Chiralne modele kwarkowe a procesy wysokoenergetyczne

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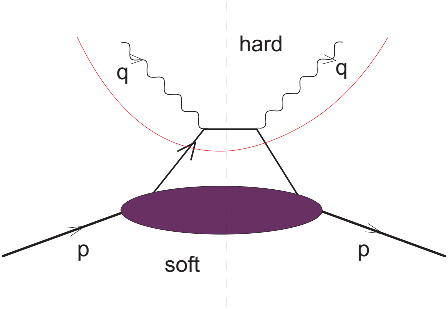
- Pion light cone wave function and pion distribution amplitude in the NJL model, Phys.Rev.D66:094016,2002, hep-ph/0207266
- Spectral quark model and low-energy hadron phenomenology, Phys.Rev.D67:074021,2003, hep-ph/0301202
- Impact parameter dependence of the GPD of the pion in chiral quark models, Phys.Lett.B574:57-64,2003, hep-ph/0307198
- Application of chiral quark models to high-energy processes, *Bled 2004, Quark dynamics* 7-10, hep-ph/0410041
- Pion transition form factor and distribution amplitudes in large-N_c Regge models, Phys.Rev.D74:034008,2006, hep-ph/0605318
- Photon DA's and light-cone wave functions in chiral quark models, EAD+WB+ERA, Phys.Rev.D74:054023,2006, hep-ph/0607171
- Pion-photon Transition Distribution Amplitudes in the Spectral Quark Model, Phys.Lett.B649:49,2007, hep-ph/0701243

– numerous references to the field

- Low-energy quark models are used to compute low-energy matrix elements of hadronic operators
- Matching to QCD at the low quark-model scale
- QCD evolution to experimental scales
- Comparison to data: DA, GPD, PDF, TDA
- At the moment for processes including pions and photons/rho mesons
- Leading twist (known QCD evolution), large number of colors (one quark loop)

Deep Inelastic Scattering – Parton Distribution Function



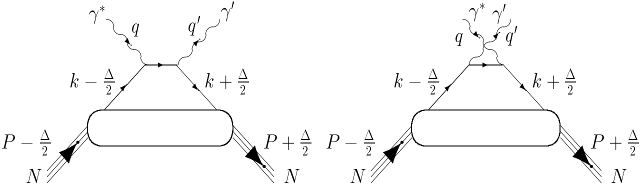
$Q^2 = -q^2, \ \ W^2 = (p+q)^2, \ \ x = \frac{Q^2}{2p \cdot q} = \frac{Q^2}{Q^2+W^2}, \ \ Q^2 \to \infty$

Factorization of soft and hard processes, Wilson's OPE, twist expansion

$$\langle J(q)J(-q)\rangle = \sum_i C_i(Q^2;\mu)\langle \mathcal{O}_i(\mu)\rangle$$

The soft matrix element can be computed in low-energy models!

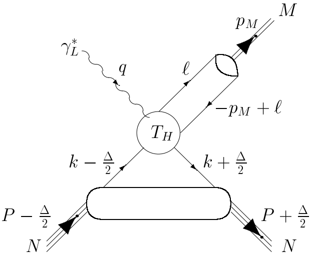
Exclusive processes in QCD



(a) (b)

Deeply Virtual Compton Scattering

non-zero momentum transfer to the target, at least one photon virtual



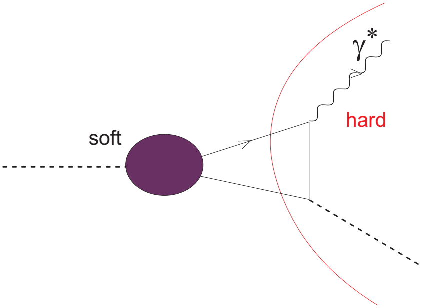
Hard Meson Production

Dictionary of matrix elements

General structure of the soft matrix elements: $\langle A \mid \mathcal{O} \mid B \rangle$

- A = B = one-particle state – Parton Distribution of A (inclusive DIS)
- A = one-particle state, B = vacuum – distribution amplitude (DA) of A (hadronic form factors, HMP)
- A, B = one-particle state of different momentum – GPD (exclusive DIS, DVCS, HMP)
- A = many-particle state, B = vacuum – GDA (transition form factors)
- A $\neq$ B (A, B – different hadronic states) – Transition Distribution Amplitude ($h\bar{h} \to \gamma\gamma^*$, Pire & Szymanowski 2004)
- ...

