Phenomenology of ultrarelativistic nuclear collisions II

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Six e-lectures for PhD students

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The pdf file with the covered material is accessible via Dropbox

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Lecture 5

Correlations

Where predominantly generated?

... carry important information on the dynamics

- At the early gluonic stage?
- In hydro/rescattering phase?
- At freeze-out?
- All over?
- Are the early fluctuations destroyed?

Various types of correlations

Pearson's coefficient

Standard measure of correlations between random variables X and Y:

$$\rho(X,Y) = \frac{\mathrm{cov}(X,Y)}{\sigma(X)\sigma(Y)}$$

The mean, covariance and variance = standard deviation ^{2} are defined as

$$\bar{z} = \frac{1}{n}\sum_{i=1}^{n} z_i$$

$$\mathrm{cov}(X,Y) = \frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y}),$$

$$\mathrm{var}(Z) = \sigma^2(Z) = \frac{1}{n}\sum_{i=1}^{n}(z_i - \bar{z})^2$$

Homework

Show that $-1 \leq \rho(X,Y) \leq 1$.

["visualization", covariance matrix, principal components - on tablet]

Link of correlations to fluctuations (example)

Consider a distribution of (identical) point-like sources in the transverse plane in a collision

$$d(x) = \sum_{i=1}^{n} \delta(x - x_i), \quad x_i - \text{2D positions of sources}$$

Normalization in the event: $\int dx\, d(x) = n$, notation for average over events: $\langle . \rangle$
Statistical distribution of n points is given by $f_n(x_1, ..., x_n)$. The marginal distributions are defined as

$$\begin{aligned} f_n^{(k)}(x_1, .., x_k) &= \int dx_{k+1} ... dx_n f_n(x_1, ..., x_n), \\ f_n^{(2)}(x_1, x_2) &= \int dx_3 ... dx_n f_n(x_1, ..., x_n), \\ f_n^{(1)}(x_1) &= \int dx_2 ... dx_n f_n(x_1, ..., x_n) = \int dx_2 f_n^{(2)}(x_1, x_2) \end{aligned}$$

Link of correlations to fluctuations, cont.

$$\langle d(x) \rangle = \langle \int dx_1 ... dx_n f_n(x_1, ..., x_n) \sum_i \delta(x - x_i) = \langle n f_n^{(1)}(x) \rangle$$

$$\langle d(x) d(y) \rangle = \langle \int dx_1 ... dx_n f_n(x_1, ..., x_n) \left(\sum_{i=j} + \sum_{i \neq j} \right) \delta(x - x_i) \delta(y - x_j) \rangle$$

$$= \langle n f_n^{(1)}(x) \rangle \delta(x - y) + \langle n(n-1) f_n^{(2)}(x, y) \rangle$$

autocorrelations + genuine correlations

$$C(x, y) \equiv \langle d(x) d(y) \rangle - \langle d(x) \rangle \langle d(y) \rangle$$

$$\int dx \, dy \, C(x, y) = \langle n^2 \rangle - \langle n \rangle^2 \qquad \text{(etc.)}$$

Interpretation: correlations of density $\leftrightarrow$ e-by-e (event-by-event) fluctuations of multiplicity

Superposition model

[drawing on tablet]

Assume the number of hadrons n, as registered in the experiment, is composed from independent production from s sources. Let m_i is the number of hadrons produced by the ith source from some universal distribution:

$$n = \sum_{i=1}^{s} m_i$$

Then for the e-by-e averages:

$$\begin{aligned} \langle n \rangle_e &= \langle s \rangle_e \langle m \rangle \\ \mathrm{var}_e(n) &\equiv \langle n^2 \rangle_e - \langle n \rangle_e^2 = \langle s \rangle_e \mathrm{var}(m) + \langle m \rangle^2 \mathrm{var}_e(s) \end{aligned}$$

- fluctuations of m and s enter

Homework

Derive the above formulas.

Fluctuations of multiplicity

The superposition model shows that even if we somehow fixed exactly S, there would still be e-by-e fluctuations of n from the fact, that the sources emit hadrons in a statistical way.

[tablet]
Imagine we fix S by measuring the (complementary to participants) number of spectators in the forward and backward detectors. In mid-rapidity we would still find e-by-e fluctuations of n.

[More material and references, e.g., in
WB, A. Olszewski, Phys.Rev.C 95 (2017) 6, 064910, arXiv:1704.01532]

Transverse momentum fluctuations

$f_n(p_1, \ldots, p_n)$ – n-particle distribution of variables p_i (transverse momentum of particle i) *within events of multiplicity* n. The full probability distribution of obtaining event of multiplicity n with momenta $p_1, \ldots, p_n$ is $P_n \rho_n(p_1, \ldots, p_n)$. The relevant moments at multiplicity n are:

$$\begin{aligned} \overline{p_n} &= \int dp f_n^{(1)}(p) p, \\ \sigma_n^2(p) &= \int dp f_n^{(1)}(p)(p - \overline{p_n})^2, \\ \mathrm{cov}_n(p_1, p_2) &= \int dp_1 dp_2 f_n^{(2)}(p_1, p_2)(p_1 - \overline{p_n})(p_2 - \overline{p_n}) \end{aligned}$$

Now, in a typical experimental setup we are interested in broader classes, containing n in the range $n_1 \leq n \leq n_2$. We denote for brevity $\sum_n = \sum_{n=n_1}^{n_2}$.

The problem it that both p fluctuate (which is what we want study) and n fluctuates (which is a nuisance effect). We need to separate these dynamical and statistical effects.

$\sigma^2_{\rm dyn}$

Consider $M_n = (p_1 + \cdots + p_n)/n$ – the average value of p in a given event. Then

$$\overline{M} = \sum_n P_n \int dp_1 \dots dp_n \rho_n(p_1,\dots,p_n) \frac{p_1+\cdots+p_n}{n} = \sum_n P_n \overline{p_n}$$

$$\overline{M^2} = \sum_n P_n \int dp_1 \dots dp_n \rho_n(p_1,\dots,p_n) \frac{1}{n^2} \sum_{i,j=1}^{n} \left[(p_i - \overline{p_n})(p_j - \overline{p_n}) + \overline{p_n}^2\right]$$

$$\sigma_M^2 = \sum_n P_n \overline{p_n}^2 - \left(\sum_n P_n \overline{p_n}\right)^2 + \sum_n P_n \frac{\sigma_n^2(p)}{n} + \sum_n P_n \frac{1}{n^2} \sum_{i \neq j = 1}^{n} {\rm cov}_n(p_i, p_j)$$

Suppose mixing [explain] of events is performed. Then, by definition, no correlations are present, *i.e.*, ${\rm cov}_n^{\rm mix}(p_i,p_j) = 0$, and

$$\sigma_M^{2,{\rm mix}} = \sum_n P_n \overline{p_n}^2 - \left(\sum_n P_n \overline{p_n}\right)^2 + \sum_n P_n \frac{\sigma_n^2(p)}{n}$$

Define

$$\sigma^2_{\rm dyn} \equiv \sigma^2_{\rm dyn} = \sigma_M^2 - \sigma_M^{2,{\rm mix}} = \sum_n P_n \frac{1}{n^2} \sum_{i\neq j} {\rm cov}_n(p_i,p_j) \simeq \frac{1}{\overline{n}^2} \sum_{i \neq j} {\rm cov}_{\overline{n}}(p_i,p_j)$$

Zoo of other measures

$F_{p_T} \equiv (\sqrt{\omega} - \sqrt{\omega_{\rm mix}})/\sqrt{\omega_{\rm mix}}$, where $\omega = \sigma_M^2/\overline{M}$.

$$F_{p_T} = \frac{\sigma_M}{\sigma_M^{\rm mix}} - 1 = \sqrt{1 + \frac{\sigma^2_{\rm dyn}}{\sigma_M^{2,\rm mix}}} - 1 \simeq \frac{1}{2}\frac{\sigma^2_{\rm dyn}}{\sigma_M^{2,\rm mix}} \simeq \frac{1}{2\overline{n}\sigma^2_{\overline{n}}(p)} \sum_{i\neq j} {\rm cov}_{\overline{n}}(p_i, p_j)$$

$\Sigma^2_{p_T} \equiv \sigma^2_{\rm dyn}/\overline{p}^2$

$$\Phi_{p_T} \equiv \sqrt{\frac{\sigma_S^2}{\overline{n}}} - \sigma_{\overline{n}}(p) \simeq \frac{\sum_{i\neq j} {\rm cov}_{\overline{n}}(p_i, p_j)}{2\overline{n}\sigma_{\overline{n}}(p)}$$

