Elliptic flow in the Cracow model

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Future of Nuclear Collisions at High Energies
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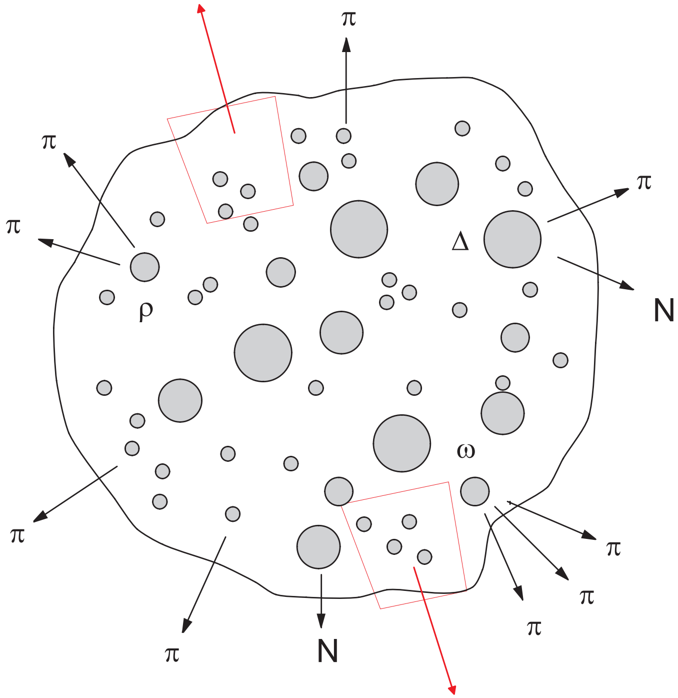


- $\sim e^{-(E-\mu)/T}$
- freezeout
- expansion
- resonances

Thermal approach

Koppe (1948), Fermi (1950), Landau, Hagedorn, Rafelski, Letessier, Torrieri, Bjorken, Gorenstein, Gaździcki, Sinyukov, Heinz, Sollfrank, Braun-Munzinger, Turko, Redlich, Prorok, Xu, Kaneta, Csörgő, Lörstad, Becattini, Cleymans, Wheaton, ...

v_2 based on PhD thesis of Anna Baran

WB + WF, **PRL 87 (2001) 272302** ($p_\perp$ spectra of pions, kaons, and protons)
WB + WF, **PRC 65 (2002) 064905** ($p_\perp$ spectra of strange particles)
WB + WF + Anna Baran, **AIP Conf. Proc. 660 (2003) 185 [nucl-th/0212053]** (v_2)
WB + WF + Brigitte Hiller, **PRC 68 (2003) 034911** (pion invariant-mass distributions)
Piotr Bożek + WB + WF, **Heavy-Ion Physics (2004)** (pion balance functions)

SHARE

G. Torrieri, W. Broniowski, W. Florkowski, J. Letessier and J. Rafelski, SHARE: Statistical HAdronization with REsonances, nucl-th/0404083.

programs put on the web

http://www.ifj.edu.pl/Dept4/share.html

or

http://www.physics.arizona.edu/∼torrieri/SHARE/share.html

Similar effort by S. Wheaton and J. Cleymans, *THERMUS: A thermal model package for ROOT*, hep-ph/0407174.

Single freeze-out / Cracow model

1. grand canonical ensamble
2. $T_{\text{chem}} = T_{\text{kin}} \equiv T$
3. Complete treatment of resonances (also for the spectra), ∼75% of secondary pions
4. Special choice of the (boost-invariant) freeze-out hypersurface, $\tau = \sqrt{t^2 - x^2 - y^2 - z^2} = \text{const}$
5. Hubble-like flow, $u^\mu = \frac{x^\mu}{\tau} = \frac{t}{\tau}(1, \frac{x}{t}, \frac{y}{t}, \frac{z}{t})$, average transverse velocity $\langle \beta_\perp \rangle \sim 0.5$
6. Only 4 parameters: T,μ_B (fixed by the ratios of the particle abundances), invariant time at freeze-out τ (controls the overall normalization), transverse size $\rho_{\max}$ ($\rho_{\max}/\tau$ controls the slopes of the $p_\perp$ spectra)

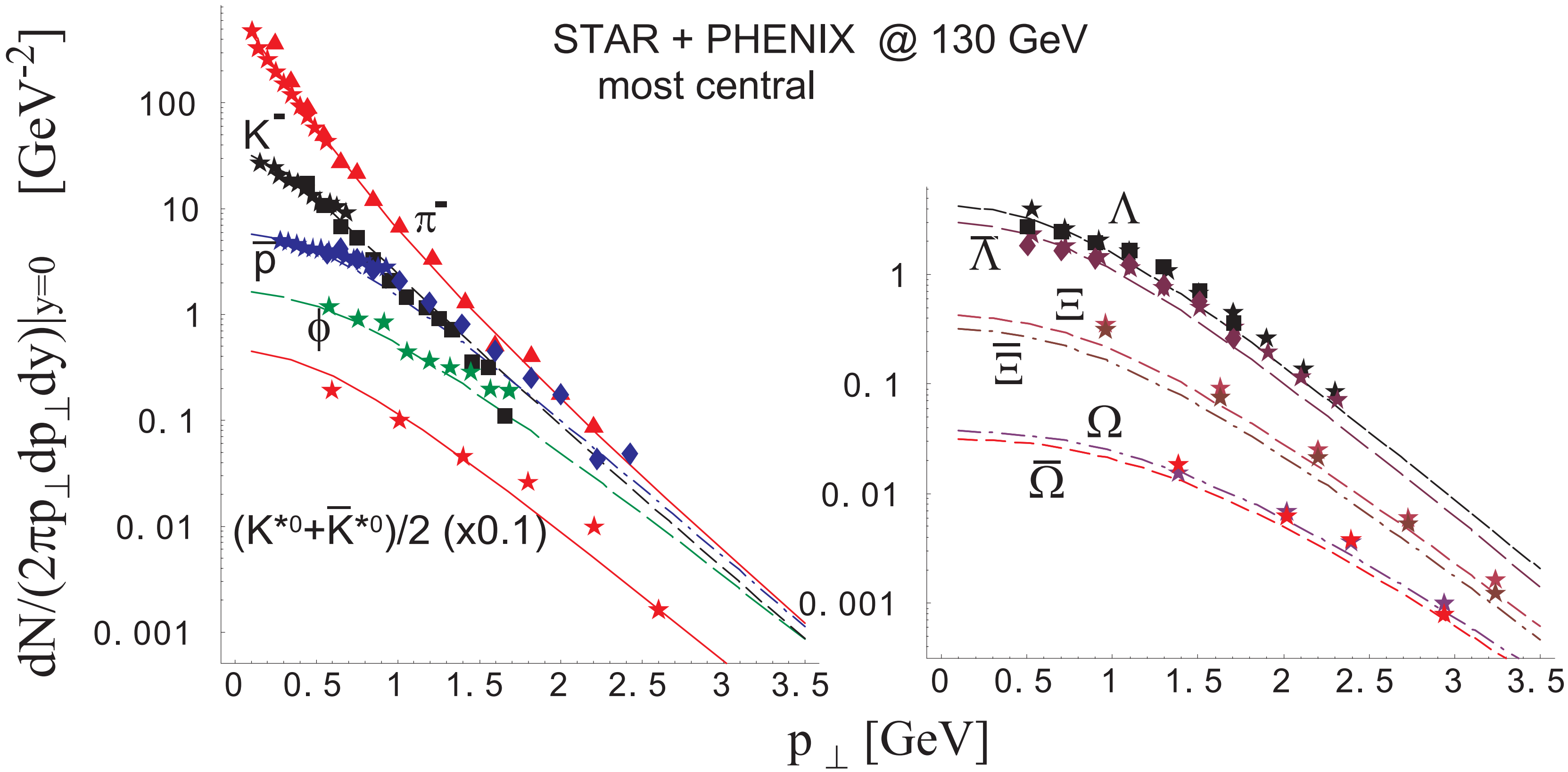
more choices of the hypersurface and flow possible – see *e.g.* papers by Rafelski and Torrieri, in particular the popular choice $t = \text{const}$ (Blast-Wave)

2 thermal parameters fitted from particle ratios

T [MeV] = 165 ±7, μ_B [MeV] = 41 ± 5, @ 130 GeV
T [MeV] = 166 ±5, μ_B [MeV] = 29 ± 4, @ 200 GeV

2 geometry parameters globally fitted from the spectra of $\pi^\pm, K^\pm, p$, and $\bar{p}$

