Rapidity-dependent chemical potentials in statistical approach*

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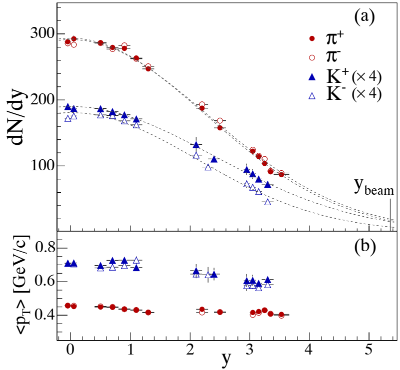
Outline

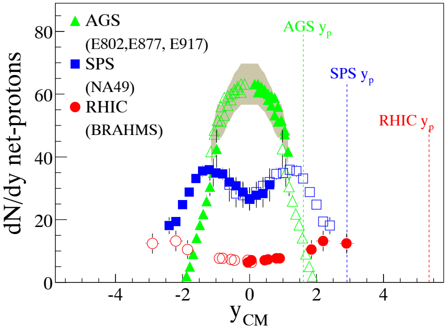
Goal:

obtain the topography of the fireball

1. Experimental facts
2. The model
   - Geometry and kinematics
   - The single freeze-out model
3. Results
   - Fit
   - Rapidity spectra
   - p_T-spectra
   - Some predictions

Typical data





This talk:

all data from BRAHMS (M. Murray, P. Staszel this morning)

4π vs. midrapidity

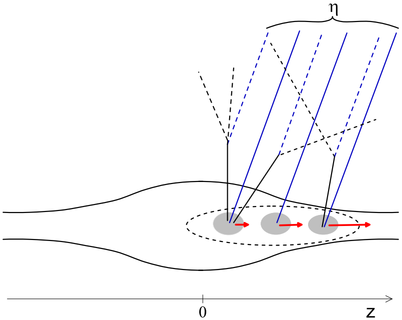
Up to now two basic categories of calculations:

- (1) 4π studies at low energies (SIS, AGS), $N_i = V \int d^3p\, f_i(\sqrt{m_i^2 + p^2}; T, \mu' s, \gamma' s)$
- (2) Studies at mid-rapidity for approximately boost-invariant systems at highest energies (RHIC) at $|y| < 1$
- Obvious fact from the boost symmetry (*e.g.* WB+Florkowski, PRL **87** (2001) 272302)

$$\frac{dN_i/dy}{dN_j/dy} = \frac{\int dy\, dN_i/dy}{\int dy\, dN_j/dy} = \frac{N_i}{N_j}.$$

- Inclusion of resonance decays simple in both above approaches
- Cooper-Frye formula → spectra $dN/(2\pi p_T dp_T dy)$ at mid-rapidity

Geometry and kinematics



- Boost non-invariant system
- Particles with the same pseudorapidity η originate from different regions
- Thermal conditions and flow in these regions are different

Boost-noninvariant calculation

- THERMINATOR [A. Kisiel, T. Tałuć, WB, WF, Comput.Phys.Commun. **174** (2006) 669-687] → Monte Carlo
- Choice of the shape of the freeze-out hypersurface Σ and collective expansion
- Dependence of thermal parameters on the position within Σ
- Parameters are fitted independently to various combinations of the data, reducing freedom

Result:

"topography" of the fireball, which forms the ground for other studies

Assumptions

1. At a certain stage thermal equilibrium between hadrons occurs (probably born that way)
2. The parameters: T, μ_B, μ_S, and μ_{I_3}. In a boost-non-invariant model these parameters depend on the position
3. The shape of the fireball is nontrivial in the longitudinal direction
4. Hubble flow $\longrightarrow$ longitudinal and transverse flow. Again, in the boost-non-invariant model the form of the velocity field may depend on the longitudinal position
5. The evolution after freeze-out includes decays of (all) resonances which may proceed in cascades
6. Elastic rescattering after the chemical freeze-out is ignored (approximation)

Hypersurface and flow

(many possibilities!)

$$x^\mu = \begin{pmatrix} t \\ x \\ y \\ z \end{pmatrix} = \begin{pmatrix} \tau \cosh\alpha_\perp \cosh\alpha_\parallel \\ \tau \sinh\alpha_\perp \cos\phi \\ \tau \sinh\alpha_\perp \sin\phi \\ \tau \cosh\alpha_\perp \sinh\alpha_\parallel \end{pmatrix}.$$

