Chiral waves in quark matter - a remake[1]

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[1] based on research with M. Kutschera, A. Kotlorz, M. Sadzikowski

Outline

A biased and incomplete introduction 21 years later ⌣̈

Pion condensation

Pion condensation in nuclear matter actively studied since the 70':
[Migdal 1971; Sawyer and Scallapino, 1973; Baym 1973; many authors followed]

Reviews: [A. B. Migdal, Rev. Mod. Phys. 50 (1978) 107; G. E. Brown, W. Weise, Phys. Rep. 27 (1976); G. Baym, D. K. Campbell, in Mesons in Nuclei, 1979]

P-wave attraction sufficiently strong, $\Delta(1232)$ included, later: detailed dynamics issues

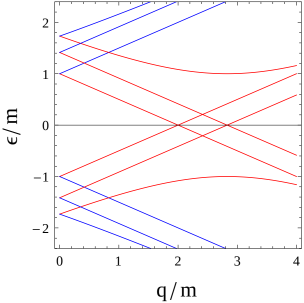
["recent work": review by Kunihiro, Muto, Tamagaki, Tatsumi, Takatsuka, Prog.Theor.Phys.Suppl.112 (1993) 307; A. Akmal, V.R. Pandharipande, PRC 56 (1997) 2261 - condensation occurs at densities $\sim 1.5 - 2$ nuclear saturation density for symmetric and neutron matter, dependence on details]

Solution of Dautry and Nyman

The work of Dautry and Nyman [Nucl. Phys. A319 (1979) 323] - pion condensation in nuclear matter in a relativistic mean-field approach

$$\mathcal{L} = \bar{\psi}\left(i\partial\!\!\!/ - Me^{i\gamma_5/f_\pi \tau\cdot\phi}\right)\psi + \dots$$

where $\tau \cdot \phi = f_\pi \tau_3 q z$ and $-Me^{i/f_\pi \tau\cdot\phi} = g(\sigma + i\tau\cdot\pi)$



A classical chiral field ansatz for the neutral pion condensation (self-consistent in the chiral limit)

$$\sigma = f\cos(qz), \quad \pi^0 = f\sin(qz), \quad \pi^{\pm} = 0$$

The Dirac spectrum is analytic:

$$\epsilon_{\pm}(k) = \sqrt{M^2 + k^2 + q^2/4 \pm \sqrt{M^2 q^2 + q\cdot k^2}}$$

(baryon density uniform)

Also: [T. Tatsumi, Prog.Theor.Phys. 63(1980)1252, “Alternating Spin Layer” – known in condensed matter from the 60’]

Pion gradient and attraction

Understanding on general grounds:

Performing the chiral rotation $\psi = e^{-i\gamma_5/(2f_\pi)\tau\cdot\phi}q$ one gets the derivative coupling

$$\mathcal{L} = \bar{q}\left(i\partial\!\!\!/ - \frac{1}{2f_\pi}\gamma_5\gamma_\mu(\partial^\mu\phi)\cdot\tau - M\right)q + \ldots$$

Attraction in some channels (with appropriately correlated spin and flavor)

$$\Sigma^z = \gamma_5\gamma^0\gamma^3$$

• The effect appears whenever the pion field is nonuniform
• Similar effect: hedgehog solitons ($|0^+\rangle = 1/\sqrt{2}(|u\downarrow\rangle - |d\uparrow\rangle)$)
• Pion condensation is driven by the Fermi sea. Kinetic (Dirac sea) terms in the Lagrangian suppress nonuniformity, hence it is a nontrivial dynamical question if the ground-state (mean-field) solution is nonuniform, but it has a chance!

