac{A}{(m+m_0)^{5/2}}\exp(\frac{m}{T})$	1000	223	154
$A\exp(\frac{m}{T})$		311	186
$\frac{A}{m}I_2(\frac{m}{T})$		249	157

- $\boxed{T_{\rm meson} > T_{\rm baryon}}$, the inequality is substantial (a factor of ~ 1.5)
- Baryons start at larger offset
- Experimental problem? — No! More than 500 additional meson states at $m = 1.8$GeV would be needed to make the meson line parallel to the baryon line

- Have we not reached asymptotics? Quite plausible, although the plots are very suggestive starting from low values on m

- One cannon quote the value of T_H: strong dependence on the "slowly varying function" $f(m)$

$$\begin{aligned} f(m)e^{m/T} &= e^{\log f(m)+m/T} = e^{\log[f(\overline{m})+f'(\overline{m})\Delta m]+(\overline{m}+\Delta m)/T} \\ &\sim e^{\left(\frac{1}{T}+\frac{f'(\overline{m})}{f(\overline{m})}\right)\Delta m} = e^{\frac{\Delta m}{T_{\text{eff}}}} \end{aligned}$$

where

$$\frac{1}{T} = \frac{1}{T_{\text{eff.}}} - \frac{f'(\overline{m})}{f(\overline{m})}$$

Since $f'(\overline{m}) < 0$ we have $T < T_{\text{eff}}$. Only at $m \to \infty$ we have the asymptotic result $T = T_{\text{eff}}$.

Conclusion: we need a theory!

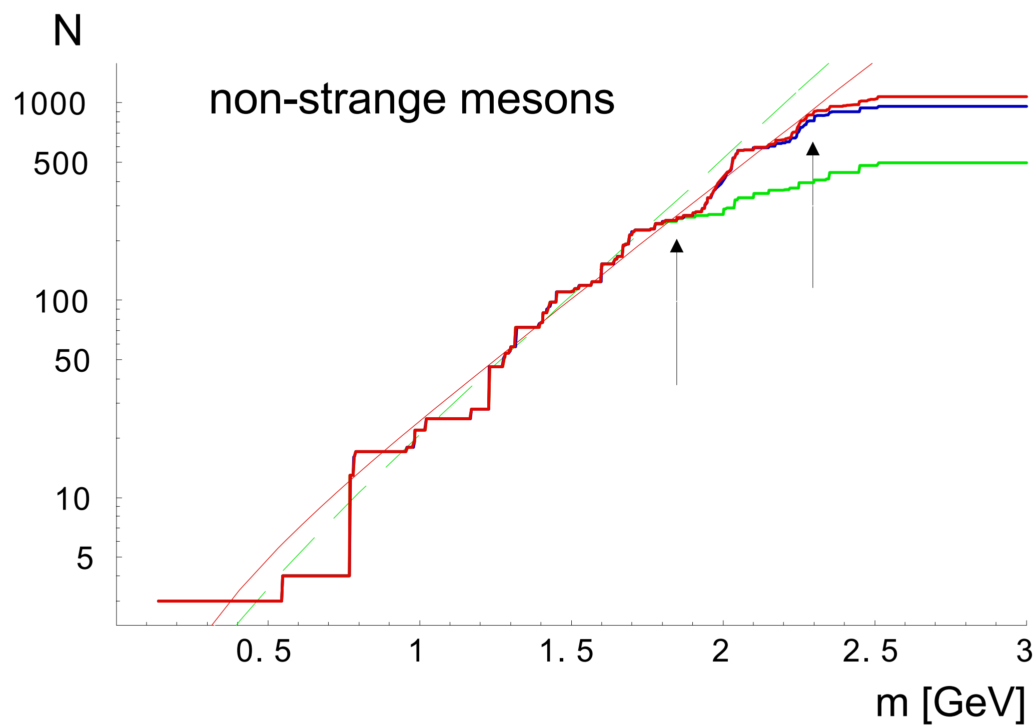
Spectra A.D. 2004

[see WB + WF + LG, Phys. Rev. **D70** (2004) 117503]

More experimental data:
A.V.Anisovich et al, Phys. Lett. **B491** (2000) 47; **B517** (2001) 261;
B542, 8 (2002); **B542** (2002) 19
D.V.Bugg, Phys. Rep., **397** (2004) 257

