Thermal description of transverse-momentum spectra at RHIC

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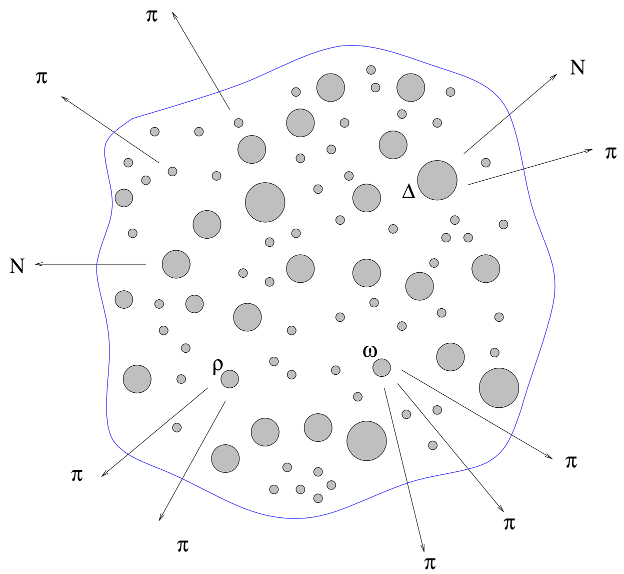
Single-Freeze-Out Model

I. chemical and thermal freeze-outs occur simultaneously

– no elastic rescattering after chemical freeze-out

– TWO thermodynamic parameters T and μ_B obtained from the analysis of the ratios of the particle multiplicities [1]



[1] Rafelski, Becattini, Gaździcki, Gorenstein, Braun-Munzinger, Stachel, Cleymans, Redlich, ...

II. complete treatment of the resonances

- all decays included exactly in a semi-analytic fashion, no approximations, no Monte-Carlo
- the same number of the resonances as that used in the study of the ratios

III. Hubble-like expansion

- definition of the freeze-out hypersurface (Bjorken, Csörgő-Lörstad, Heinz)

$$\tau = \sqrt{t^2 - r_z^2 - r_x^2 - r_y^2} = \text{const}$$

$$u^\mu = \partial^\mu \tau = \frac{x^\mu}{\tau} = \frac{t}{\tau}\left(1, \frac{r_z}{t}, \frac{r_x}{t}, \frac{r_y}{t}\right)$$

- TWO expansion parameters: τ fixes overall normalization, ρ_{max}/τ determines the shape of the spectra

$$\sqrt{r_x^2 + r_y^2} < \rho_{\text{max}}$$

- the model is boost-invariant

Parameters

I. our analysis of the particle ratios, nucl-th/0106009 (APPB 33 (2002) 761), gives two thermodynamic parameters:

$$T = 165 \pm 7 \text{ MeV}, \;\; \mu_B = 41 \pm 5 \text{ MeV}$$

- T very close to the critical temperature inferred from the lattice simulations of QCD (Karsch (173+154)/2=164)
- consistent with other calculations (Braun-Munzinger, Magestro, Redlich, and Stachel, PLB 518 (2001) 41)
- strangeness conservation gives: $\mu_S = 9$ MeV, isospin violation in the gold nuclei gives: $\mu_I = -1$ MeV

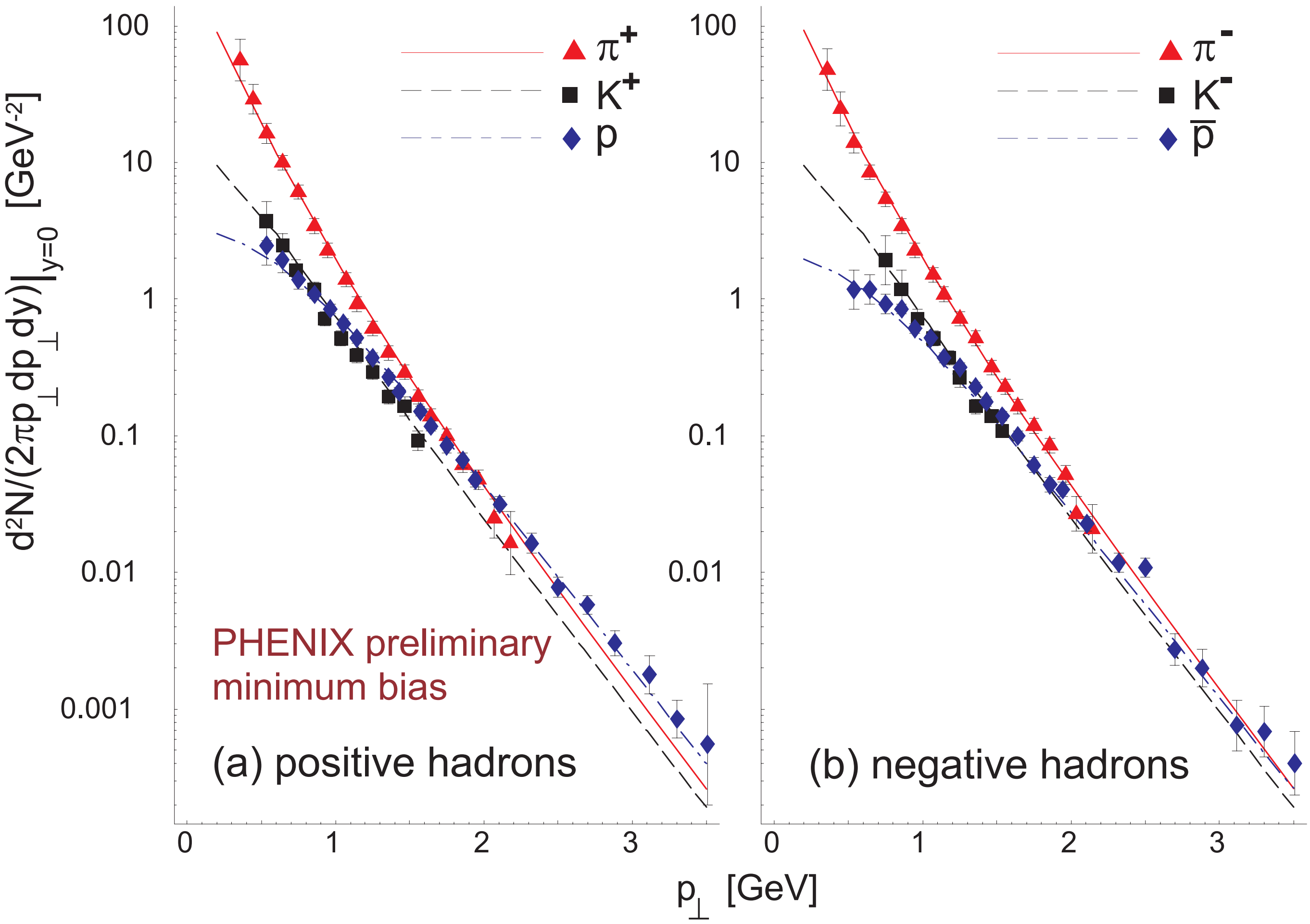
II. then, the analysis of the spectra yields two expansion parameters:

