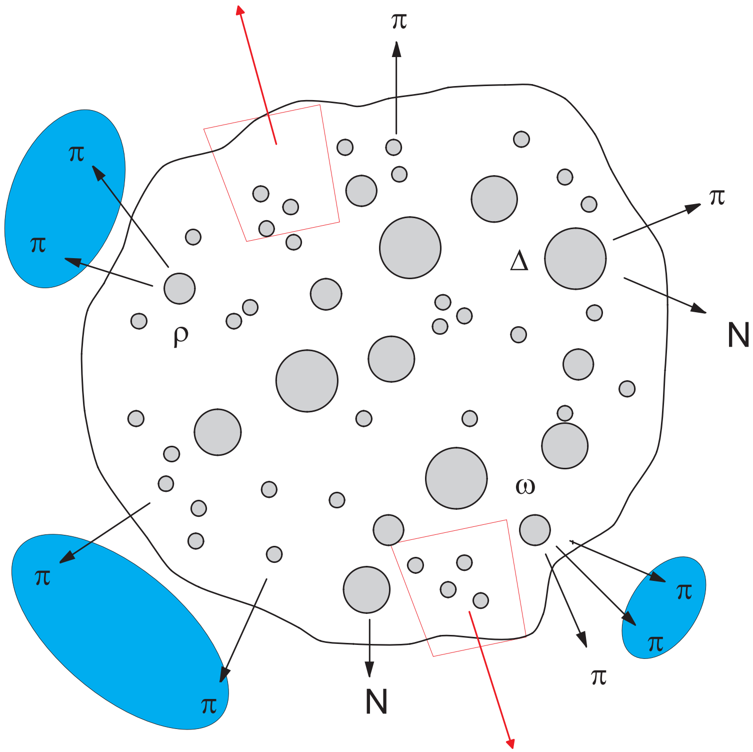
BALANCE FUNCTIONS IN A THERMAL MODEL WITH RESONANCES

Wojciech Broniowski
Institute of Nuclear Physics, PAS, Cracow, Poland

Piotr Bożek, Wojciech Florkowski, Anna Baran



Concept of balance functions

S. Bass, P. Danielewicz, and S. Pratt, PRL 85 (2000) 2689
S. Jeon and V. Koch, hep-ph/0304012
A. Białas and V. Koch, Phys. Lett. B456 (1999) 1
P. Bożek, W. Broniowski, and W. Florkowski, nucl-th/0310062

$$B(\delta, Y) = \frac{1}{2}\left\{\frac{\langle N_{+-}(\delta)\rangle - \langle N_{++}(\delta)\rangle}{\langle N_+\rangle} + \frac{\langle N_{-+}(\delta)\rangle - \langle N_{--}(\delta)\rangle}{\langle N_-\rangle}\right\}$$

N_{+-} and N_{-+} – number of the unlike-sign pairs
N_{++} and N_{--} – number of the like-sign pairs

The two members of the pair fall into the rapidity window Y, with relative rapidity

$$\delta = \Delta y = |y_2 - y_1|$$

N_+ (N_-) – number of positive (negative) particles in the interval Y

Relation to charge fluctuations

Asakawa, Heinz, and Müller, PRL 85 (2000) 2072, Jeon and Koch, PRL 85 (2000) 2076

After integration over δ

$$\int_0^Y d\delta\, B(\delta, Y) = \frac{1}{2}\left\{\frac{\langle N_+ N_-\rangle - \langle N_+(N_+ - 1)\rangle}{\langle N_+\rangle} + (+ \to -)\right\}$$

charge: $Q = N_+ - N_-$, multiplicity of charged particles: $N_{\rm ch} = N_+ + N_-$

$$\frac{\langle (Q - \langle Q\rangle)^2\rangle}{\langle N_{\rm ch}\rangle} = 1 - \int_0^Y d\delta\, B(\delta, Y)$$

For sufficiently large Y we have $\int_0^Y d\delta\, B(\delta, Y) = 1$

Physical significance

The width of B in δ gives info about the hadronization time

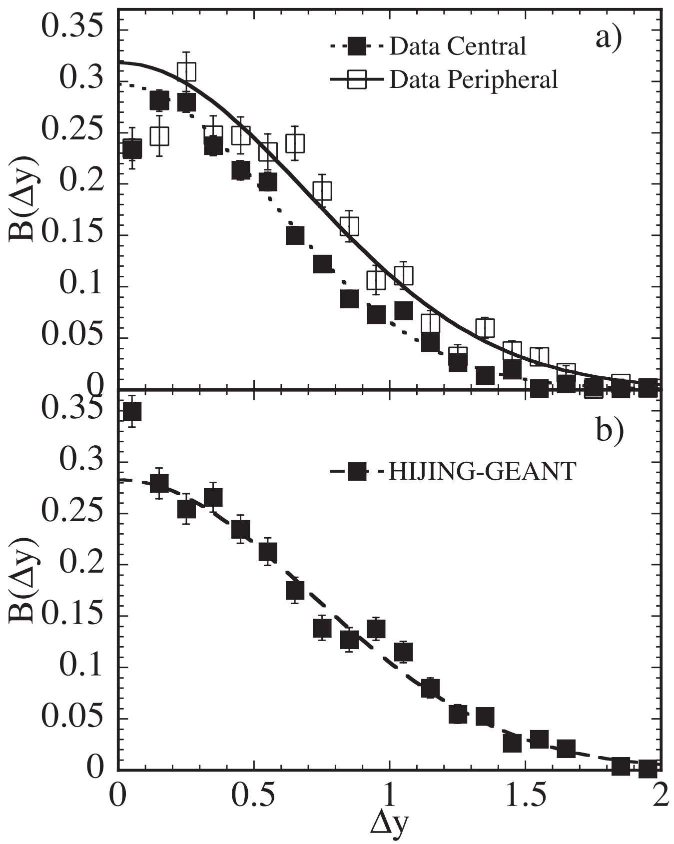
small width $\equiv$ late-stage hadronization
large width $\equiv$ production of hadrons at early stage

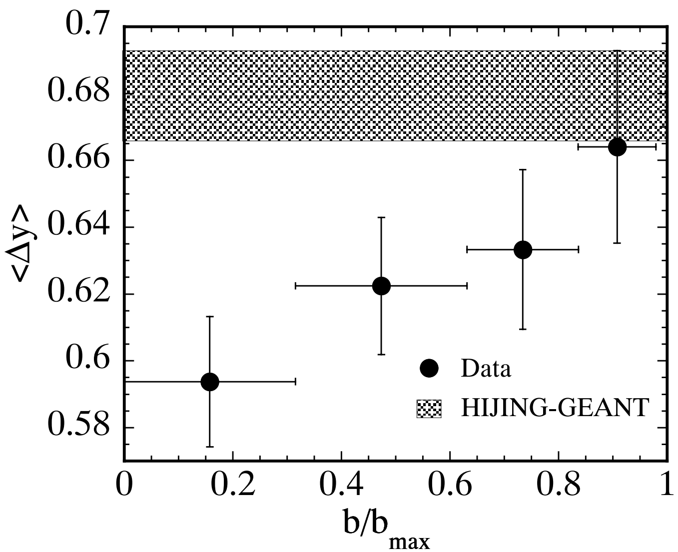
Typical scales: 1 unit of rapidity (for widths in δ), 1 fm/c (for time)

Through the balance functions we acquire insight into dynamics of hadronization

Subtraction of $++$ pairs effectively removes the uncorrelated $+-$ pairs from the distribution

Balance functions measured by STAR





$B(\delta)$ and its width for identified charged pions, $\Delta y \equiv \delta$, b – impact parameter [J. Adams et al., STAR Collaboration, PRL 90 (2003) 172301]

Specific features of our approach

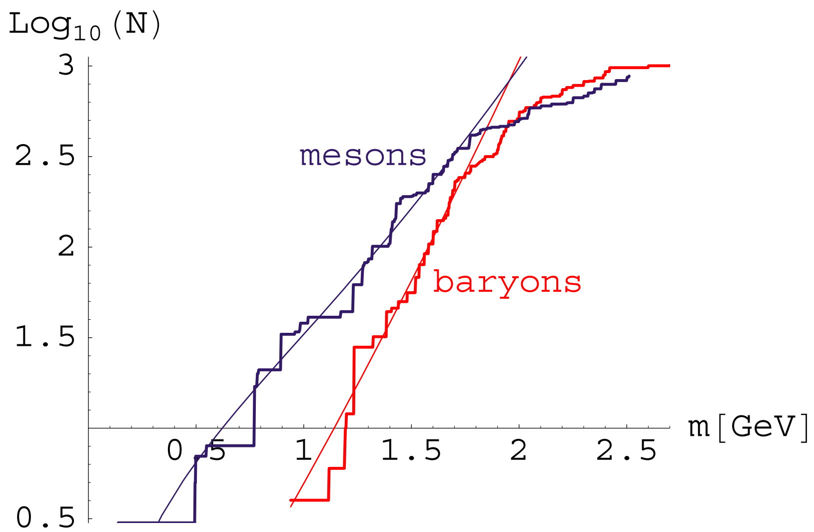
Koppe (1948), Fermi (1950), Landau, Hagedorn, Rafelski, Letessier, Bjorken, Heinz, Sollfrank, Wiedemann, Shuryak, Teaney, Gaździcki, Gorenstein, Bugaev, Braun-Munzinger, Stachel, Redlich, Cleymans, Magestro, Csörgő, Becattini, Hirano, ... (many more)

1. Single freezout approximation: $T_{\rm chem} = T_{\rm kin} \equiv T$, single freeze-out. A radical simplification, supported by the RHIC HBT results: $R_{\rm out}/R_{\rm side} \sim 1$, $R_{\rm side}(\phi)$ has out-of-plane elongation, resonances seen abundantly $\to$ short time between the freeze-outs (explosive scenario).

