Chiral quark models in high-energy processes

Wojciech Broniowski
with E. Ruiz Arriola, A. E. Dorokhov, and K. Golec-Biernat

Theory Division seminar, IFJ PAN, 29.11.07

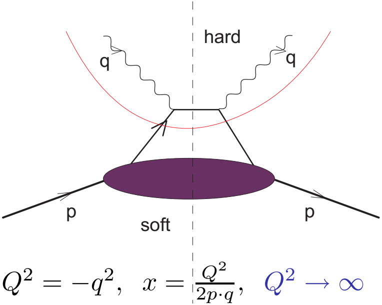
- Pion light cone wave function and pion distribution amplitude in the NJL model, Phys.Rev.D66:094016,2002, hep-ph/0207266
- Spectral quark model and low-energy hadron phenomenology, Phys.Rev.D67:074021,2003, hep-ph/0301202
- Impact parameter dependence of the GPD of the pion in chiral quark models, Phys.Lett.B574:57-64,2003, hep-ph/0307198
- Application of chiral quark models to high-energy processes, *Bled 2004, Quark dynamics* 7-10, hep-ph/0410041
- Pion transition form factor and distribution amplitudes in large-N_c Regge models, Phys.Rev.D74:034008,2006, hep-ph/0605318
- Photon DA's and light-cone wave functions in chiral quark models, **EAD+**, Phys.Rev.D74:054023,2006, hep-ph/0607171
- Pion-photon Transition Distribution Amplitudes in the Spectral Quark Model, Phys.Lett.B649:49,2007, hep-ph/0701243

– numerous references to the field

"Low energy meets high energy"

- Low-energy quark models are used to compute low-energy **matrix elements** of hadronic operators (for pions and photons)
- **Matching to QCD** at the low quark-model scale
- **QCD evolution** to experimental scales
- Comparison to (indirect) data (DA, PDF) sets the matching scale
- **Approximations**: leading twist (known QCD evolution), large number of colors (= one quark loop), LO evolution, chiral limit

Deep Inelastic Scattering – Parton Distribution Function



$Q^2 = -q^2, \;\; x = \frac{Q^2}{2p \cdot q}, \;\; Q^2 \to \infty$

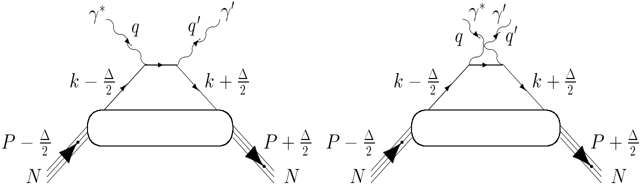
Factorization of soft and hard processes, Wilson's OPE, twist expansion

$$\langle J(q)J(-q)\rangle = \sum_i C_i(Q^2;\mu)\langle \mathcal{O}_i(\mu)\rangle, \;\; F(x,Q) = F_0(x,\alpha(Q)) + \frac{F_2(x,\alpha(Q))}{Q^2} + \ldots$$

The soft matrix element can be computed in low-energy models!

$$F_i(x,\alpha(Q_0))|_{\text{model}} = F_i(x,\alpha(Q_0))|_{\text{QCD}}, \quad Q_0 - \textbf{the matching scale}$$

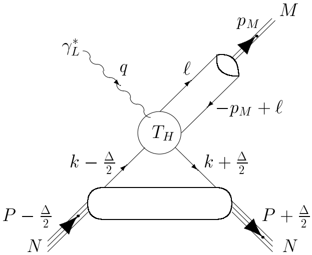
Exclusive processes in QCD



(a) (b)

Deeply Virtual Compton Scattering

non-zero momentum transfer to the target, at least one photon virtual



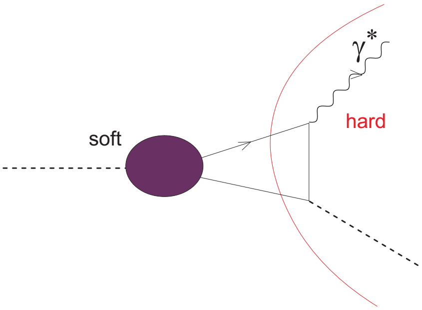
Hard Meson Production

Dictionary of matrix elements

General structure of the soft matrix elements: $\langle A \mid \mathcal{O} \mid B \rangle$

- A = B = one-particle state – Parton Distribution of A (inclusive DIS)
- A = one-particle state, B = vacuum – distribution amplitude (DA) of A (hadronic form factors, HMP)
- A, B = one-particle state of different momentum – GPD (exclusive DIS, DVCS, HMP)
- A = many-particle state, B = vacuum – GDA (transition form factors)
- A $\neq$ B (A, B – different hadronic states) – Transition Distribution Amplitude ($h\bar{h} \to \gamma\gamma^*$, Pire & Szymanowski 2004)
- ...