- minimum-bias data:
  $\tau = 5.55$ fm, $\quad \rho_{\text{max}} = 4.50$ fm
- central events:
  $\tau = 7.66$ fm, $\quad \rho_{\text{max}} = 6.69$ fm

other characteristics follow, for example, in central events:

$$\langle \beta_\perp \rangle = 0.49, \;\; \beta_\perp^{\text{max}} = 0.66$$

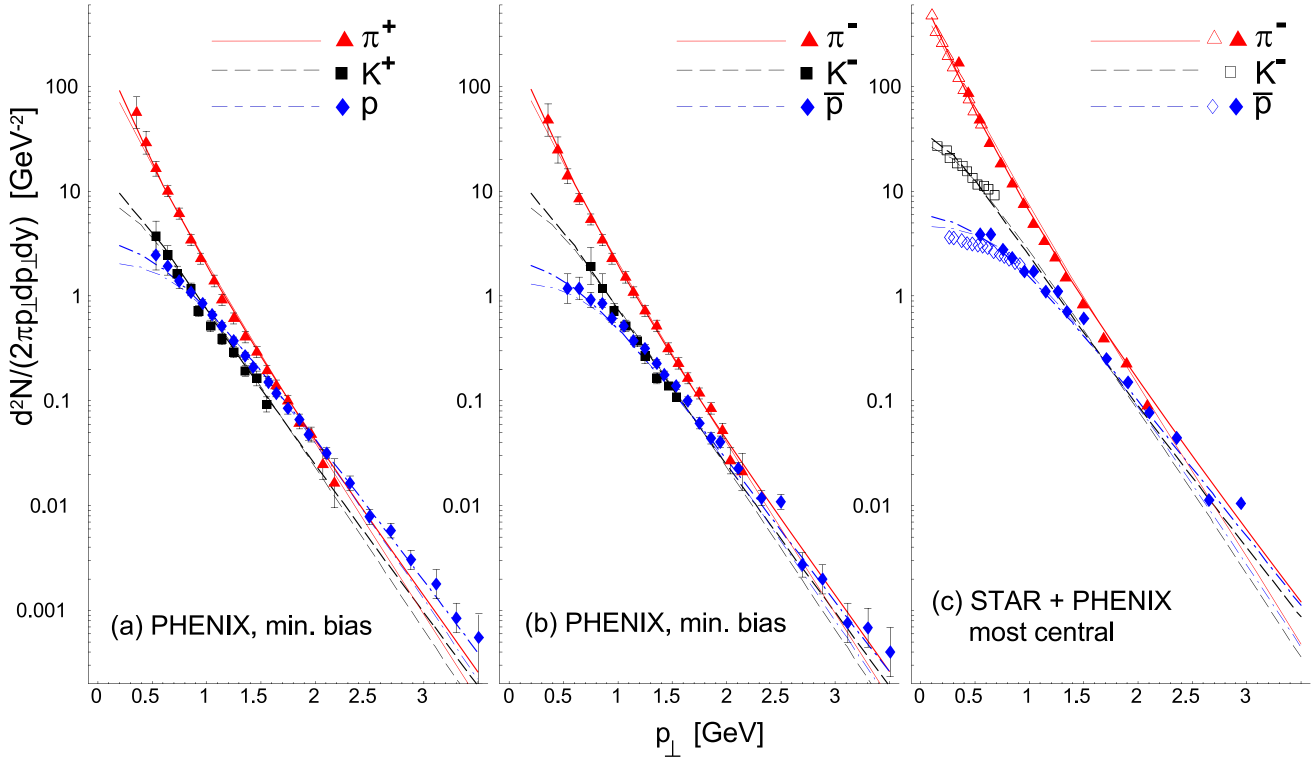
Minimum Bias Data from PHENIX
Au+Au at $\sqrt{s_{NN}} = 130$ GeV



data from: Velkovska (PHENIX) nucl-ex/0105017, QM01, Nucl. Phys. A698 (2002) 507c