Pion transition form factor - pion Distribution Amplitude



Definition (for π^+, leading twist):

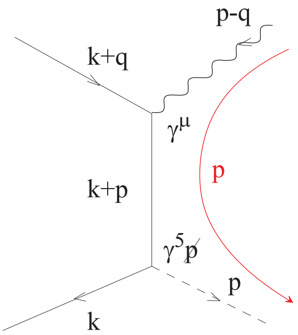
$$\langle 0|\overline{d}(z)\gamma_\mu\gamma_5 u(-z)|\pi^+(q)\rangle = i\sqrt{2}f_\pi(q^2)q_\mu\int_0^1 dx e^{i(2x-1)q\cdot z}\phi(x)$$

z is along the light cone, $z^2=0$, $f_\pi(m_\pi^2)=93$ MeV – pion decay constant, gauge link operator $[-z,z]$ omitted (in the usual light-cone gauge $[z,-z]=1$)

Normalization: $\int_0^1 dx\phi(x)=1$, since $\langle 0|A_\mu^-(0)|\pi^+(q)\rangle = if_\pi(q^2)q_\mu$

Leading-twist structure

A sample calculation of the leading-twist Dirac structure



p – hard momentum

$$\gamma_5 \not{p} \frac{1}{\not{k} + \not{p} - m} \gamma^\mu \simeq \gamma_5 \gamma^\mu + \text{higher twists}$$

(crossed diagram similar)

QM evaluation of DA

1. Invert the definition:

$$\phi(x) = -\frac{i}{\sqrt{2} f_\pi(q^2)} \int_{-\infty}^{\infty} \frac{d\tau}{\pi} e^{-i(2x-1)\tau n\cdot q} \langle 0|\bar{d}(\tau n) \not{n}\gamma_5 u(-\tau n)|\pi^+(q)\rangle$$

where $n^\mu = (n^0, n^1, n^2, n^3) = (1, 0, 0, -1)$, $n \cdot a = a^+$.

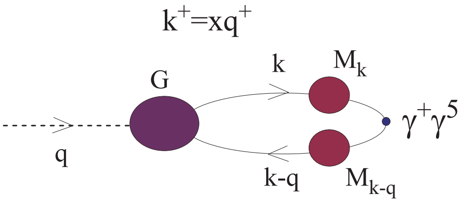
2. $\langle 0|\bar{d}(\tau n)\Gamma u(-\tau n)|\pi^+(q)\rangle =$ (LSZ reduction, one-loop QM)

$$= -\frac{N_c\sqrt{2}}{F_\pi} \int \frac{d^4k}{(2\pi)^4} e^{i\tau n\cdot(2k-q)} G_{\pi qq}(k, k-q) \mathrm{Tr}[\Gamma S_k \gamma_5 S_{k-q}]$$

1+2, evaluation of the trace → "generic" QM expression:

$$\phi(x) = -\frac{4iN_c}{f_\pi(q^2)} \int \frac{d^4k}{(2\pi)^4} \delta(k^+ - xq^+) G_{\pi qq}(k, k-q) \frac{(M_{k-q} - M_k)k\cdot n + M_k q\cdot n}{D_k D_{k-q}}$$

M_p – momentum-dependent *constituent* quark mass,
$D_p = p^2 - M_p^2 + i0$
One-loop diagram (leading $1/N_c$) with constrained integration