Pion Distribution Amplitude



Definition (for π^+, leading twist):

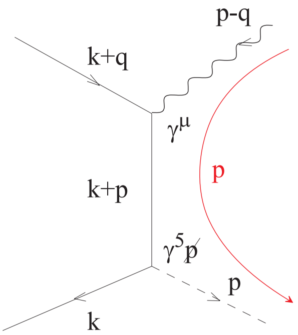
$$\langle 0|\overline{d}(z)\gamma_\mu\gamma_5[z,-z]u(-z)|\pi^+(q)\rangle = i\sqrt{2}f_\pi(q^2)q_\mu\int_0^1 dx e^{i(2x-1)q\cdot z}\phi(x)$$

z is along the light cone, $z^2=0$, $f_\pi(m_\pi^2)=93$ MeV – pion decay constant (in the usual light-cone gauge $[z,-z]=1$)

Normalization $\int_0^1 dx\phi(x)=1$, since $\langle 0|A_\mu^-(0)|\pi^+(q)\rangle = i f_\pi(q^2)q_\mu$

Leading-twist structure

A sample calculation of the leading-twist Dirac structure



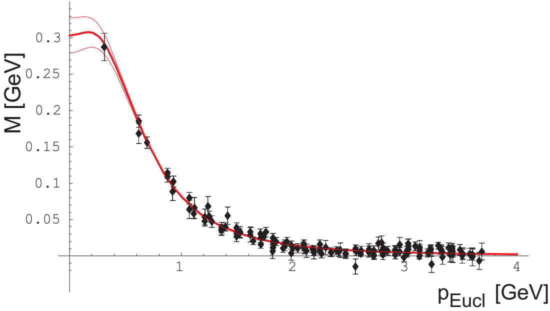
p – hard momentum

$$\gamma_5 \not{p} \frac{1}{\not{k} + \not{p} - m} \gamma^\mu \simeq \gamma_5 \gamma^\mu + \text{higher twists}$$

(crossed diagram similar)

Chiral quark models

Spontaneous chiral symmetry breaking → quark mass $M(0) \sim 300$ MeV



NJL, instanton liquid, lattices, ...$S(p) = \frac{Z(p)}{\not p - M(p)}$

(construction of interaction vertices subtle when $M = M(p^2)$)

Chiral quark models 2

Model	mass	vertex $G_{\pi qq}$
Nambu-Jona-Lasinio	M=const	$i\gamma_5/F_\pi$
instanton-liquid model	$M(p^2) = M_0 r_p^2$	$i\gamma_5 r_k r_{k-q}/F_\pi$
Pagels-Stokar model	$M(p^2) = M_0 r_p^2$	$i\gamma_5(r_k^2 + r_{k-q}^2)/(2F_\pi)$

NJL needs regularization, here Pauli-Villars subtraction
All approaches satisfy chiral symmetry constraints, WT identities

[monopole form of r_p: Praszałowicz, Rostworowski, Bzdak]

QM evaluation of DA

1. Invert the definition:

$$\phi(x) = -\frac{i}{\sqrt{2} f_\pi(q^2)} \int_{-\infty}^{\infty} \frac{d\tau}{\pi} e^{-i(2x-1)\tau n\cdot q} \langle 0|\bar{d}(\tau n) \not{n}\gamma_5 u(-\tau n)|\pi^+(q)\rangle$$

where $n^\mu = (n^0, n^1, n^2, n^3) = (1, 0, 0, -1)$, $n \cdot a = a^+$.