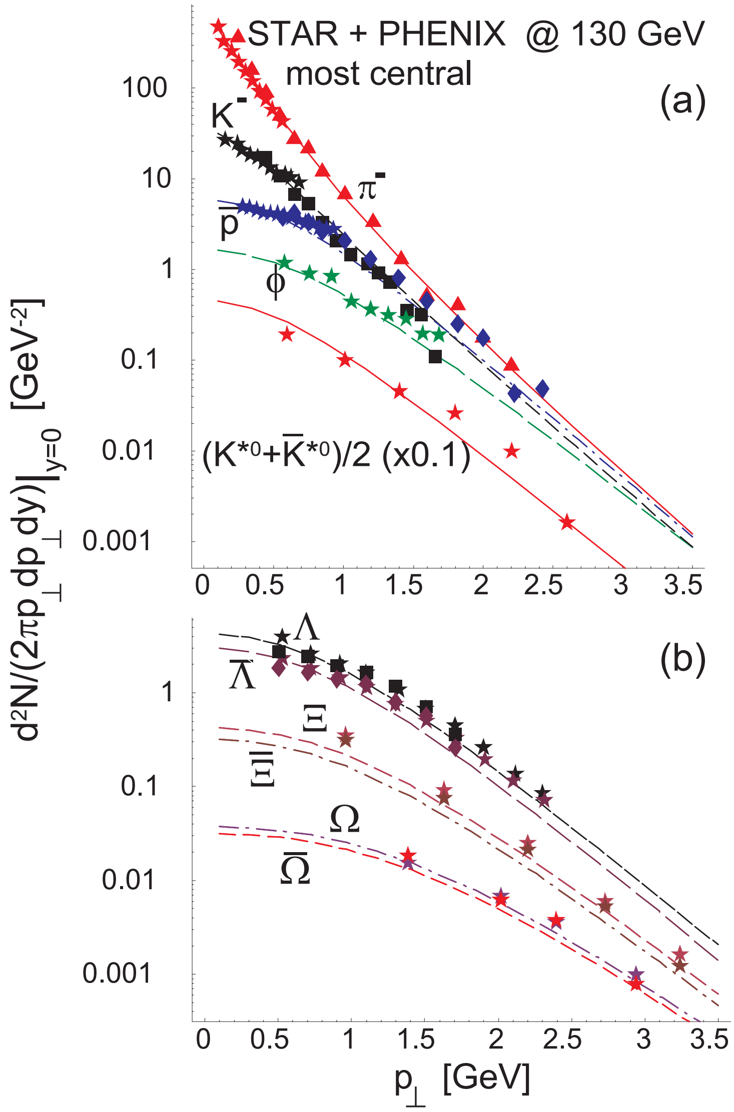
π^0 also agrees

$p_\perp$-spectra of pions, kaons, and protons



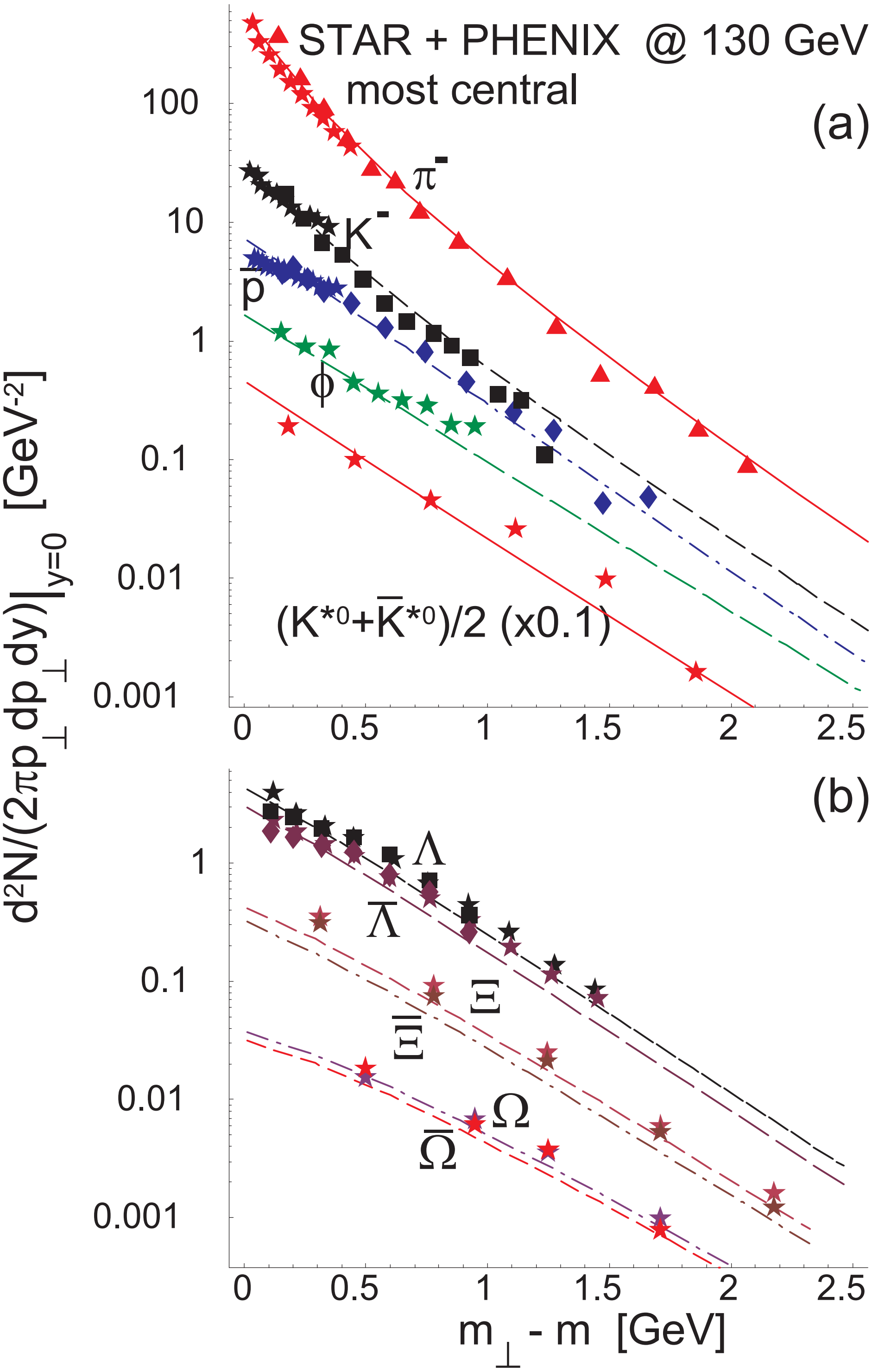
data from: Velkovska (PHENIX) + Harris (STAR) talk at QM01