T and μ_B are fitted from ratios of dN/dy at midrapidity

2. Ockham rasor: No γ-factors for strangeness (Rafelski), excluded-volume effects (Gorenstein), canonical (Redlich) or microcanonical ensemble (Becattini)

3. Hagedorn: Complete treatment of resonances (important due to the Hagedorn-like exponential growth of the number of states)

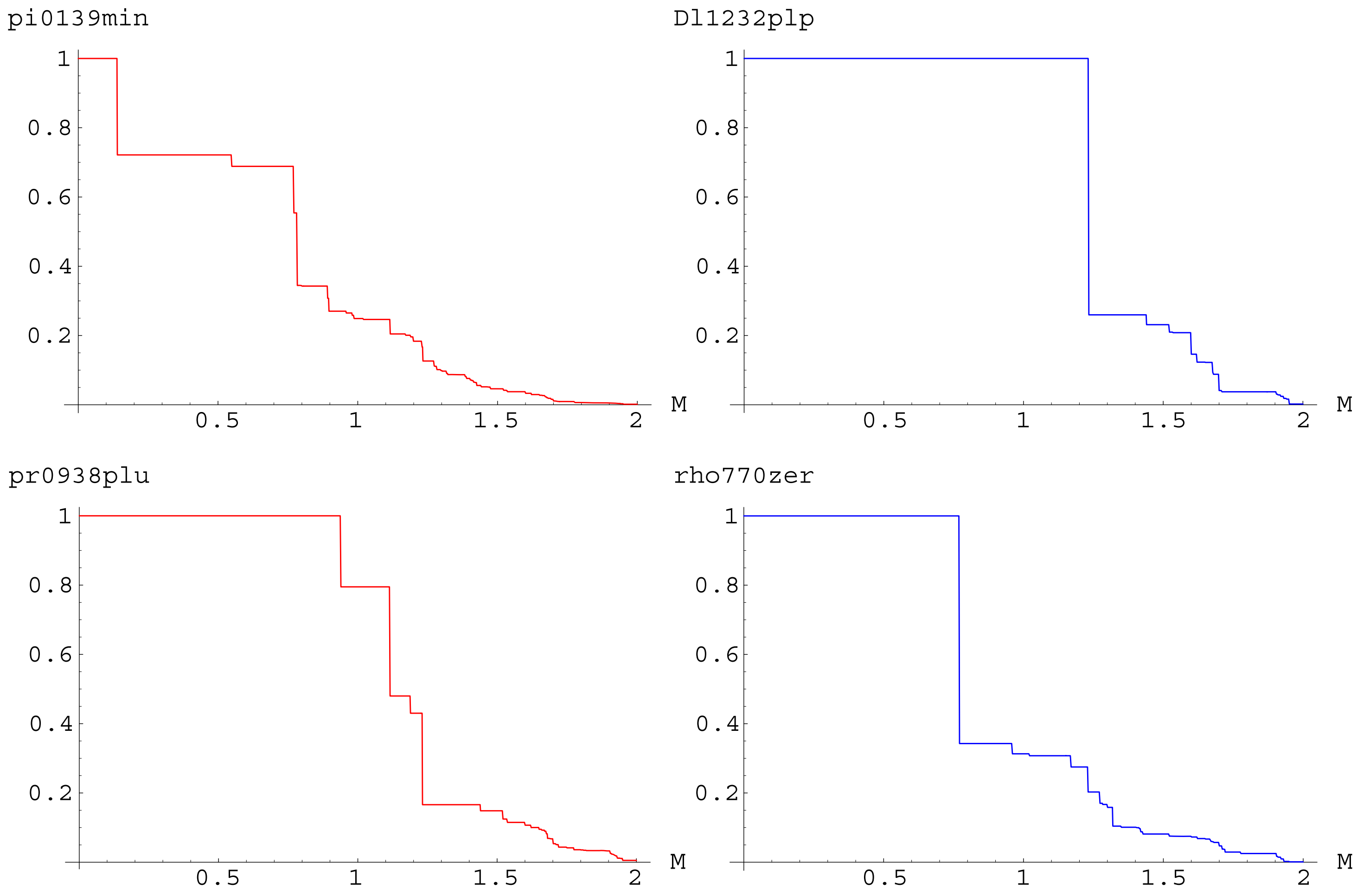


(from WB+WF, PLB 490 (2000) 223)

372 light-flavor (u, d, s) particles, ∼1500 DOF, ∼1800 decay channels! →

∼75% of pions and protons come from decays of higher states, 80% of Λ's, 60% of Ξ's, 30% of ρ_0's, . . . !

Accumulation of π^-,Δ^{++}, p, and ρ_0



4. Geometry and flow: We take the hypersurface (inspired by Bjorken and Buda-Lund models) of the form

$$\tau = \sqrt{t^2 - r_z^2 - r_x^2 - r_y^2} = \text{const}$$

and constrain the transverse size, $\rho = \sqrt{r_x^2 + r_y^2} < \rho_{\max}$. The geometric parameters τ and $\rho_{\max}$, of the order of a few fm, are fitted to the $p_\perp$-spectra (τ^3 is the overall normalization constant, $\rho_{\max}$ controls the slopes). The hydrodynamic four-velocity is (Hubble law)

$$u^\mu = \partial^\mu \tau = \frac{x^\mu}{\tau} = \frac{t}{\tau}\left(1, \frac{r_z}{t}, \frac{r_x}{t}, \frac{r_y}{t}\right)$$

Boost invariance is a good approximation for midrapidity

Other choices can be tested (Heinz+Sollfrank+Wiedemann, Torrieri+Rafelski) (*e.g.* blast wave)

Altogether 4 parameters

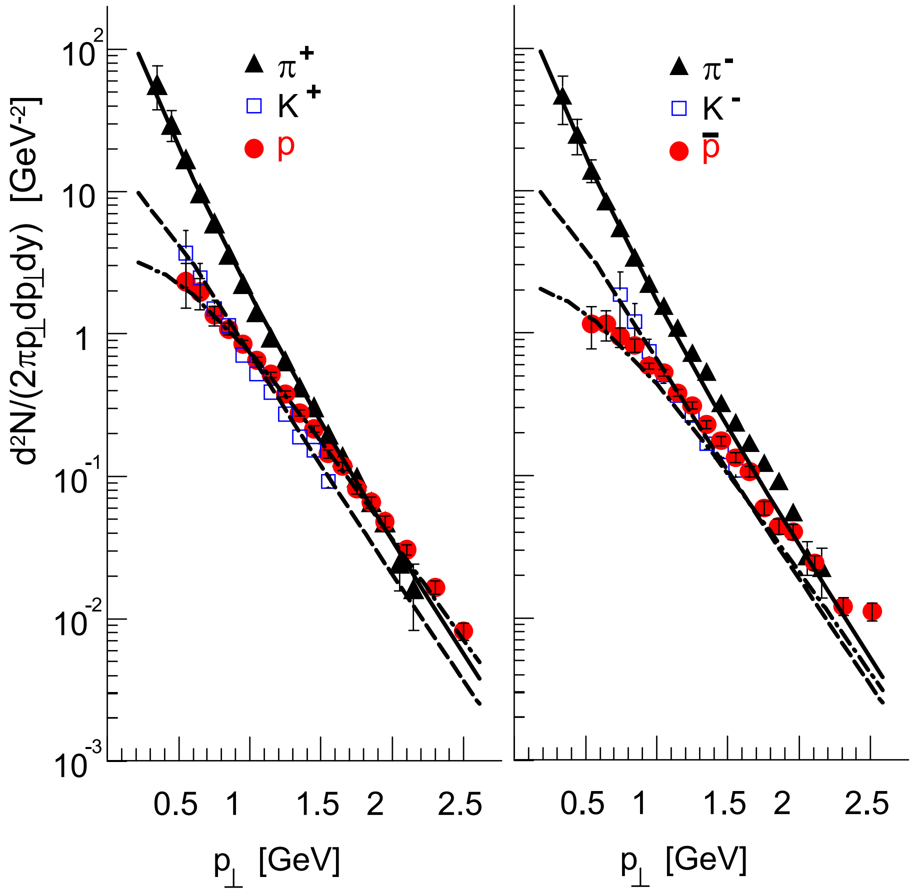
Ratios

For a boost-invariant model $\frac{dN_i/dy}{dN_j/dy} = \frac{N_i}{N_j}$ and ratios do not depend on geometry/flow.

$\sqrt{s_{NN}}$ [GeV]	130	200
T [MeV]	165 $\pm$ 7	160$\pm$5
μ_B [MeV]	41 $\pm$ 5	26$\pm$4
χ^2/DOF	1.0	1.5

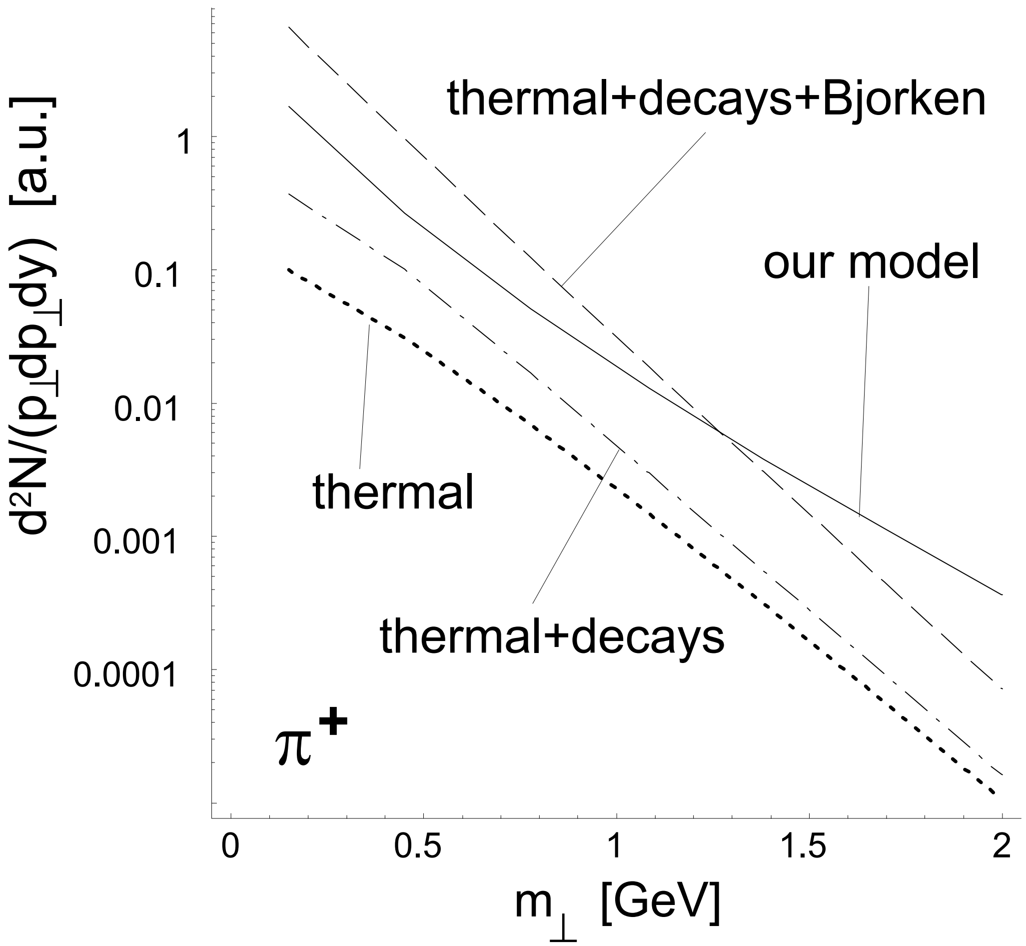
200 GeV A	Model	Experiment
π^-/π^+	1.009 ± 0.003	$1.025 \pm 0.006 \pm 0.018$ $1.02 \pm 0.02 \pm 0.10$
K^-/K^+	0.939 ± 0.008	$0.95 \pm 0.03 \pm 0.03$ $0.92 \pm 0.03 \pm 0.10$
$\overline{p}/p$	0.74 ± 0.04	$0.73 \pm 0.02 \pm 0.03$ $0.70 \pm 0.04 \pm 0.10$ 0.78 ± 0.05
$\overline{p}/\pi^-$	0.104 ± 0.010	0.083 ± 0.015
K^-/π^-	0.174 ± 0.001	0.156 ± 0.020
$\Omega/h^- \quad \times 10^3$	0.990 ± 0.120	$0.887 \pm 0.111 \pm 0.133$
$\overline{\Omega}/h^- \quad \times 10^3$	0.900 ± 0.124	$0.935 \pm 0.105 \pm 0.140$