Chiral quark models

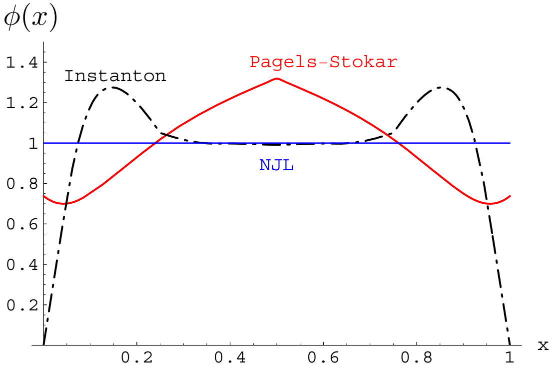
Spontaneous chiral symmetry breaking → quark mass $M \sim 300$ MeV

Model	mass	vertex
Nambu-Jona-Lasinio	M=const	$1/F_\pi$
instanton-liquid model	$M(p^2) = M_0 r_p^2$	$r_k r_{k-q}/F_\pi$
Pagels-Stokar model	$M(p^2) = M_0 r_p^2$	$(r_k^2 + r_{k-q}^2)/(2F_\pi)$

NJL needs regularization, here Pauli-Villars subtraction
All approaches satisfy chiral symmetry constraints, WT identities

[monopole form factors: Praszałowicz, Rostworowski, Bzdak]

Results



These results are at some low quark-model scale Q_0

QCD evolution

The LO evolved distribution amplitudes read (Efremov-Radyushkin, Brodsky-Lepage, Mueller 95)

$$\phi^i(x,Q^2) = \phi_{\rm as}(x) \sum_{n=0,2,4,\ldots}^{\infty} C_n^{3/2}(2x-1) a_n(Q^2),$$

$\phi_{\rm as}(x) = 6x(1-x)$, $C_n^{3/2}$ – Gegenbauer polynomials, a_n evolve with the scale:

$$a_n(Q^2) = a_n(Q_0^2) \left(\frac{\alpha(Q^2)}{\alpha(Q_0^2)} \right)^{(\gamma_n - \gamma_0)/(2\beta_0)}$$

$$a_n(Q_0^2) = \frac{2}{3} \frac{2n+3}{(n+1)(n+2)} \int_0^1 dx C_n^{3/2}(2x-1)\phi(x,Q_0^2).$$

$$\gamma_n = -\frac{8}{3}\left[3 + \frac{2}{(n+1)(n+2)} - 4\sum_{k=1}^{n+1}\frac{1}{k}\right], \quad \beta_0 = \frac{11}{3}N_c - \frac{2}{3}N_f = 9$$

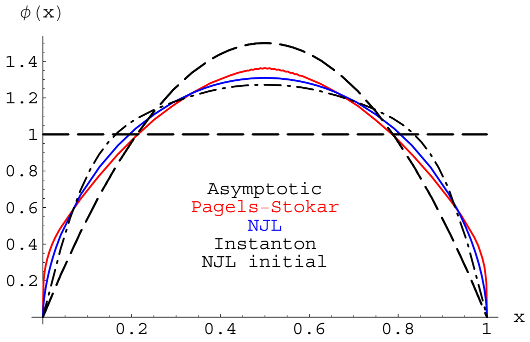
Evolved results

The analysis of Schmedding and Yakovlev, Bakulev and Mikhailov, Stefanis, of the CLEO experimental data gives $a_2(5.8\text{GeV}^2) = 0.12 \pm 0.03$ (set higher Gegenbauer coefficients to zero). Our method of determining Q_0: evolve the distribution amplitude from an arbitrary scale Q_0 to the CLEO scale $Q = 2.4$ GeV and adjust Q_0 such that $a_2 = 0.12$ is reproduced.