Working surprisingly well



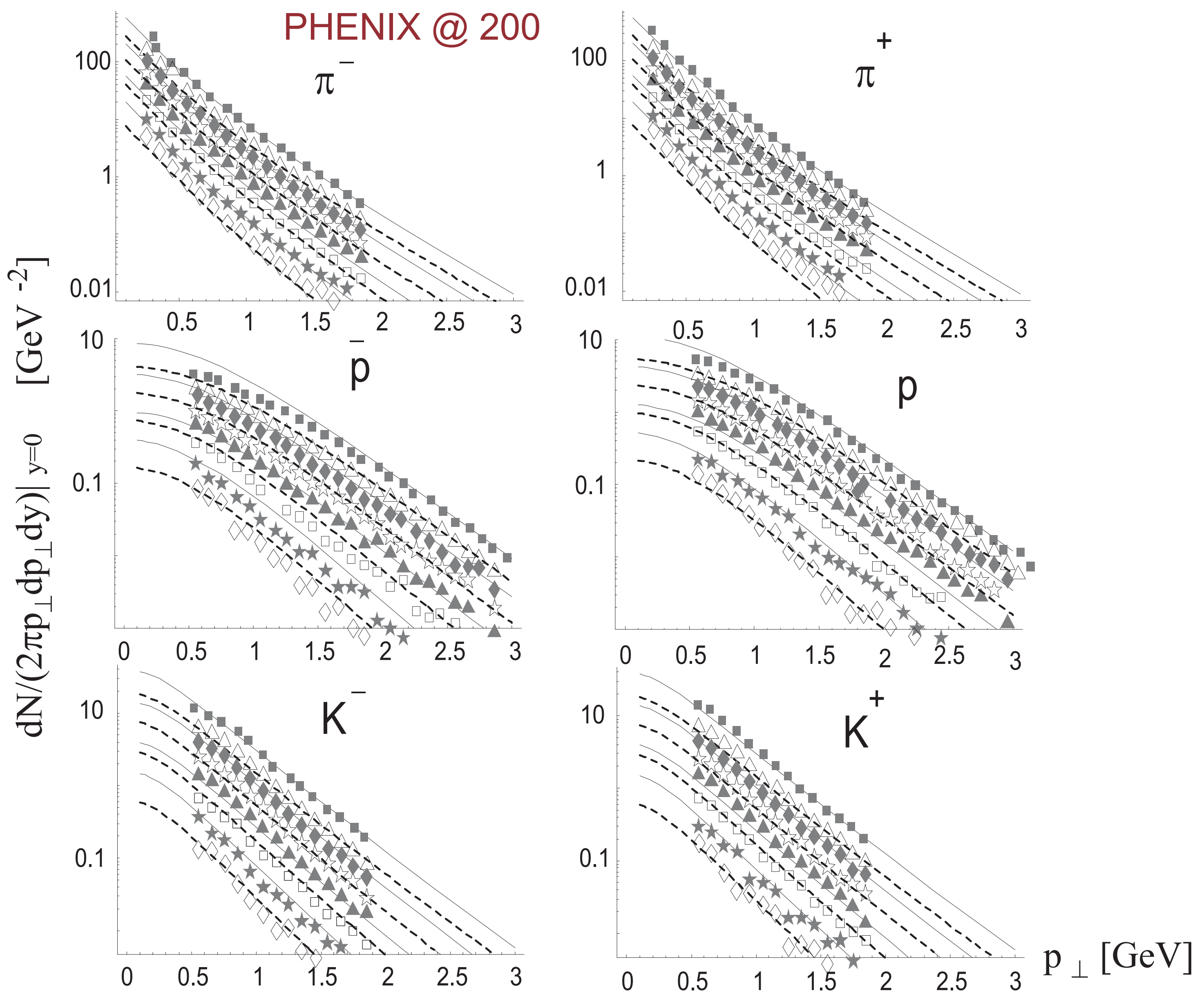
Extension for non-central collisions

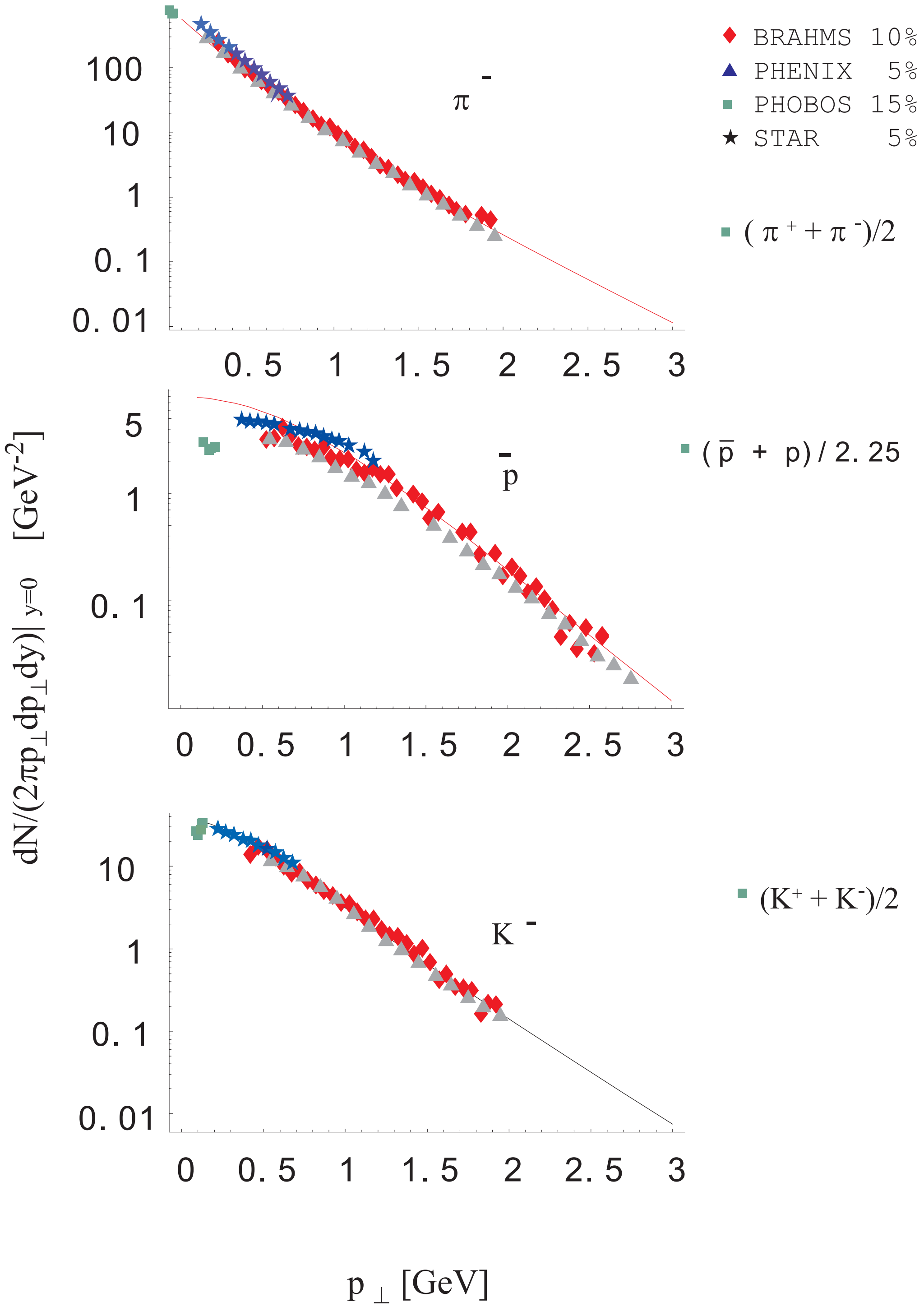
To a very good accuracy

$$c \simeq \frac{\pi b^2}{\sigma_{\rm inel}^{\rm tot}} \simeq \frac{b^2}{4R^2}$$

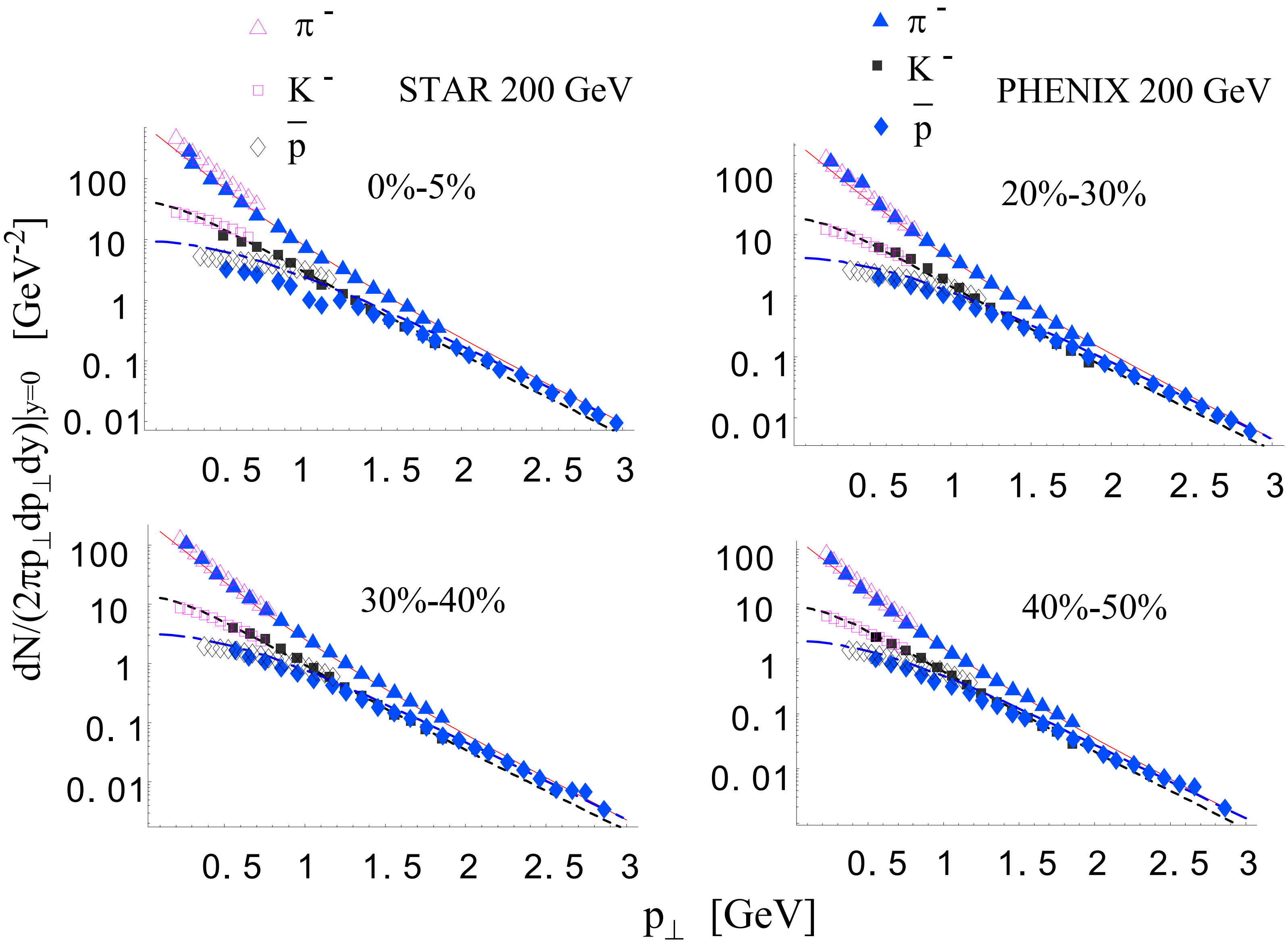
(WB+WF, PRC 65 (2002) 024905)