Most Central Events from PHENIX and STAR, $\sqrt{s_{NN}} = 130$ GeV



$\pi^-, K^-, \bar{p}$ are fitted, other curves – predictions

all theoretical curves and data are absolutely normalized!



STAR + PHENIX @ 130 GeV
most central
(a)
π⁻
K⁻
p̄
φ
(K*⁰+K̄*⁰)/2 (x0.1)
(b)
Λ
Λ̄
Ξ
Ξ̄
Ω
Ω̄
d²N/(2πp⊥dp⊥dy)|y=0 [GeV⁻²]
m⊥ - m [GeV]

input used to fix τ and $\rho^{\max}$:

$\pi^-, \bar{p}$ - Velkovska (PHENIX) nucl-ex/0105017, QM01, Nucl. Phys. A698 (2002) 507c

π^-, K^- - Harris (STAR) talk at QM01

predictions are compared to the data:

ϕ - STAR, PRC 65 (2002) 041901

K^* - STAR, nucl-ex/0205015, (*)

$\Lambda, \bar{\Lambda}$ - STAR, nucl-ex/0203016; PHENIX, nucl-ex/0204007

$\Xi, \bar{\Xi}$ - J. Castillo for STAR, talk at SQM 2001, Frankfurt

$\Omega, \bar{\Omega}$ - B. Hippolyte for STAR, talk at CRIS2002, Catania

figure shows the up-to-date data for π^-, K^- and $\bar{p}$

conclusions:

I. production of strange particles is very well reproduced by the model

II. good description of K^* supports the idea of a single freeze-out, see (*)

Fit of the Particle Ratios

WF + WB + M. Michalec, nucl-th/0106009, APPB 33 (2002) 761

	Thermal Model	Experiment
T [MeV]	165±7	
μ_B [MeV]	41±5	
χ^2/n	0.97	
π^-/π^+	1.02	1.00 ± 0.02 [PHOBOS] 0.99 ± 0.02 [BRAHMS]
$\overline{p}/\pi^-$	0.09	0.08 ± 0.01 [STAR]
K^-/K^+	0.92	0.88 ± 0.05 [STAR] 0.91 ± 0.09 [PHOBOS] 0.78 ± 0.12 [PHENIX] 0.92 ± 0.06 [BRAHMS]
K^-/π^-	0.16	0.15 ± 0.02 [STAR]
K_0^*/h^-	0.046	0.060 ± 0.012 [STAR] 0.042 ± 0.011
$\overline{K_0^*}/h^-$	0.041	0.058 ± 0.012 [STAR] 0.039 ± 0.011
$\overline{p}/p$	0.65	0.61 ± 0.07 [STAR] 0.60 ± 0.07 [PHOBOS] 0.54 ± 0.08 [PHENIX] 0.61 ± 0.06 [BRAHMS]
$\overline{\Lambda}/\Lambda$	0.69	0.73 ± 0.03 [STAR]
$\overline{\Xi}/\Xi$	0.76	0.82 ± 0.08 [STAR]

other ratios (with the feeding from all weak decays included):

$$\frac{\phi}{h^-} = 0.019, \quad \text{expt. } 0.021 \pm 0.001$$

$$\frac{\phi}{K^-} = 0.15, \quad \text{expt. } 0.10 - 0.16$$

$$\frac{\Omega^-}{\Xi^-} = 0.18, \; \frac{\Xi^-}{\Sigma^-} = 0.55, \; \frac{\Sigma^-}{\Lambda} = 0.20$$