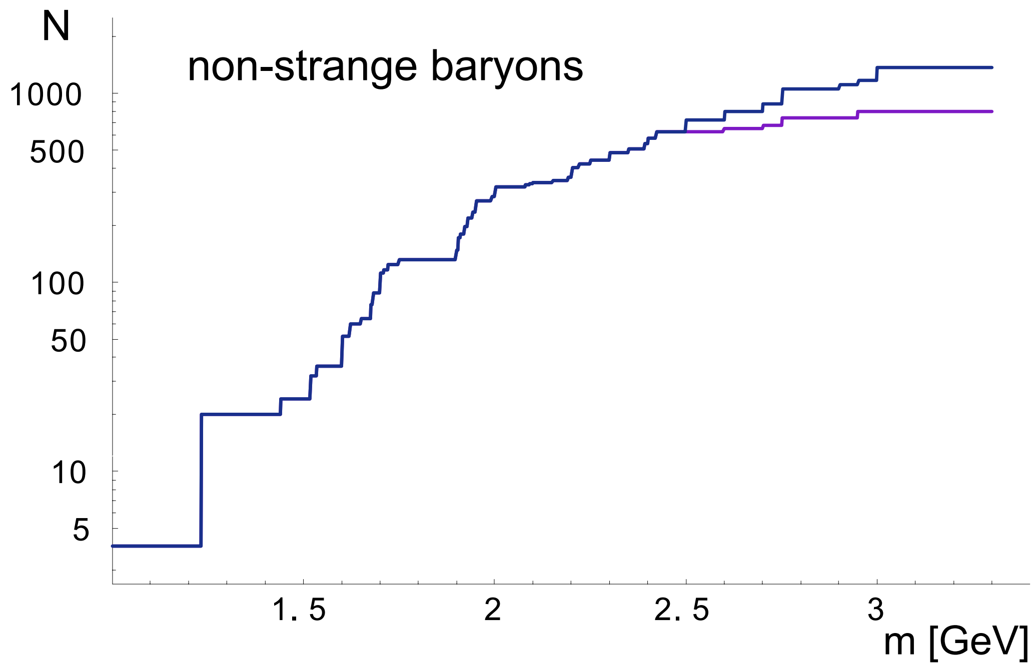
Chiral restoration of the second kind:
LG, Phys. Lett. **B475** (2000) 329
T.D.Cohen+LG, Phys. Rev. **D65** (2002) 016006; Int. J. Mod. Phys. **A17** (2002) 1327
LG, Phys. Lett. **B541** (2002) 115; **B539** (2002) 257; **B587** (2004) 6

Updated meson spectra



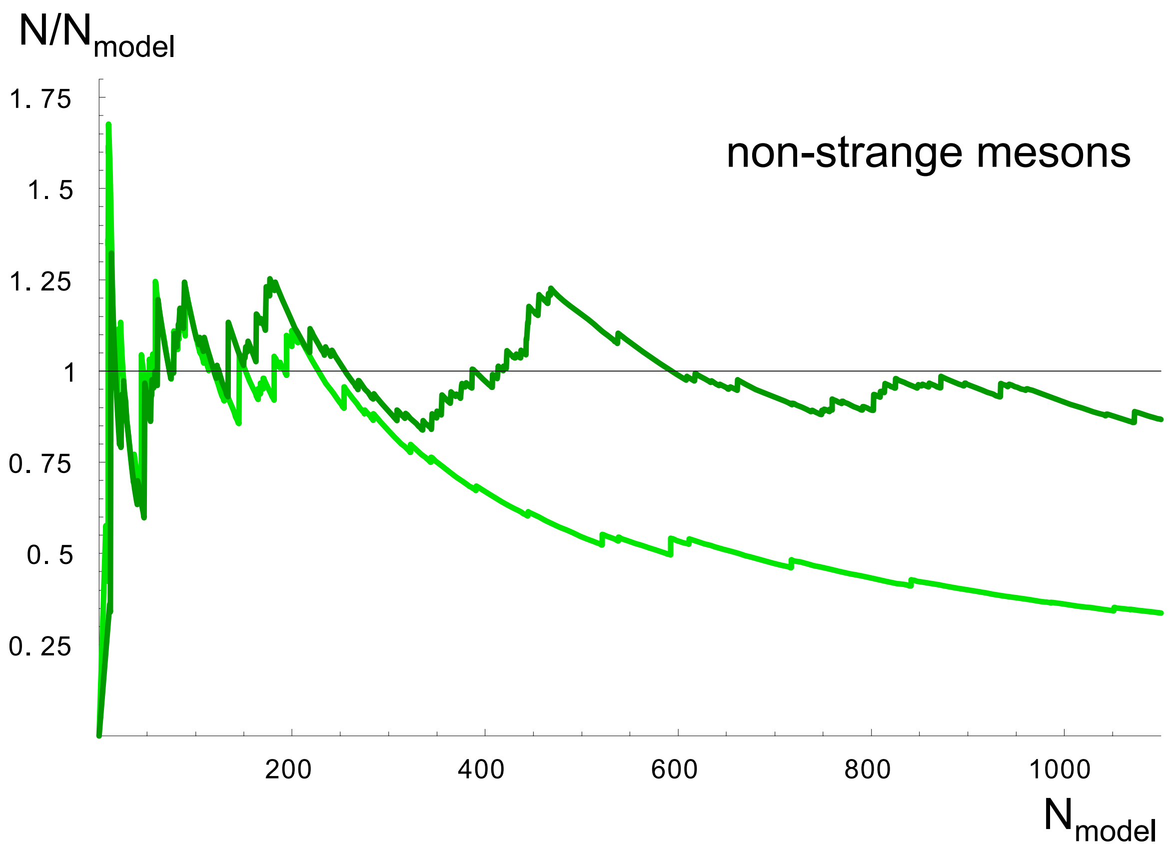
Accumulated spectrum of non-strange mesons plotted as a function of mass (step-like lines). The lower curve at high m corresponds to particles listed in the PDG Tables, the middle curve includes the states listed in new experimental papers, while the top curve adds the states with hidden strangeness. The thin dashed (solid) line corresponds to the exponential fit to the spectra of the old (new) data. The arrows indicate the approximate upper values in m of the validity of the Hagedorn hypothesis for the old and new data, respectively.