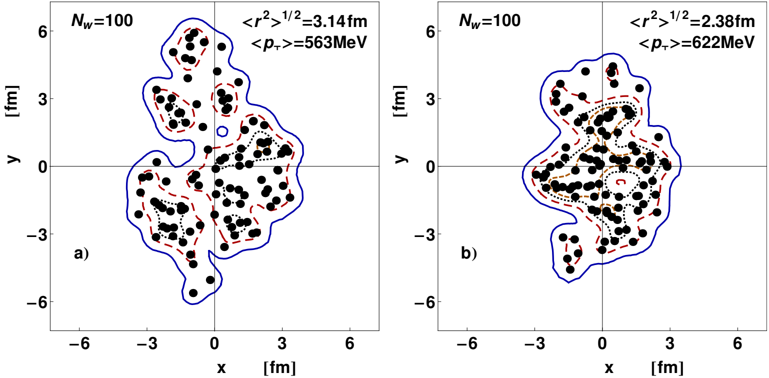
where $S_n = p_1 + \cdots + p_n$ (Φ_{p_T} is a so-called strongly-intensive measure)
The STAR measure:

$$\langle \Delta p_{T,i} \Delta p_{T,j} \rangle = \cdots = \sigma^2(M) - \left\langle \frac{\sigma_n^2(p_T)}{n} \right\rangle$$

All these measures of dynamical fluctuations are related

Initial shape fluctuations in the Glauber approach

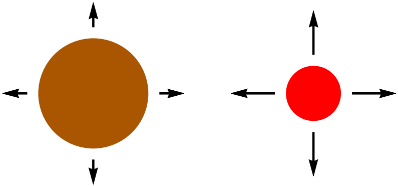
Ready to do some physics...



[Bożek, WB 2012]

Two typical configurations of wounded nucleons in the transverse plane generated with GLISSANDO, isentropes at $s = 0.05$, 0.2, and 0.4 GeV^{-3}

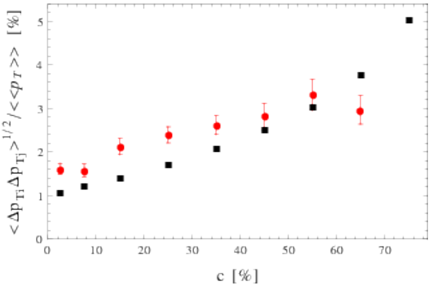
Size – radial flow transmutation



smaller size $\to$ stronger flow
larger size $\to$ weaker flow

[WB, Chojnacki, Obara 2009]

Transverse momentum fluctuations in Au+Au@200GeV



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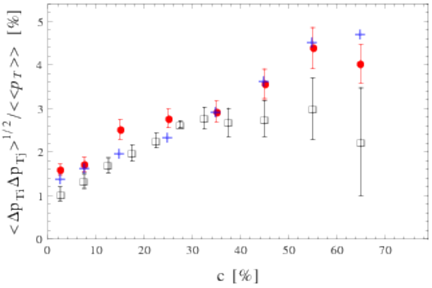


PHENIX

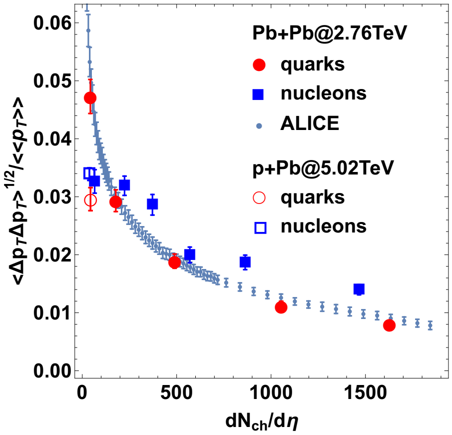
red points – model

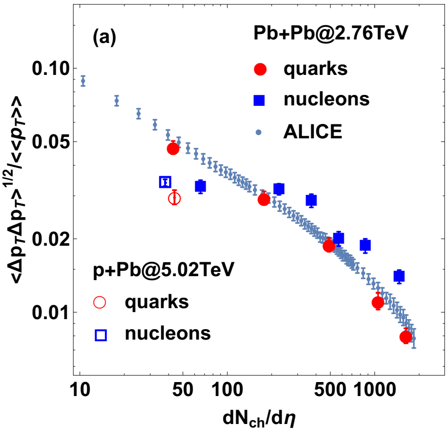
[Bożek, WB 2012]

- Measure removes trivial fluctuations from finite sampling
- Model overshoots the data by about 50% for most central collisions, need to decrease initial fluctuations
- Hydro response not much modified by: viscosity, T_f, source smearing, total momentum conservation, ... $\Delta\langle p_T\rangle/\langle\langle p_T\rangle\rangle \simeq 0.4\Delta\langle r\rangle/\langle\langle r\rangle\rangle$

Transverse momentum fluctuations with wounded quarks

Wounded quark model [tablet] as implemented in [Bożek, WB, Rybczyński 2016]: more participants → less fluctuation

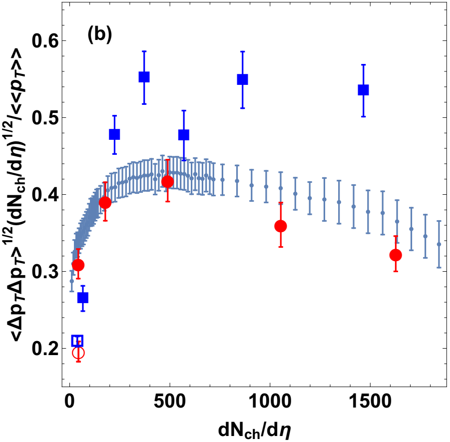




[Bożek, WB 2017]

Transverse momentum fluctuations with wounded quarks

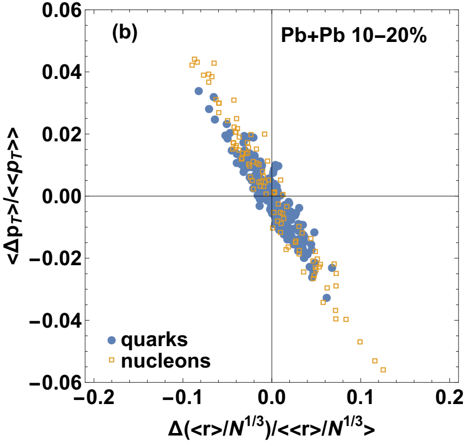
Nontrivial dependence on multiplicity



Excludes independent production from sources (would be flat)

Size – flow anti-correlation

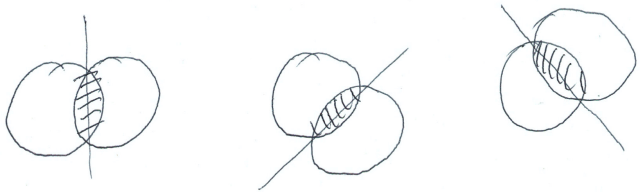
Very strong e-by-e anti-correlation of size and $\langle p_T \rangle$



- This is the mechanism for p_T fluctuations!

Lecture 6

Event-plane



Imagine for each event-plane orientation we have an elliptically deformed azimuthal distribution

$$f(\phi; \Psi) = A\left[1 + 2v_2 \cos 2(\phi - \Psi)\right],$$

where v_2 is the "true" elliptic flow coefficient Averaging over orientations yields (for a perfect, Ψ-independent, detector) $\int_0^{2\pi} \frac{d\Psi}{2\pi} f(\phi; \Psi) = A$, i.e., we loose any information on the elliptic flow. We would need to know Ψ in the event (there are methods of determining the reaction plane, not discussed here) or consider multiparticle correlations $\rightarrow$

[Borghini, Dinh, Ollitrault, PRC 64 (2001) 054901, arXiv:0105040]

2-particle measurement of harmonic flow

The two-particle distribution with a given Ψ has the generic form

$$f^{(2)}(\phi_1, \phi_2; \Psi) = f(\phi_1; \Psi) f(\phi_2; \Psi) + c^{(2)}(\phi_1 - \phi_2)$$

where $c^{(2)}$ is a correlation function not coming from harmonic flow (nonflow), e.g., jets, resonance decays, ...– it will typically depend on $\phi_1 - \phi_2$. Averaging $f^{(2)}$ over orientations Ψ yields

$$\int \frac{d\Psi}{2\pi} f^{(2)}(\phi_1, \phi_2; \Psi) = A^2 \left[1 + 2v_2^2 \cos 2(\phi_1 - \phi_2)\right] + \text{nonflow}$$

Homework

Derive the above formula and the lemma:
$\int_0^{2\pi} d\Psi/(2\pi) \cos n(\phi_1 - \Psi) \cos m(\phi_2 - \Psi) = \frac{1}{2}\delta_{nm} \cos n(\phi_1 - \phi_2)$, $n, m = 1, 2, \ldots$

Above reasoning generalizes to higher-rank flow$n = 3$ (triangular), 4, ... Taking the experimental distributions of hadron pairs in $\Delta\phi \equiv \phi_1 - \phi_2$ we can evaluate

$$c_n\{2\} \equiv v_n^2\{2\} = \int \frac{d\Delta\phi}{2\pi} \frac{dN}{d\Delta\phi} \cos n\Delta\phi = \langle v_n^2 \rangle + \text{nonflow}$$

Cumulants for harmonic flow

Flow correlations are specific: originate from "external" correlation via Ψ, and not from "direct" two-particle correlations.