$\alpha_\parallel$ - *spatial rapidity*, $\alpha_\perp$ - transverse rapidity

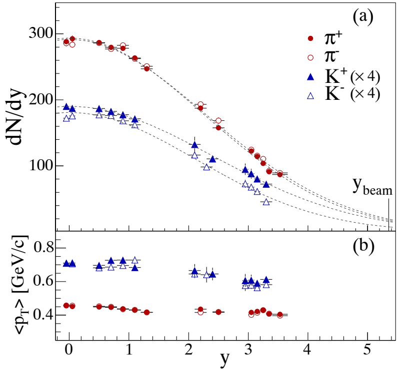
$$\alpha_\parallel = \frac{1}{2}\log\frac{t+z}{t-z}, \quad \rho = \sqrt{x^2+y^2} = \tau \sinh\alpha_\perp$$

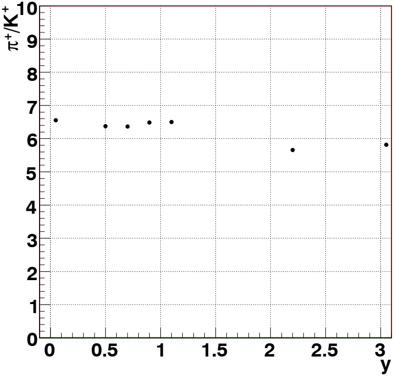
The four-velocity follows the Hubble law

$$u^\mu = x^\mu/\tau.$$

The longitudinal flow $v_z = \tanh\alpha_\parallel = z/t$ as in the Bjorken model, the transverse flow (at $z=0$) is $v_\rho = \tanh\alpha_\perp = \rho/\sqrt{1+\rho^2/\tau^2}$.

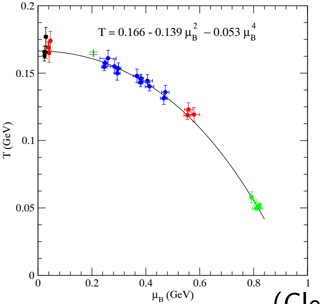
Cooler or thinner?





Yields drop with $y \to$ (1) decrease the transverse size with $|y|$, or (2) decrease T, or both. BRAHMS: $(dN_\pi/dy)/(dN_K/dy)$ is, within a few %, independent of $y \to T \sim$ const., and we must take (1)!

Approximate constancy of T at BRAHMS



(Cleymans *et al.*, 2006)

- The *universal freeze-out curve* gives from $\mu_B = 0$ to 250 MeV a slowly-varying value of T. We take $T = 165$ MeV.
- At larger rapidity and/or lower collision energies, T does depend on $\alpha_\parallel$ and decreases towards the fragmentation region, where $T \sim 0$ and $\mu_B \sim 1$ GeV

The farther, the thinner!

A new element in this work:

$$0 \le \alpha_\perp \le \alpha_\perp^{\max}(\alpha_\parallel) = \alpha_\perp^{\max}(0) \exp\left(-\frac{\alpha_\parallel^2}{2\Delta^2}\right).$$

- As we depart from the center by increasing $|\alpha_\parallel|$, we reduce $\alpha_\perp$, or $\rho_{\max}$. The rate of this reduction is controlled by a new model parameter, Δ. The farther, the thinner!
- We admit the dependence of chemical potentials on the spatial rapidity, necessary to describe the increasing density of baryon number towards the fragmentation region: $\mu_i(\alpha_\parallel) = \mu_i(0)\left[1 + A_i \alpha_\parallel^{2.4}\right]$

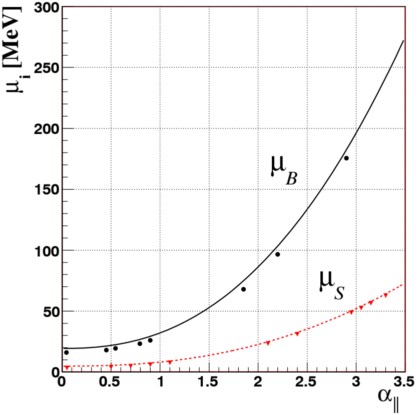
Fitting strategy

- $\tau = 9.74$ fm, $\rho_{\rm max}(z=0) = 7.74$ fm (earlier fits)
- The Δ parameter is fixed with the pion rapidity spectra $dN_{\pi^\pm}/dy$, with the optimum value $\Delta = 3.33$
- For a given set of parameters we generate THERMINATOR events
- First optimize $\mu_B(0)$ and A_B with the experimental $p/\bar{p}$ rapidity dependence
- Then fix $\mu_S(0)$ and A_S using K^+/K^-
- Iterate two above items until a fixed point is reached
- $\mu_{I_3}(0)$ and A_{I_3} are consistent with zero and thus irrelevant

Result:

$$\mu_B(0) = 19 \text{ MeV},\ \mu_S(0) = 4.8 \text{ MeV},\ A_B = 0.65,\ A_S = 0.70$$

The farther, the denser!