	Pagels-Stokar			Instanton			NJL
M_0 [GeV]	0.25	0.3	0.35	0.25	0.3	0.35	0.3
Q_0 [GeV]	0.477	0.565	0.688	0.347	0.388	0.491	0.321
a_4	0.057	0.074	0.102	0.211	0.010	-0.004	0.044
a_6	0.033	0.046	0.062	0.002	-0.006	-0.017	0.023
$\sum_{n=2,4,\ldots} a_n$	0.338	0.475	0.632	0.139	0.123	0.099	0.250

Evolved DA of the pion

Pion DA evolved to the CLEO scale from Q_0 specific to the given model



Pion-photon transition form factor

Pion-photon transition form factor

$$\Gamma^{\mu\nu}_{\pi^0\gamma^*\gamma^*}(q_1,q_2) = \epsilon_{\mu\nu\alpha\beta} e_1^\mu e_2^\nu q_1^\alpha q_2^\beta F_{\pi\gamma^*\gamma^*}(Q^2,A),$$

wrere

$$Q^2 = -(q_1^2+q_2^2),\ A = \frac{q_1^2-q_2^2}{q_1^2+q_2^2},\ -1 \le A \le 1.$$

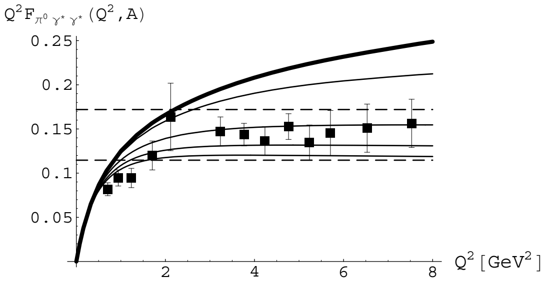
For large virtualities one finds the standard twist decomposition of the pion transition form factor (Brodsky & Lepage, 1980),

$$F_{\pi^0\gamma^*\gamma^*}(Q^2,A) = J^{(2)}(A)\frac{1}{Q^2} + J^{(4)}(A)\frac{1}{Q^4} + \ldots,$$

with

$$J^{(2)}(A) = \frac{4f_\pi}{N_c}\int_0^1 dx \frac{\phi(x)}{1-(2x-1)^2A^2}$$

Comparison to CLEO



The pion-photon transition form factor in a large-N_c Regge model. Solid lines from top to bottom: $|A| = 1$, 0.95, 0.75, 0.5, and 0. The dashed lines indicate the Brodsky-Lepage limit: $2f_\pi$ for $|A| = 1$ and $4f_\pi/3$ for $|A| = 0$. The CLEO points are for $A = 1$

Quark models overshoot the BL limit by 10-25% (LO, leading twist, large-N_c, ...)

Pion light-cone wave function

At the quark-model scale Q_0 (in the chiral limit) we find, leaving k_T unintegrated,

NJL:

$$\Psi(x,k_T) = \frac{4N_c M^2}{f_\pi^2} \sum_j c_j \frac{1}{k_T^2 + \Lambda_j^2 + M^2} \sim (\text{two subtractions}) \sim \frac{1}{k_T^6}$$

$$\langle k_T^2 \rangle = -\frac{M\langle \bar{q}q \rangle}{f_\pi^2} \sim (600\ \text{MeV})^2$$

SQM:

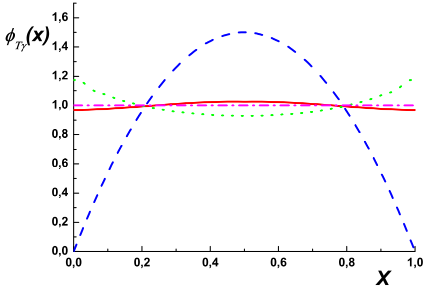
$$\Psi(x,k_T) = \frac{3m_\rho^3}{16\pi(k_T^2+m_\rho^2)^{5/2}}, \quad \langle k_T^2 \rangle = \frac{m_\rho^2}{2} = (540\ \text{MeV})^2$$