Results for the transverse-momentum spectra

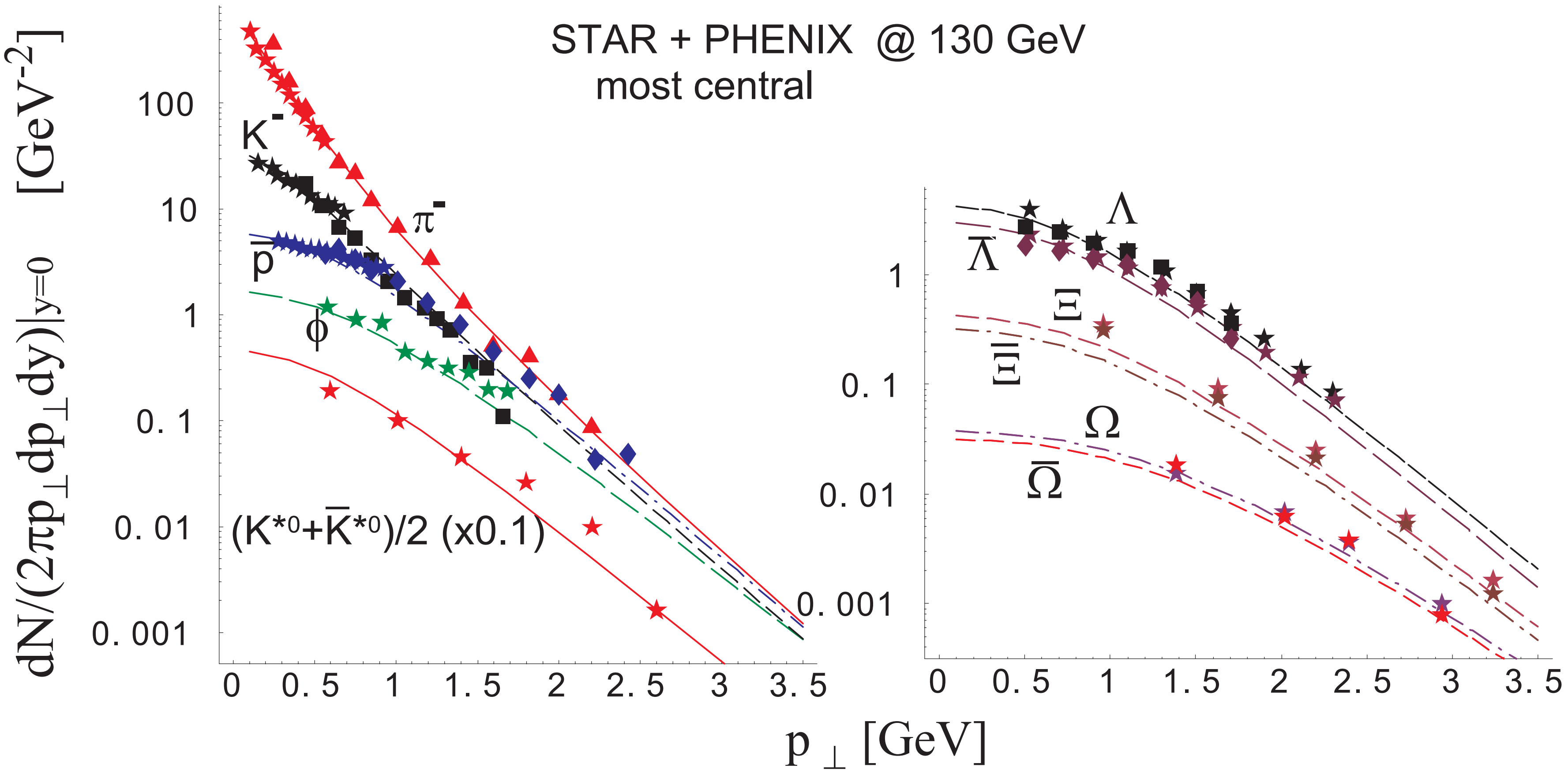


Min. bias $p_\perp$-spectra of pions, kaons, protons and antiprotons as evaluated from our model with $\tau = 6$ fm, $\rho_{\max}/\tau = 0.76$, compared to the earliest PHENIX data (Velkovska, nucl-ex/0105012). Very good agreement up to $p_\perp \sim 2\,\mathrm{GeV}$. At larger values, where hard processes enter, the model falls below the data

“Cooling” via decays



Resonance decays lower the inverse slope by about 30 MeV



($T = 165$ MeV)

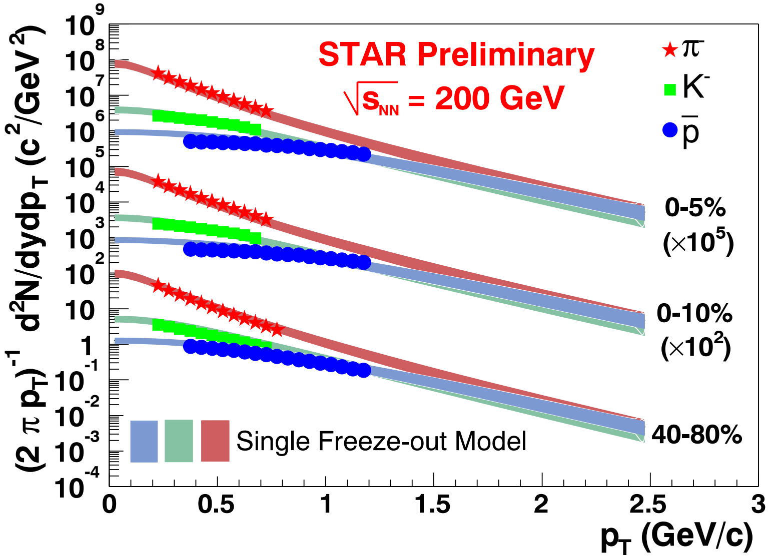
ϕ – very weak interactions, serves as a thermometer

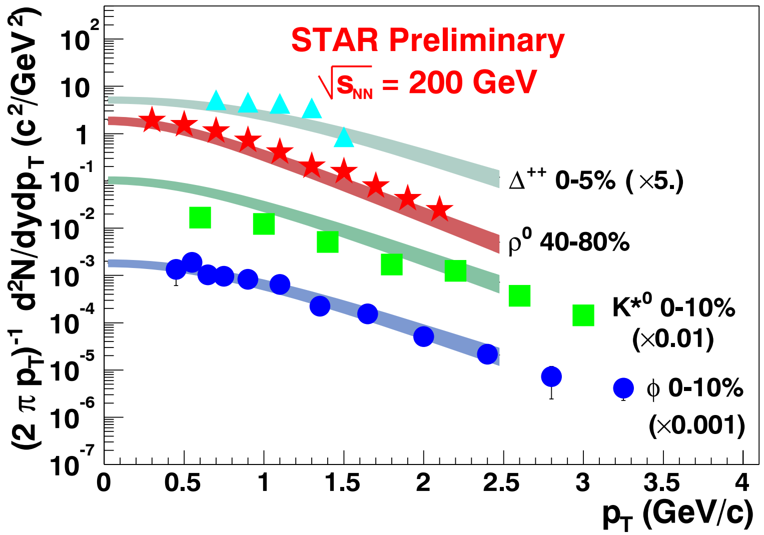
K^* – resonance, lower T would lead to much less K^*'s

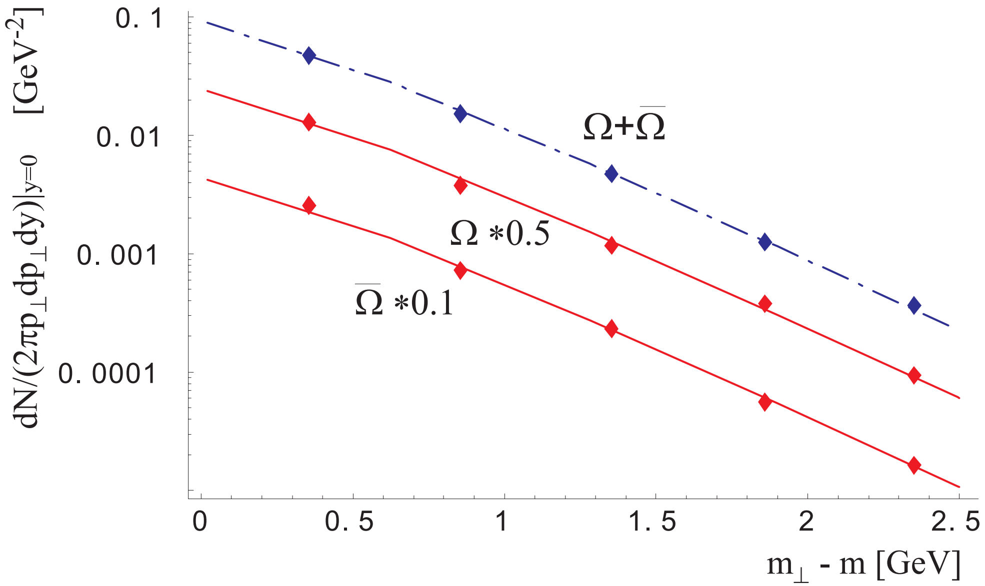
(experimental Ξ's went down by $\sim$ a factor of 2)

No special treatment of Ω's

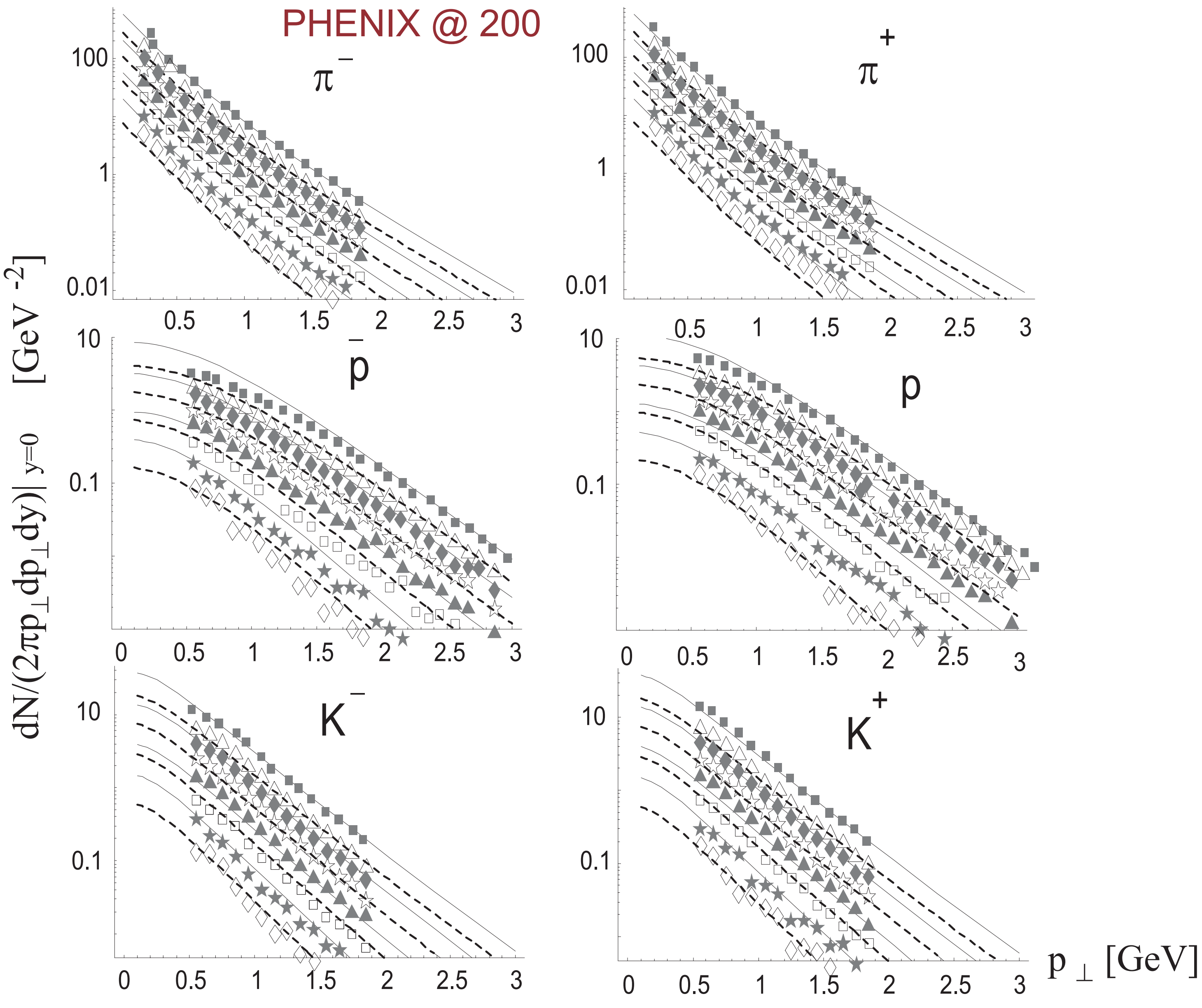
Spectra at 200 GeV A (P. Fachini, STAR)