Updated baryon spectra



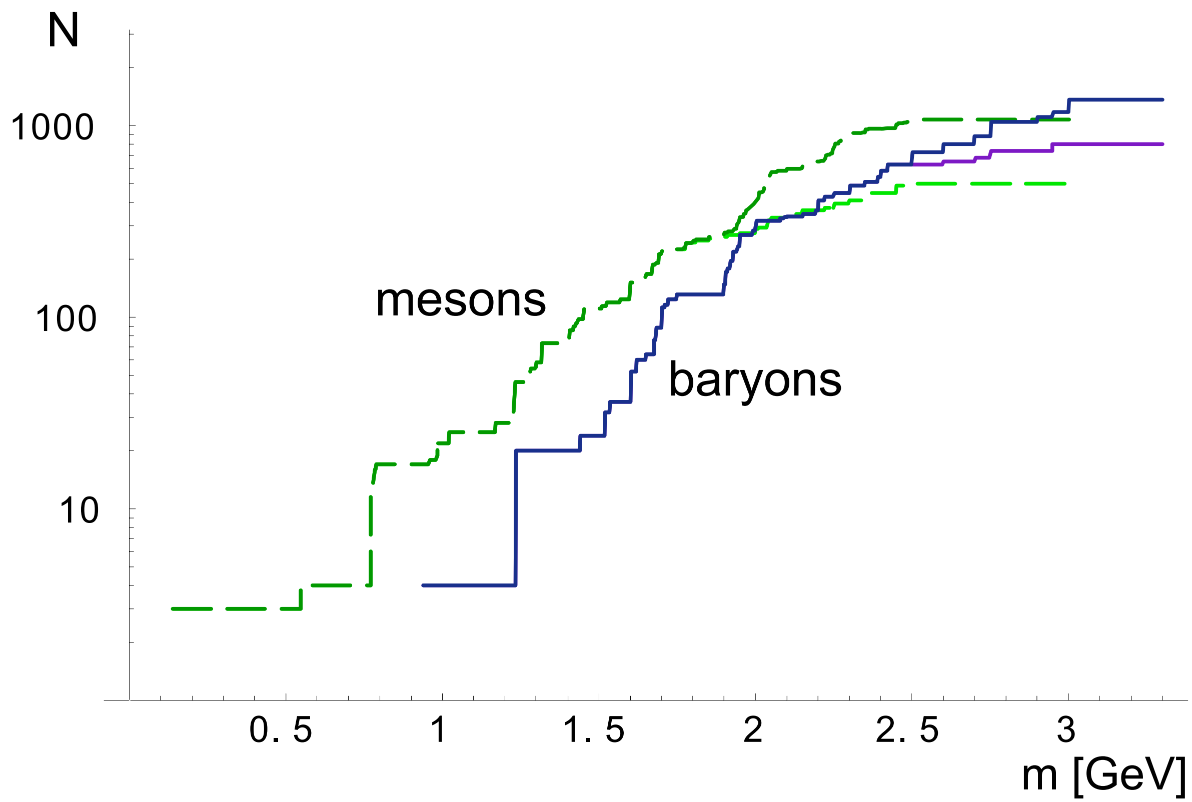
Accumulated spectrum of non-strange baryons plotted as a function of m. The lower curve at high m corresponds to the PDG data, while the higher curve includes new states, added with help of identification of chiral multiplets. In the present case we do not show the fit to the exponential formula, since it is difficult to line-up the results along one straight line in a sufficiently broad range of m. Actually, with the present data one may see a straight line up to about $m = 2$ GeV, and possibly another straight line, with a lower slope, above. However, this may be an artifact of missing data in the high-mass range

Better way of comparing the hypothesis and experiment



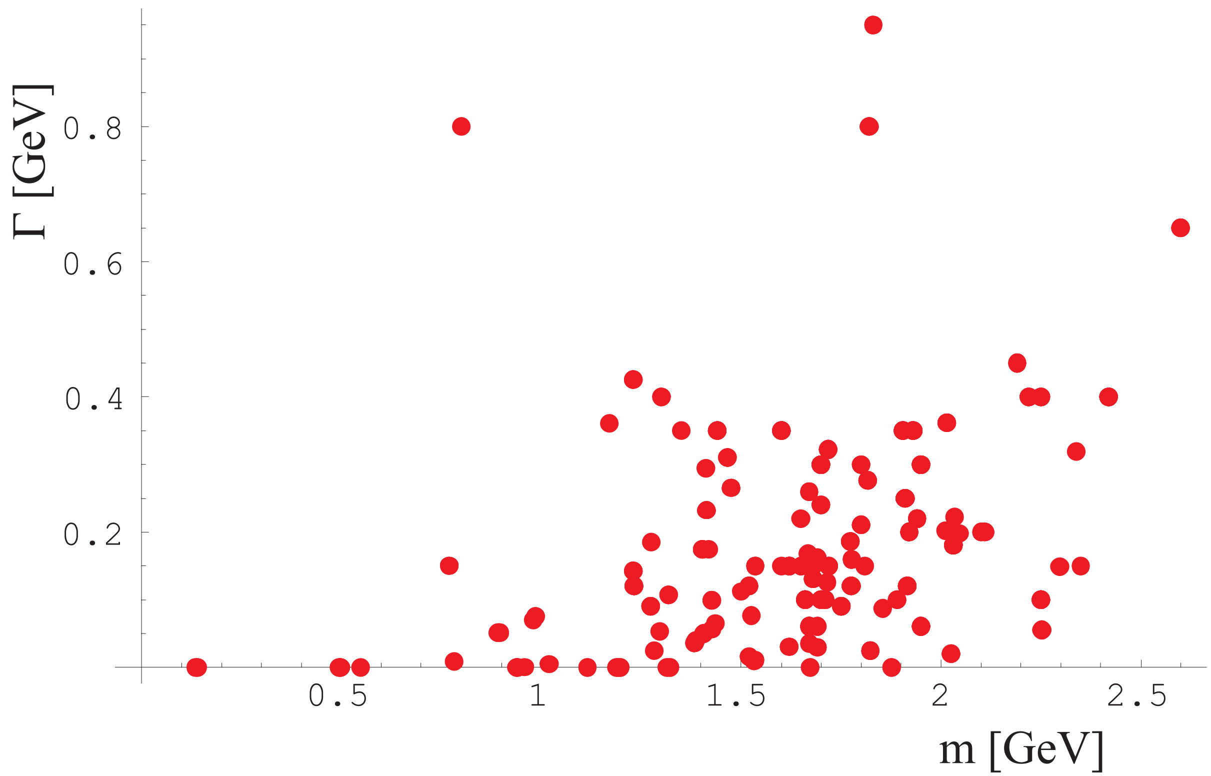
The ratio of the accumulated spectrum of non-strange mesons to the exponential fit. The lower curve at high m corresponds to the PDG data, while the higher curve includes the new states. We note a sizeable increase of the validity range of the Hagedorn hypothesis, from ∼250 to ∼1000 states!