Analogous analysis for the photon, both real and virtual (ρ-meson) [with ERA and A. E. Dorokhov] basic framework: Braun, Filianov, Ball. Example:

$$\langle 0|\overline{q}(z)\sigma_{\mu\nu}q(-z)|\gamma^{\lambda}(q)\rangle =$$

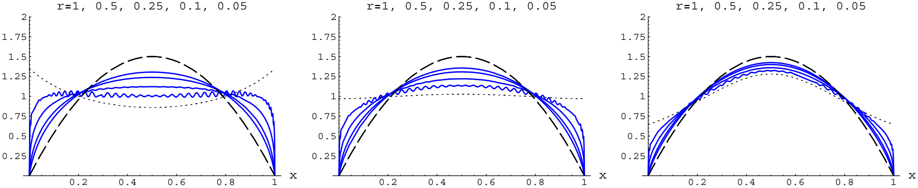
$$ie_q\langle\bar{q}q\rangle\chi_{\mathrm{m}}f^{t}_{\perp\gamma}\left(q^2\right)\left(e^{(\lambda)}_{\perp\mu}q_\nu - e^{(\lambda)}_{\perp\nu}q_\mu\right)\int_0^1 dx e^{i(2x-1)q\cdot z}\phi_{\perp\gamma}(x,q^2) +$$

$$+\text{higher twists}$$



Leading-twist photon DA in the tensor channel, $\phi_{\perp\gamma}(x, q^2 = 0)$, evaluated in the instanton model – solid line, in the NJL model $(= 1)$ – dot dashed line, its asymptotic form – dashed line, and the result of the local appr. to the instanton model – dotted line

Evolution of the photon DA



The LO ERBL evolution of $\phi^{(t)}_{\perp\gamma}(x, q^2)$ in the instanton model. Left: $q^2 = 0.25$ GeV^2, middle: $q^2 = 0$, right: $q^2 = -0.09$ GeV^2. The dashed lines: asymptotic DA, $6x(1-x)$. Initial conditions, indicated by dotted lines, are evaluated at the initial scale $Q_0^{\rm inst} = 530$ MeV. The solid lines correspond to evolved DA'a at scales $Q = 1$, 2.4, 10, and 1000 GeV. The corresponding values of the evolution ratio $r = \alpha(Q^2)/\alpha(Q_0^2)$ are given in the figures

Definition of GPD

Generalized Parton Distributions

The two isospin projections of the twist-2 GPD of the pion are defined as

$$\delta_{ab}\,\mathcal{H}^{I=0}(x,\zeta,t) = \int \frac{dz^-}{4\pi} e^{ixp^+z^-} \langle \pi^b(p+q)|\bar{\psi}(0)\gamma\cdot n\psi(z)|\pi^a(p)\rangle\Big|_{z^+=0,z^\perp=0}$$

$$i\epsilon_{3ab}\,\mathcal{H}^{I=1}(x,\zeta,t) = \int \frac{dz^-}{4\pi} e^{ixp^+z^-} \langle \pi^b(p+q)|\bar{\psi}(0)\gamma\cdot n\psi(z)\,\tau_3|\pi^a(p)\rangle\Big|_{z^+=0,z^\perp=0}$$

where $p^2 = m_\pi^2$, $q^2 = -2p\cdot q = t$, $n^2 = 0$, $p\cdot n = 1$, $q\cdot n = -\zeta$

Some background

- K. Goeke, M. V. Polyakov, and M. Vanderhaeghen, Prog. Part. Nucl. Phys. 47 (2001) 401, hep-ph/0106012
- M. Diehl, Phys. Rept. 388 (2003) 41, hep-ph/0307382
- A. V. Belitsky, A. V. Radushkin, Phys.Rept.418(2005)1, hep-ph/0504030

