Co nam mówią fluktuacje pędu poprzecznego na RHICu?

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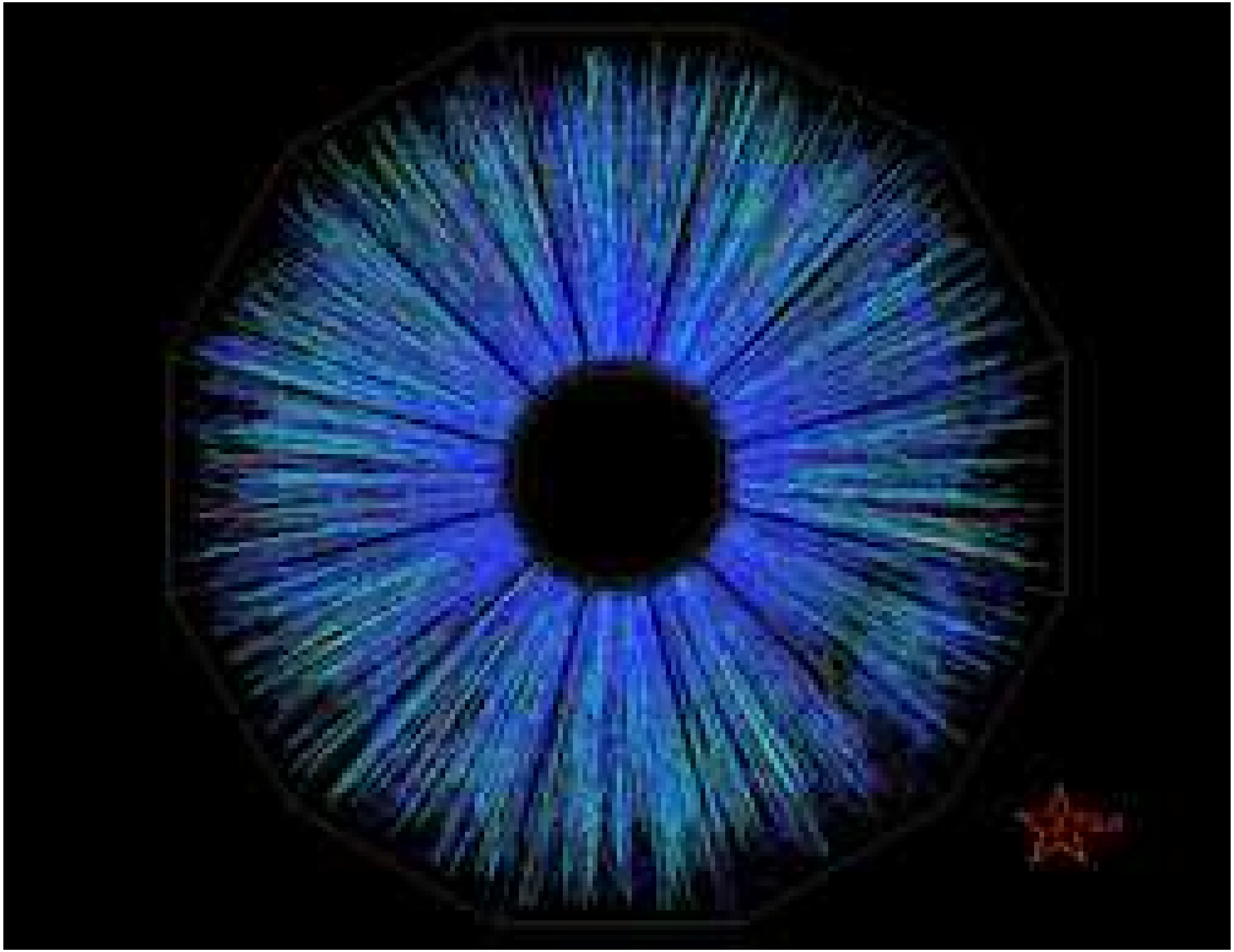
IF AŚ, 9 listopada 2005

[Gaździcki, Mrówczyński, Białas, Koch, Jeon, Voloshin, Ritter, Pruneau, Gavin, Abdel-Aziz, Liu, Trainor, Rybczyński, Włodarczyk, Wilk, Utyuzh, Brogueira, Dias de Deus, Ferreiro, de Moral, Pajares, ...
PHENIX, STAR, NA49, CERES]