Mesons vs. baryons



Comparison of mesons (dashed lines) and baryons (solid lines)

Width vs. mass



[mesons + baryons]

Some conclusions

1. The newly-observed meson states lead to a continued exponential growth on the number of states with mass, in accordance to the Hagedorn hypothesis, which now reaches up to masses of about 2.3 GeV

2. For the baryons the situation is less clear, with the exponential growth seen up to about 2 GeV, then a change of slope?

3. The inclusion of the missing states based on the identification of chiral multiplets helps to comply to the Hagedorn hypothesis at high masses

4. Certainly, more experimental data in the high-mass range are highly desired to investigate further and with greater detail the hadron spectroscopy

Why do mesons and baryons behave differently?

Models leading to the exponential growth of number of states:

- Statistical bootstrap models – equal Hagedorn temperatures for mesons and baryons
- Bag models – almost equal T_{mes} and T_{bar}
- Dual string models (fundamental strings) — equal Hagedorn temperatures for mesons and baryons only when baryons are described by the quark-diquark configuration. For other configurations, with more strings,

  $$T = T_{\mathrm{meson}}/\sqrt{\#\text{ of strings}}.$$

  In particular for the 3-string configuration $T_{\mathrm{meson}} = \sqrt{3}\, T_{\mathrm{baryon}}$. This is our favorite explanation!

Statistical bootstrap models

[S. Frautschi, PRD **3** (1971) 2821, review: R. Hagedorn, CERN-TH.7190/94 (1994)]

Pre-QCD physics: particles form clusters, clusters form clusters — idea of self-similarity

The simplest bootstrap equation (Frautschi-Yellin):

$$\rho(m) = \delta(m - m_0) + \sum_{n=2}^{\infty} \frac{1}{n!} \int_0^{\infty} dm_1 ... dm_n \delta(m - \sum_{i=1}^{n} m_i) \rho(m_1) ... \rho(m_n)$$

Solution: $\rho(m) \sim \exp(m/T)$, with

$$T = -\frac{m_0}{\log(\log \frac{4}{e})}$$

- Coupling of mesons and baryons leads to equal Hagedorn temperatures
- More elaborate/complicated bootstraps do not change this result
- Idea and results are “approximate” and asymptotic

Dual String Models

Dual String models [review: *Dual Theory*, Vol. 1 of Phys. Rep. reprint book series, ed. M. Jacob (N.H., Amsterdam, 1974)] also date back to pre-QCD times. Their greatest success is a natural explanation of the Regge trajectories. The particle spectrum for mesons is generated by the harmonic-oscillator operator describing vibrations of the string,

$$N = \sum_{k=1}^{\infty} \sum_{\mu=1}^{D} k a^{\dagger}_{k,\mu} a_{k,\mu},$$

where k labels the modes and μ labels additional degeneracy related to the number of dimensions or degrees of freedom, D. Eigenvalues of N are composed in order to get m^2 according to the Regge formula $\alpha' m^2 - \alpha_0 = n$, where $\alpha' \sim 1\mathrm{GeV}^{-2}$ and $\alpha_0 \approx 0$.

Example: take $n = 5$. The value 5 can be formed by taking the $k = 5$ eigenvalue of N (this is the leading Regge trajectory, with a maximum angular momentum), but we can also obtain the same m^2 by exciting one $k = 4$ and one $k = 1$ mode, alternatively $k = 3$ and $k = 2$ modes, and so on. The number of possibilities corresponds to partitioning the number 5 into natural components: 5, 4+1, 3+2, 3+1+1, 2+2+1, 2+1+1+1, 1+1+1+1+1. Here we have 7 possibilities – the number of partitions $P_D(n)$ grows very fast with n. Partitions with more than one component describe the sub-leading Regge trajectories (daughers).