Pion condensate in quark matter

M. Kutschera, A. Kotlorz, WB [PLB 237 (1990) 159, NPA 516 (1990) 566, Acta Phys. Polon. B22 (1991) 145]

Nuclear Physics A516 (1990) 566–588
North-Holland

QUARK MATTER WITH PION CONDENSATE IN AN EFFECTIVE CHIRAL MODEL

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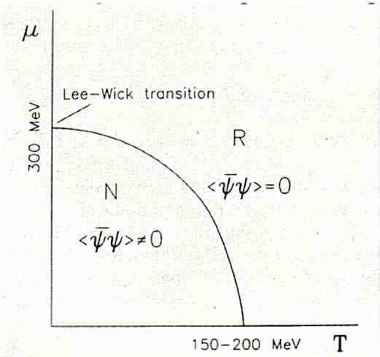
Abstract: We study the chirally broken phase of quark matter with static pion condensates. We use the framework of the sigma model with quark and meson degrees of freedom and mean-field approximation. For commonly used parameters, the pion-condensed phase is the ground state of the system already at densities of the order of nuclear saturation density. We find that for the pion-condensed phase the chiral symmetry restoration takes place at much higher densities than for the normal phase. In the case of the neutral pion condensate the up and down quark Fermi seas are oppositely polarized along the direction of the pion field wave vector. We calculate the net spin density of the up and down Fermi seas and we estimate the magnetization of the system. We obtain the equation of state and study the phase transition from nucleon matter to pion-condensed quark matter.

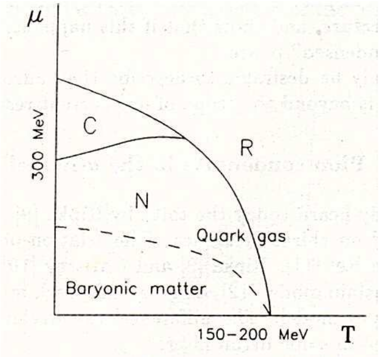
1990 chiral waves

• baryons → quarks – percolation arguments • nonrelativistic → relativistic • analytic calculation for $T = 0$ • spin properties

then accepted view



our 1990 "inhomogenious island"



nowadays quarkyonic, nonuniform ...

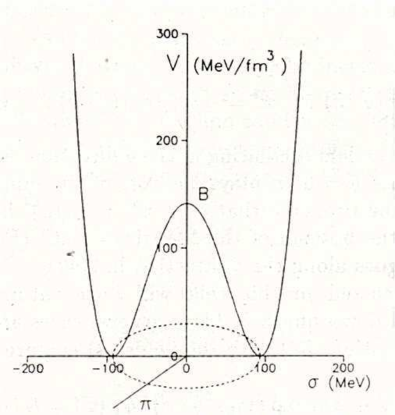
Explanation of the effect

$M \searrow$ – Fermi energy $\searrow$
– meson field energy $\nearrow$

$q \nearrow$ – Fermi energy $\searrow$
– meson kinetic energy $\nearrow$
– (Dirac sea energy $\nearrow$)

Dynamical balance possible

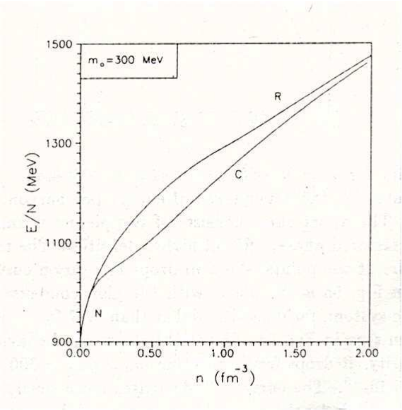
Mexican Hat

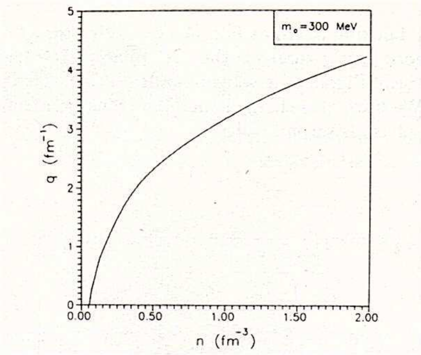


[another early work: Standing wave ground state in high density, zero temperature QCD at large N_c. Deryagin, Grigoriev, Rubakov, Int.J.Mod.Phys. A7 (1992) 659]

Results for σ-model, $T = 0$, $m = 0$

Gell-Mann–Lévy σ-model, $T = 0$, strict chiral limit, $M = 300\ MeV$, $m_\sigma = 1200\ MeV$:

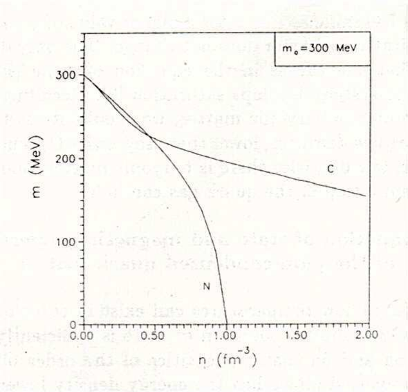




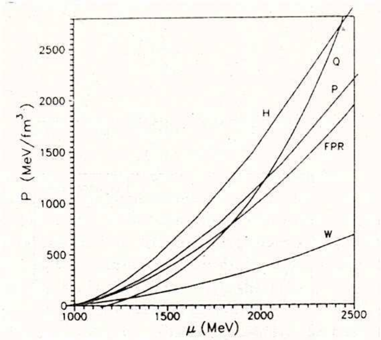
(plots vs baryon density)