[nucl-th/0510033]

Event



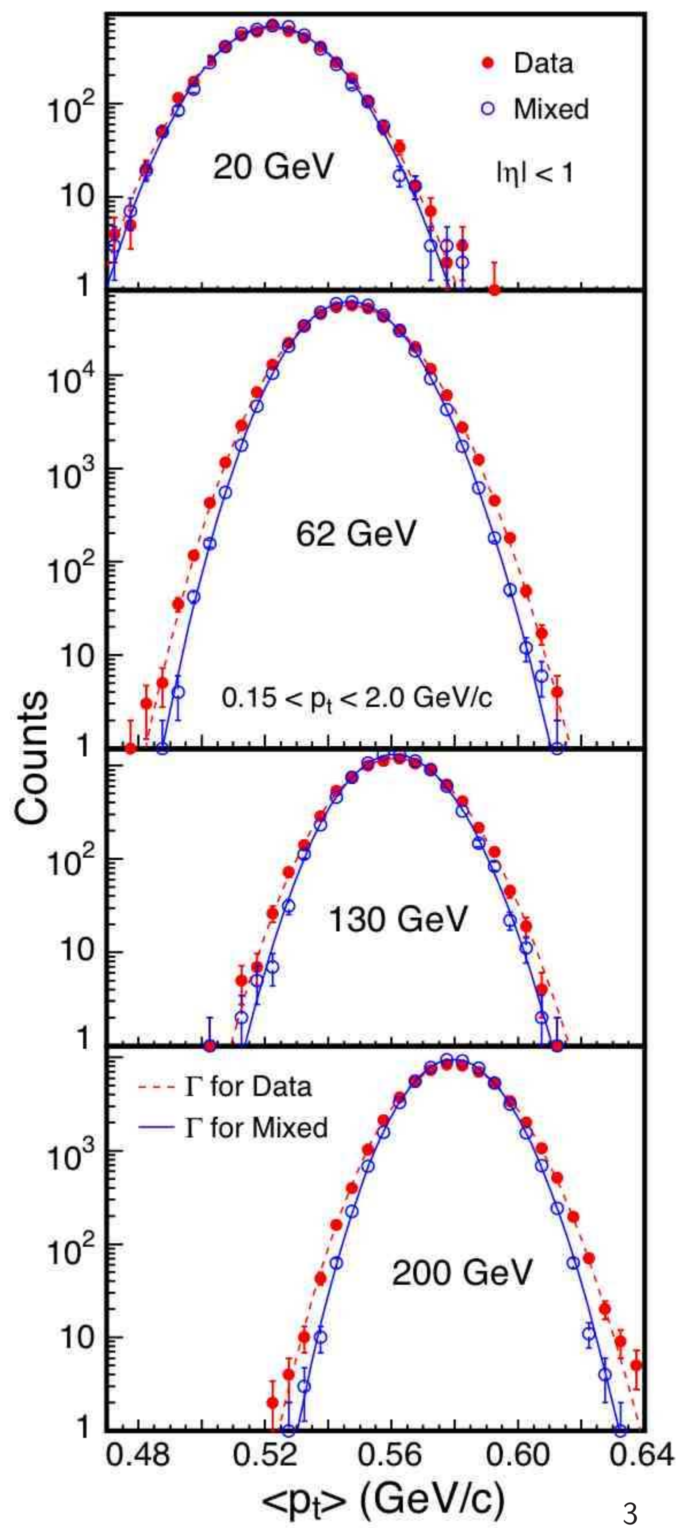
Wstęp

- Korelacje $\longleftrightarrow$ fluktuacje "od przypadku do przypadku" (event-by-event) , różnego rodzaju: liczby cząstek, ładunku w danym przedziale kinematycznym, liczby barionowej i dziwności, ...

- Źródło korelacji - dynamika, mechanizm produkcji cząstek. Mierząc fluktuacje wchodzimy głębiej w dynamikę niż z pomocą wielkości jednocząstkowych (krotności, widma). Wyzwanie dla teorii!