For large n the asymptotic formula for *partitio numerorum* gives [Huang-Weinberg, PRL **25** (1970) 895]

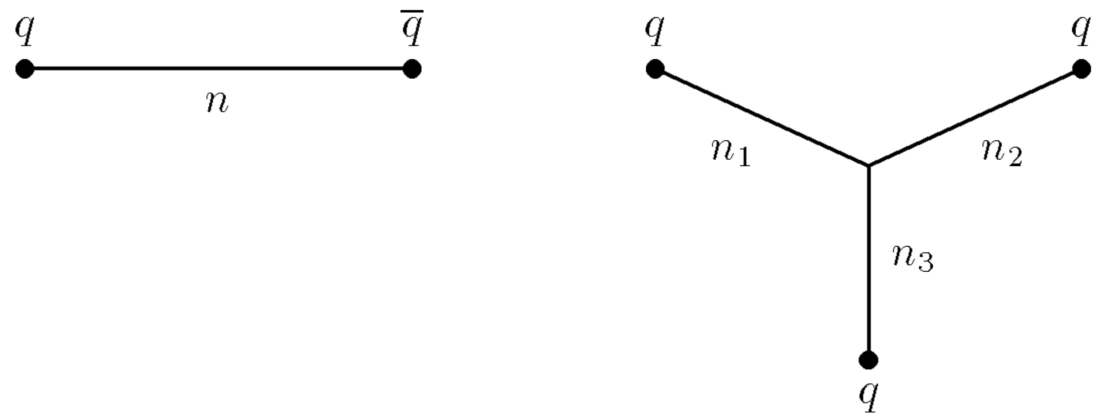
$$\rho(m) \sim 2\alpha' m P_D(n), \quad P_D(n) = \sqrt{\frac{1}{2n}} \left(\frac{D}{24n}\right)^{\frac{D+1}{4}} \exp\left(2\pi\sqrt{\frac{Dn}{6}}\right)$$

from which we read-off $\boxed{T_{\mathrm{meson}} = \frac{1}{2\pi}\sqrt{\frac{6}{D\alpha'}}}$

We stress the picture is fully consistent with the Regge trajectories. The leading Regge trajectory for baryons is generated by the excitation of a single string. The subleading trajectories come in a much larger degeneracy for baryons than for mesons, because of more combinatorial possibilities. As m^2 is increased, more degrees of freedom enter. Via combinatorics it leads to the exponential growth of the spectrum

Folding more strings

Meson and baryon string configurations:



For the Mercedes-Benz configuration for the baryon the three strings vibrate independently. Thus we simply have a partition problem with 3 times more degrees of freedom than in the meson. This imeediately gives

$$T_{\mathrm{baryon}} = \frac{1}{2\pi}\sqrt{\frac{6/3}{D\alpha'}},$$

such that $\boxed{T_{\mathrm{mes}}/T_{\mathrm{bar}} = \sqrt{3}}$

The scalar string

[K.R.Dienes, J.-R.Cudell, PRL **72**(1994)187] (at low m better than HW):

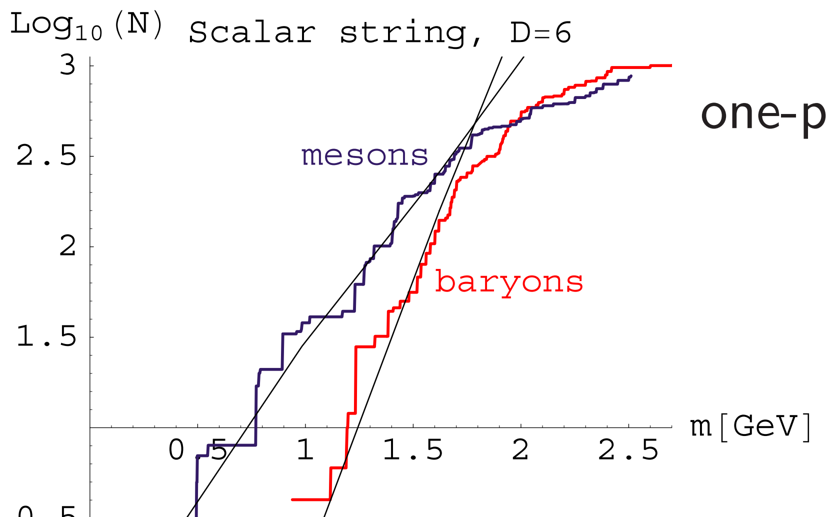
$$\rho_s(m) = \frac{2\pi}{(4\pi\alpha' T m)^{1+D/2}} I_{1+D/2}\left(\frac{m}{T}\right)$$

$$\rho_{\rm meson}(m) = C_{\rm meson}\, 2m\alpha' \rho_s(m), \quad C_{\rm meson} = (2N_F)^2 = 36$$

For baryons

$$\rho_{\rm baryon}(m) = C_{\rm baryon}\ 2m\alpha'^3 \int_0^\infty dm_1^2 dm_2^2 dm_3^2 \delta(m^2 - m_1^2 - m_2^2 - m_3^2)\rho_s(m_1)\rho_s(m_2)\rho_s(m_3)$$

$$C_{\rm baryon} = 56$$

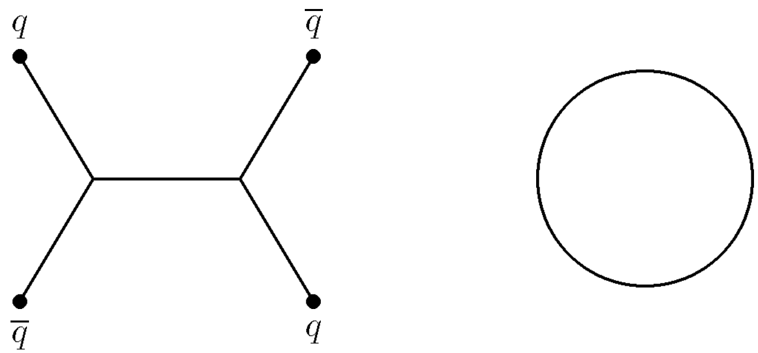


one-parameter global fit!