Second-order phase transition occurs at very low baryon density

Results for σ-model, cont.



quark mass M vs baryon density – chiral restoration inhibited



pressure vs baryon chemical potential
Q - our calculation

Magnetization

For the neutral pion condensation the interaction term in the Dirac Hamiltonian can be written as

$$-\frac{1}{2}\vec{\Sigma}\cdot\vec{q}\tau_3, \quad \Sigma^i = \gamma_5\gamma^0\gamma^i$$

i.e., attraction for $|u\downarrow\rangle$ and $|d\uparrow\rangle$, repulsion for $|u\uparrow\rangle$ and $|d\downarrow\rangle$.
Magnetization is $\mathcal{M} = g(\mu_u s_u + \mu_d s_d)$, where $g = 2$ and $\mu_u = -2\mu_d$,

$$\mathcal{M} = 2\left[\mu_u \frac{1}{2}(n_{u\uparrow} - n_{u\downarrow}) + \mu_d \frac{1}{2}(n_{d\uparrow} - n_{d\downarrow})\right]$$

In flavor-symmetric matter $n_{u\uparrow} = n_{d\downarrow}$, $n_{d\uparrow} = n_{u\downarrow}$. Then

$$\mathcal{M} = 3\mu_d(n_{d\uparrow} - n_{d\downarrow})$$

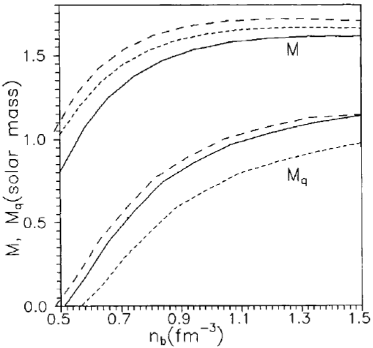
Because the E_- branch is filled more than the E_+ branch, $n_{d\uparrow} > n_{d\downarrow}$, and permanent magnetization occurs:

$$\mathcal{M} \neq 0$$

At the mean-field level $s_{u/d} = \mp\frac{1}{2}f^2 q$ (Baym-Flowers, 1974, $\langle A_z^3\rangle = 0$)

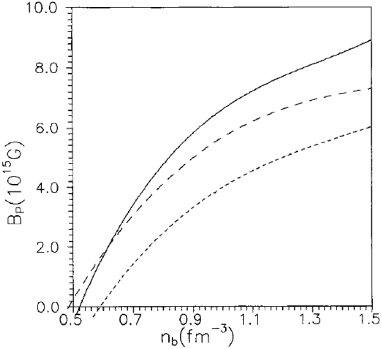
Neutron stars

[M. Kutschera and A. Kotlorz, Acta Phys. Polon. B25 (1994) 859]



M - mass of the neutron star, M_q - star with a quark core in the pion-condensed phase

magnetic field



(plots vs central baryon density)

[Blaschke, Klähn et al. 2007, see the talks by T. Klähn, A . Sedrakian]

Charged pion condensation

[Dautry and Nyman, 1979]

Ansatz for the charged pion condensation:

$$\pi_1 = f\cos(qz), \;\; \pi_2 = f\sin(qz), \;\; \pi_3 = \sigma = 0.$$

Obtained with chiral rotation $\exp\left(i\frac{\pi}{4}\gamma_5\tau_1\right)$ from the previous case, therefore the same spectrum, energy, thermodynamic features, but no interesting magnetic properties.