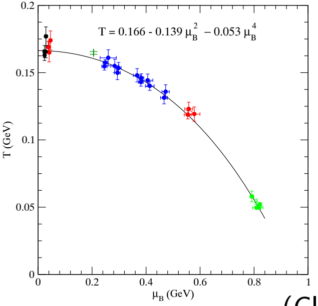


Lines - parameterization: $\mu_i(\alpha_\parallel) = \mu_i(0)\left[1 + A_i \alpha_\parallel^{2.4}\right]$

Points - approximate result: $\frac{p}{\bar{p}} \simeq \exp(2\beta\mu_B)$, $\frac{K^+}{K^-} \simeq \exp(2\beta\mu_S)$

μ_B at large rapidities

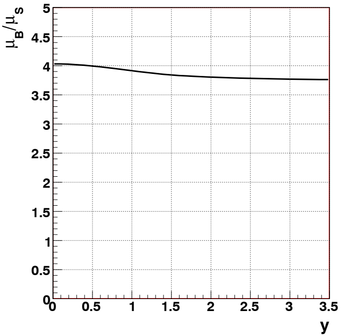
- At $\alpha_{\|} = 3$ we have μ_B around 200 MeV, more than 10 times larger than at the origin – comparable to the highest-energy SPS fit, where $\mu_B \simeq 230$ MeV



(Cleymans *et al.*, 2006)

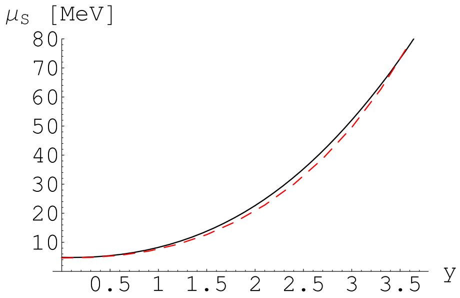
μ_B/μ_S

- $\mu_B(\alpha_\parallel)/\mu_S(\alpha_\parallel)$ is very close to a constant, $\simeq 4 - 3.5$



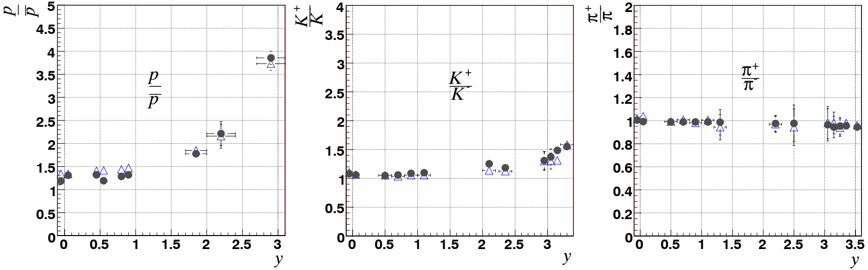
Zero strangeness density

- Results consistent with zero strangeness density



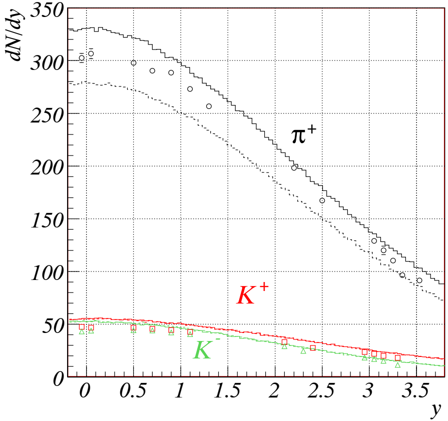
solid – μ_S from the fit to the data
dashed – from the condition of zero local strangeness density