- thermal parameter kept independent of centrality
- geometry parameters do depend on centrality


PHENIX @ 200
π⁻
π⁺
p̄
p
K⁻
K⁺
dN/(2πp⊥dp⊥dy)| y=0 [GeV -2]
p ⊥ [GeV]



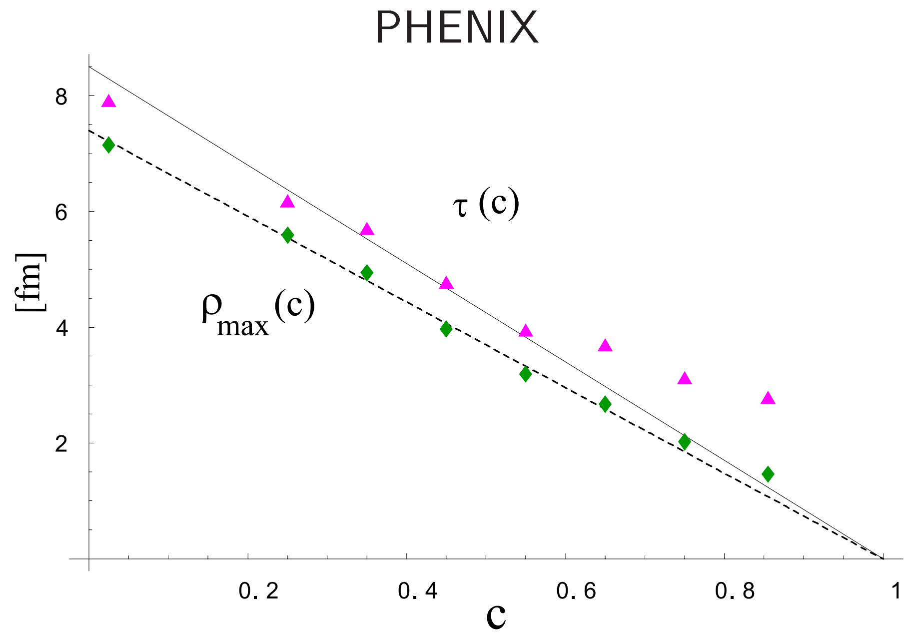
BRAHMS 10%
PHENIX 5%
PHOBOS 15%
STAR 5%
π -
(π + + π -)/2
p̄
(p̄ + p)/2.25
K -
(K+ + K-)/2
dN/(2πp⊥dp⊥dy)| y=0 [GeV-2]
p ⊥ [GeV]

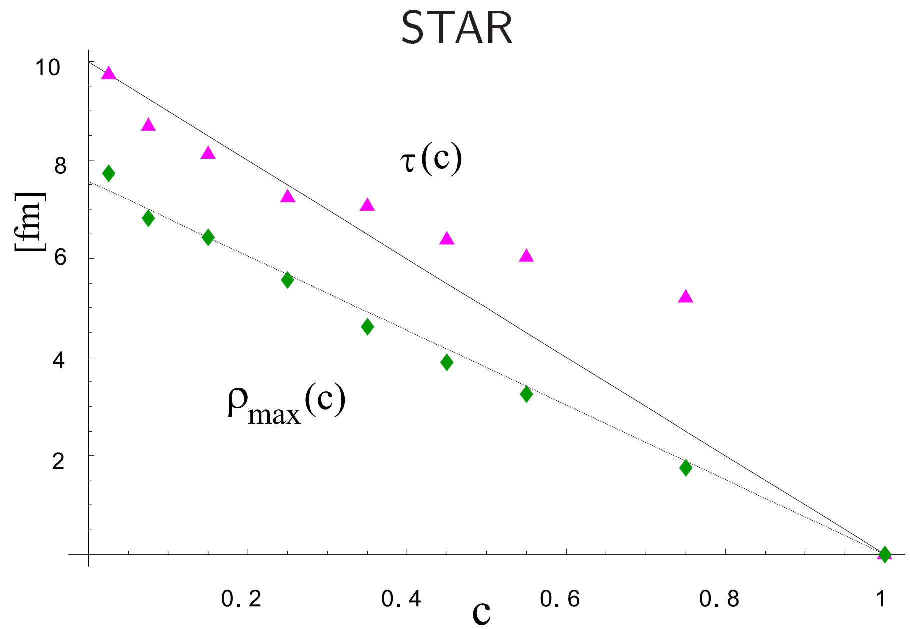


Model works for the $p_\perp$-spectra

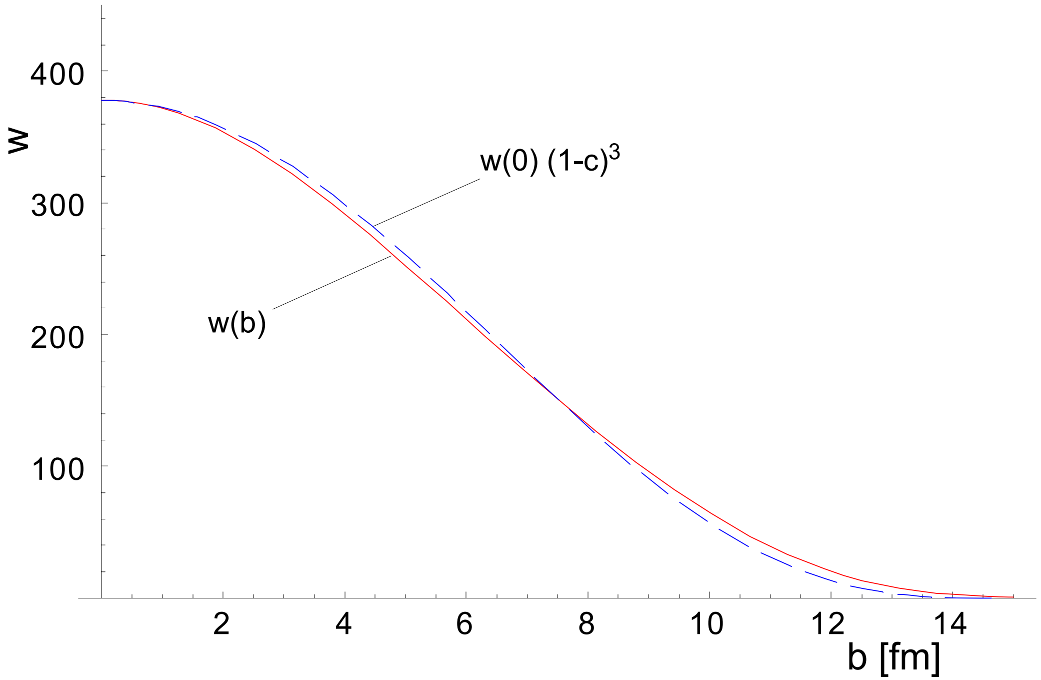
Compilation of geometric parameters (by A. Baran)

	c [%]	τ [fm] (norm)	$\rho_{\max}$ [fm] (slope)	$\langle\beta_\perp\rangle = \langle\rho/\sqrt{\tau^2+\rho^2}\rangle$
ALL	$0-5/10$	7.58 ± 0.32	7.27 ± 0.12	0.51 ± 0.02
BRAHMS	10	7.68 ± 0.19	7.46 ± 0.05	0.52 ± 0.01
STAR	$0-5$	9.74 ± 1.57	7.74 ± 0.68	0.45 ± 0.08
	$5-10$	8.69 ± 1.39	7.18 ± 0.64	0.47 ± 0.08
	$10-20$	8.12 ± 1.31	6.44 ± 0.57	0.45 ± 0.08
	$20-30$	7.24 ± 1.18	5.57 ± 0.50	0.44 ± 0.08
	$30-40$	7.07 ± 1.17	4.63 ± 0.39	0.39 ± 0.08
	$40-50$	6.38 ± 1.02	3.91 ± 0.33	0.37 ± 0.07
	$50-60$	6.19 ± 1.09	3.25 ± 0.28	0.32 ± 0.07
	$70-80$	5.48 ± 0.81	4.03 ± 0.10	0.43 ± 0.06
PHENIX	$0-5$	7.86 ± 0.38	7.15 ± 0.13	0.50 ± 0.02
	$20-30$	6.14 ± 0.32	5.62 ± 0.11	0.50 ± 0.02
	$30-40$	5.73 ± 0.16	4.95 ± 0.05	0.48 ± 0.01
	$40-50$	4.75 ± 0.28	3.96 ± 0.09	0.47 ± 0.03
	$50-60$	3.91 ± 0.23	3.12 ± 0.07	0.45 ± 0.03
	$60-70$	3.67 ± 0.12	2.67 ± 0.03	0.42 ± 0.01
	$70-80$	3.09 ± 0.11	2.02 ± 0.02	0.39 ± 0.01
	$80-91$	2.76 ± 0.20	1.43 ± 0.03	0.32 ± 0.03