($\tau = 8.3$ fm and $\rho_{\rm max} = 7.1$ fm, most central events) data from STAR (C. Suire, QM2002)



PHENIX @ 200
π⁻
π⁺
p̄
p
K⁻
K⁺
dN/(2πp⊥dp⊥dy)| y=0 [GeV -2]
p ⊥ [GeV]

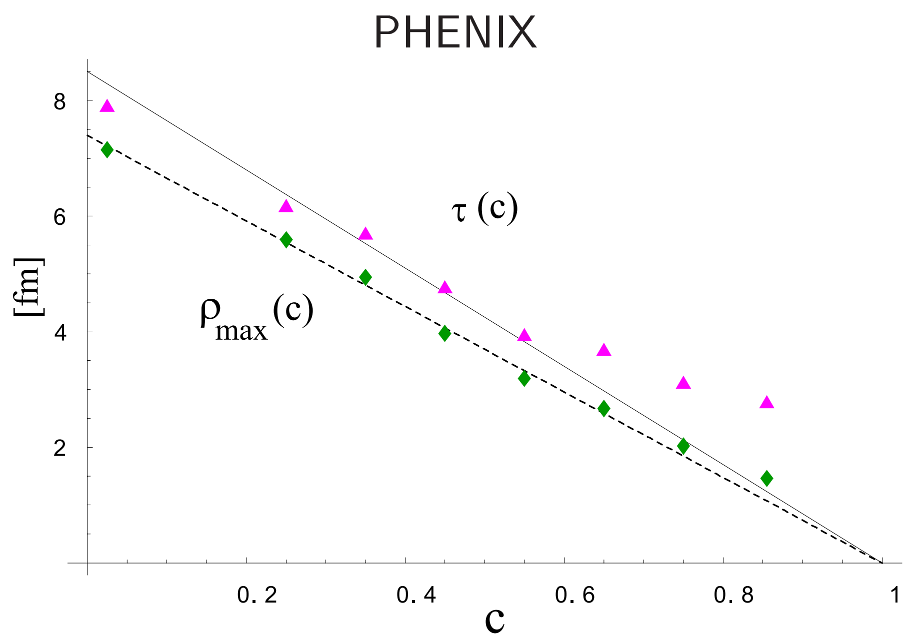
(data at different centrality, or impact parameter)

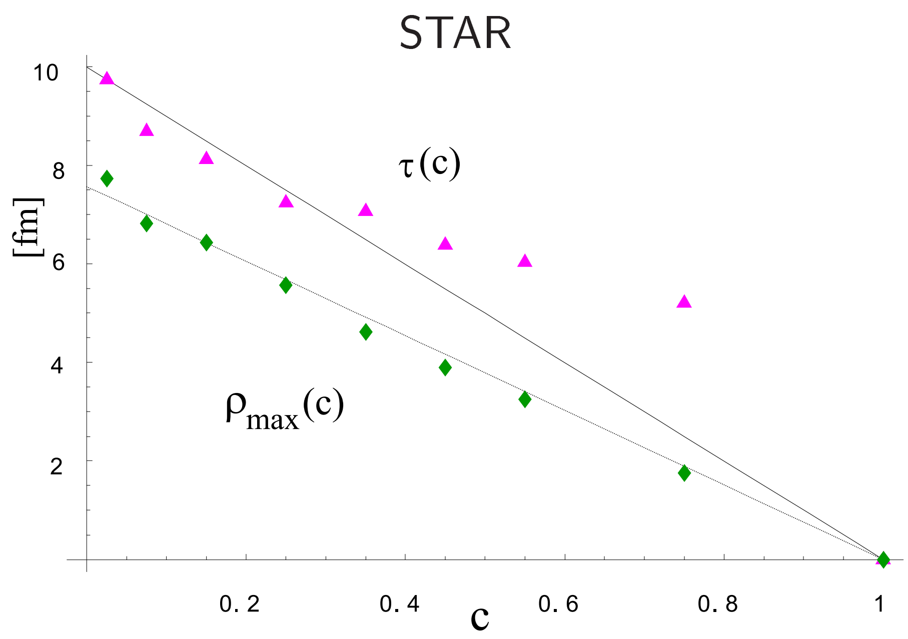
Centrality c is defined as a percentage of the most central events. To a very good accuracy

$$c \simeq \frac{\pi b^2}{\sigma_{\rm inel}^{\rm tot}} \simeq \frac{b^2}{4R^2}$$

(WB+WF, PRC 65 (2002) 024905)

Effectively parameterize RHIC with 2 thermal parameters + 2 geometric parameters for each centrality

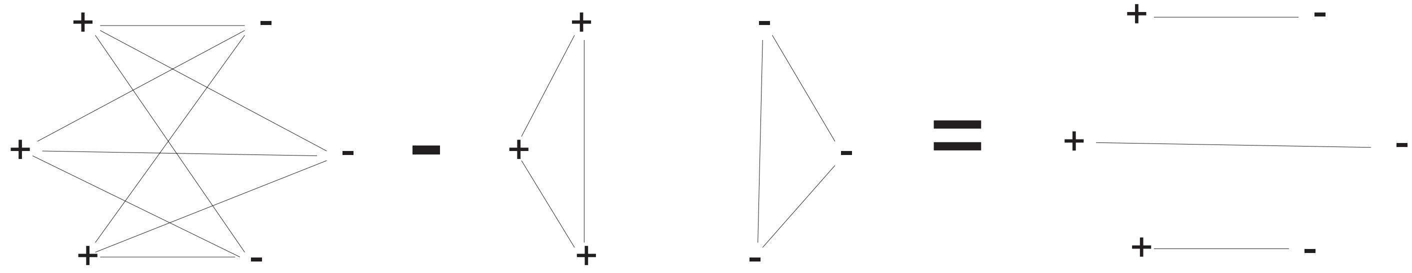

PHENIX
8
6
4
2
[fm]
τ (c)
ρmax(c)
0.2
0.4
0.6
0.8
1
c


STAR
10
8
6
4
2
[fm]
τ(c)
ρmax(c)
0.2
0.4
0.6
0.8
1
c

The Białas clusters

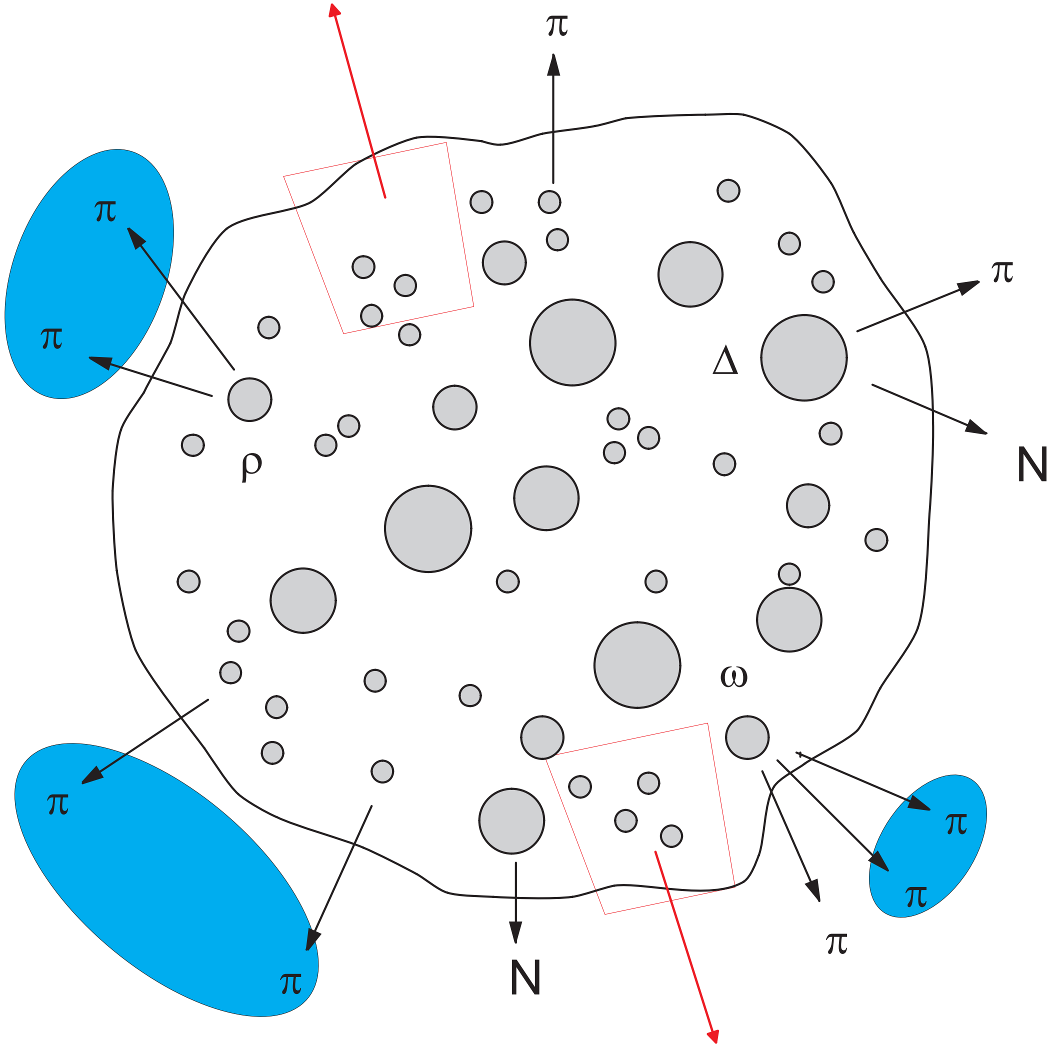
Because of the charge conservation, in the late-hadronization scenario, the opposite-charge particles may be treated as created from neutral clusters. In the calculation of the two-particle distributions one has to take into account that particles of the same charge must originate from different clusters, whereas the particles of opposite charge may come either from different cluster or from the same cluster (A. Białas, hep-ph/0308245)

In this case the difference of the two-particle distributions, $\rho_{+-}(p_1, p_2) - \rho_{++}(p_1, p_2)$, may be reduced to a two-particle distribution in a single cluster



9-6=3

QUESTION: What are the clusters in a thermal model?