In a compact notation $$c_n\{2\} \equiv v_n^2\{2\} = \left\langle e^{in(\phi_1-\phi_2)} \right\rangle$$

Idea: take proper higher-particle correlations (cumulants) to minimize nonflow, e.g.,

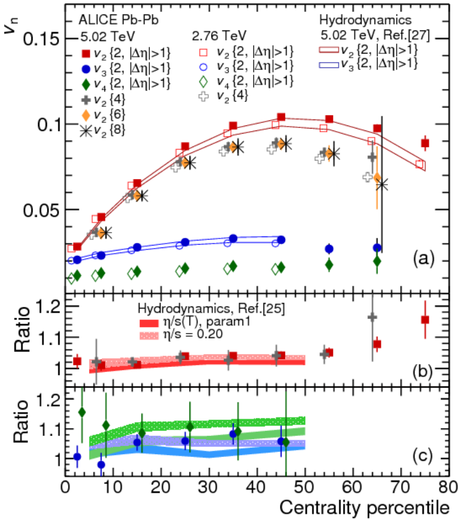
$$\begin{aligned} c_n\{4\} &= \left\langle e^{in(\phi_1+\phi_2-\phi_3-\phi_4)} \right\rangle - \left\langle e^{in(\phi_1-\phi_3)} \right\rangle \left\langle e^{in(\phi_2-\phi_4)} \right\rangle - \left\langle e^{in(\phi_1-\phi_4)} \right\rangle \left\langle e^{in(\phi_2-\phi_3} \right. \\ &= \left\langle e^{in(\phi_1+\phi_2-\phi_3-\phi_4)} \right\rangle - 2c_n^2\{2\} = \langle v_n^4 \rangle - 2\langle v_n^2 \rangle^2 \end{aligned}$$

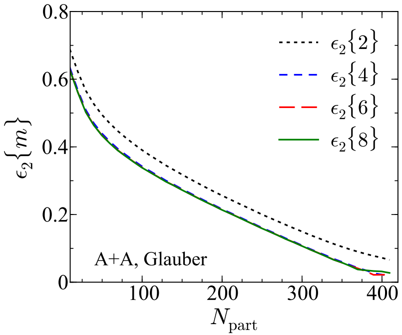
A direct evaluation shows, that all 2-body correlations cancel out!. Thus nonflow may now originate from 4-particle correlations, which are much weaker than the two-particle non-flow. One finds

$$v_n^4\{4\} = -c_n\{4\} = \langle v_n^4 \rangle - 2\langle v_n^2 \rangle^2 + 4\text{p nonflow}$$

For detailed formulas with finite sampling (number of hadrons) and for higher $\{k\}$ see [Bilandzić, Snellings, Voloshin, PRC 83 (2011) 044913, arXiv:1010.0233]

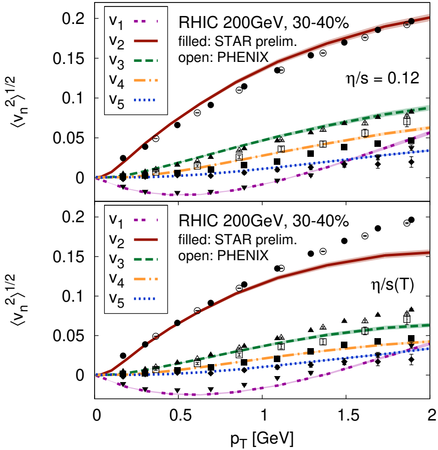
Whole "industry" of higher cumulants for harmonic flow

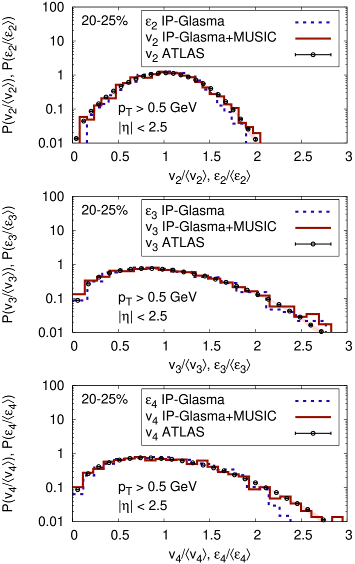




[Bzdak, Bożek, McLerran, Nucl. Phys. A927 (2014) 15]

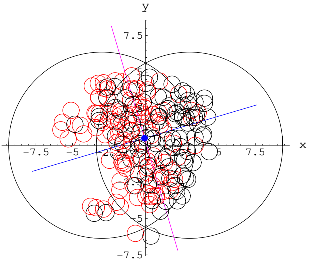
Flow fluctuations from the IP-Glasma initial conditions

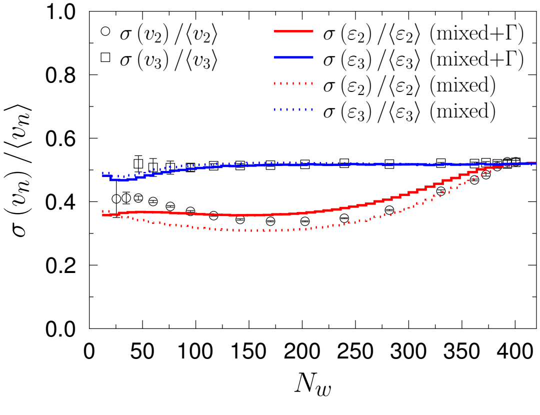




[Gale, Jeon, Schenke, Venugopalan, PRL 110 (2013) 012302]

Recall $v_n \simeq \kappa_n \epsilon_n \to \sigma(v_n)/\langle v_n \rangle \simeq \sigma(\epsilon_n)/\langle \epsilon_n \rangle$ (κ_n cancels)

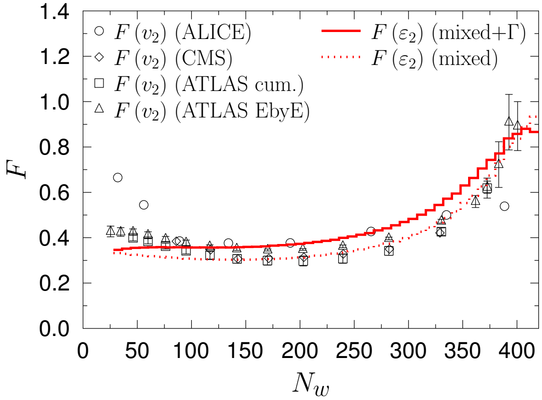




$\leftarrow \quad \sqrt{4/\pi - 1}$

[WB, Rybczyński 2016]

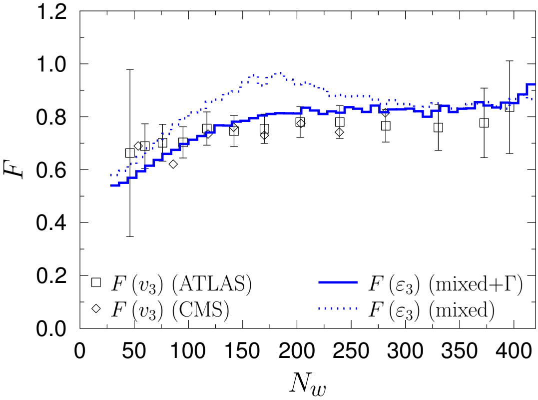
$$F_n = \sqrt{\frac{\varepsilon_n\{2\}^2 - \varepsilon_n\{4\}^2}{\varepsilon_n\{2\}^2 + \varepsilon_n\{4\}^2}}$$