GPD's provide more detailed information of the structure of hadrons, three-dimensional picture instead of one-dimensional projection of the usual PD. Information on GPD's may come from such processes as $ep \to ep\gamma$, $\gamma p \to pl^+l^-$, $ep \to epl^+l^-$, or from lattices. Small cross sections of exclusive processes require very high accuracy experiments. First results are for the nucleon coming from HERMES and CLAS, also COMPASS, H1, ZEUS

Formal features

In the *symmetric* notation one introduces $\xi = \frac{\zeta}{2-\zeta}$, $X = \frac{x-\zeta/2}{1-\zeta/2}$, where $0 \le \xi \le 1$ and $-1 \le X \le 1$. Then

$$H^{I=0,1}(X,\xi,t) = \mathcal{H}^{I=0,1}\left(\frac{\xi+X}{\xi+1}, \frac{2\xi}{\xi+1}, t\right)$$

with the symmetry properties

$$H^{I=0}(X,\xi,t) = -H^{I=0}(-X,\xi,t),\ H^{I=1}(X,\xi,t) = H^{I=1}(-X,\xi,t).$$

The following sum rules hold:

$$\int_{-1}^{1} dX\, H^{I=1}(X,\xi,t) = 2F_V(t),$$

$$\int_{-1}^{1} dX\, X\, H^{I=0}(X,\xi,t) = \theta_2(t) - \xi^2\theta_1(t),$$

where $F_V(t)$ is the electromagnetic form factor, while in large-N_c quark models $\theta_1(t)$ and $\theta_2(t)$ are the gravitational form factors of the pion.

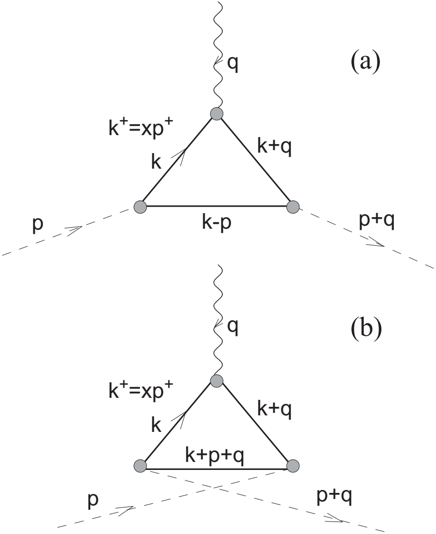
The above sum rules express the electric charge conservation and the momentum sum rule in deep inelastic scattering.
Finally, for $X \geq 0$ we have $\mathcal{H}^{I=0,1}(X,0,0) = q(X)$, which relates the GPD to PDF, $q(X)$.
The *polynomiality* conditions state that

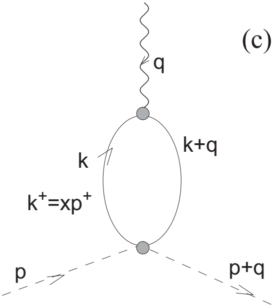
$$\int_{-1}^{1} dX\, X^{2j}\, H^{I=1}(X,\xi,t) = \sum_{i=0}^{j} A_i^{(j)}(t)\xi^{2i},$$

$$\int_{-1}^{1} dX\, X^{2j+1}\, H^{I=0}(X,\xi,t) = \sum_{i=0}^{j+1} B_i^{(j)}(t)\xi^{2i},$$

where $A_i^{(j)}(t)$ and $A_i^{(j)}(t)$ are the coefficient functions (form factors) depending on j and i. The conditions follow from the Lorentz invariance, time reversal, and hermiticity, hence are satisfied in approaches that obey these requirements

Quark-model evaluation





Direct (a), crossed (b), and contact (c) contribution to the GPD of the pion. Diagram (c) is responsible for the so-called D-term