π
π
π
π
Δ
N
ρ
ω
π
π
π
π
π
N
π

Two contributions for the $\pi^+\pi^-$ balance function

1) RESONANCE CONTRIBUTION (R) is determined by the decays of neutral hadronic resonances which have a $\pi^+\pi^-$ pair in the final state

$$K_S,\ \eta,\ \eta',\ \rho^0,\ \omega,\ \sigma,\ f_0$$

2) NON-RESONANCE CONTRIBUTION (NR) other possible correlations among the charged pions

The pion balance function is constructed as a sum of the two terms

$$B(\delta, Y) = B_{\mathrm{R}}(\delta, Y) + B_{\mathrm{NR}}(\delta, Y)$$

Resonance contribution

$$\frac{dN_R^{+-}}{dy_1 dy_2} = \int dy d^2p_\perp \int d^2p_1^\perp d^2p_2^\perp C_\pi \frac{dN_R}{dyd^2p_\perp} \rho_{R\to\pi^+\pi^-}(p, p_1, p_2)$$

C_π indicates the kinematic cuts for the pions $(|\eta| < 1.3,\ p_\perp > 100\mathrm{MeV})$ The momentum distribution of the resonance R is obtained from the Cooper-Frye formula

$$\frac{dN_R}{dyd^2p_\perp} = \int d\Sigma(x) \cdot p\, f_R\,(p \cdot u(x))$$

where f_R is the phase-space distribution function of the resonance

The two-particle pion momentum distribution in a two-body $(\pi^+\pi^-)$ resonance decay is

$$\rho_{R\to\pi^+\pi^-} = \frac{b_{\pi\pi}}{N_2}\delta^{(4)}(p - p_1 - p_2)$$

$b_{\pi\pi}$ – the branching ratio, $N_2 = \int \frac{d^3p_1}{E_1}\frac{d^3p_2}{E_2}\delta^{(4)}(p - p_1 - p_2)$ – normalization

Three-body decays $(\pi^+\pi^-\pi^0)$ are treated in an analogous way, with the assumption of a constant transition matrix element

Finally,

$$B_{\mathrm{R}}(\delta) = \frac{1}{N_\pi} \sum_R \int dy_1 dy_2 C_\pi \frac{dN_R^{+-}}{dy_1 dy_2} \delta(|y_2 - y_1| - \delta)$$

Model parameters (fixed earlier by the ratios and spectra)

$T = 165\mathrm{MeV}, \quad \langle \beta_\perp \rangle = 0.5$

Non-resonance contribution

$$\frac{dN^{+-}_{NR}}{dy_1 dy_2} = A \int d^2p_1^{\perp} d^2p_2^{\perp} C_\pi \int d\Sigma(x) p_1 \cdot u(x) f^{\pi}_{NR}\left(p_1 \cdot u(x)\right) p_2 \cdot u(x) f^{\pi}_{NR}\left(p_2 \cdot u(x)\right)$$

f^{π}_{NR} – phase-space distribution function of non-resonance pions

normalization constant A obtained from the condition $\int dy_2 \left(\frac{dN^{+-}_{NR}}{dy_1 dy_2}\right) = \frac{dN^{\pi}_{NR}}{dy_1}$

$$\tilde{B}_{\rm NR}(\delta) = \frac{1}{N_\pi} \int dy_1 dy_2 C_\pi \frac{dN^{+-}_{NR}}{dy_1 dy_2} \delta(|y_2 - y_1| - \delta)$$

R + NR contributions

$\int_0^Y d\delta B_R(\delta) = N^\pi_{\mathrm{R}}/N_\pi, \quad \int_0^Y d\delta \tilde{B}_{NR}(\delta) = N^\pi_{NR}/N_\pi$

Since some of the non-resonance pions are balanced by other charged hadrons, the final expression for the pion balance function is

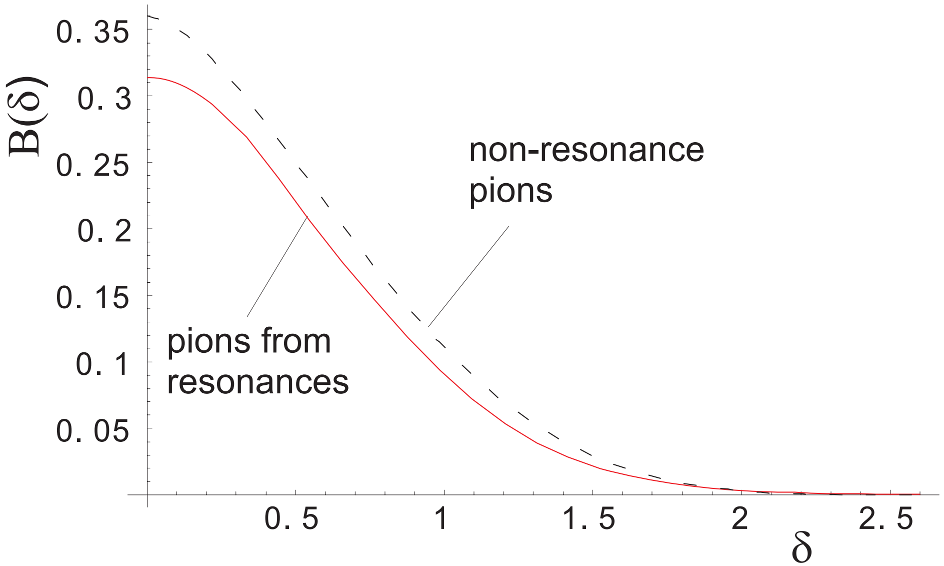
$$B(\delta) = B_{\mathrm{R}}(\delta) + \frac{N^\pi_{\mathrm{NR}}}{N_{\mathrm{charged}} - N^\pi_{\mathrm{R}}} \tilde{B}_{\mathrm{NR}}(\delta)$$

From the thermal model

$N_{\mathrm{charged}} = N^\pi_{\mathrm{R}} + N^\pi_{\mathrm{NR}} + \Delta N \;\rightarrow\; N^\pi_{\mathrm{NR}} + \Delta N = N_{\mathrm{charged}} - N^\pi_{\mathrm{R}}$

$N^\pi_{NR}/(N_{\mathrm{charged}} - N^\pi_{\mathrm{R}}) = 0.68$

Results

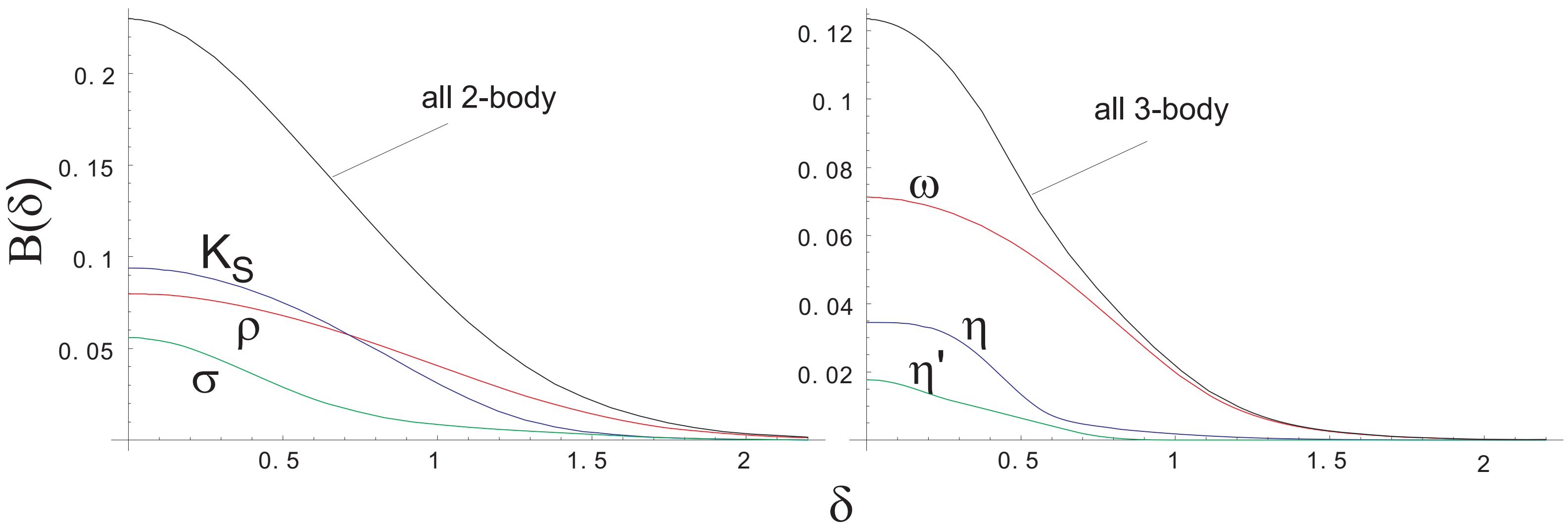


$\rho_{\max}/\tau = 0.9 \rightarrow \langle \beta_\perp \rangle = 0.5$

$\langle \delta \rangle \equiv \int_{0.2}^{2.4} \delta \, B(\delta) \, d\delta / \int_{0.2}^{2.4} B(\delta) \, d\delta$

$\langle \delta \rangle_{NR} = 0.67, \; \langle \delta \rangle_R = 0.65, \; \langle \delta \rangle_{R+NR} = 0.66, \; (\text{exp}: 0.59 - 0.66)$