PHENIX
8
6
4
2
[fm]
τ (c)
ρmax(c)
0.2
0.4
0.6
0.8
1
c


STAR
10
8
6
4
2
[fm]
τ(c)
ρmax(c)
0.2
0.4
0.6
0.8
1
c

observation: at low and moderate c the model complies to the wounded nucleon scaling

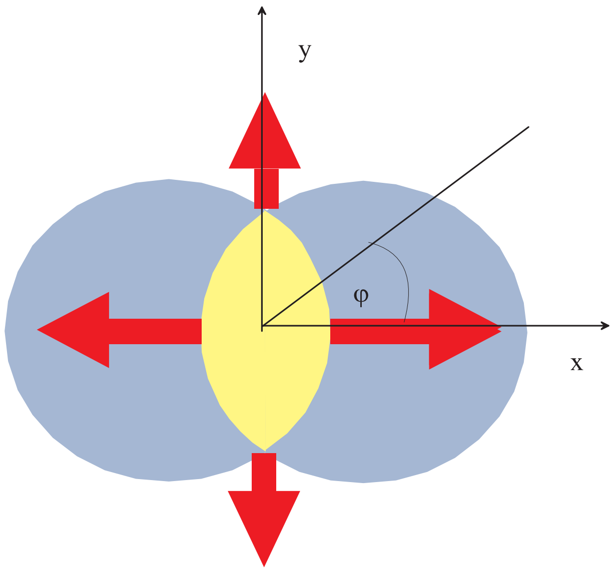


Elliptic flow

Anna Baran, PhD thesis, 2004

WB + WF + AB, Proceedings of the Coimbra Workshop on Hadron Physics, nucl-th/0212053, AIP 660 (2003) 185 [nucl-th/0212053]

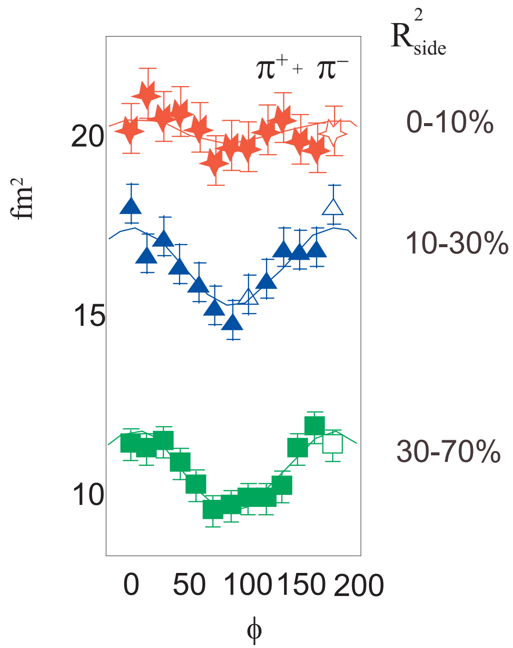
(review by Ollitrault)

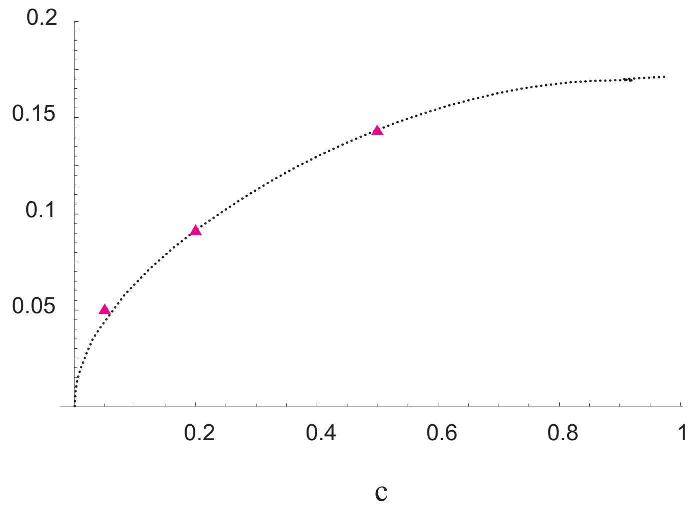


When the nuclei collide at non-zero impact parameter, $b \neq 0$, the momentum distribution of the produced particles carries azimuthal asymmetry. At mid-rapidity for same nuclei

$$\left.\frac{dN}{d^2 p_\perp dy}\right|_{y=0} = \left.\frac{dN}{2\pi p_\perp dp_\perp dy}\right|_{y=0} \left(1 + 2\, v_2 \, \cos 2\phi + 2\, v_4 \, \cos 4\phi + \ldots\right)$$

excentricity parameter, ϵ, is obtained from the measured values of $R_{\rm side}(\phi)$, STAR Collaboration, nucl-ex/0301005





$$\epsilon = \frac{\langle y\rangle^2 - \langle x\rangle^2}{\langle y\rangle^2 + \langle x\rangle^2}$$

modification of the freeze-out hypersurface (out-of-plane elongation)

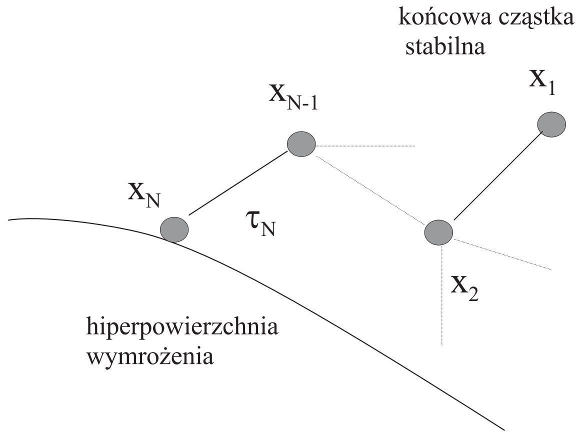
$$r_x = \rho_{\rm max}\sqrt{1-\epsilon}\,\cos\varphi, \quad r_y = \rho_{\rm max}\sqrt{1+\epsilon}\,\sin\varphi$$

modification of the flow profile (stronger in-plane)

$$u_x = \frac{r_x}{N}\sqrt{1+\delta}\,\cos\varphi, \quad u_y = \frac{r_y}{N}\sqrt{1-\delta}\,\sin\varphi, \quad u_z = \frac{r_z}{N}, \quad u_t = \frac{t}{N}$$

N obtained from the normalization condition $u^\mu u_\mu = 1$

Resonance decays



$$n(x_1,p_1) = \int \frac{d^3p_2}{E_{p_2}} B(p_2,p_1) \int d\tau_2 \Gamma_2 e^{-\Gamma_2\tau_2} \int d^4x_2 \delta^{(4)}(x_2 + \frac{p_2\tau_2}{m_2} - x_1) \times$$

$$\cdots \times \int \frac{d^3p_N}{E_{p_N}} B(p_N,p_{N-1}) \int d\tau_N \Gamma_N e^{-\Gamma_N\tau_N}$$

$$\int d\Sigma_\mu(x_N) p_N^\mu \delta^{(4)}(x_N + \frac{p_N\tau_N}{m_N} - x_{N-1}) f_N(p_N \cdot u(x_N))$$

$$B(q,k) = \frac{b}{4\pi p^*} \delta(\frac{k\cdot q}{m_R} - E^*)$$

$$E_{p_1}\frac{dN_1}{d^3p_1} = \int \frac{d^3p_2}{E_{p_2}} B(p_2,p_1) \times \ldots \times \int \frac{d^3p_N}{E_{p_N}} B(p_N,p_{N-1}) \int d\Sigma_\mu(x_N) p_N^\mu f_N[p_N \cdot u_N]$$

Independence of Fourier components

Each step of the cascade involves

$$g(q_\perp, \varphi_q) = \int k_\perp dk_\perp \int_0^{2\pi} d\varphi_k J(k_\perp, q_\perp, \varphi_q - \varphi_k) f(k_\perp, \varphi_k)$$

We introduce the Fourier decomposition

$$g(q_\perp, \varphi_q) = \sum_n \cos(n\varphi_q) g_n(q_\perp), \, f(k_\perp, \varphi_k) = \sum_m \cos(m\varphi_k) f_m(k_\perp)$$

and immediately find that the evolution is diagonal in the Fourier index n:

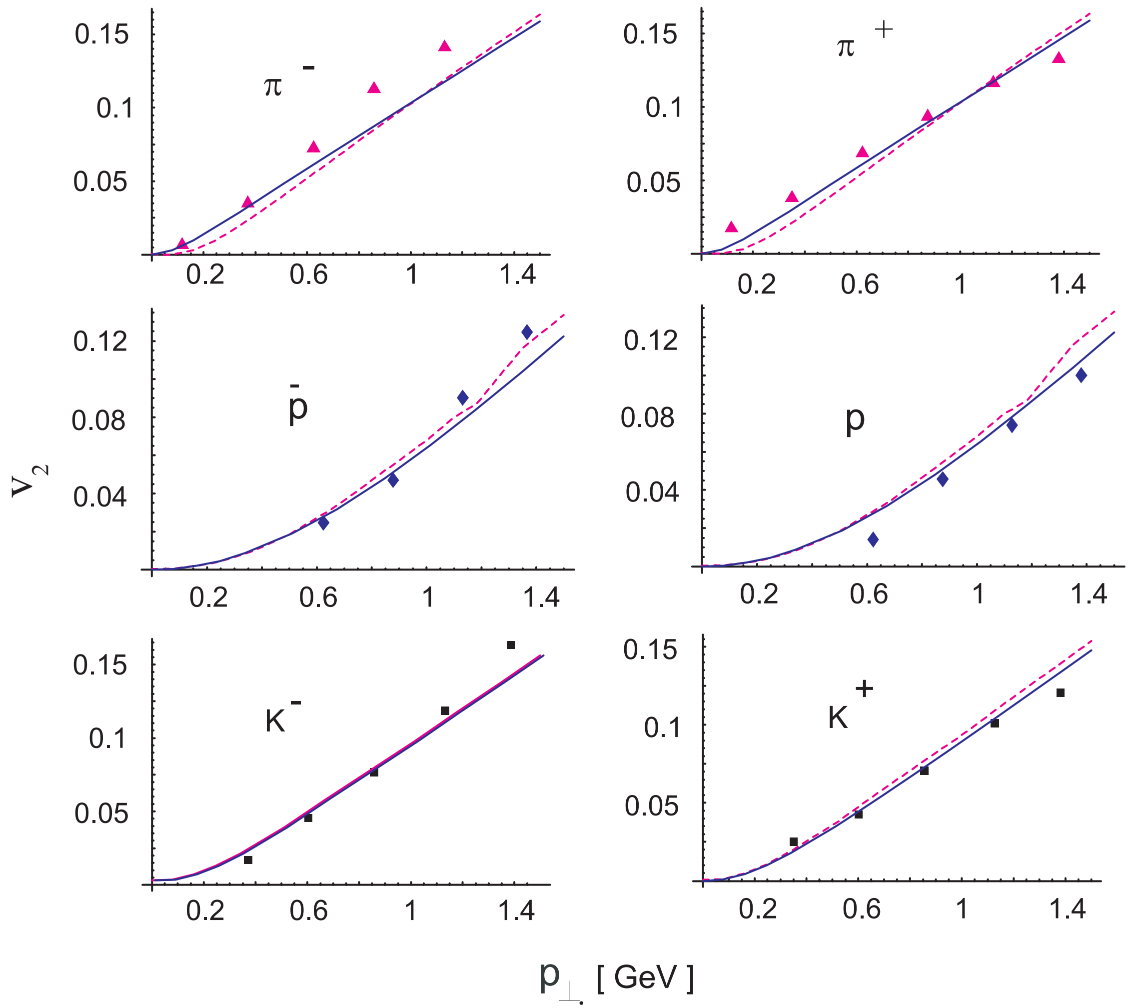
$$g_n(q_\perp) = \int k_\perp dk_\perp \int_0^{2\pi} d\varphi_k J(k_\perp, q_\perp, \varphi) \cos(n\varphi) f_n(k_\perp)$$

The whole cascade proceeds independently for each Fourier component of the spectrum

Now we apply the same method as for the φ-integrated spectra: fit the new parameter δ (asymmetry of the flow) to "good" particles and make predictions for other particles.

$$v_2 = \frac{g_2}{g_0}$$

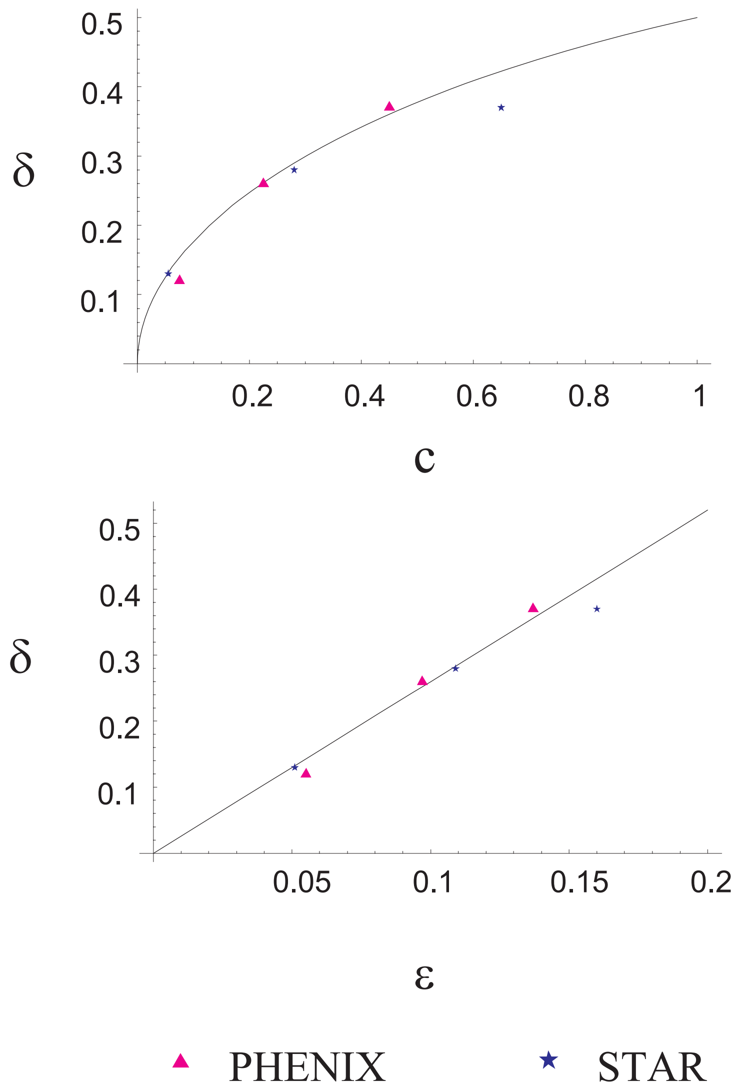
preliminary min. bias data, PHENIX @ 200 GeV, S. A. Voloshin, NP **A715** (2003) 379c



model fit: $T = 165$ MeV, $\mu_B = 26$ MeV (from the ratios), $\tau = 4.04$ fm, $\rho_{\max} = 3.70$ fm (from the spectra), $\epsilon = 0.13$ (from R_{side}), $\delta = 0.25$ (from v_2)

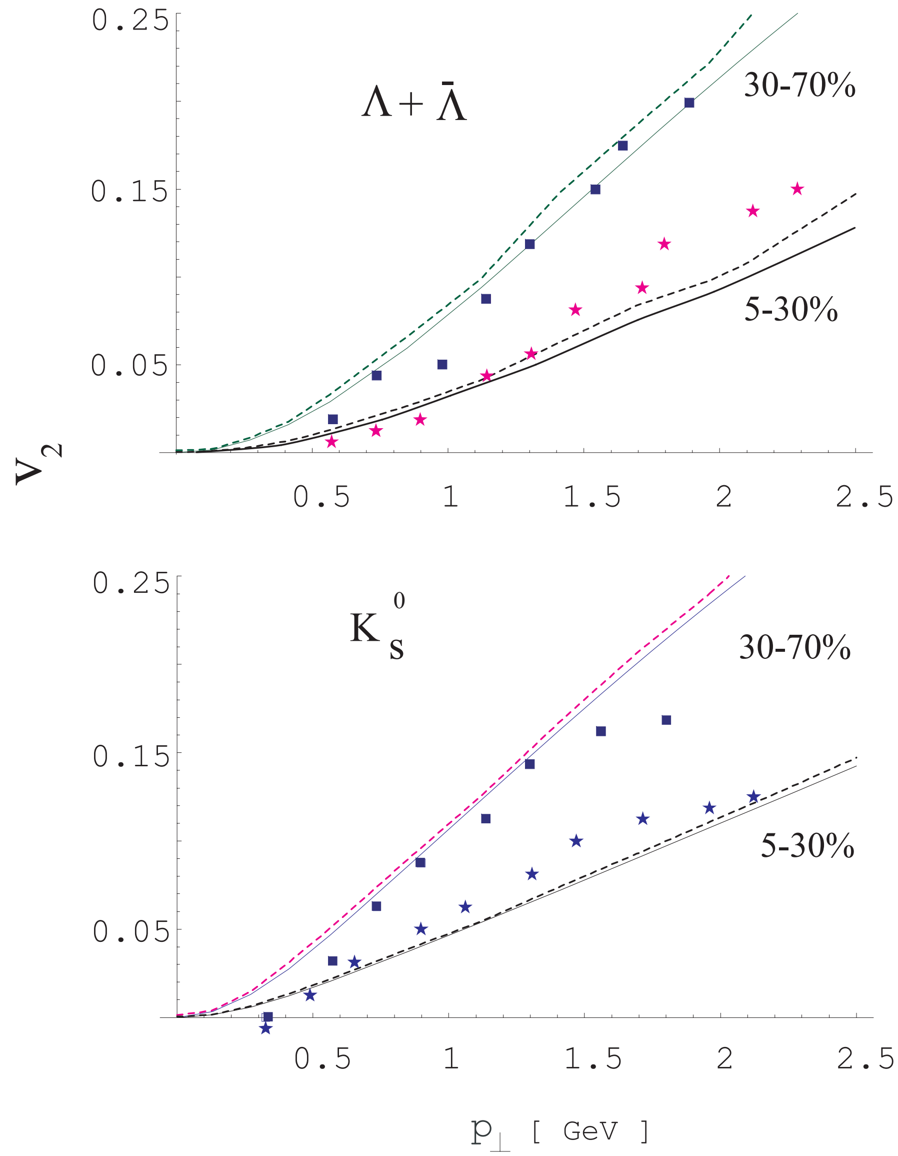
solid – no resonanses, dashed – with resonances