Słowniczek:

- Przypadek (event) - zderzenie ciężkich jonów $\longrightarrow$ zestaw liczb, np. moduł pędu poprzecznego (do kierunku wiązki) każdej rejestrowanej cząstki, $|p_T^{(k)}|$, $k = 1, \ldots, n_i$, n_i - krotność produkowanych cząstek w i-tym przypadku (kilkadziesiąt - kilkaset w zadanym oknie kinematycznym).
- Średniowanie po przypadku, np. pęd średni w danym przypadku, M_i, $i = 1, \ldots, N_{\rm ev}$, $N_{\rm ev}$ - liczba przypadków (dziesiątki, setki tysięcy).
- Histogram w zmiennej M, średnia średnich $\langle M \rangle$, wariancja, itd.
- Mieszanie - generowanie fikcyjnych przypadków, które mają krotności przypadków fizycznych, n_1,n_2, ..., ale każda cząstka w danym mieszanym przypadku brana jest z innego fizycznego przypadku. To ubija korelacje!



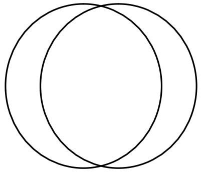
STAR, hep-ph/0504031 $c = 0 - 5\%$

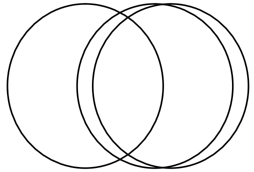
n - multiplicity of (observed) charged particles,

$|\eta| < 0.35$, $0.2 < p_T < 1.5$ GeV, $\Delta\phi = 45^o$

$M = \frac{p_1+p_2+\ldots+p_n}{n}$, p_i - magnitude of the transverse momentum

mix - mixed events ($c \simeq \frac{b^2}{4R^2}$)   centrality from multiplicity of produced particles, $\langle n \rangle$



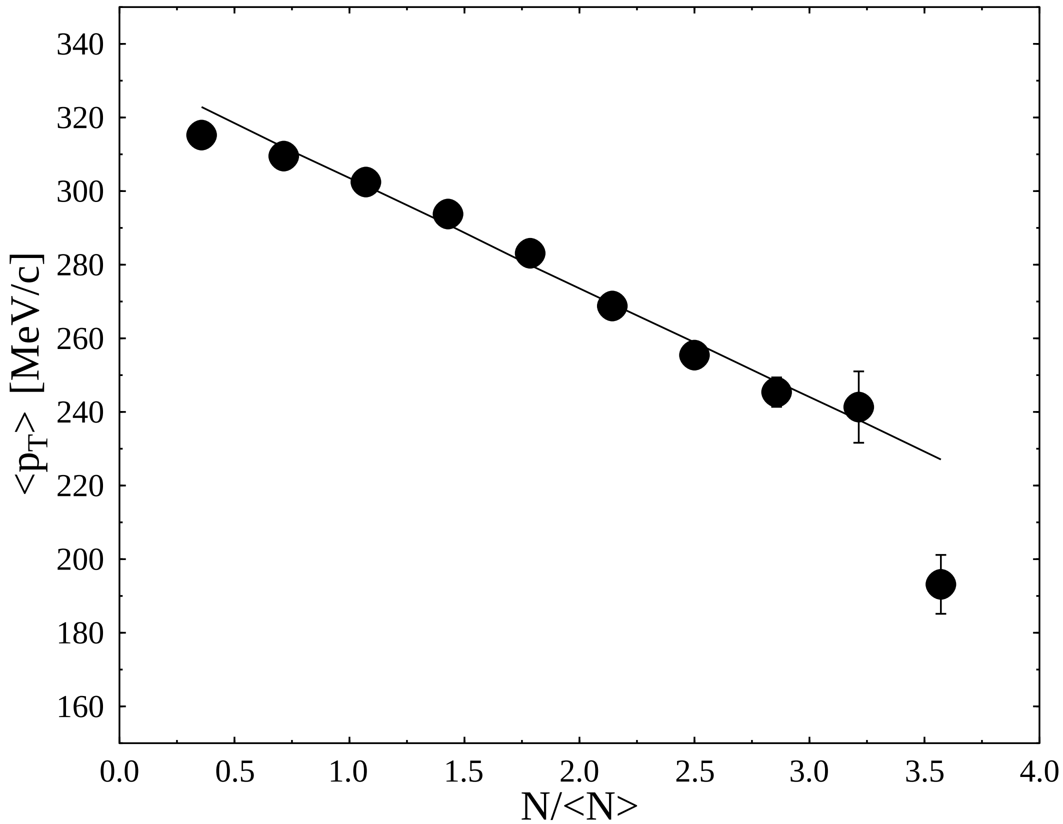


centrality	0-5%	0-10%	10-20%	20-30%
$\langle n \rangle$	59.6	53.9	36.6	25.0
σ_n	10.8	12.2	10.2	7.8
$\langle M \rangle$	523	523	523	520
σ_p	290	290	290	289
σ_M	38.6	41.1	49.8	61.1
$\langle M \rangle^{\rm mix}$	523	523	523	520
$\sigma_M^{\rm mix}$	37.8	40.3	48.8	60.0