[WB, Rybczyński 2016]

Flow fluctuations in the Glauber approach

$$F_n = \sqrt{\frac{\varepsilon_n\{2\}^2 - \varepsilon_n\{4\}^2}{\varepsilon_n\{2\}^2 + \varepsilon_n\{4\}^2}}$$

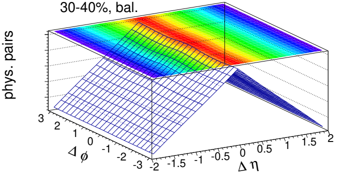


[WB, Rybczyński 2016]

Modeling in rapidity

2D two-particle correlations

$$R_2(\Delta\eta,\Delta\phi) \equiv C(\Delta\eta,\Delta\phi) = \frac{\langle N^{\rm pairs}_{\rm phys}(\Delta\eta,\Delta\phi)\rangle}{\langle N^{\rm pairs}_{\rm mixed}(\Delta\eta,\Delta\phi)\rangle}$$



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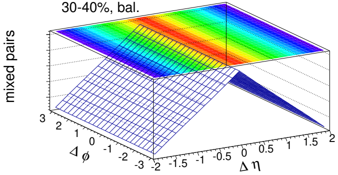


$\rightarrow$

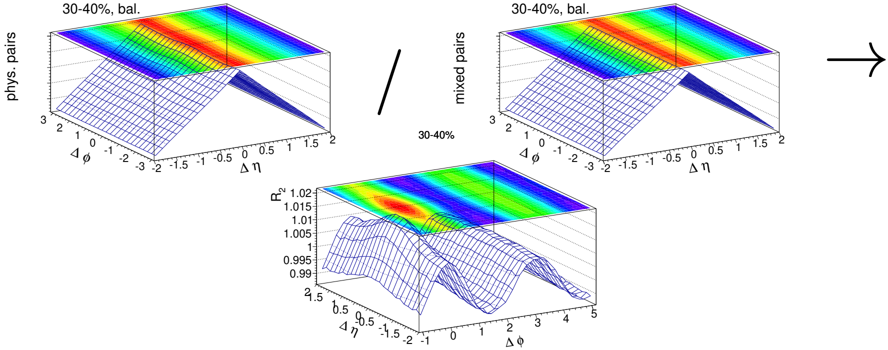
"Tent":

$$\int_{-\eta_a}^{\eta_a}\frac{d\eta_1}{2\eta_a}\int_{-\eta_a}^{\eta_a}\frac{d\eta_2}{2\eta_a}\,\delta[\Delta\eta-(\eta_1-\eta_2)] = \text{triangle in } \Delta\eta \text{ from } -2\eta_a \text{ to } 2\eta_a$$

$$\int_{0}^{2\pi}\frac{d\phi_1}{2\pi}\int_{0}^{2\pi}\frac{d\phi_2}{2\pi}\,\delta[\Delta\phi-(\phi_1-\phi_2)] = \text{flat in } \Delta\phi$$

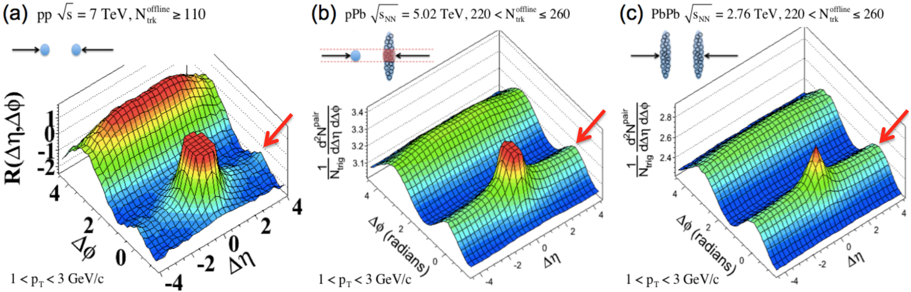
2D two-particle correlations

$$R_2(\Delta\eta, \Delta\phi) \equiv C(\Delta\eta, \Delta\phi) = \frac{\langle N^{\rm pairs}_{\rm phys}(\Delta\eta,\Delta\phi)\rangle}{\langle N^{\rm pairs}_{\rm mixed}(\Delta\eta,\Delta\phi)\rangle}$$



- free of detector acceptance bias

Near-side ridge



As we will see, the near-side ridge indicates collectivity

Total surprise in p-p!

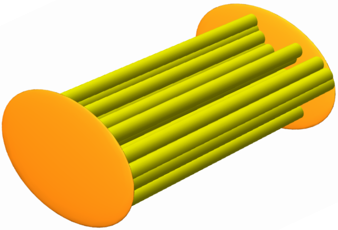
Homework

Show that for uncorrelated distributions of two particles, which are uniform in the azimuths $\phi_{1,2}$ and uniform in $A \le \eta_{1,2} \le A$, one indeed gets the "tent".

Factorization of the transverse and longitudinal distributions

Just after the collision:

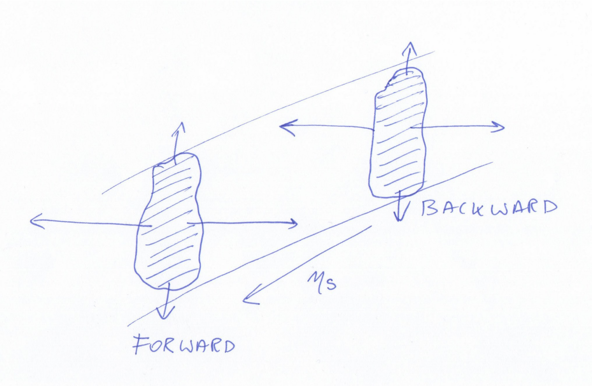
backward-moving participants strings forward-moving participants



Factorization of the transverse and longitudinal distributions

Just after the collision:

backward-moving participants strings forward-moving participants

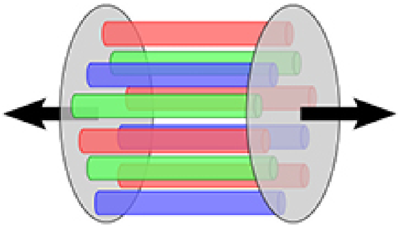


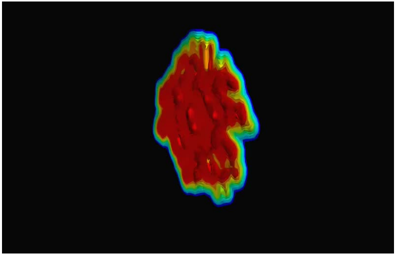
Approximate (up to fluctuations) alignment of F and B event planes

Collimation of flow at very distant longitudinal separations → ridges!

F and B transverse shapes similar

Dynamical realization – glasma tubes

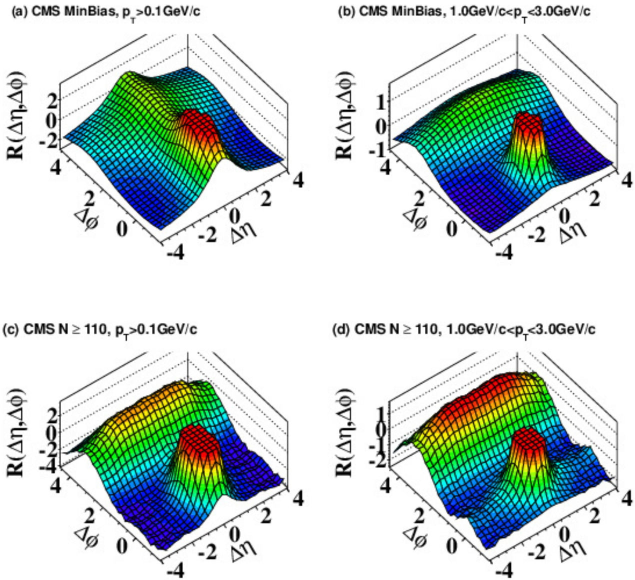




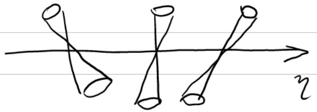
Surfers - the near-side ridge

Collimated even if separated by a mile!

Near-side ridge in $p + p$ – high multiplicity only!