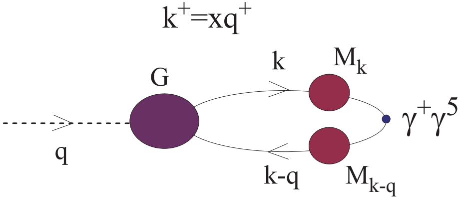
2. $\langle 0|\overline{d}(\tau n)\Gamma u(-\tau n)|\pi^+(q)\rangle =$ (LSZ reduction, one-loop QM)

$$= -\frac{N_c\sqrt{2}}{F_\pi} \int \frac{d^4k}{(2\pi)^4} e^{i\tau n\cdot(2k-q)} G_{\pi qq}(k, k-q) \mathrm{Tr}[\Gamma S_k \gamma_5 S_{k-q}]$$

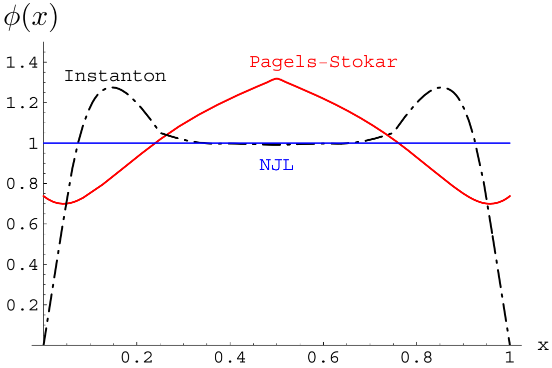
1+2, evaluation of the trace → "generic" QM expression:

$$\phi(x) = -\frac{4iN_c}{f_\pi(q^2)} \int \frac{d^4k}{(2\pi)^4} \delta(k^+ - xq^+) G_{\pi qq}(k, k-q) \frac{(M_{k-q} - M_k)k \cdot n + M_k q \cdot n}{D_k D_{k-q}}$$

M_p – momentum-dependent *constituent* quark mass,
$D_p = p^2 - M_p^2 + i0$
One-loop diagram (leading $1/N_c$) with constrained integration



Results



These results are at some low quark-model scale Q_0

QCD evolution

The LO evolved distribution amplitudes read (Efremov-Radyushkin, Brodsky-Lepage, Mueller 95)

$$\phi^i(x,Q^2) = \phi_{\rm as}(x) \sum_{n=0,2,4,\ldots}^{\infty} C_n^{3/2}(2x-1) a_n(Q^2),$$

$\phi_{\rm as}(x) = 6x(1-x)$, $C_n^{3/2}$ – Gegenbauer polynomials, a_n evolve with the scale:

$$a_n(Q^2) = a_n(Q_0^2) \left(\frac{\alpha(Q^2)}{\alpha(Q_0^2)} \right)^{(\gamma_n - \gamma_0)/(2\beta_0)}$$

$$a_n(Q_0^2) = \frac{2}{3} \frac{2n+3}{(n+1)(n+2)} \int_0^1 dx C_n^{3/2}(2x-1) \phi(x,Q_0^2).$$

$$\gamma_n = -\frac{8}{3}\left[3 + \frac{2}{(n+1)(n+2)} - 4 \sum_{k=1}^{n+1} \frac{1}{k} \right], \quad \beta_0 = \frac{11}{3} N_c - \frac{2}{3} N_f = 9$$

Evolved results

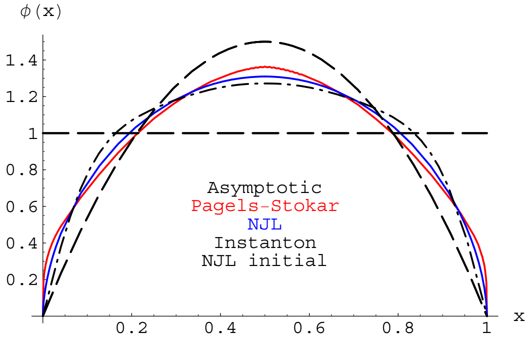
The analysis of Schmedding, Yakovlev, Bakulev, Mikhailov, Stefanis, of the CLEO experimental data gives $a_2(5.8\mathrm{GeV}^2) = 0.12 \pm 0.03$. Our method of determining Q_0: evolve the distribution amplitude from an arbitrary scale Q_0 to the CLEO scale $Q = 2.4$ GeV and adjust Q_0 such that $a_2 = 0.12$

