# Rapidity Fluctuations in the Initial State of Ultra-Relativistic Heavy-Ion Collisions

#### Wojciech Broniowski<sup>1,2</sup> and Piotr Bożek<sup>3</sup>

<sup>1</sup>Institute of Nuclear Physics PAN, Cracow <sup>2</sup>Jan Kochanowski U., Kielce <sup>3</sup>AGH University of Science and Technology, Cracow

Opportunities for Exploring Longitudinal Dynamics RIKEN BNL, 20-22 January 2016

## Motivation/new data

- Old story ...
- New data from the LHC, new methodology (ATLAS notes 2015)
- Longitudinally-extended source model

Goal: understand key elements from an analytic model anatomy of the correlations

Physics issues: production mechanism in the eary stage, degrees of freedom,...

Based on [WB+ Piotr Bożek, arXiv:1512.01945]

## 3-stage approach

#### Generation and propagation of e-by-e fluctuations



Concept of sources: wounded nucleons, quarks, flux tubes, … Hydro: provides mapping  $\eta_S \to \eta$ 

3 N.

#### New data

$$C(\eta_1, \eta_2) = \frac{\langle N(\eta_1, \eta_2) \rangle}{\langle N(\eta_1) \rangle \langle N(\eta_2) \rangle} = \frac{S(\eta_1, \eta_2)}{B(\eta_1, \eta_2)}$$
  

$$C_N(\eta_1, \eta_2) = \frac{C(\eta_1, \eta_2)}{C_p(\eta_1)C_p(\eta_2)}, \quad C_p(\eta_1) = \int d\eta_2 C(\eta_1, \eta_2), \ C_p(\eta_2) = \dots$$

 $\eta_1$  and  $\eta_2$  – pseudorapidities of different hadrons





$$\begin{split} \langle N(\eta_1)N(\eta_2)\rangle &= \langle N_A\rangle\langle f_A(\eta_1,\eta_2)\rangle + \langle N_A(N_A-1)\rangle\langle f_A(\eta_1)\rangle\langle f_A(\eta_2)\rangle \\ &+ \langle N_B\rangle\langle f_B(\eta_1,\eta_2)\rangle + \langle N_B(N_B-1)\rangle\langle f_B(\eta_1)\rangle\langle f_B(\eta_2)\rangle \\ &+ \langle N_AN_B\rangle \left[\langle f_A(\eta_1)\rangle\langle f_B(\eta_2)\rangle + \langle f_B(\eta_1)\rangle\langle f_A(\eta_2)\rangle\right] \end{split}$$

 $f_{A,B}(\eta_i)$  and  $f_{A,B}(\eta_1,\eta_2)$  – probabilities of emission from a single source



$$\begin{split} \langle N(\eta_1)N(\eta_2) \rangle &= \langle N_A \rangle \langle f_A(\eta_1,\eta_2) \rangle + \langle N_A(N_A-1) \rangle \langle f_A(\eta_1) \rangle \langle f_A(\eta_2) \rangle \\ &+ \langle N_B \rangle \langle f_B(\eta_1,\eta_2) \rangle + \langle N_B(N_B-1) \rangle \langle f_B(\eta_1) \rangle \langle f_B(\eta_2) \rangle \\ &+ \langle N_A N_B \rangle \left[ \langle f_A(\eta_1) \rangle \langle f_B(\eta_2) \rangle + \langle f_B(\eta_1) \rangle \langle f_A(\eta_2) \rangle \right] \end{split}$$



$$\begin{split} \langle N(\eta_1)N(\eta_2) \rangle &= \langle N_A \rangle \langle f_A(\eta_1,\eta_2) \rangle + \langle N_A(N_A-1) \rangle \langle f_A(\eta_1) \rangle \langle f_A(\eta_2) \rangle \\ &+ \langle N_B \rangle \langle f_B(\eta_1,\eta_2) \rangle + \langle N_B(N_B-1) \rangle \langle f_B(\eta_1) \rangle \langle f_B(\eta_2) \rangle \\ &+ \langle N_A N_B \rangle \left[ \langle f_A(\eta_1) \rangle \langle f_B(\eta_2) \rangle + \langle f_B(\eta_1) \rangle \langle f_A(\eta_2) \rangle \right] \end{split}$$