Ratios



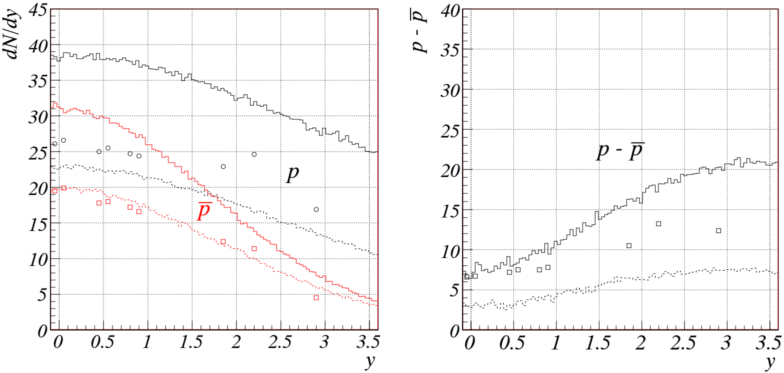
triangles – BRAHMS data
dots – model with fitted dependence of μ's on $\alpha_\perp$

π and K

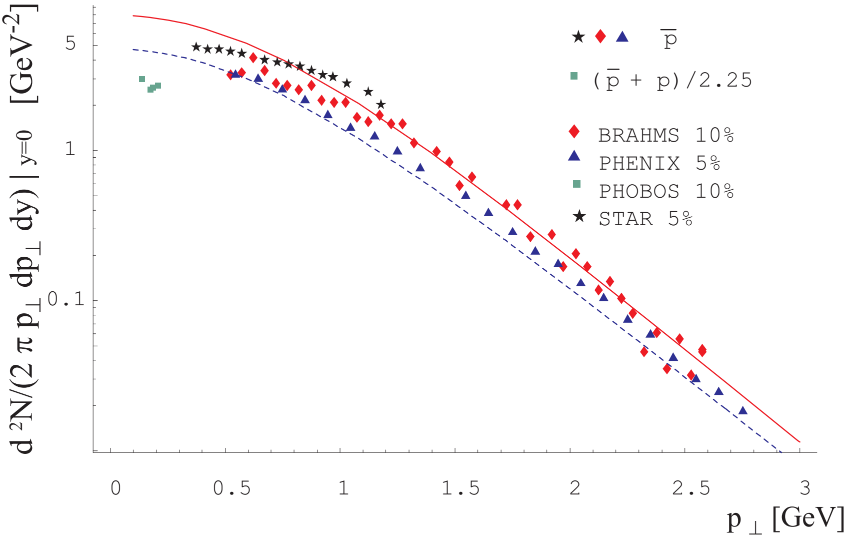


π^+ with full feeding from the weak decays (upper curve) and without the feeding (lower curve), kaons with full feeding, the data for pions corrected for the weak decays

p and $\bar{p}$

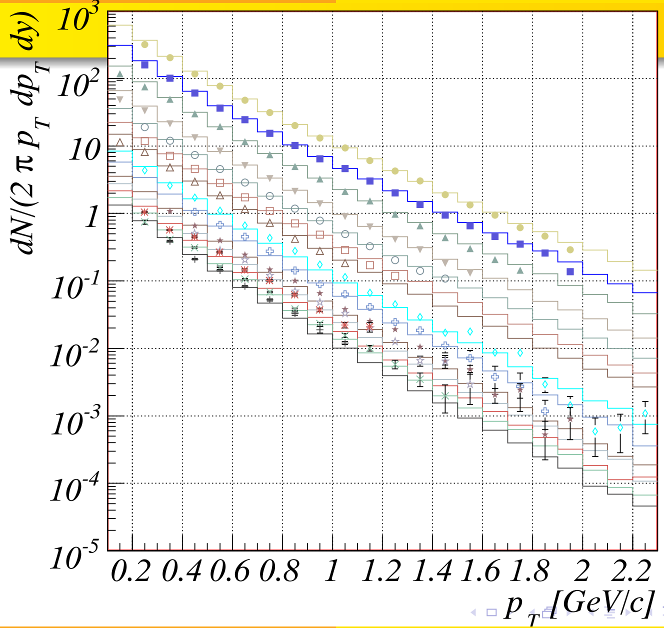


Left: full feeding from the weak decays (black curves) and no feeding (red curves) - experimental points are without feeding. Potential problem with baryon stopping (poorly understood)