	Pagels-Stokar	Instanton	NJL/SQM
Q_0 [GeV]	0.5	0.39	0.32
a_4	0.074	0.010	0.044
a_6	0.046	-0.006	0.023
$\sum_{n=2,4,\ldots} a_n$	0.475	0.123	0.250

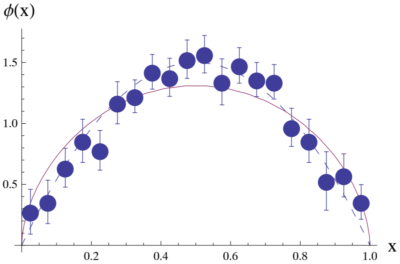
[momentum-fraction analysis gives very similar value of Q_0]

Evolved DA of the pion

Pion DA evolved to the scale $Q = 2.4$ GeV from Q_0 specific to the given model



Comparison to experimental and lattice data



E791 data from di-jet production in $\pi + A$
lines: NJL at $Q = 2$ GeV and the asymptotic form

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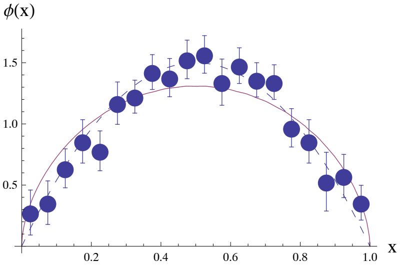

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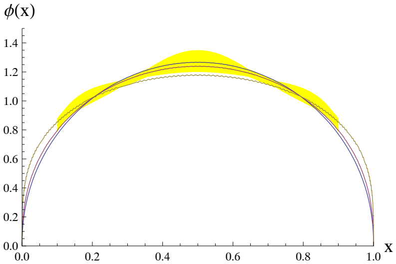

band: transverse lattice data
lines: QM at $Q = 0.5$, 0.75, 1 GeV

Definition of GPD

Generalized Parton Distributions

The two isospin projections of the twist-2 GPD of the pion are defined as

$$\delta_{ab}\,\mathcal{H}^{I=0}(x,\zeta,t)=\int\frac{dz^-}{4\pi}e^{ixp^+z^-}\,\langle\pi^b(p+q)|\bar{\psi}(0)\gamma\cdot n\psi(z)|\pi^a(p)\rangle\big|_{z^+=0,z^\perp=0}$$

$$i\epsilon_{3ab}\,\mathcal{H}^{I=1}(x,\zeta,t)=\int\frac{dz^-}{4\pi}e^{ixp^+z^-}\langle\pi^b(p+q)|\bar{\psi}(0)\gamma\cdot n\psi(z)\,\tau_3|\pi^a(p)\rangle\big|_{z^+=0,z^\perp=0}$$

where $p^2=m_\pi^2$, $q^2=-2p\cdot q=t$, $n^2=0$, $p\cdot n=1$, $q\cdot n=-\zeta$
ζ - momentum transferred along the light cone

Some background

- K. Goeke, M. V. Polyakov, and M. Vanderhaeghen, Prog. Part. Nucl. Phys. 47 (2001) 401, hep-ph/0106012
- M. Diehl, Phys. Rept. 388 (2003) 41, hep-ph/0307382
- A. V. Belitsky, A. V. Radushkin, Phys.Rept.418(2005)1, hep-ph/0504030

GPD's provide more detailed information of the structure of hadrons, three-dimensional picture instead of one-dimensional projection of the usual PD. Information on GPD's may come from such processes as $ep \to ep\gamma$, $\gamma p \to pl^+l^-$, $ep \to epl^+l^-$, or from lattices. Small cross sections of exclusive processes require very high accuracy experiments. First results are for the nucleon coming from HERMES and CLAS, also COMPASS, H1, ZEUS

Formal features

In the *symmetric* notation one introduces $\xi = \frac{\zeta}{2-\zeta}$, $X = \frac{x-\zeta/2}{1-\zeta/2}$, where $0 \leq \xi \leq 1$ and $-1 \leq X \leq 1$. Then

$$H^{I=0,1}(X,\xi,t) = \mathcal{H}^{I=0,1}\left(\frac{\xi+X}{\xi+1}, \frac{2\xi}{\xi+1}, t\right)$$

with the symmetry properties

$$H^{I=0}(X,\xi,t) = -H^{I=0}(-X,\xi,t), \; H^{I=1}(X,\xi,t) = H^{I=1}(-X,\xi,t).$$