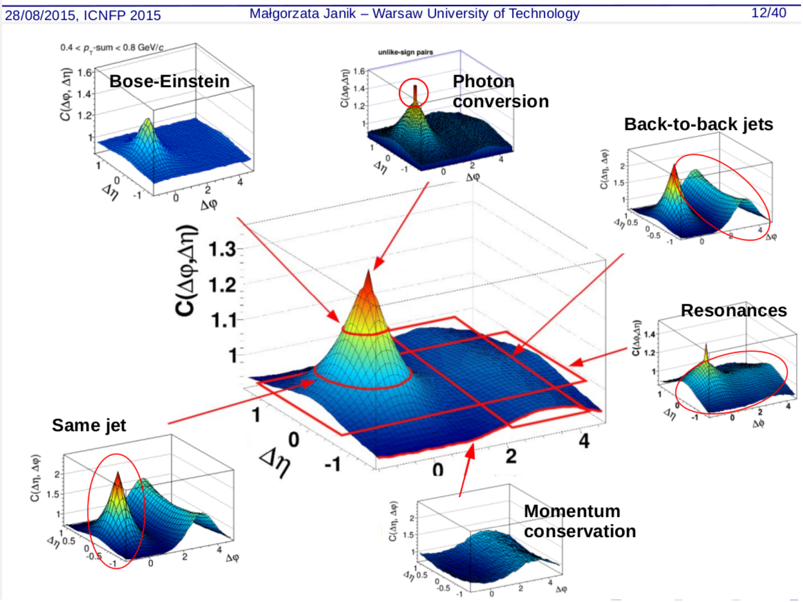
Jets



2 particles from the same jet $\rightarrow$ central peak ($\Delta\phi \sim 0$, $\Delta\eta \sim 0$)

from the opposite jets $\rightarrow$ away ridge ($\Delta\phi \sim \pi$, $\Delta\eta$ - washed out)

Other sources of correlations



Flow measures with rapidity gap

The big technical issue and physical is to separate flow effects from non-flow (jets, resonances, ...). One can use the flow vector in rapidity bin around η:

$$q_n(\eta) = \frac{1}{m}\sum_{k=1}^{m} e^{in\phi_k} \equiv \langle e^{in\phi}\rangle \equiv v_n(\eta)e^{in\Psi_n(\eta)}$$

where $n = 2, 3, \ldots$ is the rank (ellipticity, triangularity), m is the number of particles in the bin, v_n is the flow coefficient in the event and Ψ_n in the event-plane angle. Define the average over events as

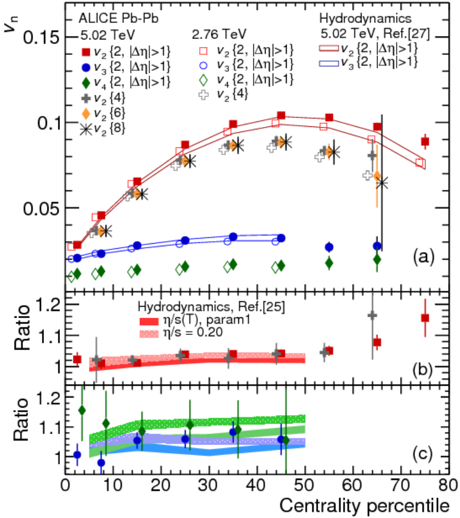
$$V_{n\Delta} = \langle q_n(\eta_1)q_n^*(\eta_2)\rangle_{\rm ev} = \langle\langle\cos n(\phi_1-\phi_2)\rangle\rangle_{\rm ev} = \langle v_n(\eta_1)v_n(\eta_2)e^{in[\Psi_n(\eta_1)-\Psi_n(\eta_2)]}\rangle_{\rm ev}$$

with bins at η_1 and η_2 sufficiently separated, $|\eta_1-\eta_2| > \delta$ (typically, $\delta \geq 1$). Then

$$\frac{dN^{\rm pair}_{|\eta_1-\eta_2|>\delta}}{d\Delta\phi} = \frac{N^{\rm pair}_{|\eta_1-\eta_2|>\delta}}{2\pi}\left[1+2\sum_n V_{n\Delta}\cos n\Delta\phi\right]$$

For symmetric bins ($\eta_1 = -\eta_2 = \eta$) and $\Psi_n(\eta) = \Psi_n(-\eta)$ we have $V_{n\Delta} = \langle v_n(\eta)^2\rangle_{\rm ev}$

(as seen before)



Wounded nucleon model

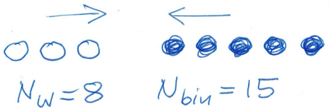
[Białas, Błeszyński, Czyż, Nucl.Phys.B 111 (1976)]

In the wounded nucleon model of soft dynamics, a nucleon that interacted inelastically more than once (i.e., interacted inelastically at all) emits a number of particles independently on how many nucleons it collided with! This somewhat counterintuitive feature finds explanation from string models (see the following)

Professor Czyż:

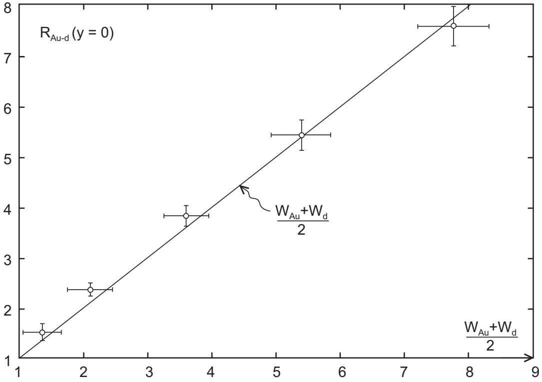
"Once the nucleon lost all its teeth in the first collision, it has no more teeth to loose"

This is juxtaposed to the binary counting, where each inelastic (hard) collision produces particles. The two scenarios are very different when the colliding nucleons fly one after another, as in $A - A$ collisions:



(mixed model)

Wounded nucleon model for d+Au at $\sqrt{s_{NN}} = 200$ GeV



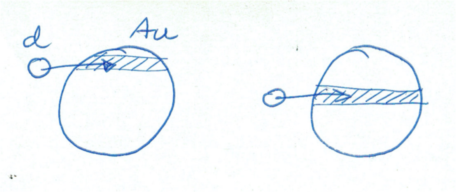
[Białas, Czyż 2004]

Emission profile in pseudorapidity

It is natural to assume that the emission profile in rapidity, $f(\eta)$, "follows" its wounded nucleon. Then the simple equation follows for the hadron spectrum (in NN CM frame):

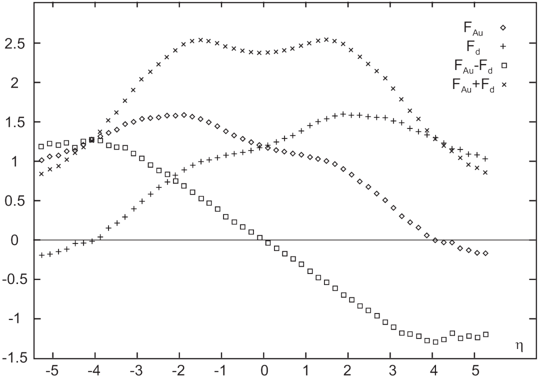
$$\frac{dN(\eta)}{d\eta} = W_F f(\eta) + W_B f(-\eta)$$

where $W_{F,B}$ are the numbers of wounded nucleons moving forward or backward. The weighting depends on centrality, which allows to get both the symmetric and antisymmetric part of $f(\eta)$.



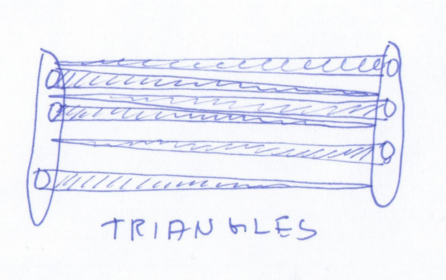
Triangles and fluctuating strings

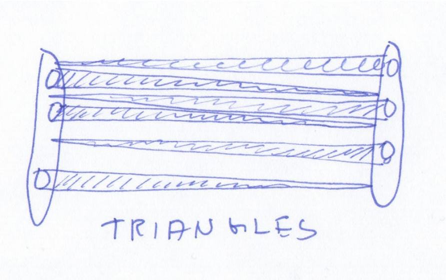
The emission profiles extracted from the d-Au data at RHIC:

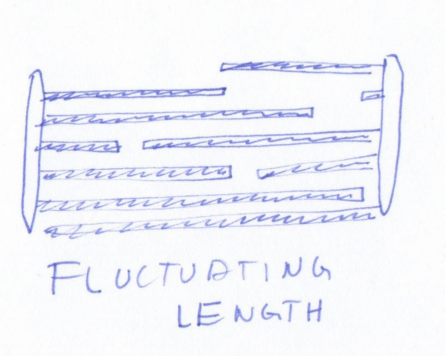


[Białas, Czyż 2004]