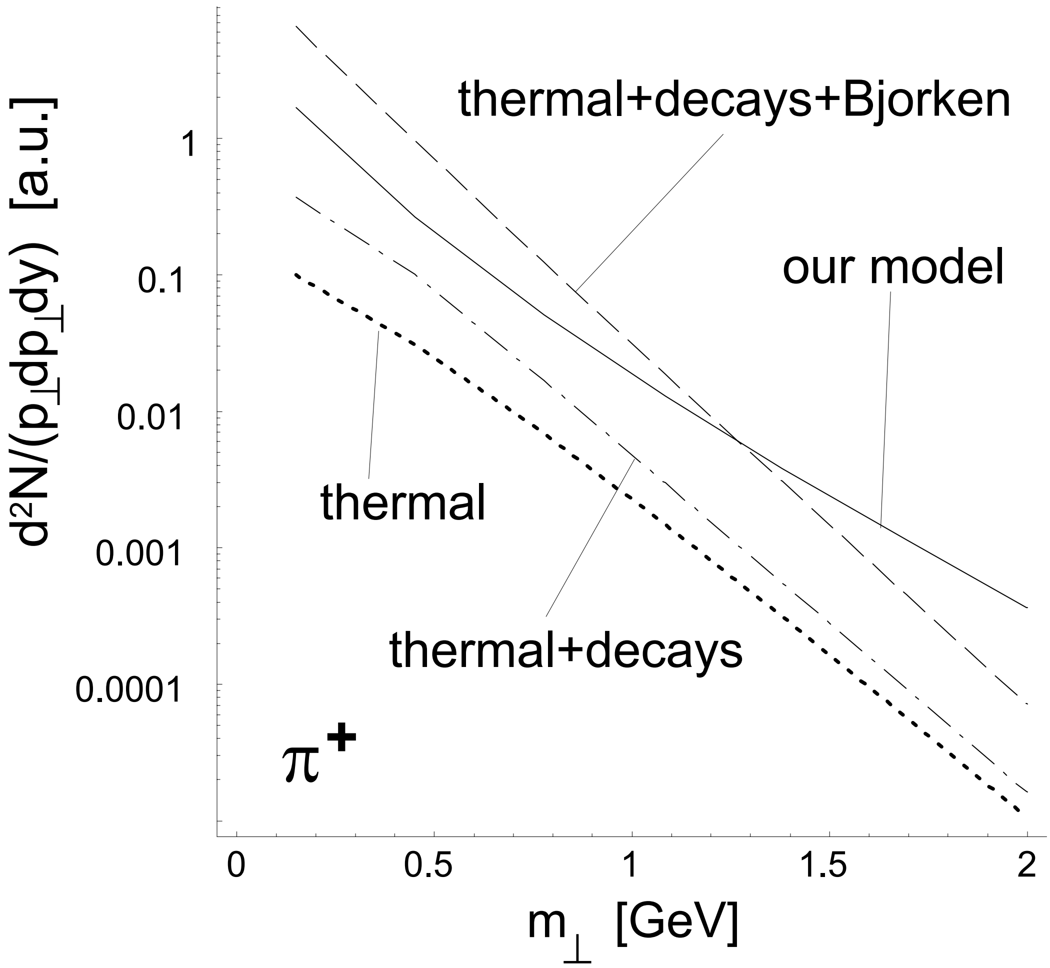
$$\frac{\Lambda}{p} = 0.47, \quad \text{expt. } 0.49 \pm 0.03$$

$$\frac{\Omega^+}{\Omega^-} = 0.85, \; \frac{\Xi^+}{\Xi^-} = 0.76, \; \frac{\Sigma^+}{\Sigma^-} = 1.02$$

expt. = Yamamoto (STAR) hep-ph/0112017 + (STAR) nucl-ex/0203016 + (STAR) PRL 87 (2001) 262302

- besides the shape of the spectra, we reproduce (at the moment) 11 independent ratios of the hadron multiplicities!
- normalization of each spectrum is fixed by the model
- at midrapidity ($y = 0$), for the boost-invariant systems $\frac{dN_i/dy}{dN_j/dy} = \frac{N_i}{N_j}$ = ratio in a static fireball
- no 4π acceptance is required, contrary, 4π-measurements obscure the thermodynamic picture, since the ratios in the fragmentation regions are different
- if the freeze-out hypersurface is defined, all experimental cuts can be easily included in the model

Redshifts and Blueshifts



contributions of various effects to the $p_\perp$-spectrum of π^+

convex shape of the spectrum originates only from the transverse flow

Heinz: we deal with LITTLE BANG

Spectra do not reveal the true temperature!

a) hydrodynamic longitudinal/transverse flow

b) decays of resonances – lowering of the inverse slopes by 30 - 40 MeV (nucl-th/0106009, APPB 33 (2002) 761) for pions, kaons, protons, and lambdas!