The following **sum rules** hold:

$$\forall\xi: \quad \int_{-1}^{1} dX\, H^{I=1}(X,\xi,t) = 2F_V(t),$$

$$\forall\xi: \quad \int_{-1}^{1} dX\, X\, H^{I=0}(X,\xi,t) = \theta_2(t) - \xi^2\theta_1(t),$$

where $F_V(t)$ is the electromagnetic form factor, while in large-N_c quark models $\theta_1(t)$ and $\theta_2(t)$ are the gravitational **form factors** of the pion.

The above sum rules express the electric charge conservation and the momentum sum rule in deep inelastic scattering.
For $X \geq 0$ we have $\mathcal{H}^{I=0,1}(X,0,0) = q(X)$, which relates the GPD to **PDF**, $q(X)$.
The **polynomiality** conditions state that

$$\int_{-1}^{1} dX\, X^{2j}\, H^{I=1}(X,\xi,t) = \sum_{i=0}^{j} A_i^{(j)}(t)\xi^{2i},$$

$$\int_{-1}^{1} dX\, X^{2j+1}\, H^{I=0}(X,\xi,t) = \sum_{i=0}^{j+1} B_i^{(j)}(t)\xi^{2i},$$

where $A_i^{(j)}(t)$ and $A_i^{(j)}(t)$ are the coefficient functions (form factors) depending on j and i. The conditions follow from the Lorentz invariance, time reversal, and hermiticity, hence are satisfied in approaches that obey these requirements

Positivity bound states that

$$|H_q(X,\xi,t)| \leq \sqrt{q(x_{\rm in})q(x_{\rm out})}, \qquad \xi \leq X \leq 1.$$

where $x_{\rm in} = (x+\xi)/(1+\xi)$, $x_{\rm out} = (x-\xi)/(1-\xi)$.
Finally, a **low-energy theorem** holds $H_{I=1}(2z-1,1,0) = \phi(z)$

The above relations and bounds form severe constraints for the form of the pion GPD

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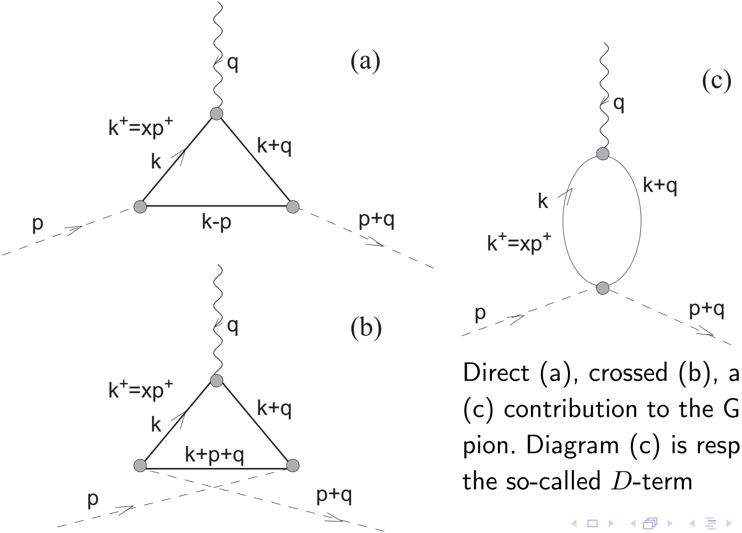
All are satisfied in our QM calculation

Quark-model evaluation



Direct (a), crossed (b), and contact (c) contribution to the GPD of the pion. Diagram (c) is responsible for the so-called D-term

PDF

In the special case of $\zeta = t = 0$ GPD becomes the PDF. The NJL result is (Davidson & Arriola, 1995)

$$q(x) = 1$$

LO DGLAP QCD evolution of the non-singlet part

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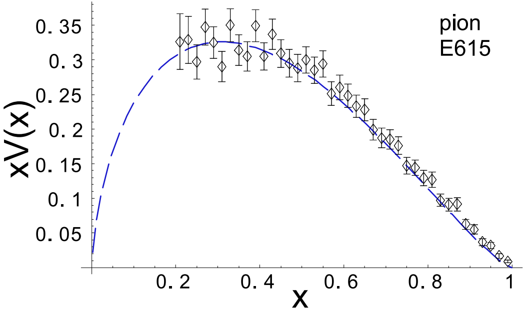
LO DGLAP QCD evolution of the non-singlet part to the scale $Q^2 = (4\ \mathrm{GeV})^2$ of the E615 Fermilab experiment:



Digression:

LO DGLAP - **the full code**

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```

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5. int[x_,r_,Q_]=2(I Exp[I Pi/4]/(2Pi I)x^(-z-1) mu[z]*
   (a[Q]/a[.313])^(Gamma0[z]/(2 beta0)) /. z-> 0.5+I Exp[I Pi/4]r)

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6. V[x_,Q_]:=NIntegrate[Re[int[x,r,Q]] ,{r,0,-100/Log[x]}]

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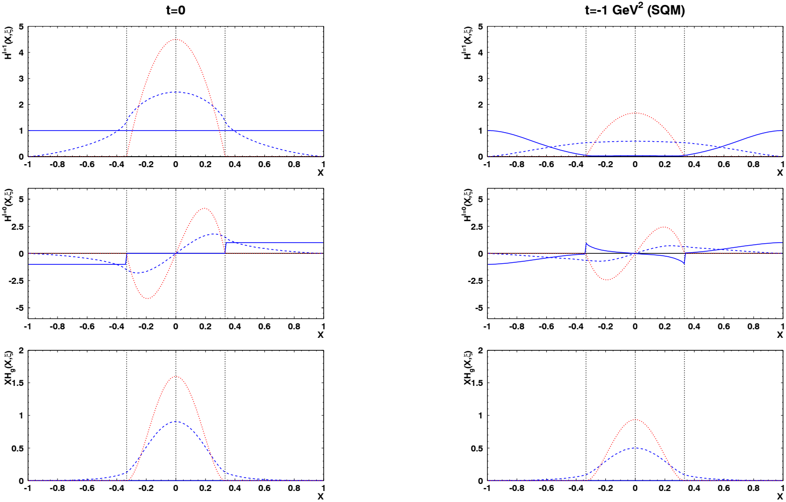
For the Kwieciński evolution (one-loop CCFM in the b-representation) the same method, only the anomalous dimensions depend on b) [WB+ERA, PRD70 (2004) 034012] – many formal features shown

GPD in chiral quark models

[with K. Golec-Biernat]

GPD in chiral quark models

[with K. Golec-Biernat]
analytic formulas derived for NJL and SQM, all formal properties satisfied, formulas fit in two lines, no factorization of the t-dependence, *etc.*



LO QCD evolution for SQM with $\xi = 1/3$. Solid - initial condition, dashed - evolved to $Q^2 = (4\mathrm{GeV})^2$, dotted - asymptotic form

GPD and lattices

[WB+ERA'03]

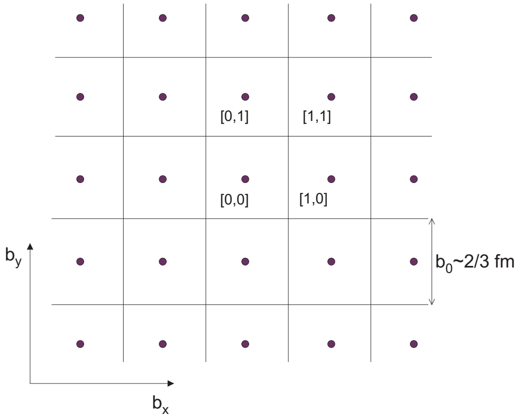
$$H_{\mathrm{SQM}}(x,0,t)=\frac{m_\rho^2(m_\rho^2+(1-x)^2t)}{(m_\rho^2-(1-x)^2t)^2}\theta(x)\theta(1-x)$$

$$F(t)=\int_0^1 dx H_{\mathrm{SQM}}(x,0,t)=\frac{m_\rho^2}{m_\rho^2+t}$$

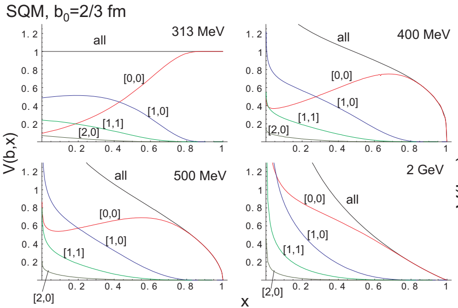
which shows the built-in vector-meson dominance in the model. We pass to the impact-parameter space by the Fourier-Bessel transformation and get