$$\begin{split} \langle N(\eta_1)N(\eta_2)\rangle &= \langle N_A\rangle\langle f_A(\eta_1,\eta_2)\rangle + \langle N_A(N_A-1)\rangle\langle f_A(\eta_1)\rangle\langle f_A(\eta_2)\rangle \\ &+ \langle N_B\rangle\langle f_B(\eta_1,\eta_2)\rangle + \langle N_B(N_B-1)\rangle\langle f_B(\eta_1)\rangle\langle f_B(\eta_2)\rangle \\ &+ \langle N_AN_B\rangle \left[\langle f_A(\eta_1)\rangle\langle f_B(\eta_2)\rangle + \langle f_B(\eta_1)\rangle\langle f_A(\eta_2)\rangle\right] \end{split}$$

< A



$$\langle N(\eta_1)N(\eta_2)\rangle = \langle N_A\rangle \operatorname{cov}_A(\eta_1,\eta_2) + \langle N_A^2\rangle \langle f_A(\eta_1)\rangle \langle f_A(\eta_2)\rangle + \langle N_B\rangle \operatorname{cov}_B(\eta_1,\eta_2) + \langle N_B^2\rangle \langle f_B(\eta_1)\rangle \langle f_B(\eta_2)\rangle + \langle N_A N_B\rangle [\langle f_A(\eta_1)\rangle \langle f_B(\eta_2)\rangle + \langle f_B(\eta_1)\rangle \langle f_A(\eta_2)\rangle]$$

Production is independent/uncorrelated, unless from the same source. However, even if  $cov_{A,B}(\eta_1, \eta_2) = 0$  we have nontrivial  $C(\eta_1, \eta_2)$  from the fluctuation of  $N_A$  and  $N_B$  [Bzdak+Teaney 2013]

W. Broniowski (IFJ PAN & UJK)

Rapidity fluctuations

Average number of particles:

$$\langle N(\eta) \rangle = \langle N_A \rangle \langle f_A(\eta) \rangle + \langle N_B \rangle \langle f_B(\eta) \rangle$$

Symmetric and antisymmetric parts (see next slide):

$$\langle f_A(\eta) \rangle = f_s(\eta) + f_a(\eta), \quad \langle f_B(\eta) \rangle = f_s(\eta) - f_a(\eta)$$

After elementary transformations  $(N_+ = N_A + N_B, N_- = N_A - N_B)$ 

$$C(\eta_1, \eta_2) = 1 + \frac{1}{\langle N(\eta_1) \rangle \langle N(\eta_2) \rangle} \times \left\{ \langle N_A \rangle \operatorname{cov}_A(\eta_1, \eta_2) + \langle N_B \rangle \operatorname{cov}_B(\eta_1, \eta_2) \right. \\ \left. + \operatorname{var}(N_+) f_s(\eta_1) f_s(\eta_2) + \operatorname{var}(N_-) f_a(\eta_1) f_a(\eta_2) \right. \\ \left. + \left[ \operatorname{var}(N_A) - \operatorname{var}(N_B) \right] \left[ f_s(\eta_1) f_a(\eta_2) + f_a(\eta_1) f_s(\eta_2) \right] \right\}$$

### Białas-Czyż triangles

Charged hadron spectra in d+Au @ RHIC [Białas+Czyż 2004]



$$\langle f_A(\eta) \rangle = h(\eta) \frac{y_b + \eta}{2y_b}, \ \langle f_B(\eta) \rangle = h(\eta) \frac{y_b - \eta}{2y_b}$$

Asymmetric profiles used ever since ....

W. Broniowski (IFJ PAN & UJK)

Rapidity fluctuations

### Observation by Bzdak and Teaney

Vanishing covariance, symmetric system, profiles from the previous slide:

$$\langle N_A \rangle = \langle N_B \rangle, \quad \operatorname{var}(N_A) = \operatorname{var}(N_B)$$

Then

$$C(\eta_1, \eta_2) = 1 + \frac{1}{\langle N_+ \rangle^2} \left[ \operatorname{var}(N_+) + \operatorname{var}(N_-) \frac{\eta_1}{y_b} \frac{\eta_2}{y_b} \right]$$
[Bzdak+Teaney 2013]