Exotics

If exotics are described via a multi-string configuration then the spectrum will grow exponentially with a Hagedorn temperature inversely proportional to the square root of the number of strings: $\boxed{T_{\mathrm{exotic}} = \frac{1}{\sqrt{\#\text{of strings}}} T_{\mathrm{mes}}}$.

For instance, $T_{q\overline{q}q\overline{q}} = \frac{1}{\sqrt{5}} T_{\mathrm{meson}}$, or $T_{\mathrm{glue}} = T_{\mathrm{mes}}$.



$\overline{q}q\overline{q}q$ and glueball configurations

Thus, according to the string model, the $q\overline{q}q\overline{q}$ grow more rapidly than non-exotic mesons and baryons, and glueballs grow at the same rate as mesons.

Pentaquarks: $\boxed{T_{qqqq\overline{q}\overline{q}} = \frac{1}{\sqrt{7}} T_{\mathrm{mes}}}$ (grow rapidly!)

Thermodynamic implications

Limiting temperature

Hagedorn: a system of hadrons cannot exhibit a higher T than T_H.
Indeed, try to compute the (canonical) partion function

$$Z \sim \int d^3k \int dm \rho(m) e^{-\sqrt{k^2+m^2}/T} \sim \int dm e^{m\left(\frac{1}{T_H}-\frac{1}{T}\right)}$$

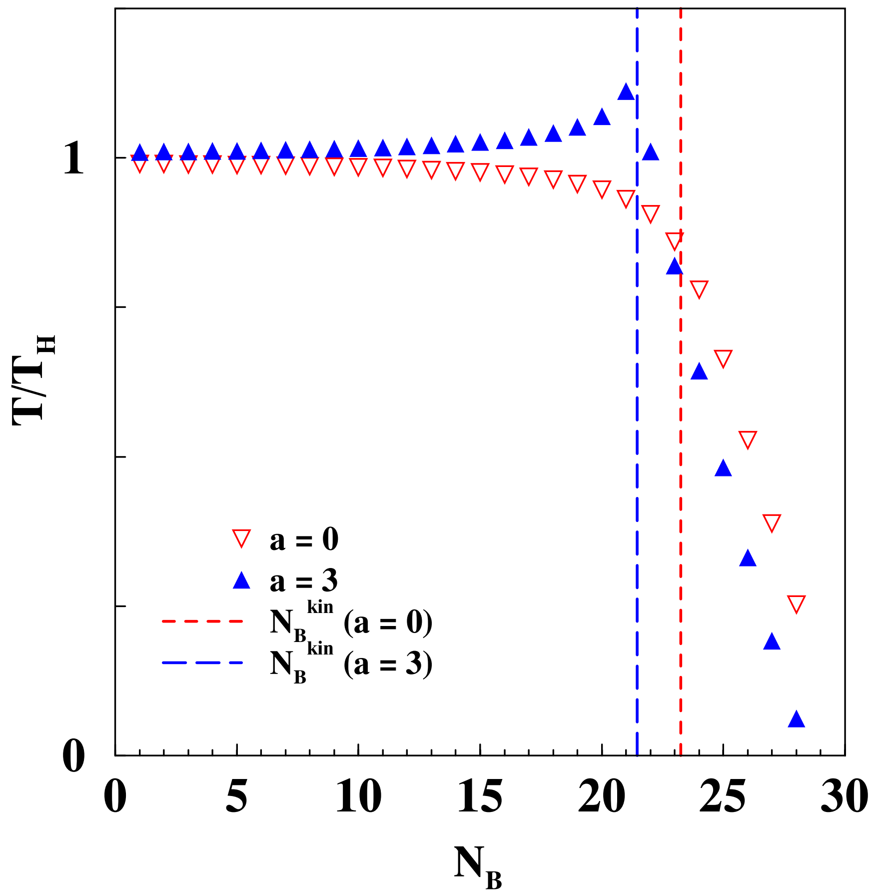
At $T > T_H$ it diverges! T_H is the Hagedorn limiting temperature

Perfect thermostat

[Moretto, Bugaev, Elliott, Phair, hep-ph/0504010, 0504011]
The Hagedorn system is a perfect thermostat. Example of the harmonic oscillator coupled to a Hagedorn system (microcanonical):

$$P(\epsilon) \sim \rho(E-\epsilon)\rho_{\mathrm{HO}}(\epsilon) = \rho(E) e^{-\epsilon/T_H}$$