c) quantum statistics (pions)

d) slowly varying functions, for example, for the Boltzmann statistics we have

$$\frac{dN}{2\pi m_\perp dm_\perp dy}(y=0) \sim m_\perp \exp\left(-\frac{m_\perp}{T}\right)$$

no simple parametrizations of our results is possible

in particular, a commonly used formula

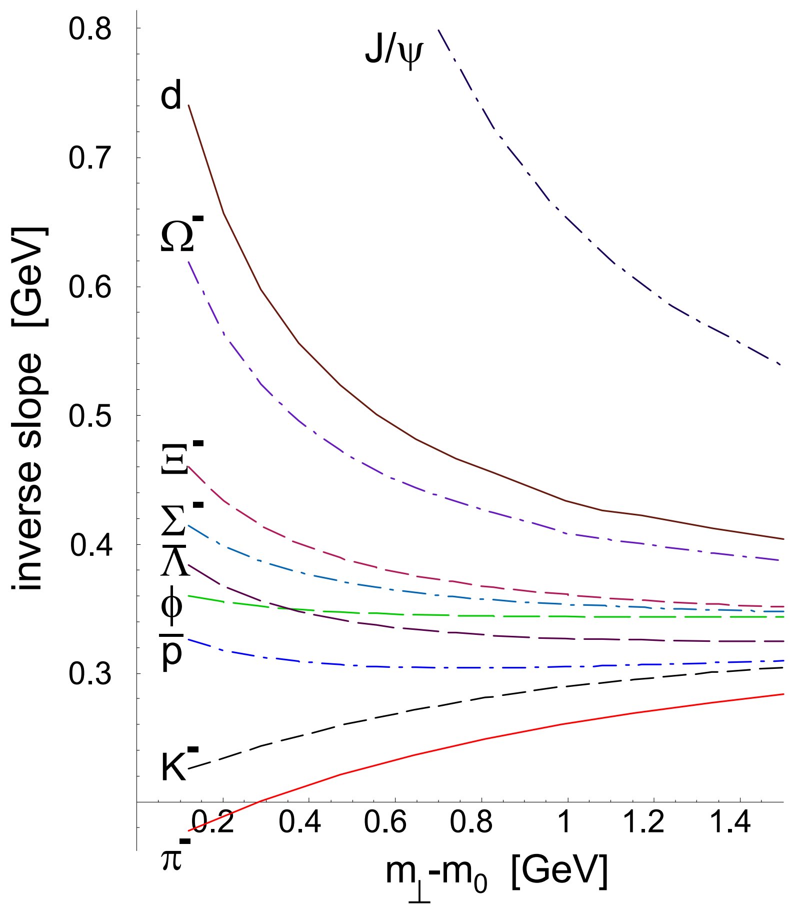
$$\lambda_i = T + \text{const}\, m_i \langle \beta_\perp \rangle$$

does not hold in our model

local inverse slopes

$$\lambda_i = -\left[\frac{d}{dm_\perp} \ln\left(\frac{dN_i}{2\pi m_\perp dm_\perp dy}\Big|_{y=0}\right)\right]^{-1}$$

$$\lambda_\infty \simeq \frac{T}{\sqrt{1+\rho^2_{\mathrm{max}}/\tau^2} - \rho_{\mathrm{max}}/\tau} = 363 \text{ MeV}$$



Parametrization of the Freeze-Out

parametrization of the hyper-surface

$$
\begin{aligned}
t &= \tau \cosh \alpha_{\|} \cosh \alpha_{\perp}, \quad r_z = \tau \sinh \alpha_{\|} \cosh \alpha_{\perp}, \\
r_x &= \tau \sinh \alpha_{\perp} \cos \phi, \quad r_y = \tau \sinh \alpha_{\perp} \sin \phi,
\end{aligned}
$$

- $\alpha_{\|}$ – rapidity of the fluid element

  $v_z = z/t = \tanh \alpha_{\|}$

- $\alpha_{\perp}$ – transverse size of the system

  $\rho = \tau \sinh \alpha_{\perp}$

parametrization of the particle four-momentum

$$
p^{\mu} = (m_{\perp} \cosh y, m_{\perp} \sinh y, p_{\perp} \cos \varphi, p_{\perp} \sin \varphi)
$$

- y – rapidity

- $p_{\perp}$ – transverse momentum

Cooper-Frye formula

$$\frac{dN_i}{d^2p_\perp dy} = \int d\Sigma_\mu \, p^\mu f_i \left(p \cdot u\right)$$

$$\begin{aligned}\frac{dN_i}{d^2p_\perp dy} &= \tau^3 \int_{-\infty}^{+\infty} d\alpha_\parallel \int_0^{\rho_{\text{max}}/\tau} \sinh \alpha_\perp d\left(\sinh \alpha_\perp\right) \\ &\times \int_0^{2\pi} d\xi \;\, p \cdot u \;\, f_i \left(p \cdot u\right)\end{aligned}$$

rapidity distribution is boost invariant

$$p \cdot u = m_\perp \cosh\left(y - \alpha_\parallel\right) \cosh \alpha_\perp - p_\perp \cos \xi \sinh \alpha_\perp$$

where $\xi = \phi - \varphi$

our standard thermal fit can be used!