Source emits mostly in its own froward hemisphere


TRIANGLES

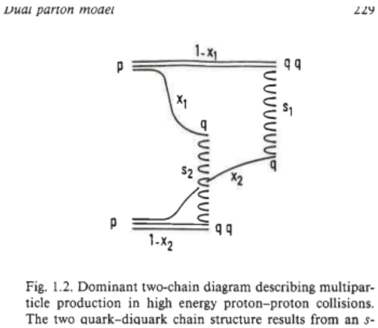




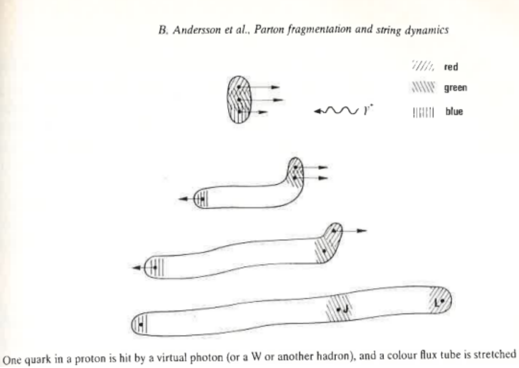
[. . . Bierlich, Gustafson, Lönnblad 2016, Monnai, Schenke 2015, Schenke, Schlichting 2016 . . . Brodsky, Gunion, Kuhn, 1977]

String models 1970's

Dual Parton Model (Capella et al.)



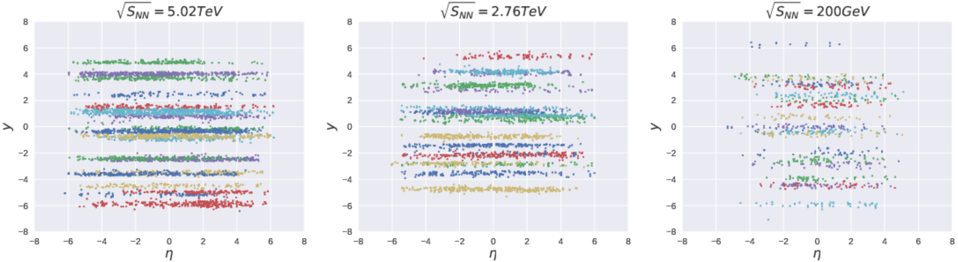
Lund model (Anderson et al.)



Basis of many successful codes (Pythia, HIJING, AMPT, EPOS, ...)

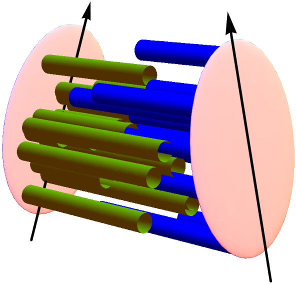
Strings are spatial objects

AMPT [Wu et al. 2018]



String end-points fluctuate in (here: space-time rapidity) η, uniform production of particles along the string (same thickness)

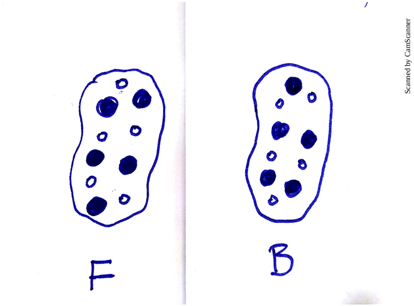
Fluctuating strings

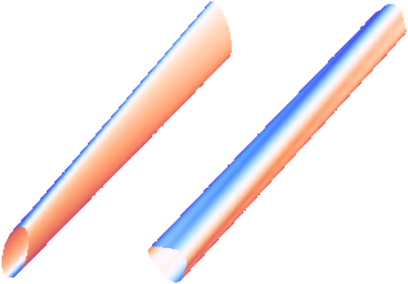


Torque effect (event-by-event)

Transverse sections with triangles

e-by-e longitudinal twist (a few degrees)

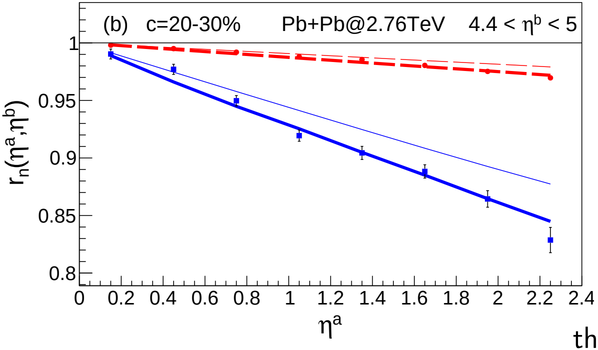




- Both e-by-e fluctuations and longitudinal asymmetry of the emission profile needed

[prediction in PB, WB, Moreira 2010 & PB, WB, Olszewski 2015, PB, WB 2016]

Torque in Pb+Pb

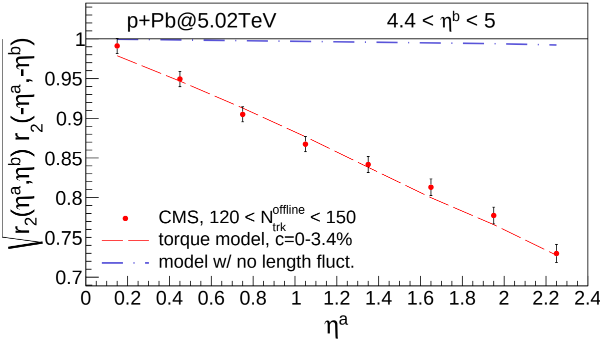


thin - triangles
thick - string breaking
v_2 and v_3

$$r_n(\eta_a, \eta_b) = \frac{\langle\langle \cos(n[\phi_i(-\eta_a) - \phi_j(\eta_b)]) \rangle\rangle_{\rm ev}}{\langle\langle \cos(n[\phi_i(\eta_a) - \phi_j(\eta_b)]) \rangle\rangle_{\rm ev}}$$

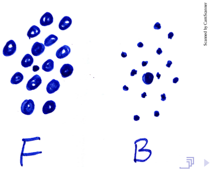
(3-bin measure by CMS)

Torque in p-Pb

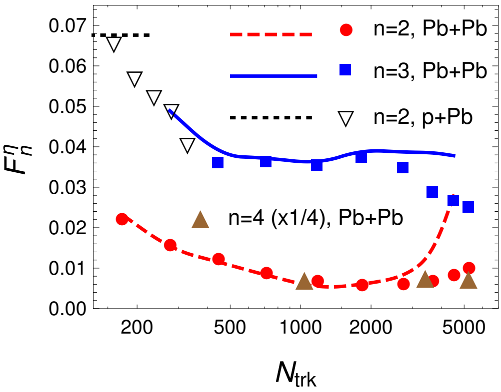


- String breaking essential to describe torque in p-Pb

With triangles almost perfect correlation



Slope of r_n at $\eta_a = 0$



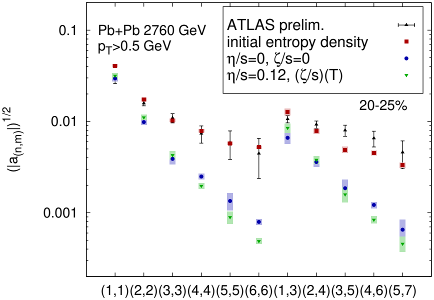
- Fair description of mid-central collisions
- Way too much decorrelation in central collisions
- $F_4 \simeq 4F_2$ [Bożek, WB, PLB 752 (2016) 206, arXiv:1506.02817]

η_1-η_2 correlations and a_{nm} coefficients

$$C(\eta_1,\eta_2) = \frac{N(\eta_1,\eta_2) - N(\eta_1)\delta(\eta_1-\eta_2)}{N(\eta_1)N(\eta_2)}$$

Coefficients proposed by [Bzdak, Teaney, 2012]

$$a_{nm} = \int_{-Y}^{Y} \frac{d\eta_1}{Y} \int_{-Y}^{Y} \frac{d\eta_2}{Y} \frac{1}{\mathcal{N}_C} C(\eta_1,\eta_2) T_n\left(\frac{\eta_1}{Y}\right) T_m\left(\frac{\eta_2}{Y}\right)$$



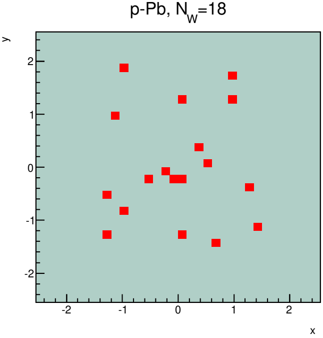
[Monnai, Schenke, PLB 752 (2016) 317]

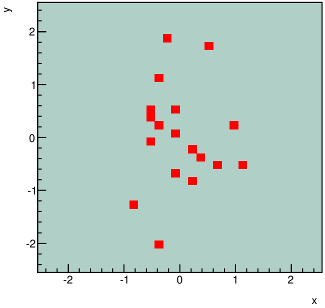
Lecture ∞

Small systems

Snapshots of initial Glauber condition in central p-Pb

Typical transverse-plane configuration of centers of the participant nucleons in a p+Pb collision generated with GLISSANDO
5% of collisions have more than 18 participants, rms ~ 1.5 fm – large!