Results of fitting of δ to v_2 for pions, kaons, and protons

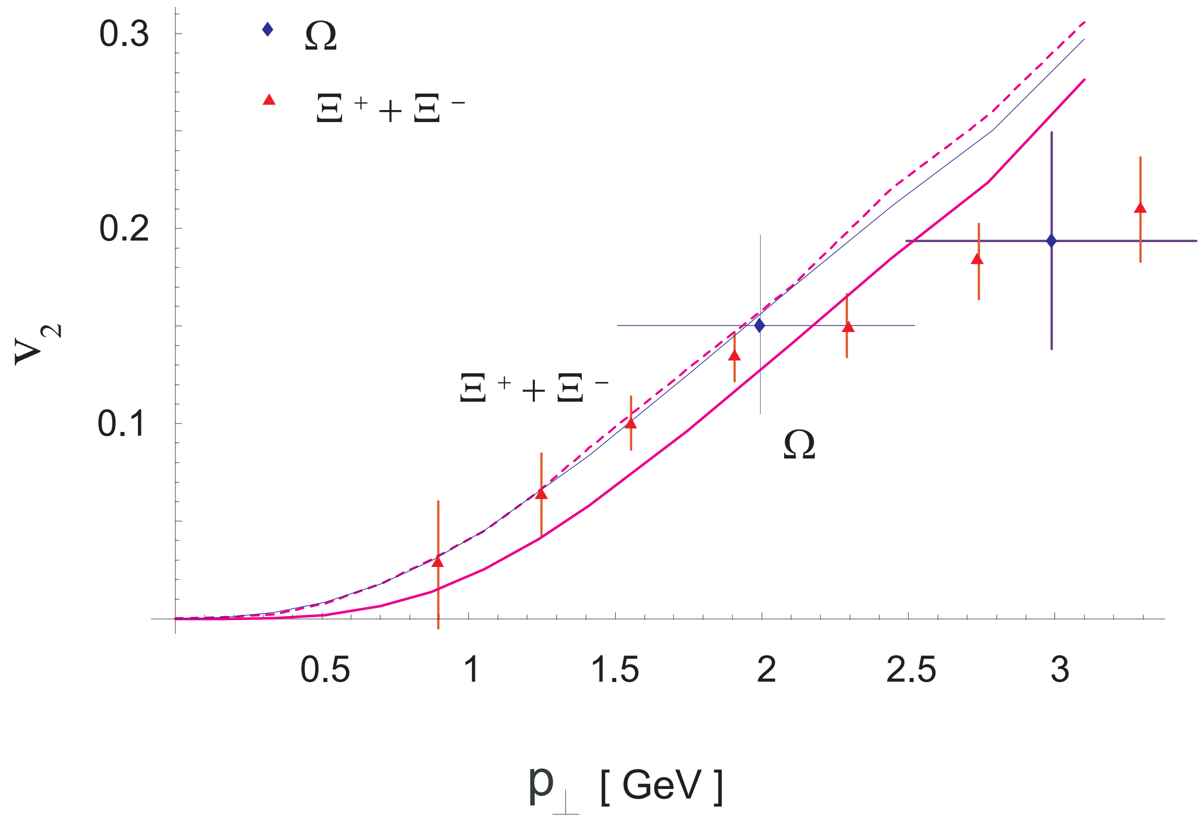


Verification / predictions

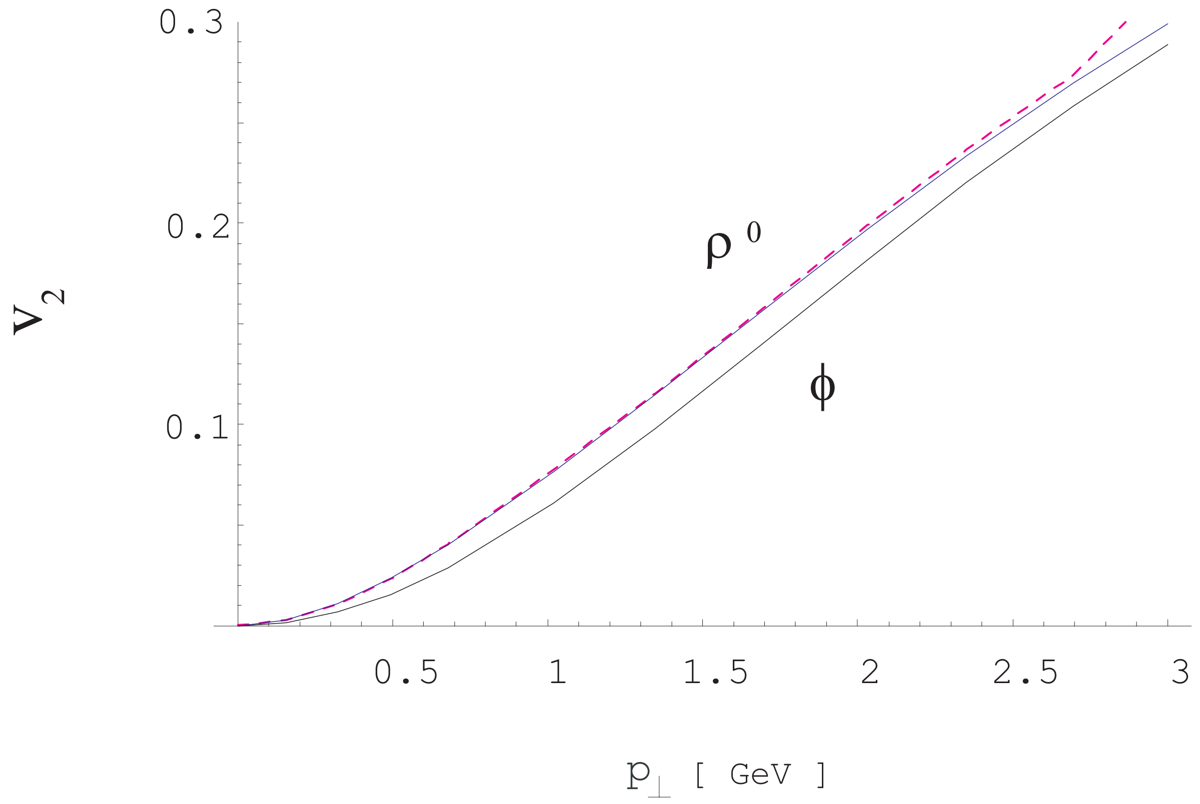
minimum bias (0-80%) data from STAR @ 200 GeV, PRL 92 (2004) 052302



the first measurement of the elliptic flow for multistrange baryons, J. Castillo, contribution to QM04, J. Phys. G30 (2004) S1207



predictions for ρ and ϕ



Summary of v_2: model works for not-too-large $p_\perp$ (no saturation at large momenta), results similar to hydro, works well for hyperons, supports the flow explanation of azimuthal asymmetry

Departure from boost invariance

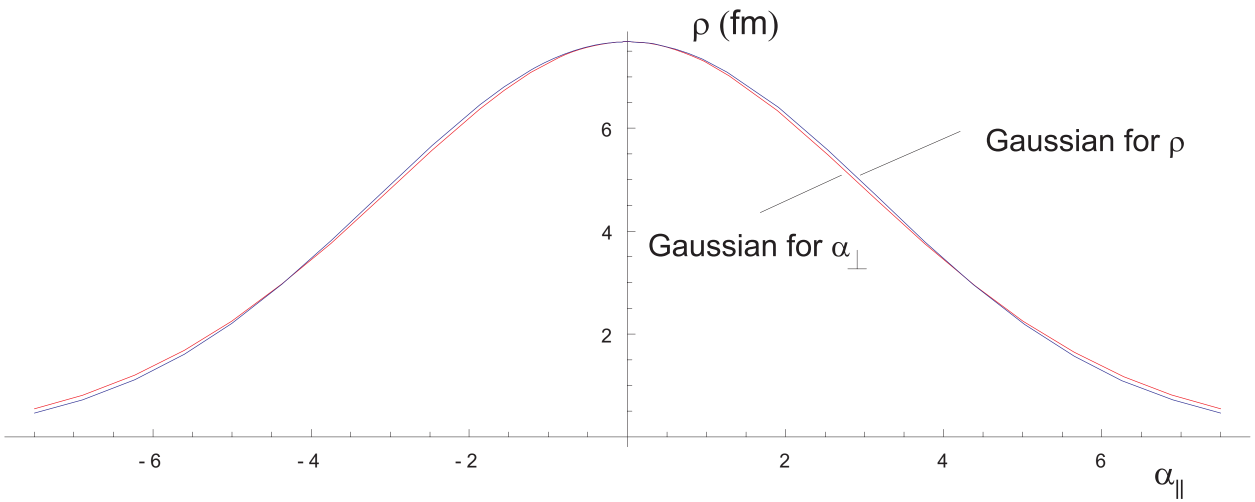
$$x = \tau \sinh \alpha_\perp \cos \phi, \quad y = \tau \sinh \alpha_\perp \sin \phi$$
$$z = \tau \sinh \alpha_\parallel \cosh \alpha_\perp, \quad t = \tau \cosh \alpha_\parallel \cosh \alpha_\perp,$$

Now we take $\alpha_\perp \in [0, \alpha_2 \exp[-\alpha_\parallel^2/(2\Delta^2)]$.