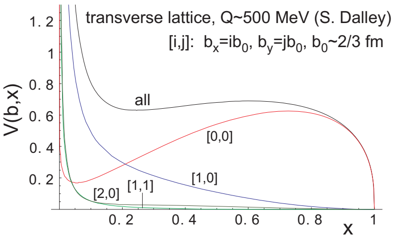
$$q_{\mathrm{SQM}}(b,x)=\frac{m_\rho^2}{2\pi(1-x)^2}\left[K_0\left(\frac{bm_\rho}{1-x}\right)-\frac{bm_\rho}{1-x}K_1\left(\frac{bm_\rho}{1-x}\right)\right]$$

Lattice



labeling of lattice plaquettes





model

lattice data

qualitative agreement

Summary

- Soft hadronic matrix elements of quark bilinears (**input for the evolution**) can be straightforwardly evaluated for pions and photons at the one-quark-loop level (large N_c, leading teist) at the **quark-model scale** Q_0. All formal features satisfied
- DA, light-cone wave functions, GPD, PDF, GDA, TDA [Pire+Szymanowski]...
- **QCD evolution necessary**, it allows to determine Q_0
- The quark-model scale Q_0 is very low or low, $300 - 500$ MeV, depending on the particular model
- Some differences between evolved results of variants of chiral quark models models, overall agreement with (scarce) available data very **reasonable**, despite the low value of Q_0
- **Nucleon**: much more challenging (Bochum, Tübingen) but more rewarding (data!)

Pion-photon transition form factor

Pion-photon transition form factor

$$\Gamma^{\mu\nu}_{\pi^0\gamma^*\gamma^*}(q_1,q_2) = \epsilon_{\mu\nu\alpha\beta} e_1^\mu e_2^\nu q_1^\alpha q_2^\beta F_{\pi\gamma^*\gamma^*}(Q^2,A),$$

wrere

$$Q^2 = -(q_1^2+q_2^2),\ A = \frac{q_1^2-q_2^2}{q_1^2+q_2^2},\ -1 \le A \le 1.$$

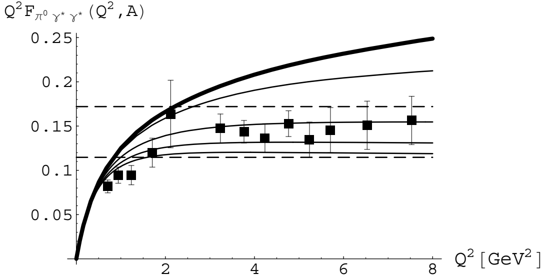
For large virtualities one finds the standard twist decomposition of the pion transition form factor (Brodsky & Lepage, 1980),

$$F_{\pi^0\gamma^*\gamma^*}(Q^2,A) = J^{(2)}(A)\frac{1}{Q^2} + J^{(4)}(A)\frac{1}{Q^4} + \dots,$$

with

$$J^{(2)}(A) = \frac{4f_\pi}{N_c}\int_0^1 dx \frac{\phi(x)}{1-(2x-1)^2A^2}$$

Comparison to CLEO



The pion-photon transition form factor in a large-N_c Regge model. Solid lines from top to bottom: $|A| = 1$, 0.95, 0.75, 0.5, and 0. The dashed lines indicate the Regge model calculations at various values of A

The Brodsky-Lepage limit for $J^{(2)}$ is obtained with the asymptotic DA $6x(1-x)$ and equals $2f_\pi$ for $|A| = 1$.

The CLEO experimental points are somewhat below this limit.