Anatomy of the resonance contribution



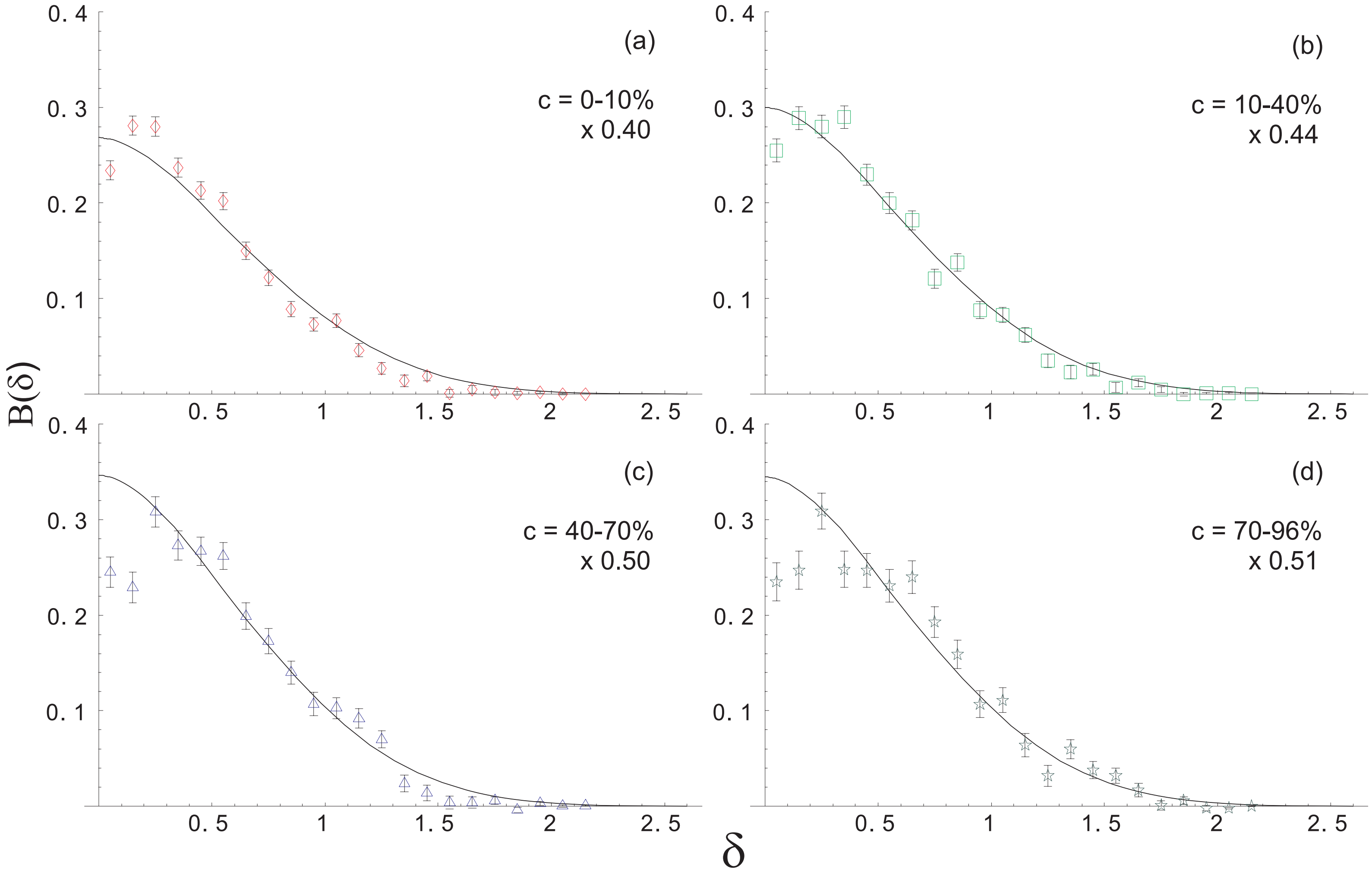
heavier resonance – wider (more phase space)

two-body wider than three-body

higher T – wider

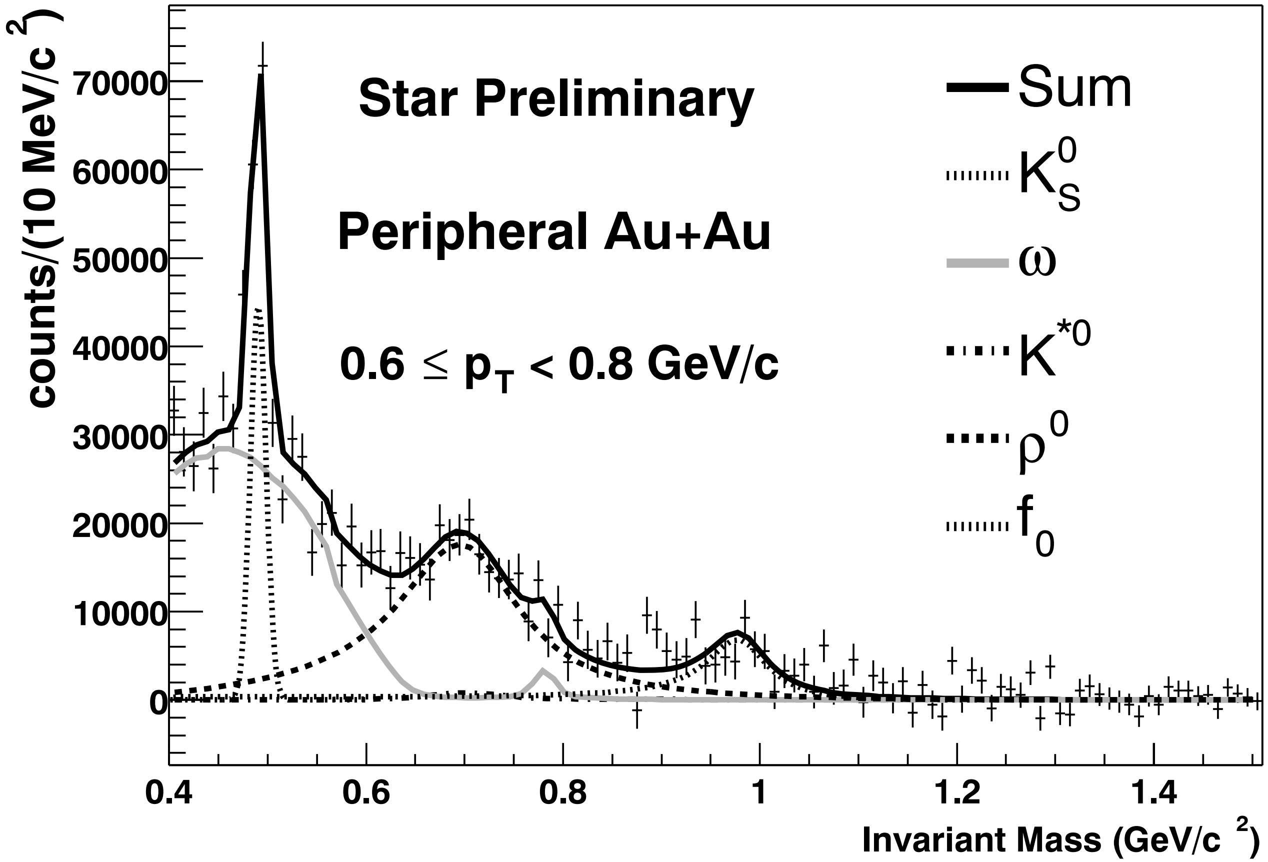
higher flow – narrower

Fit to the STAR data



Rescaling factors (from χ^2 fits) are poor man's way of taking into account the detector acceptance and efficiency

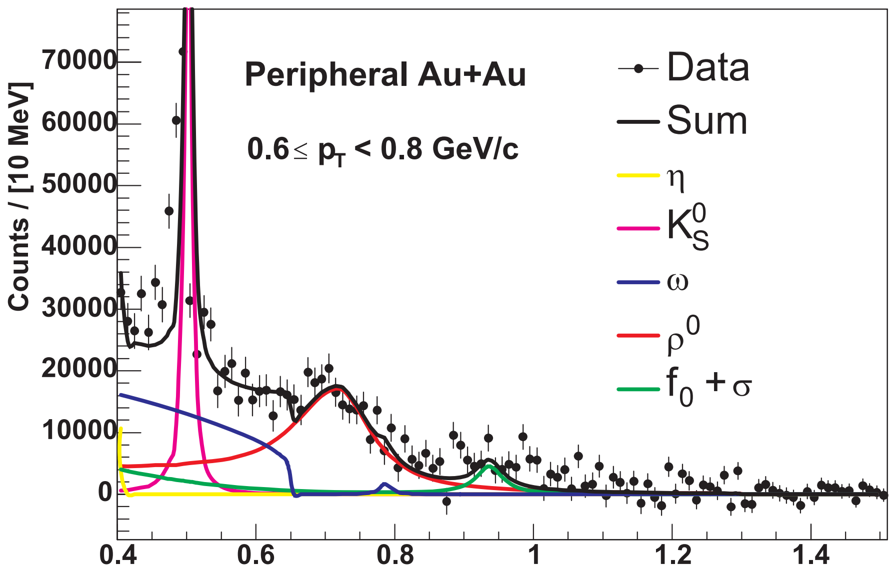
$\pi^+\pi^-$ pairs from STAR

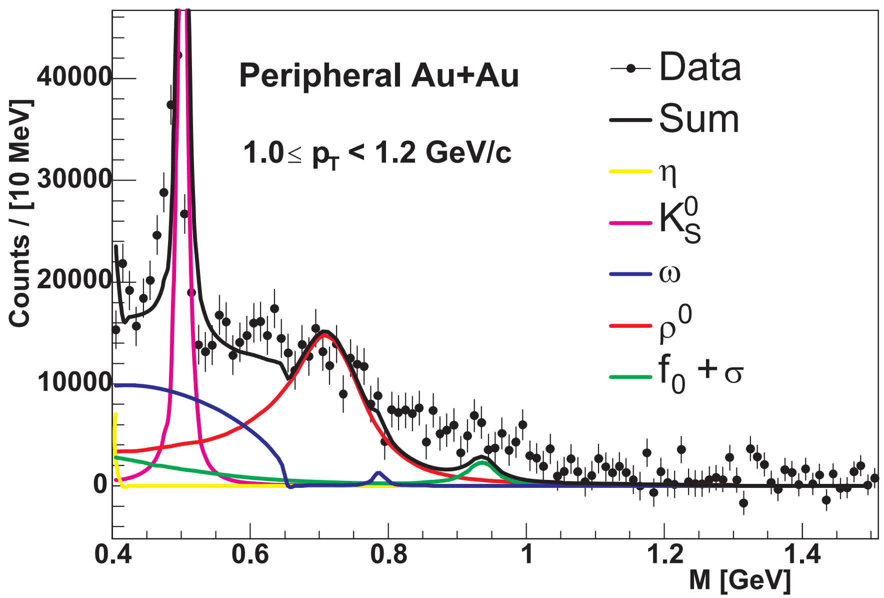


(from J. Adams et al., nucl-ex/0307023; P. Fachini, nucl-ex/0305034)