PHENIX, PRC66 (2002) 024901, nucl-ex/0203015

$\langle M \rangle$ and σ_p are practically constant in the "fiducial" centrality range $c = 0 - 30\%$ **(1)**

not the case in $p-p$ collisions!



[NA49, PRC70 (2004) 034902, SPS at 158 GeV]

Some statistics

Multiplicity n and the momenta $p_1, p_2, \ldots, p_n$ vary randomly from event to event. The probability of a given configuration is $P(n)\rho_n(p_1, \ldots, p_n)$, where $P(n)$ is the multiplicity distribution and $\rho_n(p_1, \ldots, p_n)$ is the conditional probability distribution of occurrence of $p_1, \ldots, p_n$ provided we have multiplicity n. In general ρ depends functionally on n. The normalization is

$$\sum_n P(n) = 1, \quad \int dp_1 \ldots dp_n \rho_n(p_1, \ldots, p_n) = 1$$

The *marginal* probability densities are defined as

$$\rho_n^{(n-k)}(p_1, \ldots, p_{n-k}) \equiv \int dp_{n-k+1} \ldots dp_n \rho_n(p_1, \ldots, p_n),$$

with $k = 1, \ldots, n-1$. These are also normalized to 1. We introduce

$$\begin{aligned} \langle p \rangle_n &\equiv \int dp \rho_n(p) p, \quad \mathrm{var}_n(p) \equiv \int dp \rho_n(p) \left(p - \langle p \rangle_n\right)^2, \\ \mathrm{cov}_n(p_1, p_2) &\equiv \int dp_1 dp_2 \left(p_1 - \langle p \rangle_n\right) \left(p_2 - \langle p \rangle_n\right) \rho_n(p_1, p_2). \end{aligned}$$

The subscript n indicates that the averaging is taken in samples of a given multiplicity n

For the variable $M = \sum_{i=1}^{n} p_i/n$ we find immediately

$$\begin{aligned}
\langle M\rangle &= \sum_n P(n)\int dp_1\ldots dp_n M\rho_n(p_1,\ldots,p_n) = \sum_n P(n)\langle p\rangle_n, \\
\langle M^2\rangle &= \sum_n P(n)\int dp_1\ldots dp_n M^2\rho_n(p_1,\ldots,p_n) \\
&= \sum_n \frac{P(n)}{n}\langle p^2\rangle_n + \sum_n \frac{P(n)}{n^2}\left[\sum_{i,j=1,j\neq i}^{n} \mathrm{cov}_n(p_i,p_j) + n(n-1)\langle p\rangle_n^2\right]
\end{aligned}$$

(1) allows us to replace $\langle p\rangle_n$ with $\langle M\rangle$ and $\sigma_{p,n}^2 = \langle p^2\rangle_n - \langle p\rangle_n^2$ with $\sigma_{p,\langle n\rangle}^2$,

$$\sigma_M^2 = \sigma_{p,\langle n\rangle}^2 \sum_n \frac{P(n)}{n} + \sum_n \frac{P(n)}{n^2}\left[\sum_{i,j=1,j\neq i}^{n} \mathrm{cov}_n(p_i,p_j)\right]$$

In mixed events, by construction, particles are not correlated, hence the covariance term vanishes and

$$\sigma_M^{2,\mathrm{mix}} = \sigma_{p,\langle n\rangle}^2 \sum_n \frac{P(n)}{n} \simeq \sigma_{p,\langle n\rangle}^2 \left(\frac{1}{\langle n\rangle} + \frac{\sigma_n^2}{\langle n\rangle^3} + \ldots \right) \qquad \mathbf{(2)}$$

where we have used the fact that $P(n)$ is narrow and expanded $1/n = 1/[\langle n\rangle + (n - \langle n\rangle)]$ to second order in $(n - \langle n\rangle)$

centrality	0-5%	0-10%	10-20%	20-30%
$\langle n\rangle$	59.6	53.9	36.6	25.0
σ_n	10.8	12.2	10.2	7.8
σ_p	290	290	290	289
σ_M^{mix}	37.8	40.3	48.8	60.0
$\sigma_p \sqrt{\frac{1}{\langle n\rangle} + \frac{\sigma_n^2}{\langle n\rangle^3}}$	38.2	40.5	49.8	60.8