Pion light-cone wave function

At the quark-model scale Q_0 (in the chiral limit) we find, leaving k_T unintegrated,
NJL:

$$\Psi(x, k_T) = \frac{4N_c M^2}{f_\pi^2} \sum_j c_j \frac{1}{k_T^2 + \Lambda_j^2 + M^2} \sim (\text{two subtractions}) \sim \frac{1}{k_T^6}$$

$$\langle k_T^2 \rangle = -\frac{M \langle \bar{q}q \rangle}{f_\pi^2} \sim (600 \text{ MeV})^2$$

SQM:

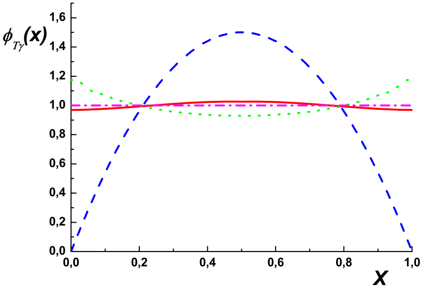
$$\Psi(x, k_T) = \frac{3m_\rho^3}{16\pi (k_T^2 + m_\rho^2)^{5/2}}, \quad \langle k_T^2 \rangle = \frac{m_\rho^2}{2} = (540 \text{ MeV})^2$$

Analogous analysis for the photon, both real and virtual (ρ-meson) [with ERA and A. E. Dorokhov] basic framework: Braun, Filianov, Ball. Example:

$$\langle 0|\overline{q}(z)\sigma_{\mu\nu}q(-z)|\gamma^{\lambda}(q)\rangle =$$

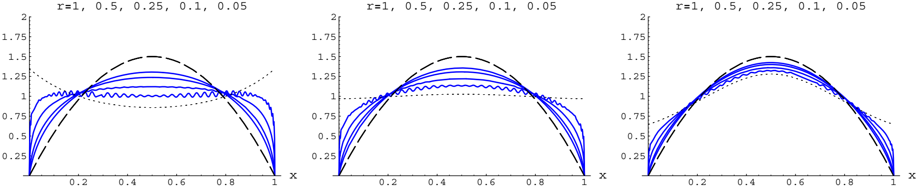
$$ie_q\langle\bar{q}q\rangle\chi_{\rm m}f^{t}_{\perp\gamma}\left(q^2\right)\left(e^{(\lambda)}_{\perp\mu}q_\nu - e^{(\lambda)}_{\perp\nu}q_\mu\right)\int_0^1 dx e^{i(2x-1)q\cdot z}\phi_{\perp\gamma}(x,q^2)+$$

$$+\text{higher twists}$$



Leading-twist photon DA in the tensor channel, $\phi_{\perp\gamma}(x,q^2=0)$, evaluated in the instanton model – solid line, in the NJL model $(=1)$ – dot dashed line, its asymptotic form – dashed line, and the result of the local appr. to the instanton model – dotted line

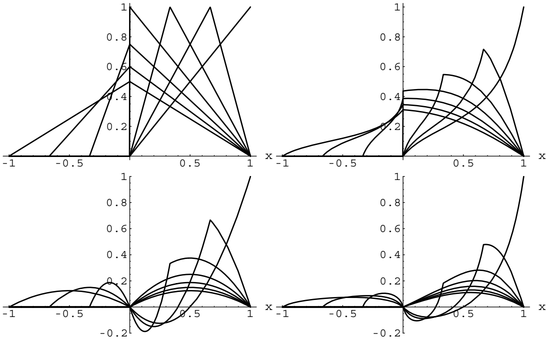
Evolution of the photon DA



The LO ERBL evolution of $\phi^{(t)}_{\perp\gamma}(x, q^2)$ in the instanton model. Left: $q^2 = 0.25$ GeV^2, middle: $q^2 = 0$, right: $q^2 = -0.09$ GeV^2. The dashed lines: asymptotic DA, $6x(1-x)$. Initial conditions, indicated by dotted lines, are evaluated at the initial scale $Q_0^{\rm inst} = 530$ MeV. The solid lines correspond to evolved DA'a at scales $Q = 1$, 2.4, 10, and 1000 GeV. The corresponding values of the evolution ratio $r = \alpha(Q^2)/\alpha(Q_0^2)$ are given in the figures

Pion-photon TDA

[Pire and Szymanowski](as GPD, but between the π and γ states)



Top: vector TDA for $t = 0$ (left) and $t = -0.4$ GeV (right) several values of ζ: -1, $-2/3$, $-1/3$, 0, $1/3$, $2/3$, and 1. Bottom: the same for the axial TDA, SQM atthe scale Q_0