(combination of condensates, disoriented chiral condensates)

One-loop physics

[WB and M. Kutschera, PLB 234 (1990) 449; PRD 41 (1990) 3800]

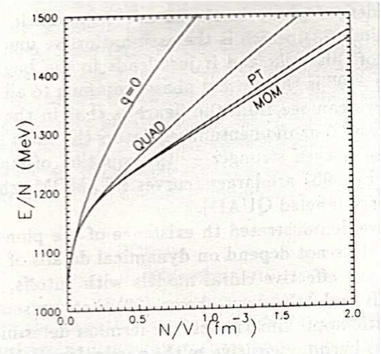
Fermion and meson one-loop corrections in the σ-model are analytic in the "chiral wave" background. For instance,

$$
\begin{aligned}
\mathcal{E}_{\rm meson\ \ loop} &= \frac{\lambda^4 f^4}{16\pi^2} \sum_{n=0}^{\infty} B_n \\
B_0 &= \ldots, \quad B_2 = \ldots \\
B_{2j+2} &= \frac{[\alpha(1-Z)]^{2j}}{2j(2j+1)(2j+2)} {}_2F_1(2j, 1/2; j+3/2; Z) \\
\alpha &= \frac{f^2}{f_\pi^2}, \quad Z = \frac{q^2}{q^2 + \lambda^2(2f^2 - f_\pi^2)}
\end{aligned}
$$

Tests of gradient expansions, convergence radii, etc.
→ expansion parameter is $\frac{q}{2M}$, should request $q < 2M$ in the mean-field treatment

Effective action

Compared to the σ-model, in the NJL model the Mexican Hat is modified. Moreover, the Dirac sea generates the kinetic terms for the chiral meson fields, as well as higher terms in the gradient expansion. For the "chiral wave" background it correspond to generating the energy density $\frac{1}{2}q^2 f^2 + \sum_{n=2}^{\infty} c_n q^{2n}$. Details matter.

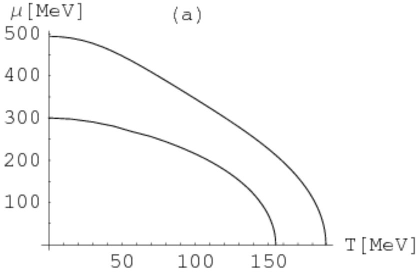


NJL "softer" than the σ-model – easier to form the nonuniform background

[WB, M. Kutschera, PLB 242 (1990) 133]

NJL in the strict chiral limit

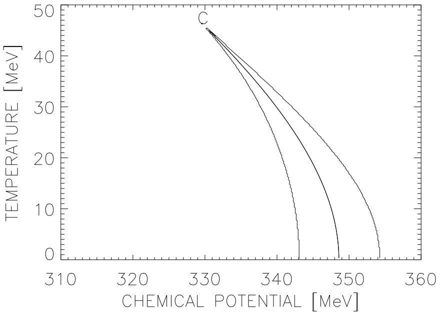
[M. Sadzikowski and WB, PLB 488 (2000) 63]
First-order phase transition to the pion-condensed phase (here $m_\pi = 0$)



(details of the diagram depend on the model parameters)
[Schon, Thies, 2000; Nakano, Tatsumi, 2005; Bringoltz 2007; Partyka, Sadzikowski, 2009; Nickel 2009; Carignano, Nickel, Buballa, 2010; Frolov, Zhukovsky, Klimenko, 2010; Ebert, Gubina, Klimenko, Kurbanov, Zhukovsky, 2011;]

Off the chiral limit - uniform phase

(figure from O. Scavenius, A. Mocsy, I.N. Mishustin, D.H. Rischke, PRC 64 (2001) 045202)



σ-model with the physical pion mass – the m_π effect is important

Numerous papers on application of chiral quark models to the phase diagram, Polyakov-NJL, see, e.g., the review by Schaefer, Pawlowski, and Wambach [PRD 76 (2007) 074023]

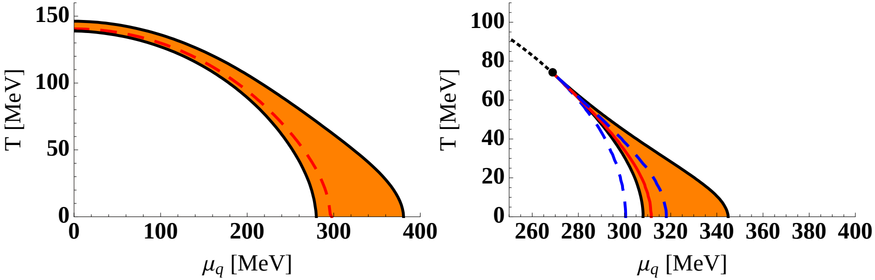
[talks by Fischer, Kojo, Pawłowski, Schramm, Friman, Glozman]