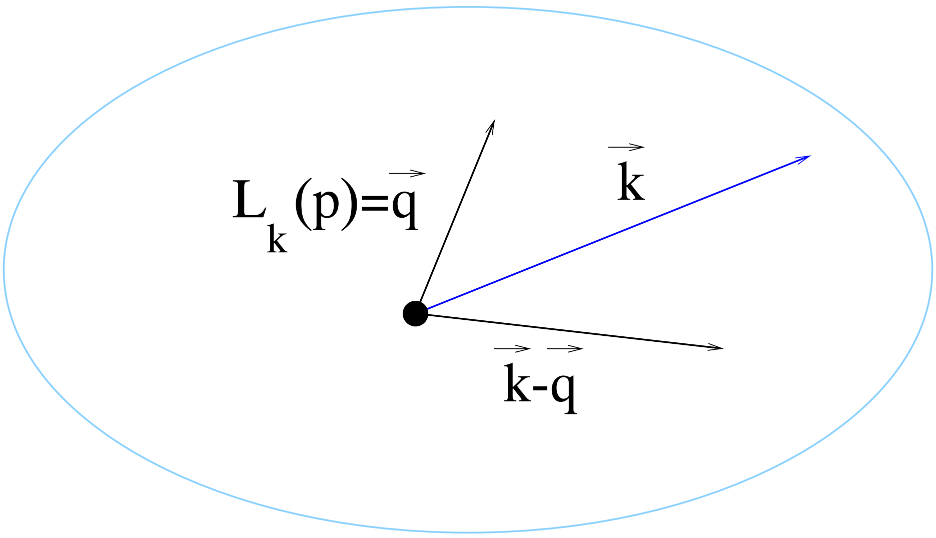
$$\begin{aligned}\frac{dN_i}{dy} &= \int d^2p_\perp \frac{dN_i}{d^2p_\perp dy} \\ &= \pi \rho_{\text{max}}^2 \tau \int d^3p' \;\, f_i \left(\sqrt{p'^{\,2} + m_i^2}\right)\end{aligned}$$

Spectra of Secondaries

for two-body decays, the spectrum of the emitted particle is

$$\delta f_1(|\mathbf{q}|) = b\,\frac{2J_R+1}{2J_1+1}\int d^3k\ f(k)\int \frac{d^3p}{4\pi p^{*2}} \times \delta\left(|\mathbf{p}|-p^*\right)\delta^{(3)}\left(\hat{L}_k\mathbf{p}-\mathbf{q}\right)$$

isotropic distribution of particle 1 in the resonance rest frame is boosted to the fireball frame, and there folded with the resonance distribution



for three-body decays extra integration over p^*

Sollfrank, Koch, Heinz transform $dN/(dyd^2p_\perp)$ more general but also more complicated approach

$\mathbf{p}$ – momentum of the emitted particle in the rest frame of the resonance, $\hat{L}_k$ – Lorentz transformation to the fireball rest frame

$$\hat{L}_k \mathbf{p} = \mathbf{p} + [(\gamma_k - 1)\ v_k^2\ \mathbf{v}_k \cdot \mathbf{p} + \gamma_k\ E^*]\mathbf{v}_k$$

$\mathbf{k}$ – momentum of the resonance in the fireball

$$\mathbf{v}_k = \frac{\mathbf{k}}{\sqrt{k^2 + m_R^2}}, \qquad \gamma_k = \left(1 - v_k^2\right)^{-1/2}$$

$$p^* = \frac{((m_R^2 - (m_1 - m_2)^2)(m_R^2 - (m_1 + m_2)^2))^{1/2}}{2m_R}$$

$$E^* = \sqrt{m_1^2 + p_*^2}$$

m_R – mass of the resonance, m_1 and m_2 masses of the emitted particles, b is the branching ratio for the channel, J_R and J_1 are the spins of the resonance and particle 1

$$\delta f_1(q) = b\frac{2J_R+1}{2J_1+1}\frac{m_R}{2E_q p^* q}\int_{k_-(q)}^{k_+(q)} k\,dk\,f(k)$$

limits of integration

$$k_\pm(q) = \frac{m_R|E^* q \pm p^* E_q|}{m_1^2}$$

our freeze-out hypersurface satisfies the condition

$$d\Sigma^\mu \sim u^\mu$$

all decays may be calculated in the rest frame

$$u^\mu = (1,0,0,0)$$

no need to use Monte-Carlo methods, sequential decays can be easily done, nucl-th/0112043 (PRC 65 (2002) 064905)

HBT Radii

our system is too small, such behavior is typical for thermal models, one uses the excluded volume corections: Hagedorn, Gorenstein, Braun-Munzinger

$$\frac{dN_i}{d^2p_\perp dy} = \frac{\tau^3}{(2\pi)^3}\int_{-\infty}^{+\infty} d\alpha_\parallel \int_0^{\rho_{\max}/\tau} \sinh\alpha_\perp d\left(\sinh\alpha_\perp\right) \times \int_0^{2\pi} d\xi\ p_i\cdot u\ \exp\left(-\beta p_i\cdot u\right) S_i^{-3}$$

in the self-consistent treatment of Gorenstein et al.