$$\bar{\epsilon} = T_H \left[1 - \frac{E/T_H}{\exp(E/T_H)-1}\right]$$

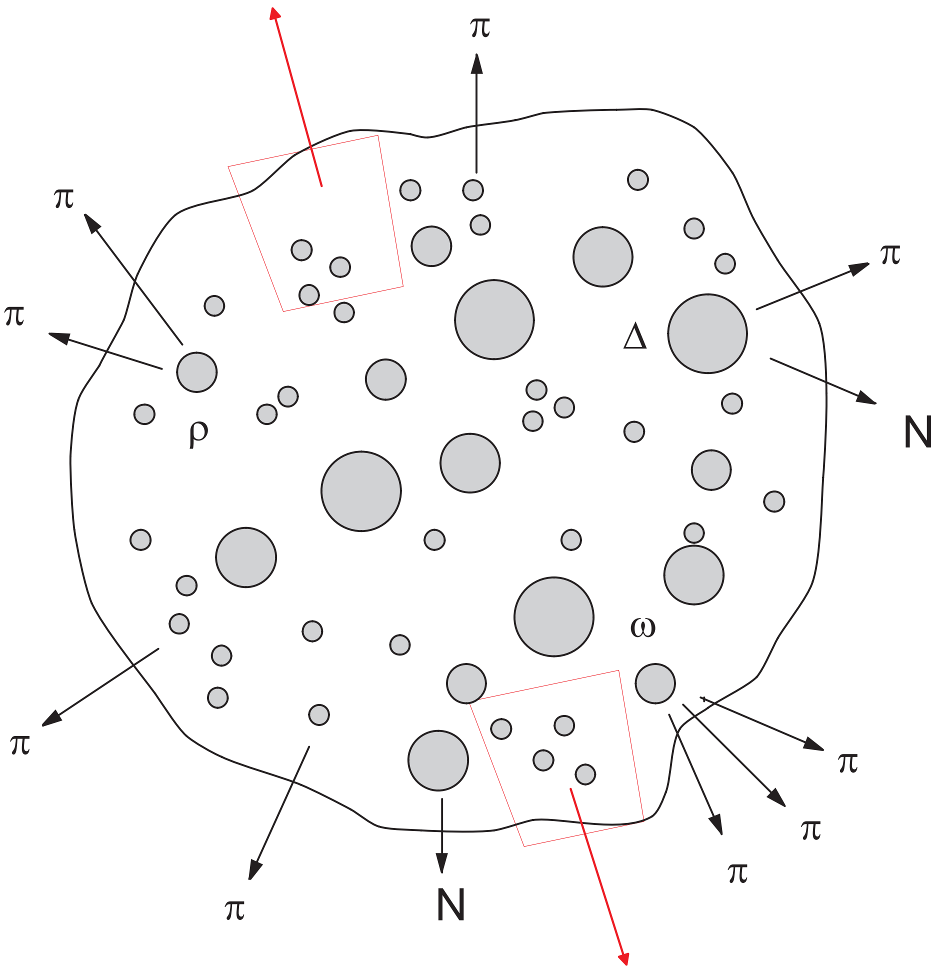


[Bugaev et. al, hep-ph/0504011]

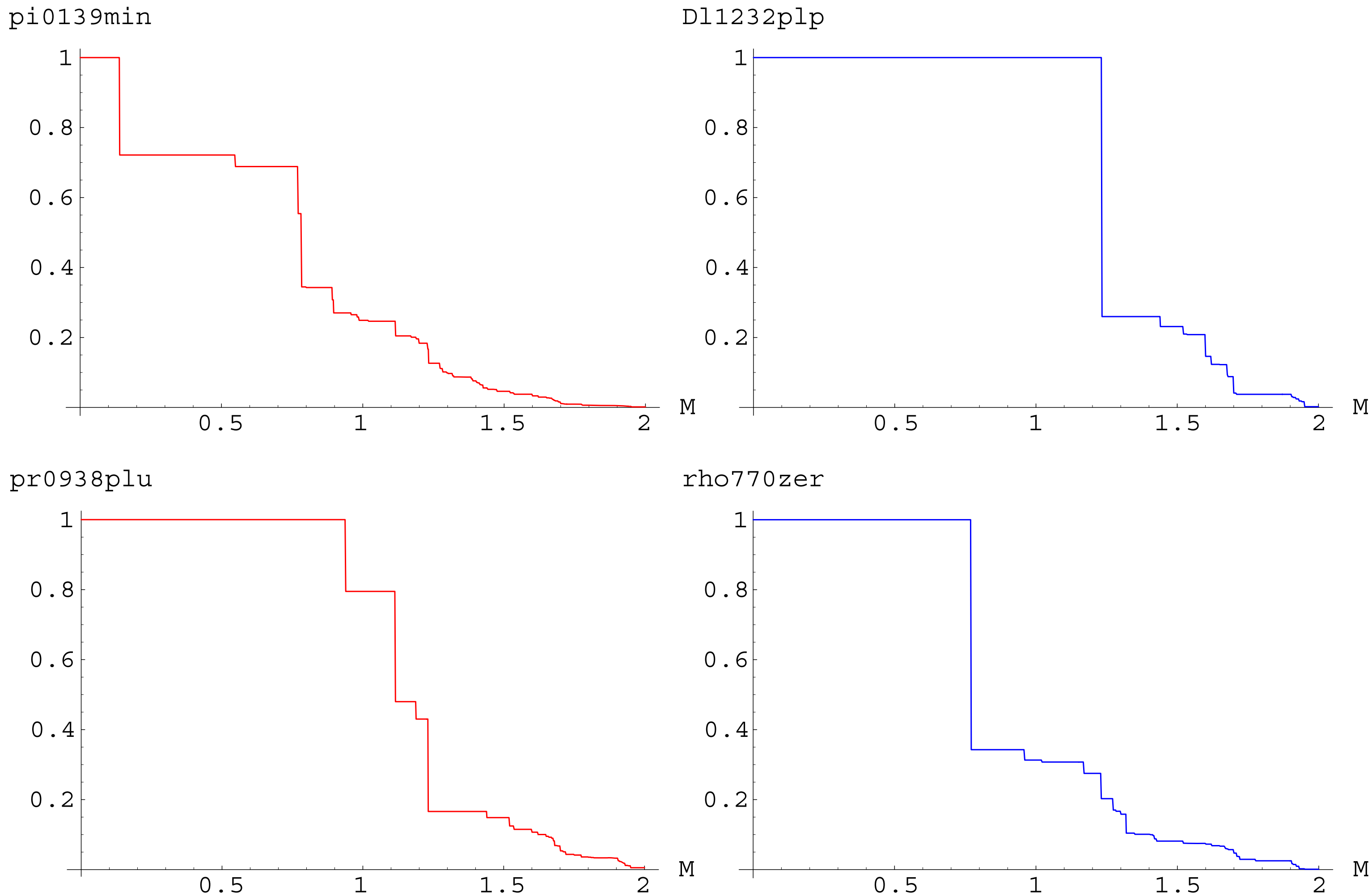
Temperature of a system of N_B Boltzmann particles coupled to a Hagedorn thermostat. Microcanonical ensemble with the total energy of $30m_B$ is used. T is nearly constant up to the kinematic cut-off. a is a parameter of the "slowly varying function".