Another nonuniform phase

• physical m_π, but $\pi^a = 0$, σ modulated • lattice of domain-wall solitons (implanted from the Gross-Neveu model [Schnetz, Thies, Urlichs, 2005]) [D. Nickel, PRD 80 (2009) 074025; DN and M. Buballa, PRD 79 (2009) 054009; S. Carignano, DN, MB, PRD 82 (2010) 054009] (talk by M. Buballa)

[figure from D. Nickel, σ model, $m_\sigma = 600$ MeV, domain-wall solitons]

$m_\pi = 0 \quad \rightarrow \quad m_\pi = 139$ MeV



• CEP=Lifshitz P • self-consistency (!) • modulation of $q^\dagger q$

Self consistency for chiral waves, stability

Self-consistency is highly nontrivial. For instance, with the chiral-wave ansatz

$$\langle\bar{\psi}\psi\rangle \sim \cos(qz), \;\; \langle\bar{\psi}i\gamma_5\tau_3\psi\rangle \sim \sin(qz)$$

in the σ-model off the chiral limit $U = V(\sigma^2 + \pi^2) + f_\pi m_\pi^2 \sigma$. Then the mean-field Euler-Lagrange equation is

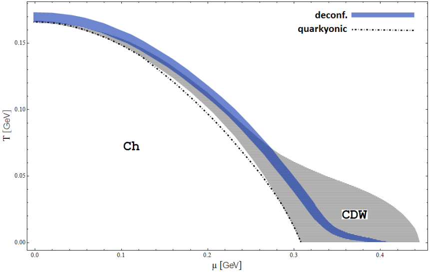
$$\Box\sigma = g\langle\bar{\psi}\psi\rangle + 2V'\sigma + f_\pi m_\pi^2$$

The last term breaks the ansatz, hence we do not have a self-consitent mean-field solution

• Instability of one-dimensional structures for $T > 0$ wrt thermodynamic fluctuations
[G. Baym, B. Friman and G. Grinstein, Nucl. Phys. B 210 (1982) 193]

Chiral waves saving the quarkyonic phase

[K. Fukushima, *Phase diagram of hot and dense QCD constrained by the Statistical Model*, PLB 695 (2011) 387]
→ the window for the quarkyonic phase is very narrow (or none)

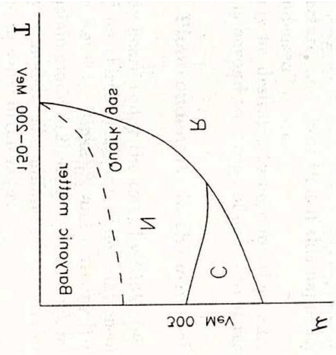


allowing for the nonuniform phase broadens the quarkyonic window [T. Partyka, M. Sadzikowski, arXiv:1011.0921]

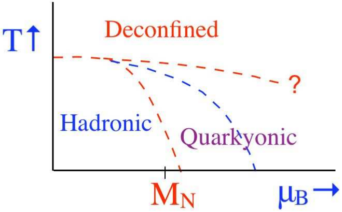
Concluding remarks

1. A phase with the pion condensation (alternating spin layer structure, chiral waves, chiral spirals, domain-wall soliton lattice, nonuniform chiral field) may occur at high-density quark matter as a simple dynamical effect (P-wave pion attraction)
2. Softening of the quark equation of state, net magnetization, neutron stars with magnetic quark core
3. Chiral restoration occurs at a much higher density in the nonuniform phase compared to the uniform phase
4. Dependence of the phase diagram on the details of the model (σ-model, NJL, PNJL) and their parameters
5. The wave vector q (or $q/(2M)$) of the chiral wave should not be too large for the mean-field treatment. Otherwise quantum corrections are necessary.
6. Instability of a one-dimensional structure, transition to a yet lower state $\rightarrow$ nonuniformity in more than one spatial dimension ($\rightarrow$ Skyrmion crystal [Goldhaber, Manton, 1987])

Concluding remarks 2



(from McLerran and Pisarski, 2007)



1. Our analysis holds for quark matter, since quarks and not baryons are the degrees of freedom. This is fine at large baryon densities. We must stick to quarkyonic systems!
2. Geometric/percolation arguments [Castorina, Gavai, Satz, 2010]
3. 2SC, CFL, . . . also may develop nonuniform phases [talks by M. Buballa, S. Carignano, V. de la Incera]