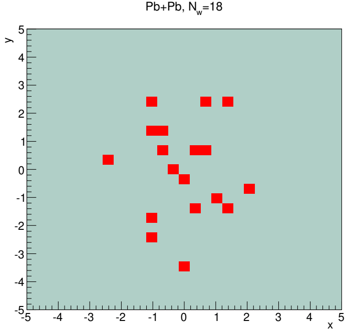




Snapshot of peripheral Pb+Pb

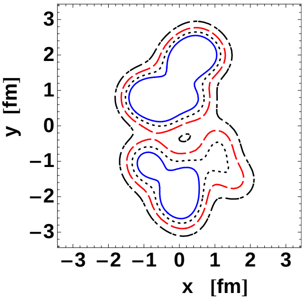
Most central values of N_w in p-Pb would fall into the 60-70% or 70-80% centrality class in Pb+Pb

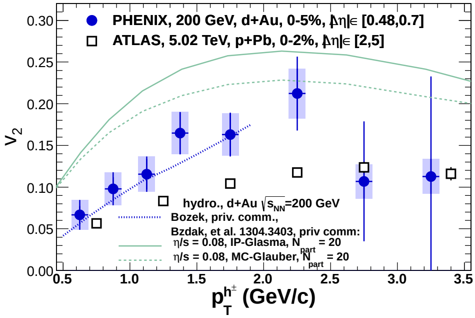
Pb+Pb: c=60-70% $\equiv 22 \leq N_w \leq 40$, c=70-80% $\equiv 11 \leq N_w \leq 21$



d has an intrinsic dumbbell shape with a large deformation: rms $\simeq 2$ fm

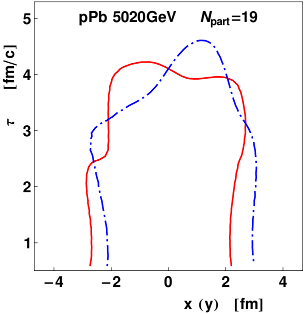
Initial entropy density in a d-Pb collision with $N_{\text{part}} = 24$ [Bożek 2012]





Resulting large elliptic flow confirmed with the later RHIC analysis
(geometry + fluctuations)

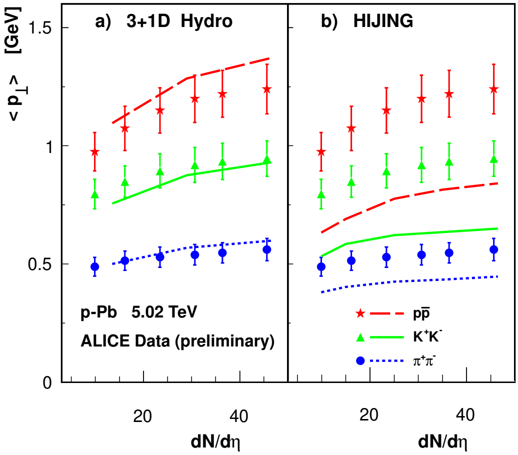
Size of the p-Pb fireball



isotherms at freeze-out $T_f = 150$ MeV
(for two sections in the transverse plane)

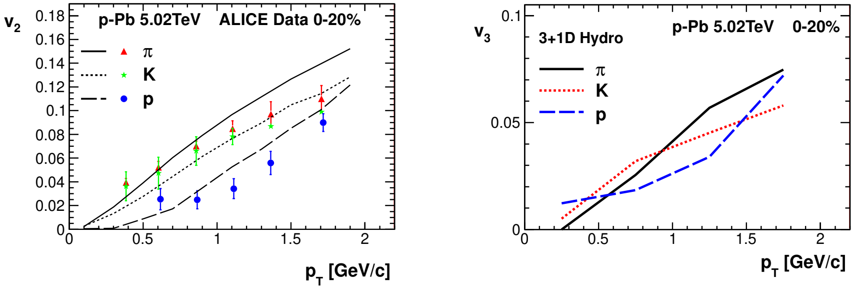
evolution lasts about 4 fm/c – shorter but more rapid than in A+A

Mass hierarchy in p-A



[P. Bożek, WB, G. Torrieri, PRL 111 (2013) 172303]

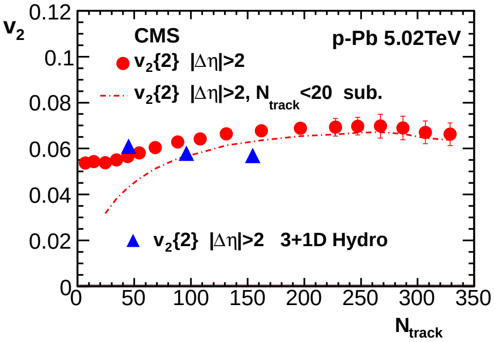
Mass hierarchy in p-A



[P. Bożek, WB, G. Torrieri, PRL 111 (2013) 172303]

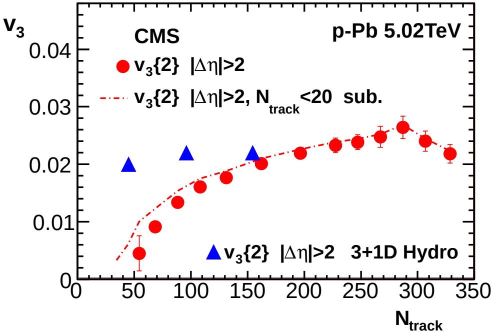
Harmonic flow in p-A

no geometry, only fluctuations



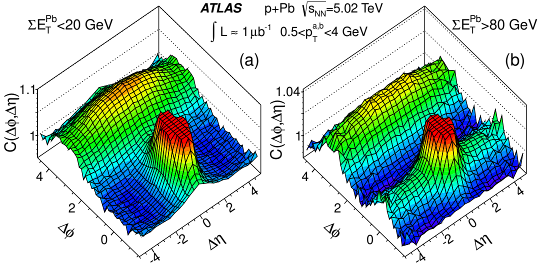
[P. Bożek, WB, PRC 88 (2013) 014903]

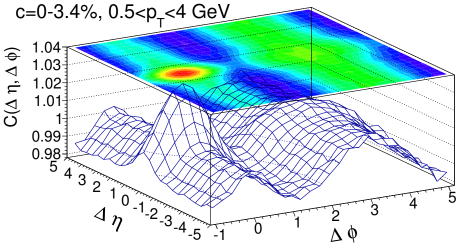
no geometry, only fluctuations



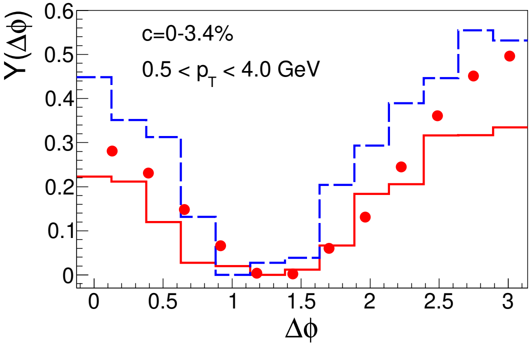
[P. Bożek, WB, PRC 88 (2013) 014903]

Ridge in p-Pb, ATLAS





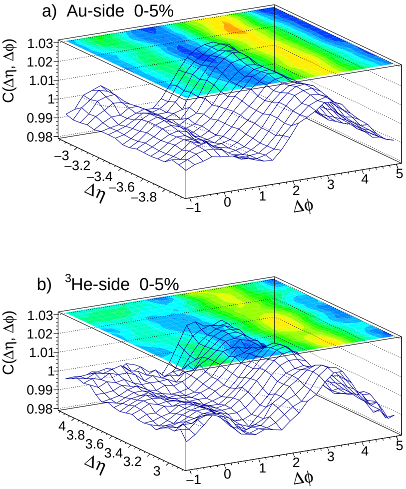
Near-side ridge, $2 \leq |\Delta\eta| \leq 5$