Fluctuations of  $N_A$  vs  $N_B$  cause the  $\eta_1\eta_2$  structure in  $C(\eta_1,\eta_2)$ 

## Fluctuating length

- Idea: entropy deposition from wounded nucleons originates from string-like objects whose other end-point is randomly distributed in  $\eta$  (related to [Brodsky+Gunion+Kuhn 1977])
- "Soft particle production in hadronic collisions is dominated by multiple gluon exchanges between partons from the colliding hadrons, followed by radiation of ... partons distributed uniformly in rapidity" [Białas+Jeżabek 2004]
- Torque in p-A collisions (see talk by PB) [PB+WB+Moreira 2011, PB+WB, arXiv:1506.02817]
- Similar ideas in [Monnai+Schenke, arXiv:1509.04103]



What it yields?



$$f_A(\eta; y) = \theta(y < \eta < y_b), \quad f_B(\eta; y) = \theta(-y_b < \eta < -y)$$

(uniform string fragmentation function) Random end y is uniformly selected from  $[-y_b, y_b]$ , where  $y_b$  is the beam rapidity Averaging over  $y \rightarrow$  "triangles":

$$\langle f_{A,B}(\eta) \rangle = \int_{-y_b}^{y_b} \frac{dy}{2y_b} f_{A,B}(\eta;y) = \frac{y_b \pm \eta}{2y_b}$$

Correlations from length fluctuations:

$$\langle f_{A,B}(\eta_1,\eta_2) \rangle = \int_{-y_b}^{y_b} dy f_{A,B}(\eta_1;y) f_{A,B}(\eta_2;y) = \frac{y_b \pm \min(\eta_1,\eta_2)}{2y_b} \\ \operatorname{cov}_{A,B}(\eta_1,\eta_2) = \frac{y_b^2 - \eta_1 \eta_2 - y_b |\eta_1 - \eta_2|}{4y_b^2}$$

Results for C

$$\bar{C}(\eta_1, \eta_2) = \frac{C(\eta_1, \eta_2)}{\int_{-Y}^{Y} \frac{d\eta_1}{2Y} \int_{-Y}^{Y} \frac{d\eta_2}{2Y} C(\eta_1, \eta_2)} \quad \text{(normalization to 1)}$$



Generation of the ridge (structure from  $-|\eta_1 - \eta_2|$ ) Fluctuating length affects both short- and long-range components Results for C $\bar{C}(\eta_1, \eta_2) = \frac{C(\eta_1, \eta_2)}{\int_{-V}^{Y} \frac{d\eta_1}{2V} \int_{-V}^{Y} \frac{d\eta_2}{2V} C(\eta_1, \eta_2)} \quad \text{(normalization to 1)}$  $C(\eta_1, \eta_2)$  Pb+Pb@2.76TeV, 30-40% 1.001 no length fluct. 1.000 with length fluct. 0.999 0.998 -2  $\eta_2$  $\eta_1$ 2

Generation of the ridge (structure from  $-|\eta_1 - \eta_2|$ ) Fluctuating length affects both short- and long-range components

W. Broniowski (IFJ PAN & UJK)

Rapidity fluctuations

OELD 11 / 1



Generation of the ridge (structure from  $-|\eta_1 - \eta_2|$ ) Fluctuating length affects both short- and long-range components

W. Broniowski (IFJ PAN & UJK)

Rapidity fluctuations

OELD 11 / 1



Generation of the saddle in the ridge (seen in experiment) is trivial Fluctuating length essential

W. Broniowski (IFJ PAN & UJK)

OELD 12 / 1



Generation of the saddle in the ridge (seen in experiment) is trivial Fluctuating length essential

W. Broniowski (IFJ PAN & UJK)

OELD 12 / 1



Generation of the saddle in the ridge (seen in experiment) is trivial Fluctuating length essential