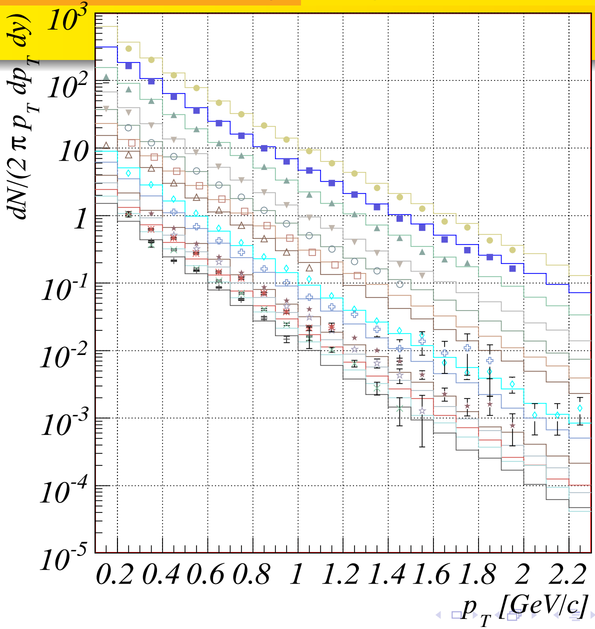


$\bar{p}$ @ 200 GeV compiled by Anna Baran, Ph.D. thesis, 2004

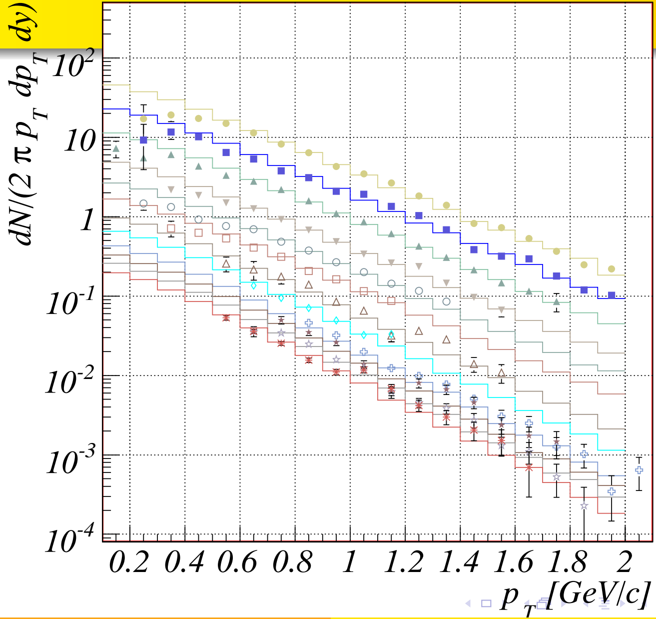
π^+


dN/(2 π p_T dp_T dy)
10^3
10^2
10
1
10^-1
10^-2
10^-3
10^-4
10^-5
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
2.2
p_T [GeV/c]

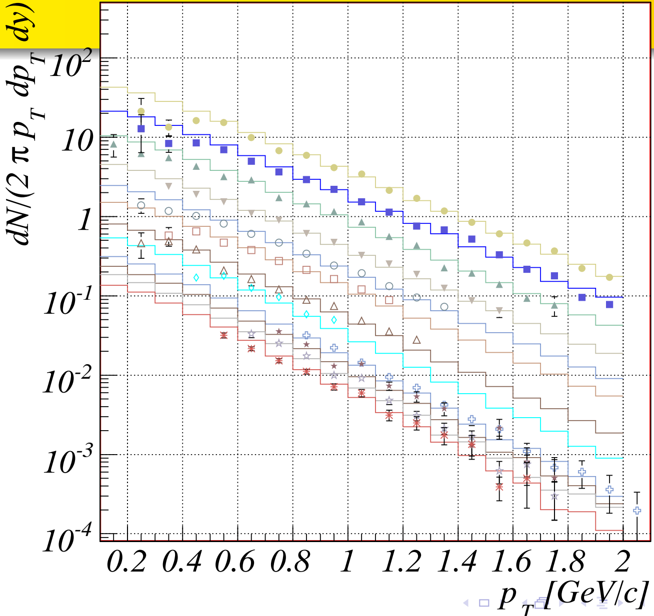
π^-


dN/(2 π p_T dp_T dy)
10^3
10^2
10
1
10^-1
10^-2
10^-3
10^-4
10^-5
0.2 0.4 0.6 0.8 1 1.2 1.4 1.6 1.8 2 2.2
p_T [GeV/c]