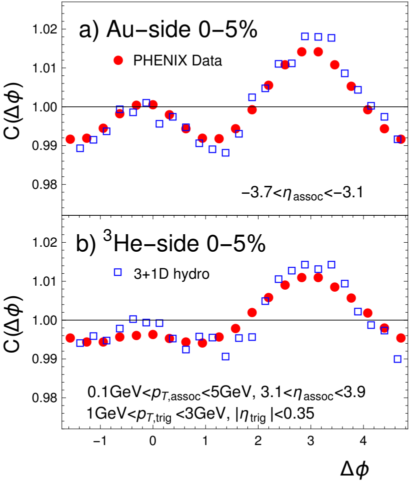


two variants of the Glauber model:
red – $< R^2 >^{1/2} = 1.5$ fm, blue – $< R^2 >^{1/2} = 0.9$ fm, dots – ATLAS

see also CGC-based calculation: [K. Dusling, R. Venugopalan, PRD 87 (2013) 094034]

Ridge in ^{3}He-Au at RHIC

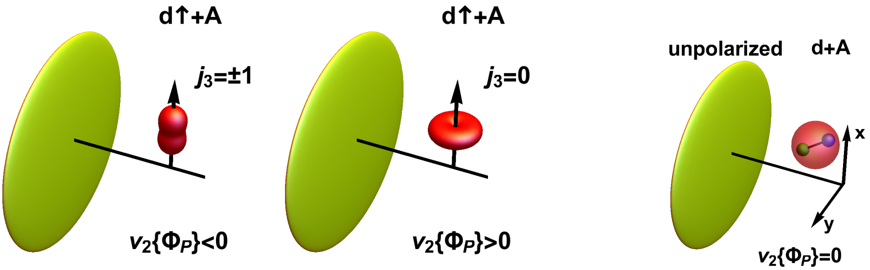




(seen on both pseudorapidity sides)

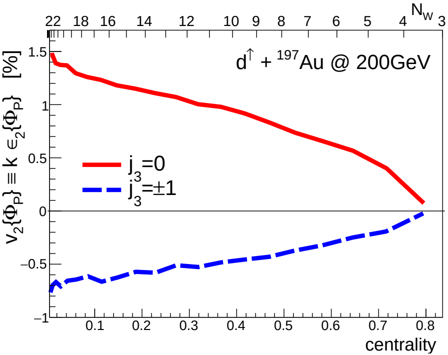
[Bożek, WB 2015]

Polarized d+A collisions



[P. Bożek, WB, PRL 121 (2018) 202301]

Predictions



$$\frac{dN}{d\phi} \propto 1 + 2v_2 \cos\left[2(\phi - \Phi_P)\right]$$

Φ_P fixed!

$$v_2 \simeq k\epsilon_2, \quad k \sim 0.2$$

For $j = 1$ nuclei the *tensor polarization* is

$$P_{zz} = n(1) + n(-1) - 2n(0)$$
$$v_2\{\Phi_P\} \simeq k\,\epsilon_2^{j_3=\pm1}\{\Phi_P\} P_{zz}$$

$$-0.5\% \lesssim v_2\{\Phi_P\} \lesssim 1\%$$

One-particle distribution - can be measured precisely !

Prospects for AFTER@LHC

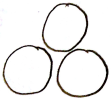
^{12}C-Pb – role of α clusters

Nuclear structure from ultra-relativistic collisions!

Probe to what degree ^{12}C is made of three α's

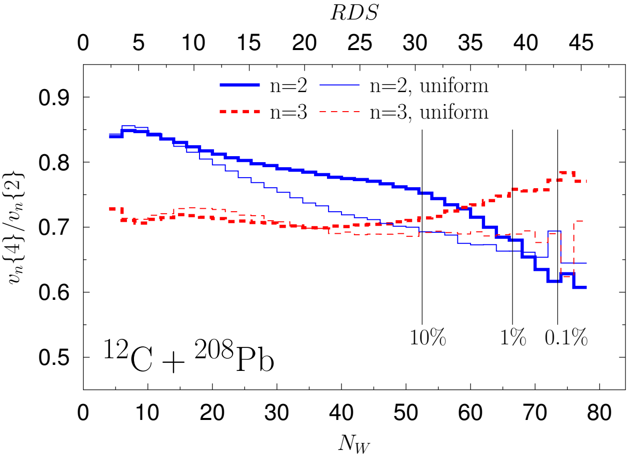
Specific features of the ^{12}C collisions with a "wall":

The cluster plane parallel or perpendicular to the transverse plane:

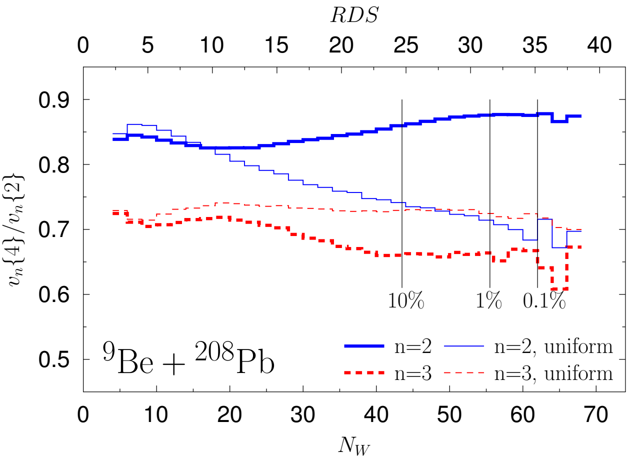


higher multiplicity	lower multiplicity
higher triangularity	lower triangularity
lower ellipticity	higher ellipticity

Ellipticity and triangularity vs multiplicity



Ellipticity and triangularity vs multiplicity



Summary

"Circumstantial" but strong evidence

- Multiplicities $\rightarrow$ thermal parameters
- p_T spectra $\rightarrow$ radial flow
- harmonic flow $\rightarrow$ initial geometry and fluctuations
- fluctuations of $< p_T >$ $\rightarrow$ fluctuations of the initial size
- Ridge $\rightarrow$ flow
- Interferometry $\rightarrow$ size and flow (not covered)

Conclusions

The approach with collectivity (copious rescattering in the intermediate stage) works as a generic framework of the reaction mechanism

- Collectivity from rescattering in A+A commonly accepted
- Explanation of the near-side ridge
- Mechanism for p_T fluctuations
- Torque (event-plane angle decorrelation)
- Small systems (p-Pb, d-Pb) also exhibit collectivity
- Torque in p-Pb $\rightarrow$ longitudinal fluctuations (string breaking)
- Shape-flow transmutation in small systems
- Polarized deuteron
- α-clustered small nuclei

Not mentioned

- Jet quenching by the medium
- Early probes
- Femtoscopy
- Chiral magnetic effect
- Vorticity and Λ polarization
- ...

Recommended literature (and references therein)

- *Phenomenology of Ultra-Relativistic Heavy-Ion Collisions*, Wojciech Florkowski, World Scientific 2010 **(with exercises!)**
- *Ultra-relativistic Heavy-Ion Collisions*, Ramona Vogt, Elsevier 2007
- *Relativistic hydrodynamics for heavy-ion collisions*, Jean-Yves Ollitrault, Lectures given at the Advanced School on Quark-Gluon Plasma, Indian Institute of Technology, Bombay, 3-13 July 2007, Eur.J.Phys. 29(2008)275, arXiv:0708.2433 **(with exercises!)**
- *Nearly perfect fluidity: from cold atomic gases to hot quark gluon plasmas*, Thomas Schäfer, Derek Teaney, Rept. Prog. Phys. 72 (2009) 126001
- *New theories of relativistic hydrodynamics in the LHC era*, Wojciech Florkowski, Michal P. Heller, Michal Spaliński, Rept. Prog. Phys. 81 (2018) 046001
- *Collective flow and viscosity in relativistic heavy-ion collisions*, Ulrich Heinz, Raimond Snellings, Ann. Rev. Nucl. Part. Sci. 63 (2013) 123
- *Initial state fluctuations and final state correlations: Status and open questions*, Andrew Adare, Matthew Luzum, Hannah Petersen, Phys.Scripta 87(2013)048001, Phys.Scripta 04(2013)048001
- . . .

THANKS!