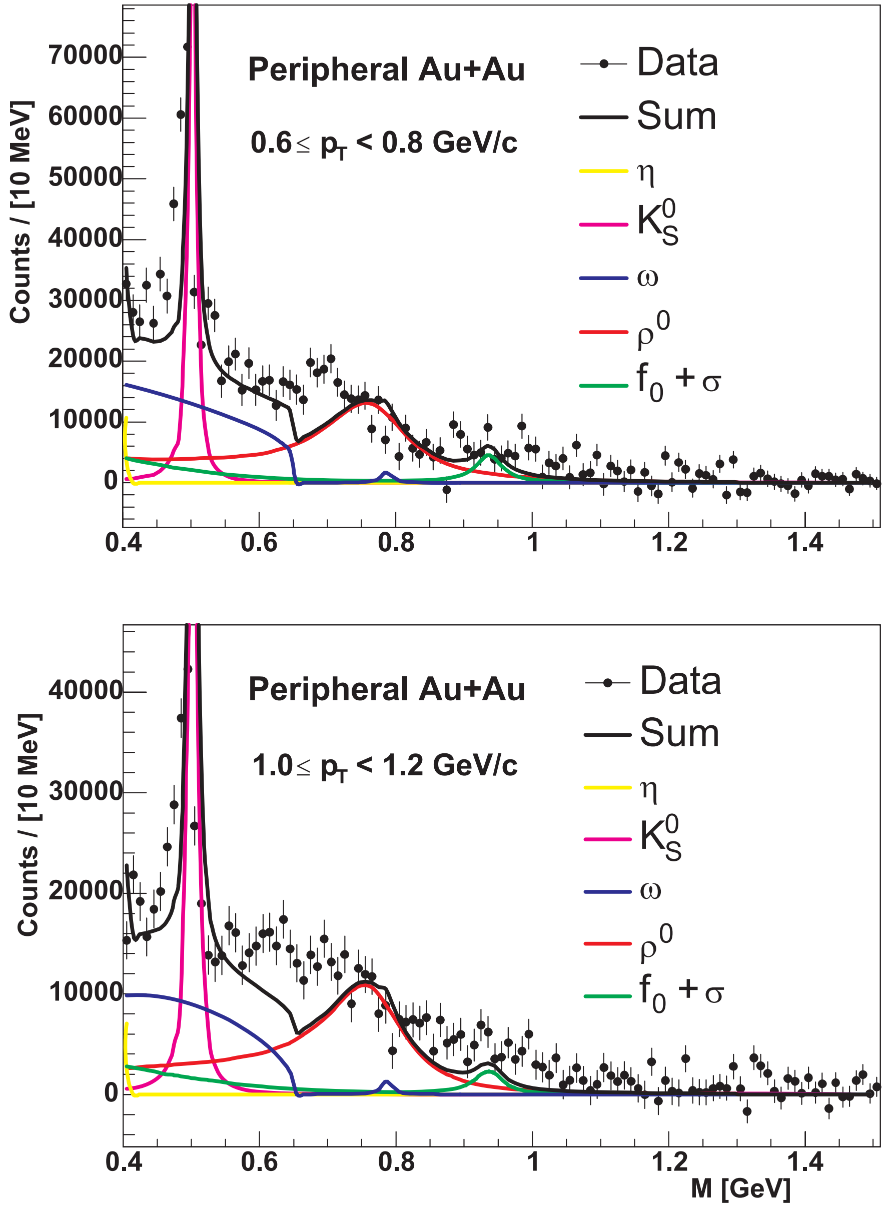
(Brown+Shuryak, Kolb-Prakash, Rapp, Pratt+Bauer)

STAR vs. thermal model, lowered ρ





(prepared by P. Fachini)



vacuum ρ

(worse agreement)

Conclusions

1. Thermal model works for soft ($p_\perp < 1.5$ GeV) physics The model reproduces the ratios, spectra, $R_{\rm out}/R_{\rm side} \sim 1$, $\pi^+\pi^-$ invariant mass distributions, and the balance functions)
2. Hadrons reach thermal equilibrium. How? Cooked up in quarks and gluons?
3. The analysis of balance functions brings a further indication that the single-freeze-out model is a very reasonable first-order approximation of the final state in ultra-relativistic heavy-ion collisions
4. Balance functions can be calculated in the thermal model: the resonance contribution (R) is determined in the unique way, the form of the non-resonance contribution (NR) involves additional assumptions
5. The two calculated contributions (R+NR) have similar δ-dependence, the width of the sum is somewhat larger than the width measured by STAR (0.66 vs. 0.59-0.66). The shape for the very small values of δ is affected by the Bose-Einstein correlations
6. The overall normalization is fitted in order to take into account the effect of the efficiency and acceptance of the detector, which brings a relatively large factor $\sim 1/2$. The effects of the detector should be more accurately incorporated in a more detailed analysis. It may affect the normalization, as well as the shape in δ (GEANT) (cf. a recent paper by S. Cheng et al., nucl-th/0401008). Experiment with larger window in rapidity highly desired!
7. No significant dependence on centrality can be produced in the thermal approach

BACKUP SLIDES

Dependence on temperature

T [MeV]	$\langle \delta \rangle_{\rm NR}$
100	0.629
125	0.653
150	0.670
200	0.695

Relation to inclusive distributions

A. Białas and V. Koch, Phys. Lett. B456 (1999) 1

Event-by-event fluctuations are related to inclusive distributions, balance functions may be expressed in terms of one-particle and two-particle observables

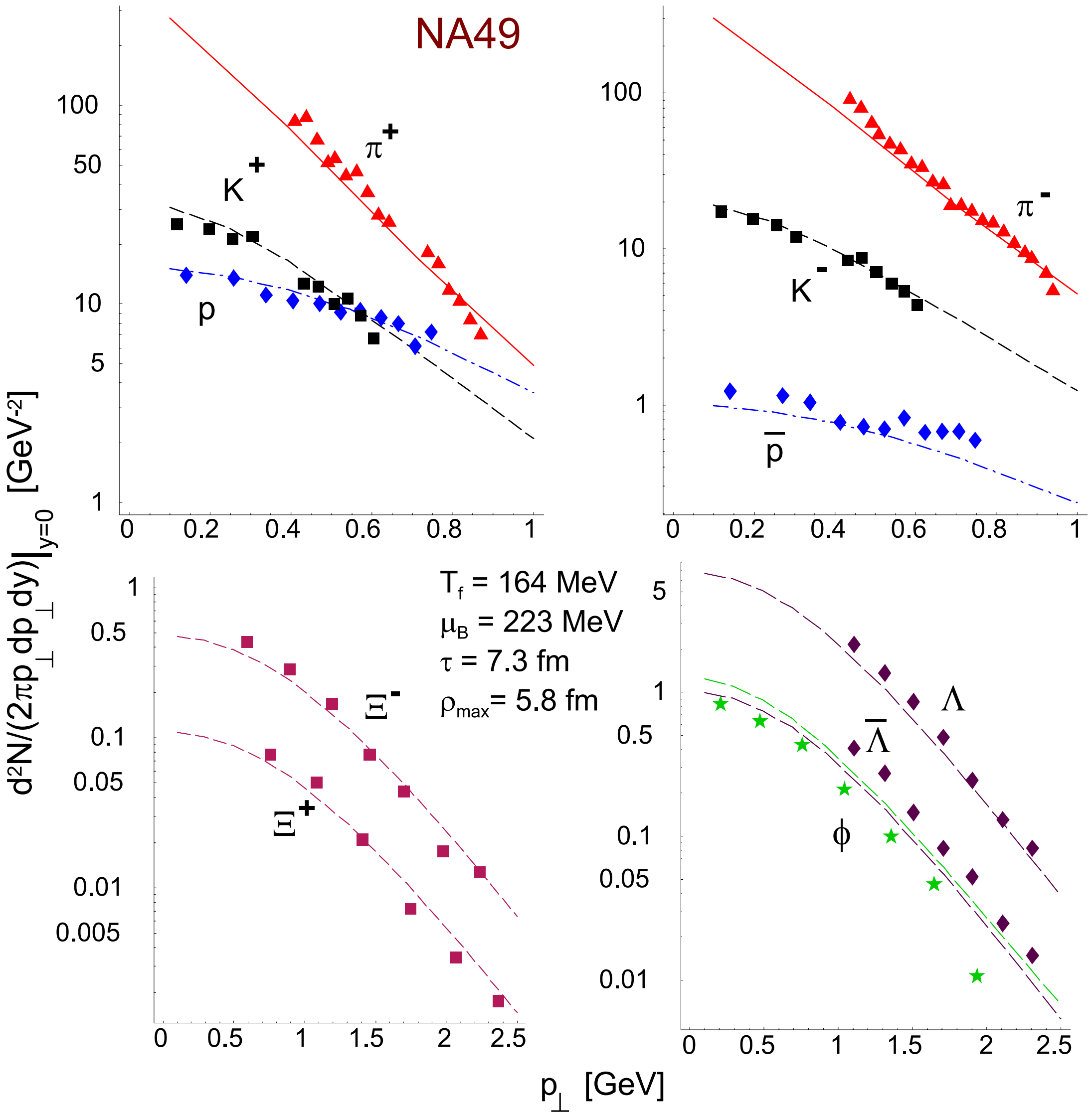
$$\langle N_{\pm} \rangle = \int d^3p \rho_{\pm}(p)$$

pairs of identical (positive or negative) particles – $\langle N_{\pm}(N_{\pm} - 1) \rangle = \int d^3p_1 d^3p_2 \rho_{\pm\pm}(p_1, p_2)$
pairs of non-identical (with opposite charge) particles – $\langle N_{\pm} N_{\mp} \rangle = \int d^3p_1 d^3p_2 \rho_{\pm\mp}(p_1, p_2)$

The balance function essentially measures the difference

$$\rho_{+-}(p_1, p_2) - \rho_{++}(p_1, p_2)$$

How was it at SPS?

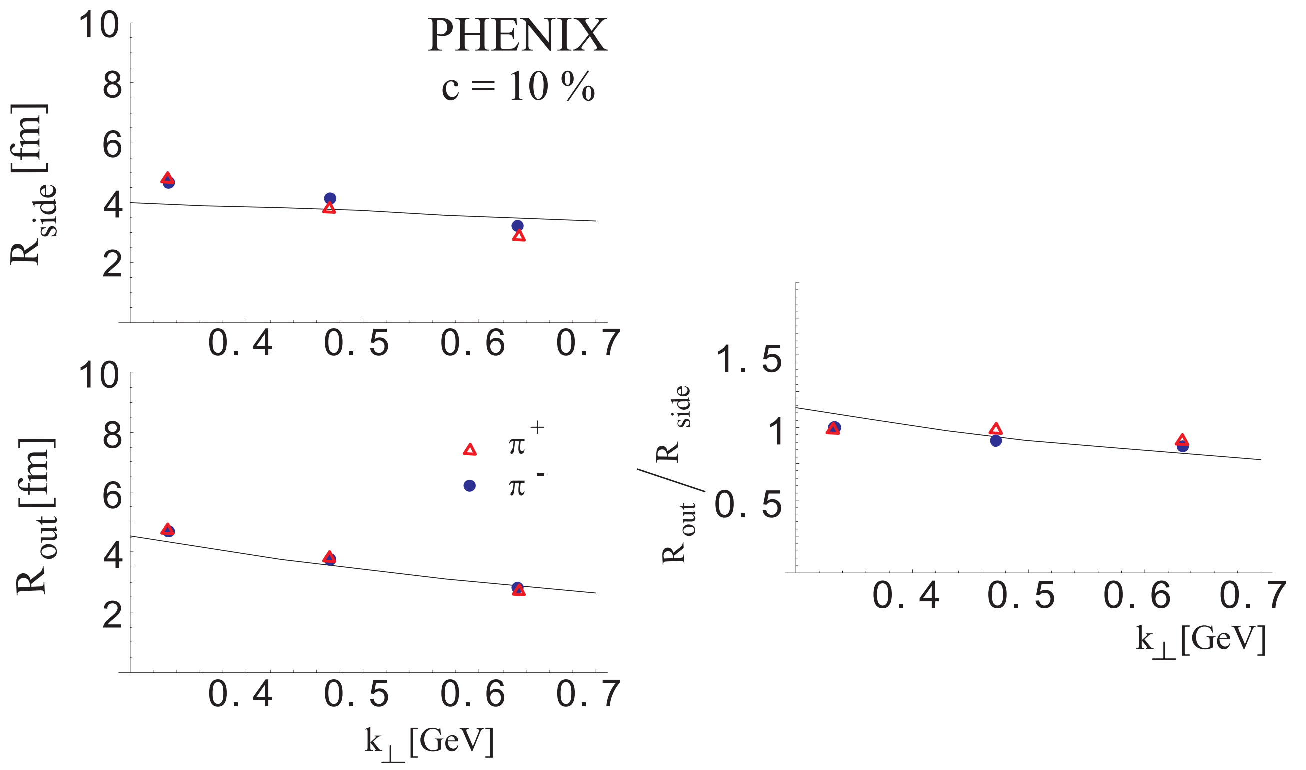


(Ω^- did not work, exp. much steeper)