PDF

In the special case of $\zeta = t = 0$ GPD becomes the PDF. The NJL result is (Davidson & Arriola, 1995)

$$q(x) = 1$$

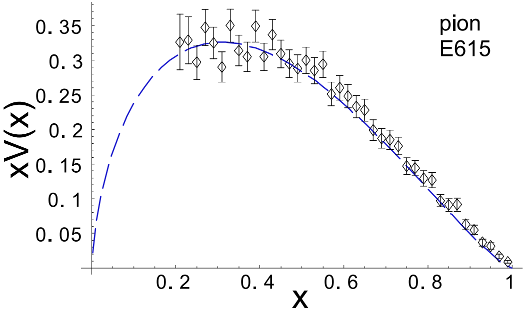
LO DGLAP QCD evolution

PDF

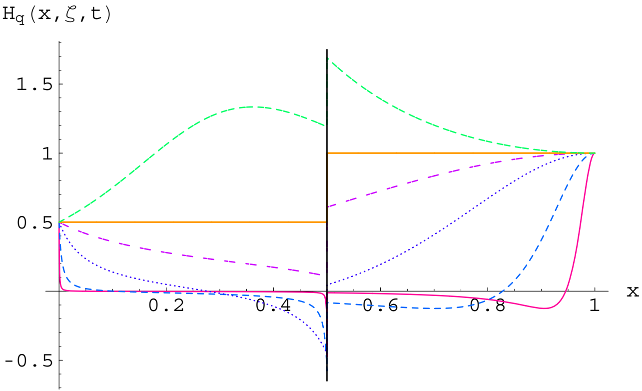
In the special case of $\zeta = t = 0$ GPD becomes the PDF. The NJL result is (Davidson & Arriola, 1995)

$$q(x) = 1$$

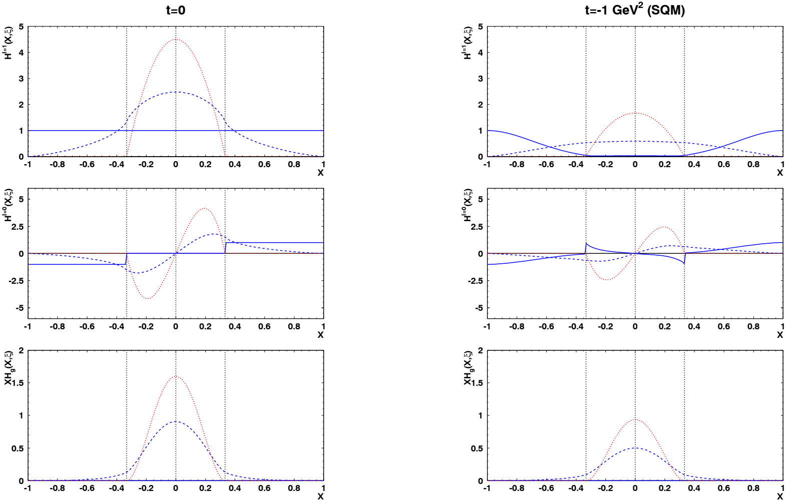
LO DGLAP QCD evolution to the scale $Q^2 = (4\ \mathrm{GeV})^2$ of the E615 Fermilab experiment:



GPD in the spectral quark model



The SQM results for the quark GPD of the pion, $\mathcal{H}_q$, for $\zeta = 1/2$ and $t = 0.2, 0, -0.2, -1, -10, -100\ \mathrm{GeV}^2$, from top to bottom (at $x = 0.9$).
Similar results by the Valencia group



LO QCD evolution for $\xi = 1/3$. Solid - initial condition, dashed - evolved to $Q^2 = (4\mathrm{GeV})^2$, dotted - asymptotic form