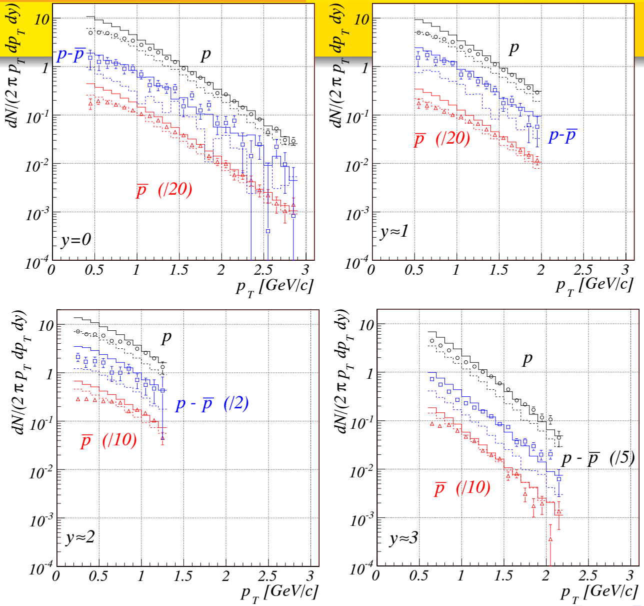
K^+


dN/(2 π p_T dp_T dy)
10^2
10
1
10^-1
10^-2
10^-3
10^-4
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
p_T [GeV/c]

K^-

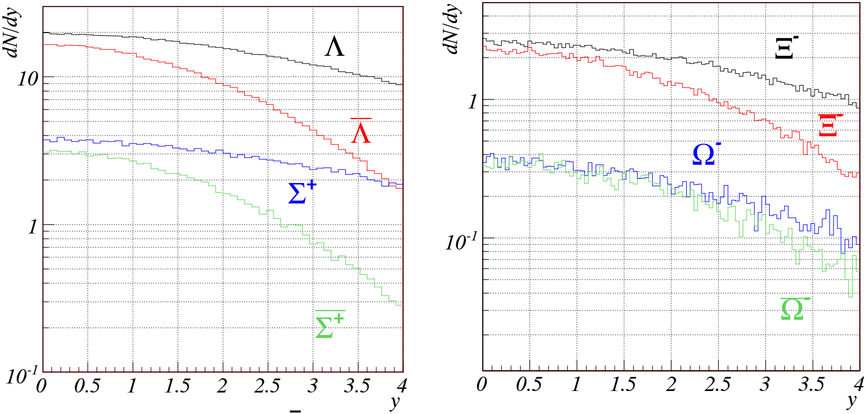

$dN/(2\pi p_T\, dp_T\, dy)$
10^2
10
1
10^{-1}
10^{-2}
10^{-3}
10^{-4}
0.2 0.4 0.6 0.8 1 1.2 1.4 1.6 1.8 2
p_T [GeV/c]

p and $\bar{p}$



Hyperons

We have accomplished the goal of fixing the fireball topography!



Small splitting of Ω and $\bar{\Omega}$

Summary

- Although for $y \neq 0$ one should run the full simulation, the naive extraction of μ_B and μ_S from $p/\bar{p}$ and K^+/K^- works surprisingly well at RHIC, $\frac{p}{\bar{p}} \simeq \exp(2\beta\mu_B)$, *etc*
- $\mu_B$and μ_S grow with $\alpha_\parallel$, reaching at $y \sim 3$ values close to those of the highest SPS energies (D. Roehrich at Florence, M. Murray, J. Cleymans here), or: chemical potentials follow the universal freeze-out curve in the fireball
- At mid-rapidity the values of $\mu' s$ are somewhat lower than derived from the previous thermal fits to the data averaging over $|y| \leq 1$, with our values taking $\mu_B(0) = 19\ \mathrm{MeV}$ and $\mu_S(0) = 5\ \mathrm{MeV}$
- The local strangeness density in the fireball is compatible with zero at all values of $\alpha_\parallel$
- μ_B/μ_S varies very weakly with rapidity, ranging from ~ 4 at midrapidity to ~ 3.5 at larger rapidities

Summary 2

- The $d^2N/(2\pi p_\perp dp_\perp dy)$ spectra of pions and kaons are well reproduced
- The rapidity shape of the spectra of p and $\bar{p}$ is described properly, while the model overpredicts the yields by about 50%. This suggests perhaps a lower value of T at increased rapidity, presence of the Rafelski γ factors, or non-thermal mechanisms behing the baryon stopping (data have systematic uncertainty)
- Increasing yield of the net protons with rapidity is obtained naturally, explaining the shape of the rapidity dependence on purely statistical grounds
- Study of NA49 data under way
- Many ways of modelling boost-noninvariant systems: cooler, thinner, more dilute, HBT data will be useful