W. Broniowski (IFJ PAN & UJK)

OELD 12 / 1

#### C and $C_N$ for p-Pb collisions



문▶ ⊀ 문

< A

$$a_{nn} = \frac{\frac{\operatorname{var}(N_{-})}{\langle N_{+} \rangle} + \frac{\operatorname{var}(\omega)}{\langle \omega \rangle^{2}}}{6\langle N_{+} \rangle} \frac{Y^{2}}{y_{p}^{2}} \delta_{n1} - \frac{\frac{\operatorname{var}(\omega)}{\langle \omega \rangle^{2}} + 1}{6\langle N_{+} \rangle} \frac{Y^{2}}{y_{p}^{2}} \delta_{n1} + \frac{\frac{\operatorname{var}(\omega)}{\langle \omega \rangle^{2}} + 1}{(2n-1)(2n+3)\langle N_{+} \rangle} \frac{Y}{y_{p}}}{a_{n,n+2}} = -\frac{\frac{\operatorname{var}(\omega)}{\langle \omega \rangle^{2}} + 1}{2(2n+3)\sqrt{(2n+1)(2n+5)}\langle N_{+} \rangle} \frac{Y}{y_{p}}} \quad \text{(length fluct.)}$$

other =0

(日) (四) (三) (三) (三)

$$a_{nn} = \frac{\frac{\operatorname{var}(N_{-})}{\langle N_{+} \rangle} + \frac{\operatorname{var}(\omega)}{\langle \omega \rangle^{2}}}{6\langle N_{+} \rangle} \frac{Y^{2}}{y_{p}^{2}} \delta_{n1} - \frac{\frac{\operatorname{var}(\omega)}{\langle \omega \rangle^{2}} + 1}{6\langle N_{+} \rangle} \frac{Y^{2}}{y_{p}^{2}} \delta_{n1} + \frac{\frac{\operatorname{var}(\omega)}{\langle \omega \rangle^{2}} + 1}{(2n-1)(2n+3)\langle N_{+} \rangle} \frac{Y}{y_{p}}}{a_{n,n+2}} = -\frac{\frac{\operatorname{var}(\omega)}{\langle \omega \rangle^{2}} + 1}{2(2n+3)\sqrt{(2n+1)(2n+5)}\langle N_{+} \rangle} \frac{Y}{y_{p}}} \quad \text{(length fluct.)}$$

$$\begin{array}{lll} a_{nm} & = & \int_{-Y}^{Y} \frac{d\eta_1}{Y} \int_{-Y}^{Y} \frac{d\eta_2}{Y} C(\eta_1, \eta_2) T_n\left(\frac{\eta_1}{Y}\right) T_m\left(\frac{\eta_1}{Y}\right) \\ T_n(x) & = & \sqrt{2+1/2} P_n(x) \qquad \text{[Bzdak+Teaney 2013, Jia 2015]} \\ & \text{(play analogous role to flow coefficients in harmonic flow)} \end{array}$$

$$a_{nn} = \frac{\frac{\operatorname{var}(N_{-})}{\langle N_{+} \rangle} + \frac{\operatorname{var}(\omega)}{\langle \omega \rangle^{2}}}{6\langle N_{+} \rangle} \frac{Y^{2}}{y_{p}^{2}} \delta_{n1} - \frac{\frac{\operatorname{var}(\omega)}{\langle \omega \rangle^{2}} + 1}{6\langle N_{+} \rangle} \frac{Y^{2}}{y_{p}^{2}} \delta_{n1} + \frac{\frac{\operatorname{var}(\omega)}{\langle \omega \rangle^{2}} + 1}{(2n-1)(2n+3)\langle N_{+} \rangle} \frac{Y}{y_{p}}}{a_{n,n+2}} = -\frac{\frac{\operatorname{var}(\omega)}{\langle \omega \rangle^{2}} + 1}{2(2n+3)\sqrt{(2n+1)(2n+5)}\langle N_{+} \rangle} \frac{Y}{y_{p}}} \quad \text{(length fluct.)}$$

 $\omega$  – overlaid strength

< 行い

글 🕨 🖌 글

$$\begin{aligned} a_{nm} &= \int_{-Y}^{Y} \frac{d\eta_1}{Y} \int_{-Y}^{Y} \frac{d\eta_2}{Y} C(\eta_1, \eta_2) T_n\left(\frac{\eta_1}{Y}\right) T_m\left(\frac{\eta_1}{Y}\right) \\ T_n(x) &= \sqrt{2 + 1/2} P_n(x) \end{aligned} \qquad \begin{bmatrix} \mathsf{Bzdak+Teaney 2013, Jia 2015} \\ (\mathsf{play analogous role to flow coefficients in harmonic flow) \end{bmatrix}$$