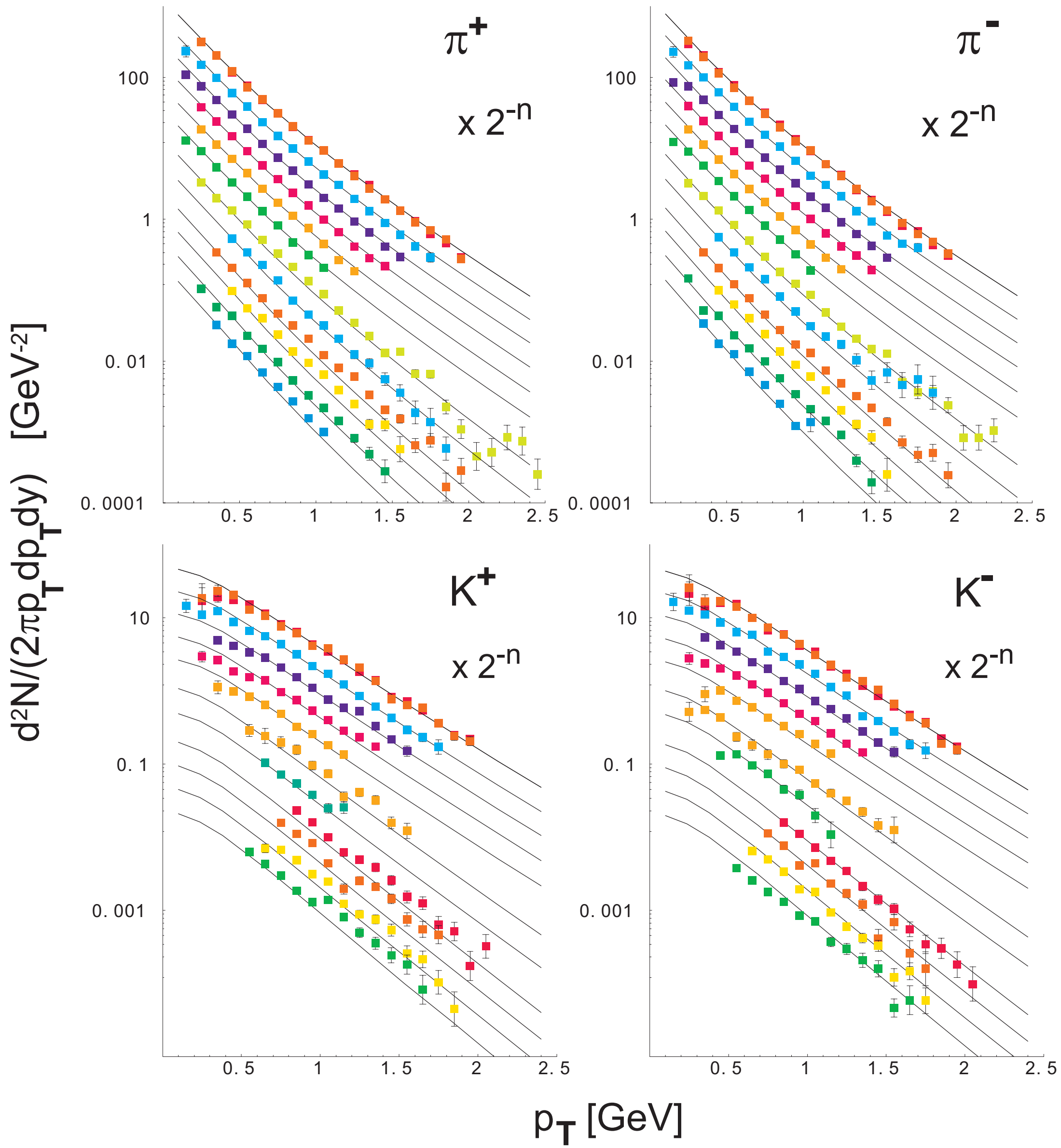
The optimum values of parameters from fits to the BRAHMS spectra are

$$\tau = 8.33 \text{ fm}, \quad \alpha_2 = 0.825, \quad \Delta = 3.33.$$

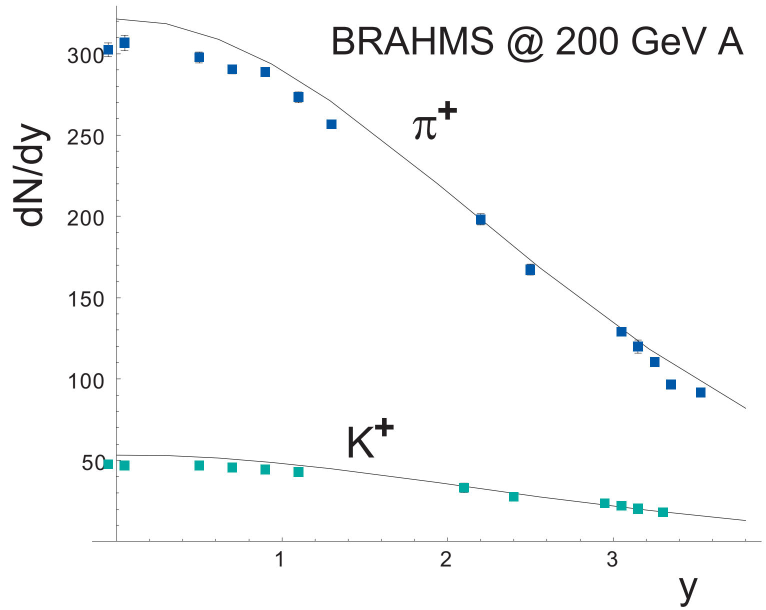
ρ vs. $\alpha_\parallel$



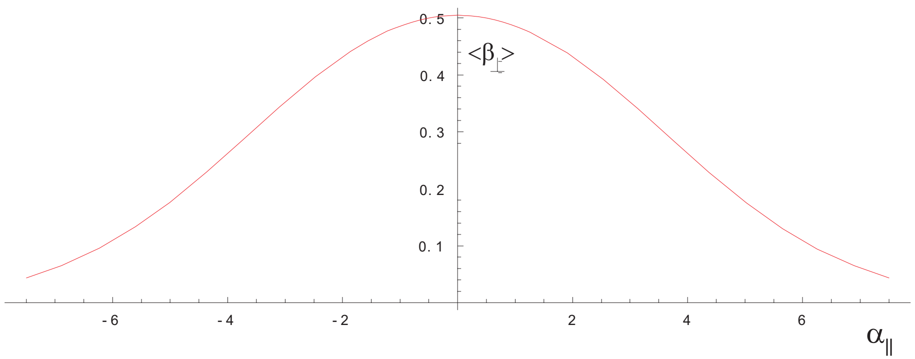
BRAHMS @ 200 GeV A



dN/dy from BRAHMS vs. the model



Average transverse velocity vs. $\alpha_{||}$



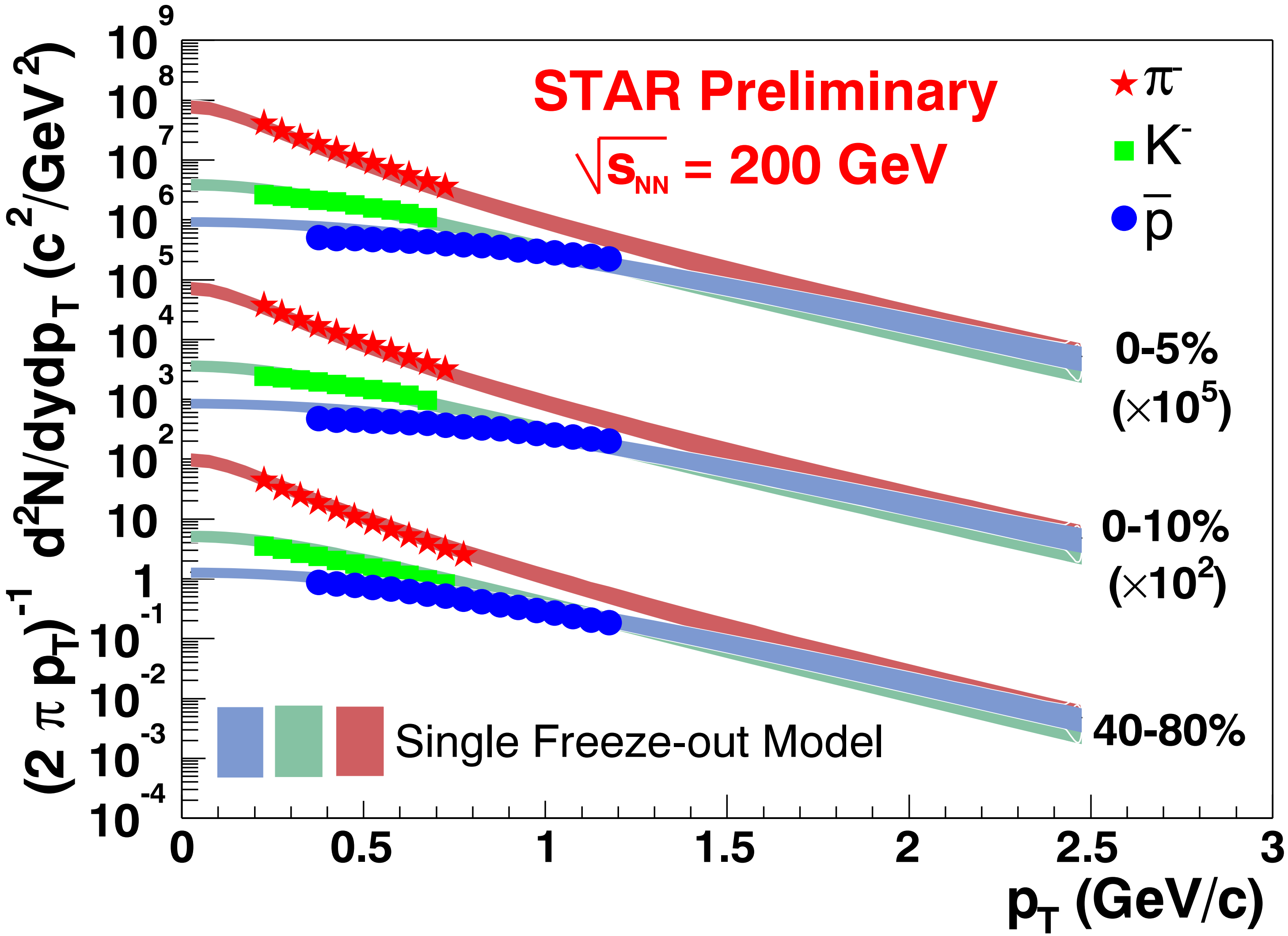
Summary

Statistical models are great!