GPD and lattices

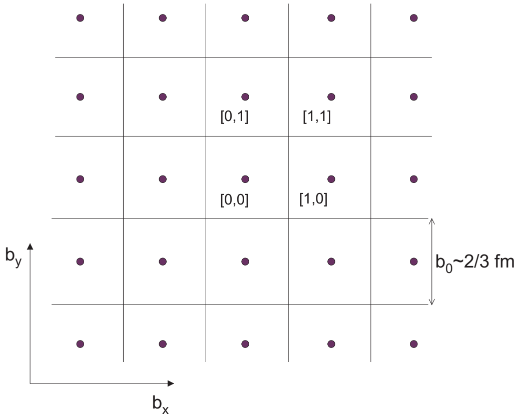
$$H_{\mathrm{SQM}}(x,0,t) = \frac{m_\rho^2(m_\rho^2+(1-x)^2t)}{(m_\rho^2-(1-x)^2t)^2}\theta(x)\theta(1-x)$$

$$F(t) = \int_0^1 dx H_{\mathrm{SQM}}(x,0,t) = \frac{m_\rho^2}{m_\rho^2+t}$$

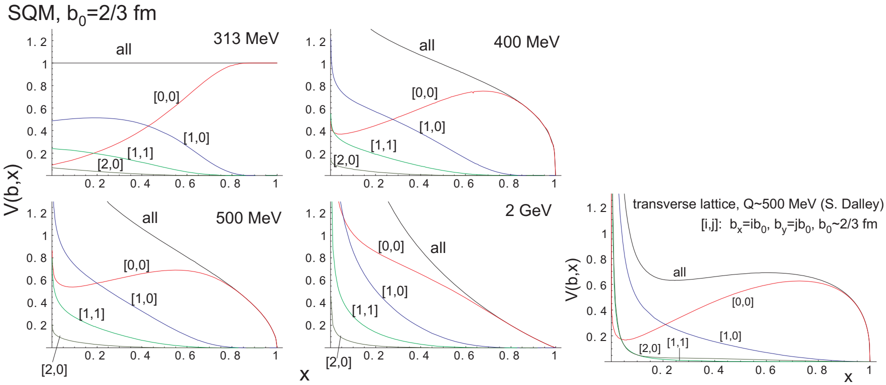
which shows the built-in vector-meson dominance in the model. We pass to the impact-parameter space by the Fourier-Bessel transformation and get

$$q_{\mathrm{SQM}}(b,x) = \frac{m_\rho^2}{2\pi(1-x)^2}\left[K_0\left(\frac{bm_\rho}{1-x}\right) - \frac{bm_\rho}{1-x}K_1\left(\frac{bm_\rho}{1-x}\right)\right]$$

Lattice



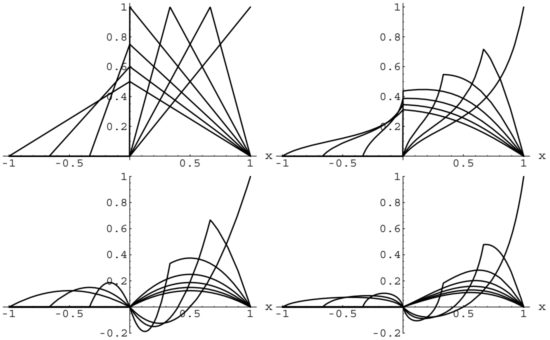
labeling of lattice plaquettes



qualitative agreement

Pion-photon TDA

[Pire and Szymanowski](as GPD, but between the π and γ states)



Top: vector TDA for $t = 0$ (left) and $t = -0.4$ GeV (right) several values of ζ: -1, $-2/3$, $-1/3$, 0, $1/3$, $2/3$, and 1. Bottom: the same for the axial TDA, SQM atthe scale Q_0

Summary

- Soft hadronic matrix element can be straightforwardly evaluated at the one-quark-loop level (large N_c) at the quark-model scale Q_0
- DA, light-cone wave functions, GPD, PDF, TDA, GDA (not discussed), ...
- QCD evolution is a necessary component
- The quark-model scale Q_0 is low, $300 - 500$ MeV, depending on the particular model
- Some (not large) differences between evolved results of variants of chiral quark models models, overall agreement with available data very reasonable
- Nucleon: more challenging but much more data (Bochum, Tübingen)