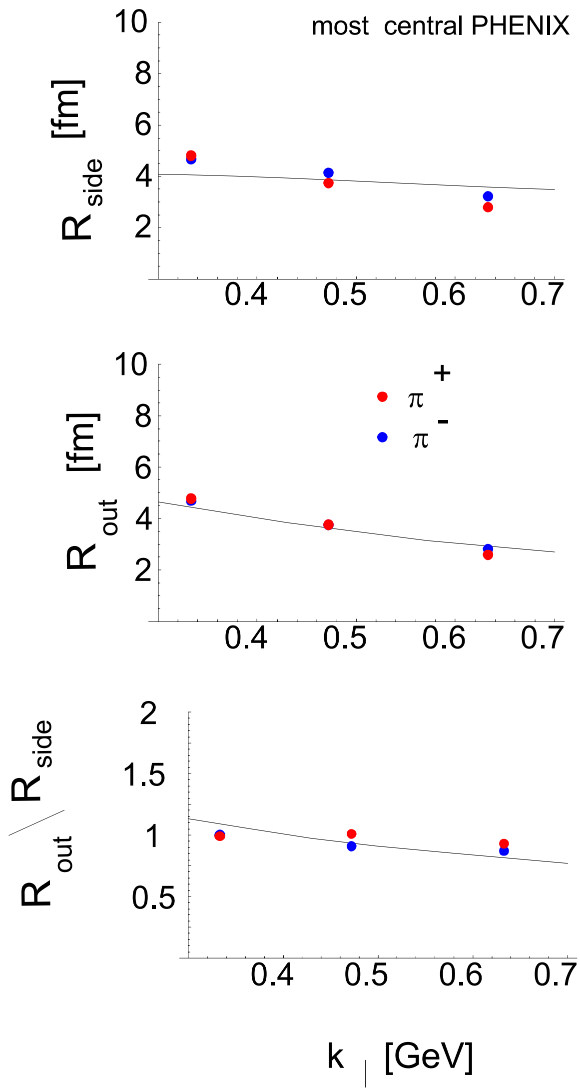
$$S_i^{-3} = \frac{e^{-Pv_i/T}}{1+\sum_j v_j e^{-Pv_j/T} n_j}$$

spectra do not change if: $\tau \to S_i\tau$ and $\rho_{\max} \to S_i\rho_{\max}$

for our values of the thermodynamic parameters

$S = 1.3$ for $r =$0.6 fm, $v_i = 4\frac{4}{3}\pi(r_i)^3$

we take v_i universal for all hadrons (ratios do not change)



approximation: instantaneous decays of the resonances

similarity to the Blast-Wave parametrizations, Hardtke (STAR)

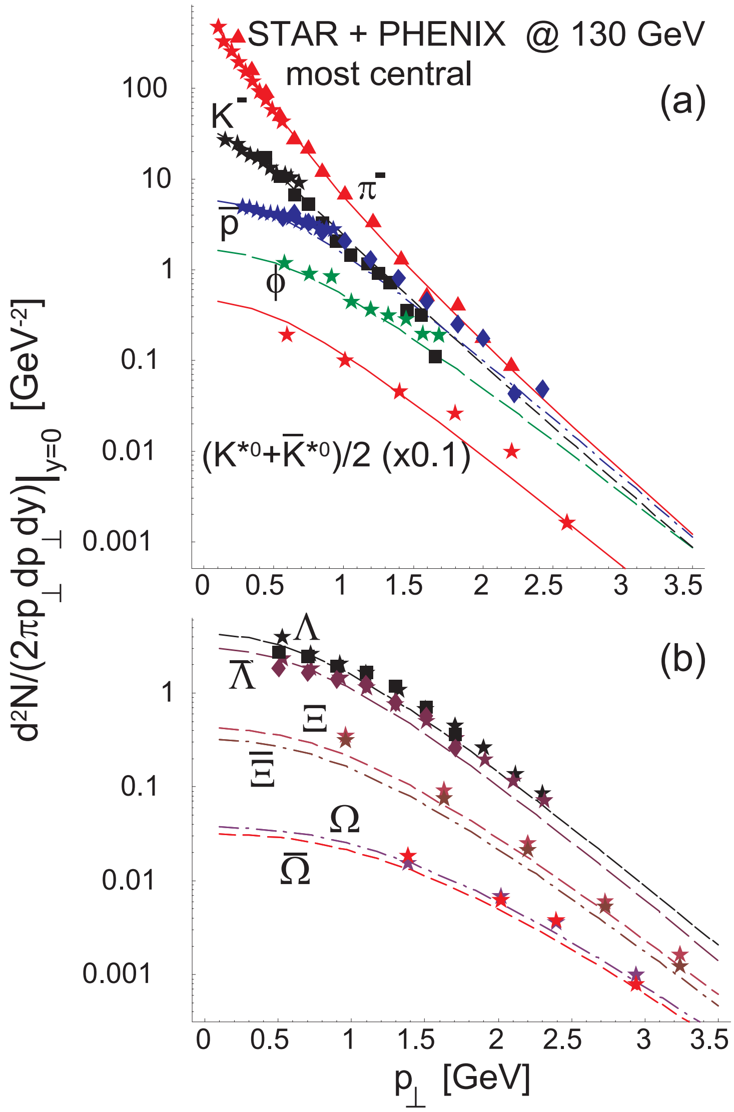
information about the freeze-out hypersurface

Conclusions

- simple explanation of the $p_\perp$ spectra of all measured hadrons with a surprising accuracy $(\rho_{\max}, \tau)$
- model fits also the particle ratios (T, μ_B) and yields $R_{\rm side} \approx R_{\rm out}$
- small sizes of the hadronic system, compatible with the measured HBT radii, the size of the system is affected by the excluded-volume corrections (v)
- the hydrodynamical evolution is omitted, the shape of the freeze-out surface is the input
- the inclusion of the resonances is crucial for the success of the model, semi-analytic approach, no need for Monte-Carlo methods
- other observables can be calculated easily (rapidity dependence, eliptic flow)
- the model works also for SPS

Most Central Events from PHENIX and STAR, $\sqrt{s_{NN}} = 130$ GeV

single-freeze-out model, Broniowski + Florkowski, PRL 87 (2001) 272302, PRC 65 (2002) 064905



all theoretical curves and data are absolutely normalized - exp. ratios are reproduced