Back-up slides

STAR spectra @ 200 GeV vs. single freeze-out model

compiled by Patricia Fachini



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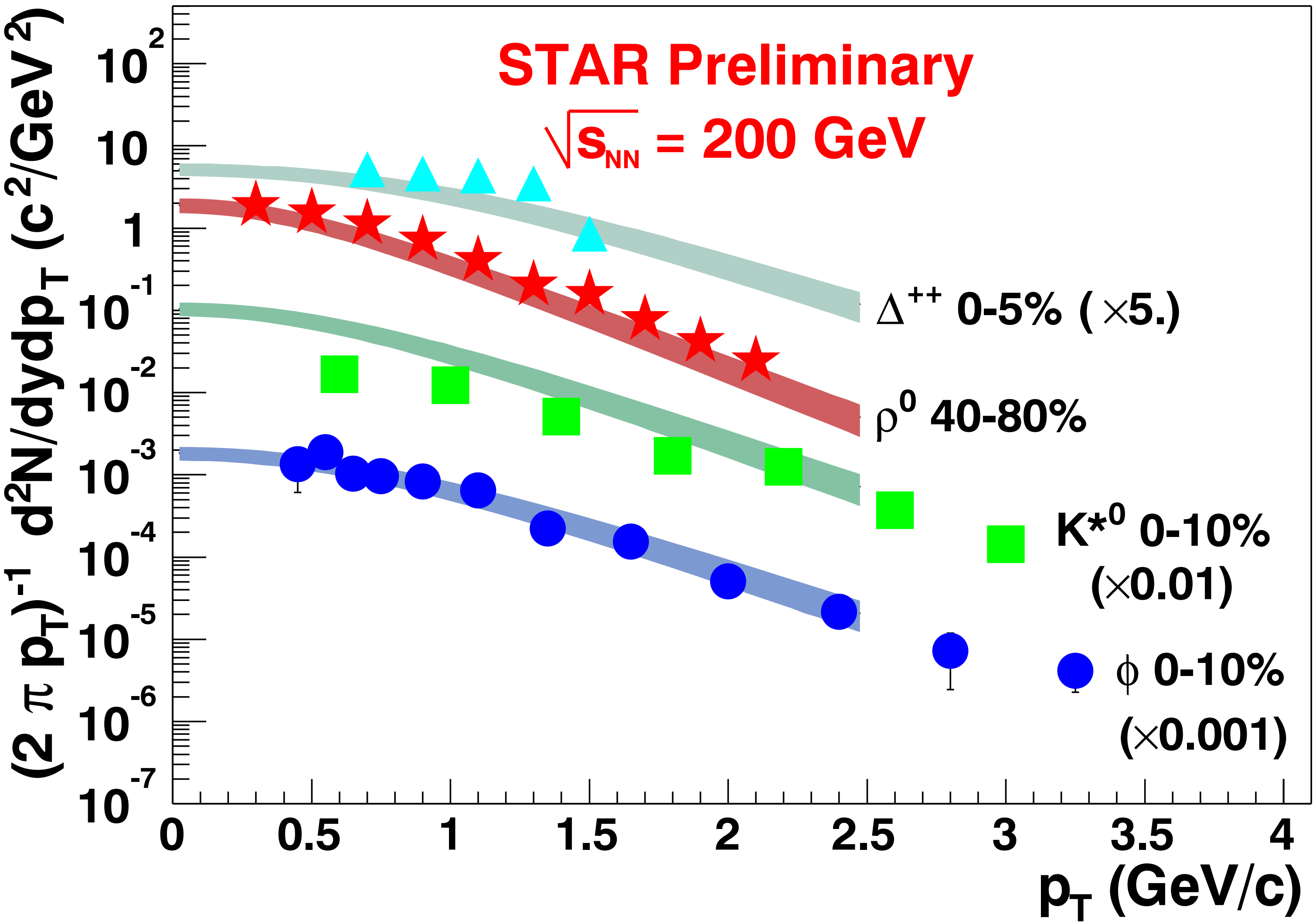

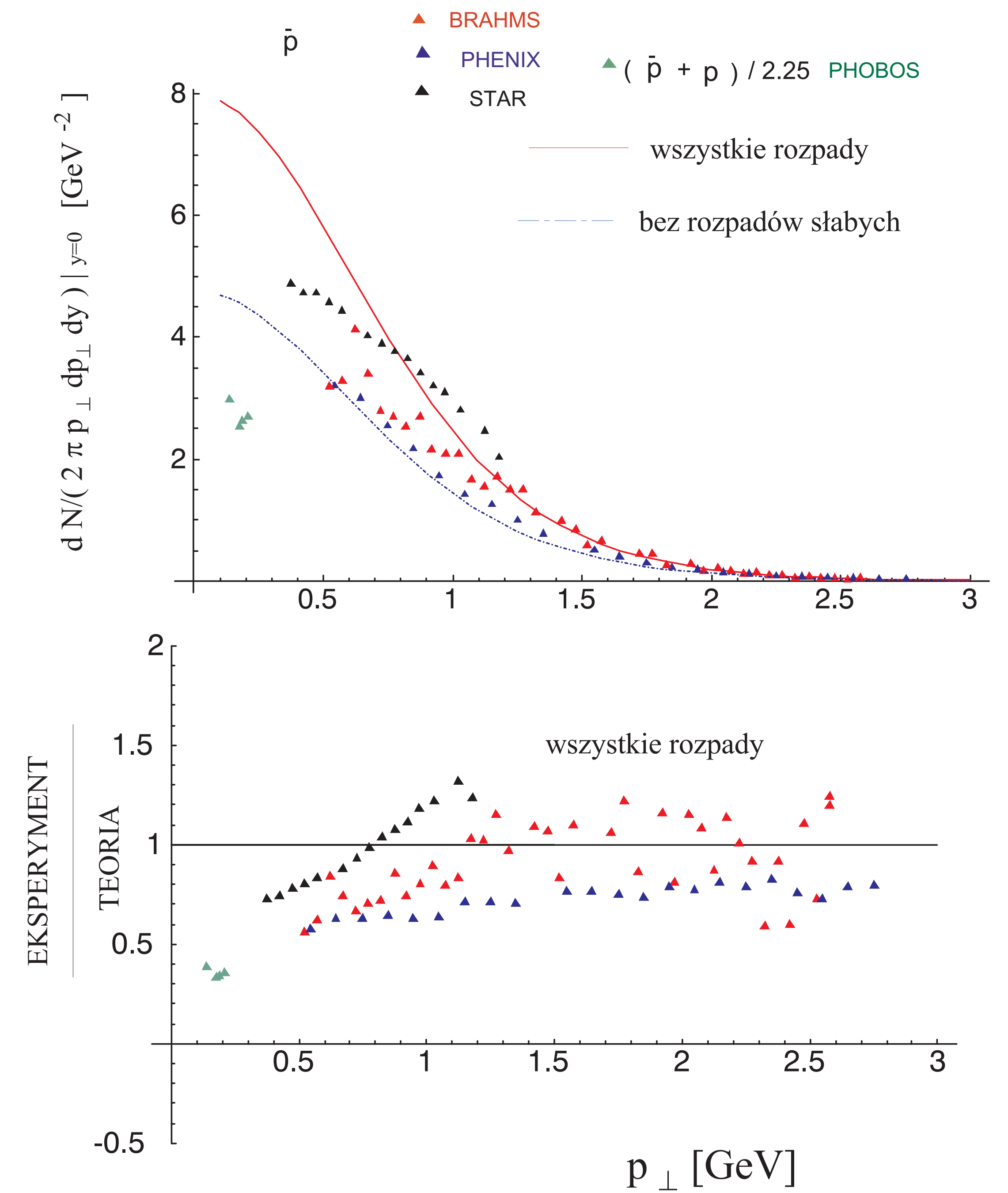
p̄
BRAHMS
PHENIX
STAR
(p̄ + p) / 2.25 PHOBOS
wszystkie rozpady
bez rozpadów słabych
d N/(2 π p ⊥ dp⊥ dy) | y=0 [GeV -2]
8
6
4
2
0.5
1
1.5
2
2.5
3
wszystkie rozpady
EKSPERYMENT
TEORIA
2
1.5
1
0.5
-0.5
p ⊥ [GeV]

BRAHMS @ 200 GeV