HBT radii

$$S(x,p) = \int d\Sigma_\mu p^\mu \delta(x'-x) f(x',p)$$

$$C(\vec{q},\vec{P}) = 1 + \frac{\left|\int d\Sigma(x)\cdot u(x) e^{iq\cdot x} S(P\cdot u(x))\right|^2}{\int d\Sigma\cdot u S((P+\frac{q}{2})\cdot u(x)) \int d\Sigma \cdot u S((P-\frac{q}{2})\cdot u(x))}$$



The pionic HBT radii for most-central collisions 30 GeV, and their ratio, as predicted by the model + excluded volume corrections (∼30% enhancement of model radii) and measured by PHENIX

Excluded-volume (Van der Waals) corrections

Such effects were found important in previous studies of the particle multiplicities in ultra-relativistic heavy-ion collisions, leading to a significant dilution of system. They bring in a factor (Gorenstein)

$$\frac{e^{-Pv_i/T}}{1+\sum_j v_j e^{-Pv_j/T} n_j},$$

into phase-space integrals, where P denotes the pressure, $v_i = 4\frac{4}{3}\pi r_i^3$ is the excluded volume, and n_i is the density of particles of species i. The pressure is calculated self-consistently from the equation

$$P = \sum_i P_i^0(T, \mu_i - Pv_i/T) = \sum_i P_i^0(T, \mu_i) e^{-Pv_i/T}$$

where P_i^0 is the partial pressure of the ideal gas of hadrons of species i. If $r_i = r$, $v_i = v$, the excluded-volume correction produces a common scale factor, S^{-3}. Then

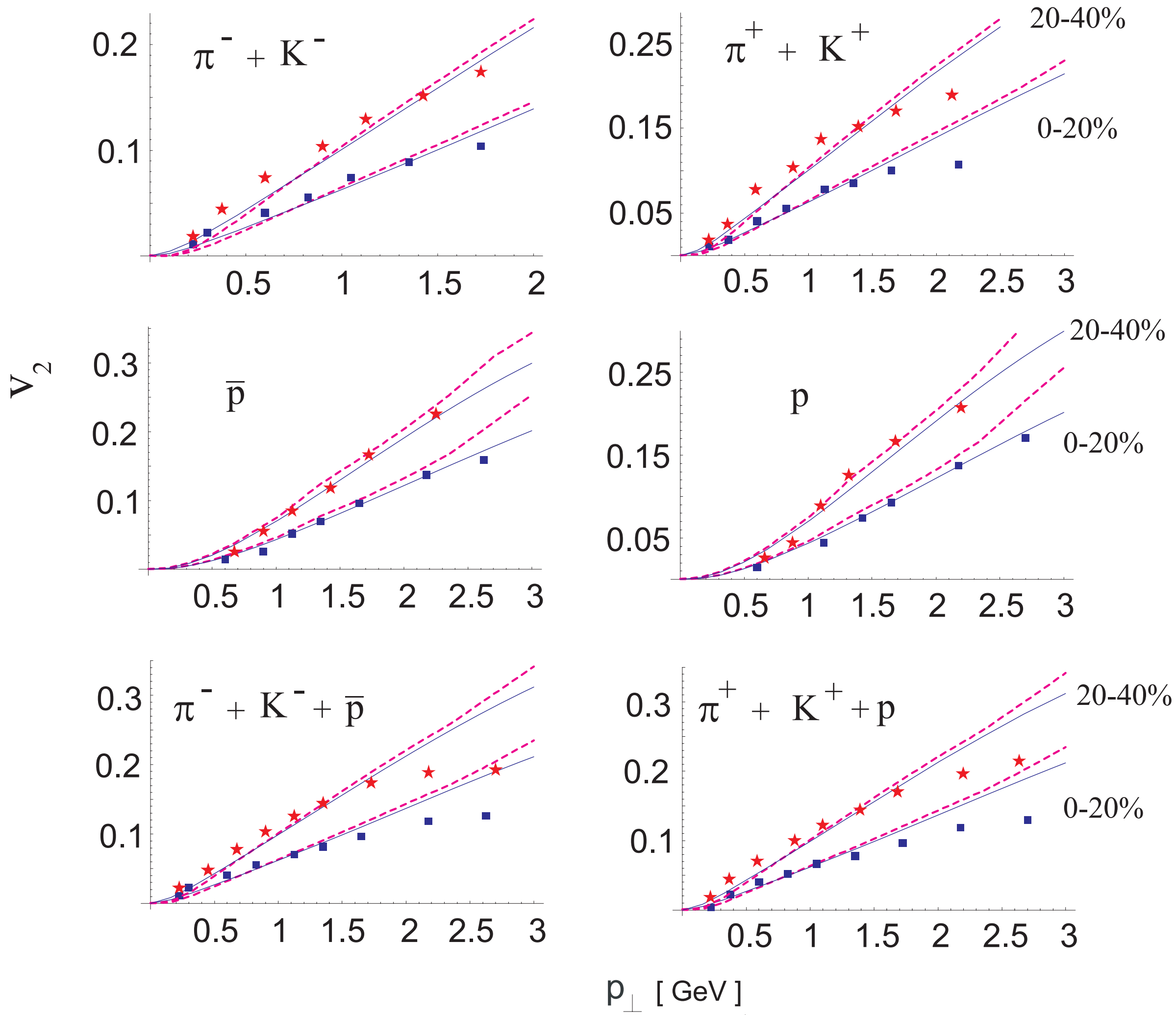
$$\frac{dN_i}{d^2p_\perp dy} = \tau^3 \int_{-\infty}^{+\infty} d\alpha_\parallel \int_0^{\rho_{\max}/\tau} \sinh\alpha_\perp d\left(\sinh\alpha_\perp\right) \int_0^{2\pi} d\xi \, p\cdot u \, S^{-3} f_i\left(p\cdot u\right)$$

The presence of the factor S^{-3} is compensated by rescaling ρ and τ by the factor S. That way, we retain all the previously obtained results for the particle abundances and the momentum spectra. However, now the system is more dilute and larger in size.

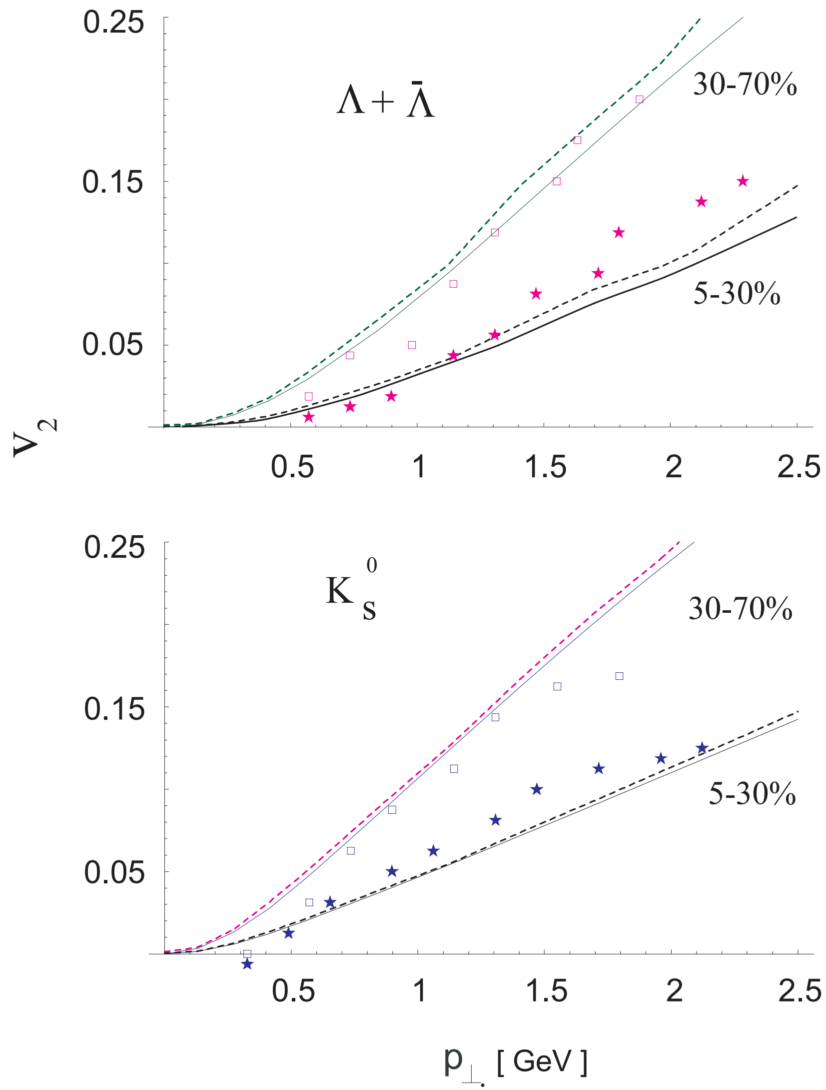
With our values of the thermodynamic parameters we have $\sum_i P_i^0(T, \mu_i) = 80\text{MeV/fm}^3$, which leads to $S = 1.3$ with $r = 0.6\text{fm}$. Values of this order have been typically obtained in other works. Thus, the excluded-volume corrections can increase the size parameters at freeze-out by about 30% and help to alleviate the problem with the HBT radii. Hadrons have sizes!

Elliptic flow

(Anna Baran, to be published) Idea: fix azimuthal asymmetry of shape/flow with the data on pions, kaons, ... and then make predictions for other particles. solid (dashed): with (without) resonance decays. (data from PHENIX 00 GeV)



v_2 for strange particles



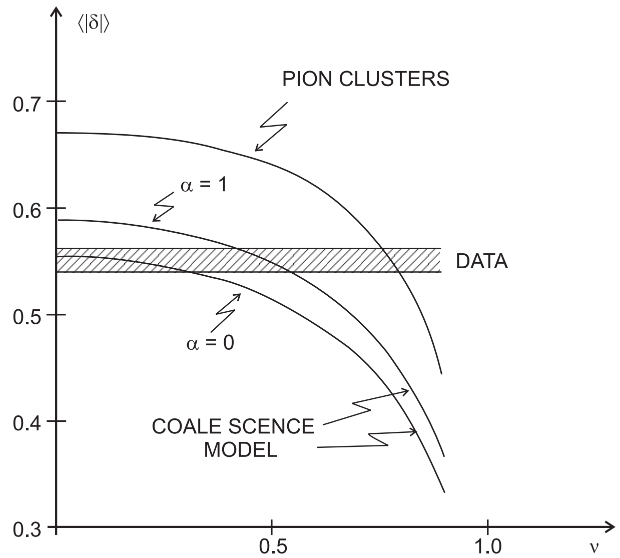
(data from STAR)

Balance functions in coalescence model

A. Białas, hep-ph/0308245

results of the pion-cluster model are confronted with the quark-antiquark coalescence mechanism for pion production

conclusion: the coalescence mechanism implies a substantial reduction of the width of the balance functions



coalescence model: T. S. Biro, P. Levai, and J. Zimanyi, Phys. Lett. B347 (1995)