$$a_{nn} = \frac{\frac{\operatorname{var}(N_{-})}{\langle N_{+} \rangle} + \frac{\operatorname{var}(\omega)}{\langle \omega \rangle^{2}}}{6\langle N_{+} \rangle} \frac{Y^{2}}{y_{p}^{2}} \delta_{n1} - \frac{\frac{\operatorname{var}(\omega)}{\langle \omega \rangle^{2}} + 1}{6\langle N_{+} \rangle} \frac{Y^{2}}{y_{p}^{2}} \delta_{n1} + \frac{\frac{\operatorname{var}(\omega)}{\langle \omega \rangle^{2}} + 1}{(2n-1)(2n+3)\langle N_{+} \rangle} \frac{Y}{y_{p}}}{a_{n,n+2}} = -\frac{\frac{\operatorname{var}(\omega)}{\langle \omega \rangle^{2}} + 1}{2(2n+3)\sqrt{(2n+1)(2n+5)}\langle N_{+} \rangle} \frac{Y}{y_{p}}} \quad \text{(length fluct.)}$$

 $a_{nm} \sim 1/\langle N_+ 
angle$ – fall-off similarly as in experiment if  $N_+ \sim N_{
m ch}$ 

W. Broniowski (IFJ PAN & UJK)

OELD 14 / 1

. . . . . . .

$$a_{nn} = \frac{\frac{\operatorname{var}(N_{-})}{\langle N_{+} \rangle} + \frac{\operatorname{var}(\omega)}{\langle \omega \rangle^{2}}}{6\langle N_{+} \rangle} \frac{Y^{2}}{y_{p}^{2}} \delta_{n1} - \frac{\frac{\operatorname{var}(\omega)}{\langle \omega \rangle^{2}} + 1}{6\langle N_{+} \rangle} \frac{Y^{2}}{y_{p}^{2}} \delta_{n1} + \frac{\frac{\operatorname{var}(\omega)}{\langle \omega \rangle^{2}} + 1}{(2n-1)(2n+3)\langle N_{+} \rangle} \frac{Y}{y_{p}}}{a_{n,n+2}} = -\frac{\frac{\operatorname{var}(\omega)}{\langle \omega \rangle^{2}} + 1}{2(2n+3)\sqrt{(2n+1)(2n+5)}\langle N_{+} \rangle} \frac{Y}{y_{p}}} \quad \text{(length fluct.)}$$

$$\begin{array}{lll} a_{nm} & = & \displaystyle \int_{-Y}^{Y} \frac{d\eta_1}{Y} \int_{-Y}^{Y} \frac{d\eta_2}{Y} C(\eta_1, \eta_2) T_n\left(\frac{\eta_1}{Y}\right) T_m\left(\frac{\eta_1}{Y}\right) \\ T_n(x) & = & \displaystyle \sqrt{2+1/2} P_n(x) \qquad \qquad \mbox{[Bzdak+Teaney 2013, Jia 2015]} \\ & & \mbox{(play analogous role to flow coefficients in harmonic flow)} \end{array}$$

$$a_{nn} = \frac{\frac{\operatorname{var}(N_{-})}{\langle N_{+} \rangle} + \frac{\operatorname{var}(\omega)}{\langle \omega \rangle^{2}}}{6\langle N_{+} \rangle} \frac{Y^{2}}{y_{p}^{2}} \delta_{n1} - \frac{\frac{\operatorname{var}(\omega)}{\langle \omega \rangle^{2}} + 1}{6\langle N_{+} \rangle} \frac{Y^{2}}{y_{p}^{2}} \delta_{n1} + \frac{\frac{\operatorname{var}(\omega)}{\langle \omega \rangle^{2}} + 1}{(2n-1)(2n+3)\langle N_{+} \rangle} \frac{Y}{y_{p}}}{a_{n,n+2}} = -\frac{\frac{\operatorname{var}(\omega)}{\langle \omega \rangle^{2}} + 1}{2(2n+3)\sqrt{(2n+1)(2n+5)}\langle N_{+} \rangle} \frac{Y}{y_{p}}} \quad \text{(length fluct.)}$$

## Results for $a_{nm}$

Pb-Pb@2.76TeV



# Results for $a_{nm}$

#### Pb-Pb@2.76TeV



(filled - from Fig. 7 of ATLAS-CONF-2015-020, open - model)