Phenomenology of heavy-ion collisions

Particle ratios and momentum spectra at RHIC are remarkably well reproduced with a thermal approach, where $T \sim 165 - 175$ MeV. Feeding from high-lying resonances is essential.

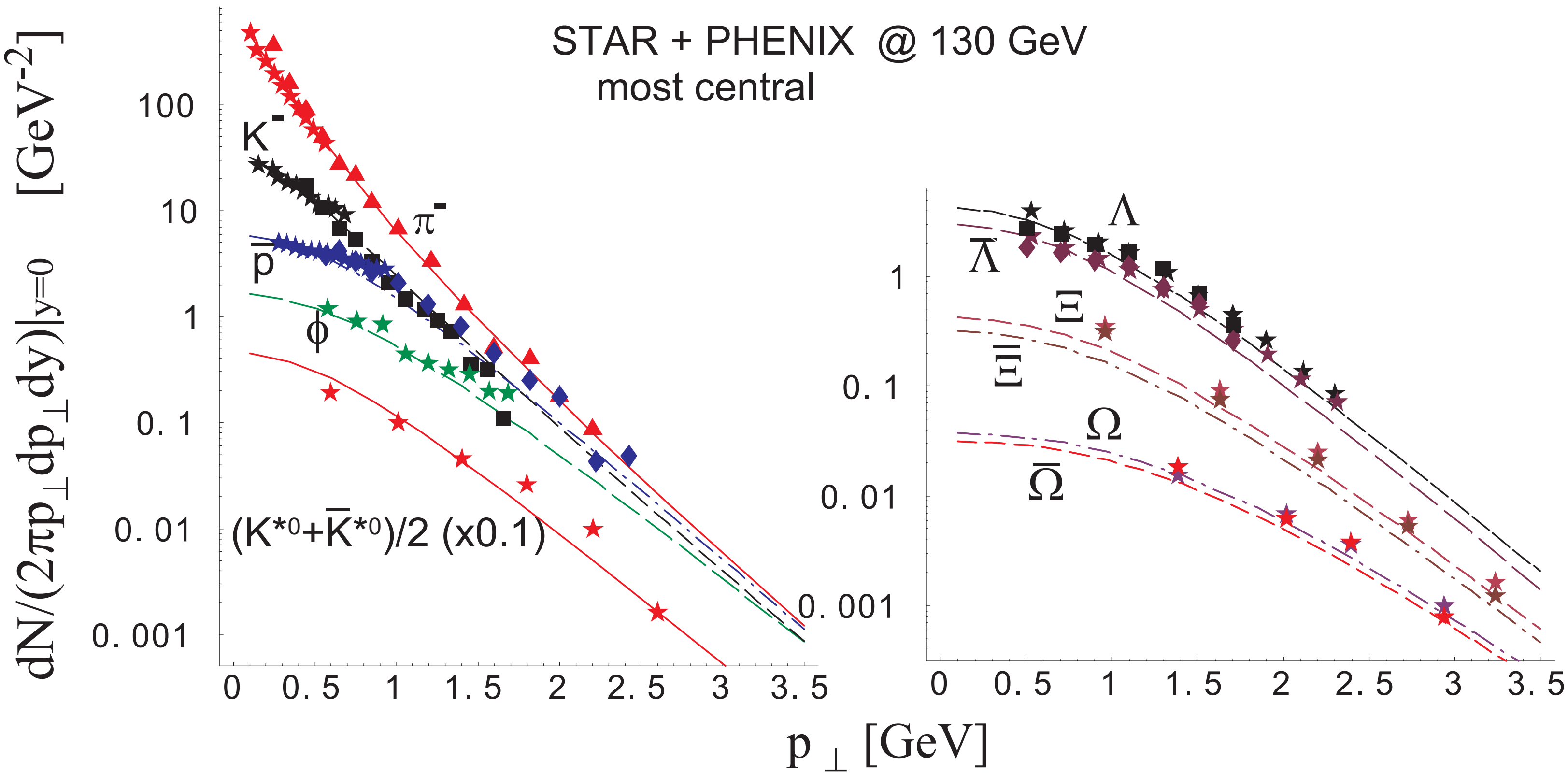


Resonance feeding of π^-,Δ^{++}, p, and ρ_0



Approximately 75% of pions come from decays of higher states, 80% of protons and Λ's, 60% of Ξ's, 30% of Δ's and ρ_0's, . . . , 3% of ϕ, 0% of Ω's

Transverse-momentum spectra



[from WB + W. Florkowski, Phys. Rev. **C65** (2002) 064905]

Summary

• Recent data support the Hagedorn hypothesis, range in mass extended
• Chiral multiplets very helpful
• More data highly desired
• Mesons and baryons grow at different rate
• Dual string models describe this behavior!
• Flavor universality
• Relevance of the slowly-varying function $\rightarrow$ one cannot quote one number for T_H
• Limiting temperature and perfect thermostat
• Relevance for heavy-ion physics, thermal models very successful