 $\begin{array}{l} a_{11} \mbox{ fitted by adjusting:} \\ N_{\rm ch} = 4.7 N_+ \mbox{ for Pb-Pb}@2.76 {\rm TeV} \longrightarrow dN_{\rm ch}/d\eta \simeq 1 \times N_+ \\ ({\rm exp:} \ dN_{\rm ch}/d\eta \simeq (3-4) \times N_W \mbox{ with wounded nucleons} \rightarrow {\rm need more participants,} \\ & \mbox{ wounded constituents? - used in [Monnai+Schenke 2015])} \\ N_{\rm ch} = 5.1 N_A \mbox{ for p-Pb}@5.02 {\rm TeV} \\ N_{\rm ch} = 8.1 N_+ \mbox{ for p-p}@13 {\rm TeV} - {\rm requires \ sources \ at \ partonic \ level} \end{array}$ 

## Smearing of $\eta$

There is some "communication" between adjacent bins in  $\eta,$  so the mapping  $\eta_S \to \eta$  should be smeared

Gaussian smearing:

$$S_{\rm sm}(\eta_1, \eta_2) = \int d\eta_1' \int d\eta_2' g(\eta_1, \eta_1') g(\eta_2, \eta_2') S(\eta_1', \eta_2')$$

$$|\eta_1 - \eta_2| \rightarrow \frac{2\sigma_\eta e^{-\frac{(\eta_1 - \eta_2)^2}{4\sigma_\eta^2}}}{\sqrt{\pi}} + (\eta_1 - \eta_2) \operatorname{erf}\left(\frac{\eta_1 - \eta_2}{2\sigma_\eta}\right)$$

with the other terms ( $\eta_1$ ,  $\eta_2$ ,  $\eta_1\eta_2$ ) unchanged ( $\rightarrow$  Bzdak-Teaney term not modified)

⊒⊳ ∢∃⊳

## Smearing of $\eta$

 $(\rightarrow$  Bzdak-Teaney term not modified)



≣ ► ≣ ৩৭৫ OELD 16/1

∃ → ( ∃ →

< 行い

# Smearing of $\eta$

#### $(\rightarrow$ Bzdak-Teaney term not modified)



OELD 16 /

▶ ∢ ≣

#### From initial state to final hadrons

- $\eta_S 
  ightarrow \eta$ , some extra hydro push ightarrow reduction of  $a_{nm}$
- resonance decays relevant effect [PB+WB+Adam Olszewski 2015]  $\rightarrow$  increase
- charge conservation, other correlations not included
- removal of the short-range effects (not simple in a model)

#### From initial state to final hadrons

- $\eta_S 
  ightarrow \eta_s$  some extra hydro push ightarrow reduction of  $a_{nm}$
- resonance decays relevant effect [PB+WB+Adam Olszewski 2015]  $\rightarrow$  increase
- charge conservation, other correlations not included
- removal of the short-range effects (not simple in a model)

Hydro (no length fluctuations included here):

 $C_{N}(\eta_{1},\eta_{2})$  c=30-40%



## Conclusions

- Data: need for extra longitudinal fluctuations in the initial state (see also the talk by Piotr Bożek: torque in p+Pb)
- Analytic expressions in the simple model, grasping features of more involved approaches
- The correlations from the early production mechanism contribute to shortand long-range components in  $\eta$ , no flow needed – robust effect
- Anatomy:
  - Early stage:  $N_A$  vs  $N_B$  fluctuations, length fluctuations
  - Late stage: resonance decays (significant effect), conservation laws
- $1/\sqrt{N_{ch}}$  scaling of  $\sqrt{a_{11}} \rightarrow \text{linear relation } N_{ch} = \kappa N_{\text{sources}}$ , with the value of  $\kappa$  suggesting wounded constituents as degrees of freedom
- Universality of Pb-Pb, p-Pb, and (possibly) p-p can be understood if sources are partons (e.g., wounded quarks)
- Intricate procedures:  $C \rightarrow C_N$ , normalization, removing short-range correlations necessary in modeling for 1-1 comparisons
- To eliminate short-range component use the cumulant method [Bzdak+Bożek 2015]

W. Broniowski (IFJ PAN & UJK)