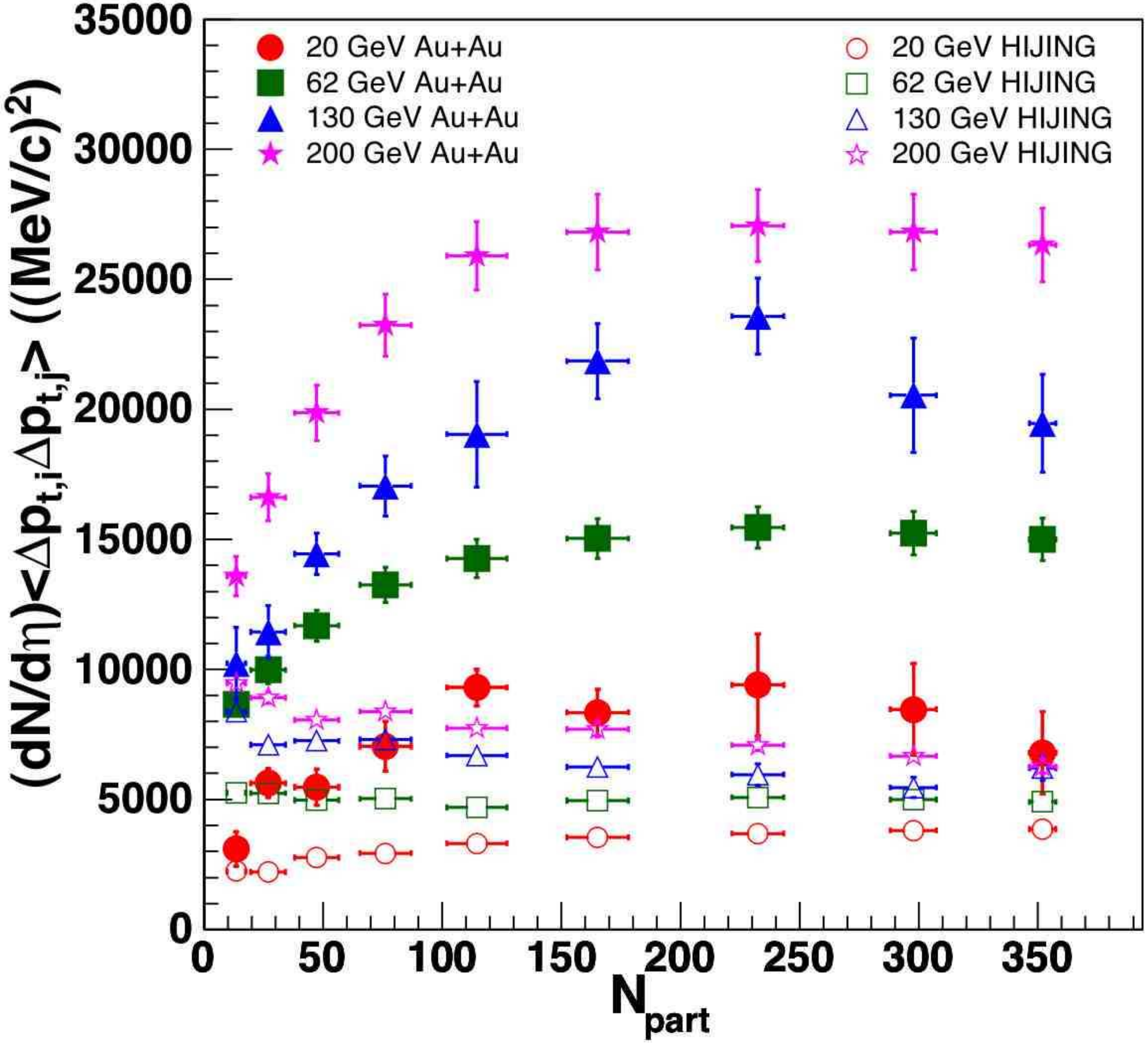
(2) works within 1%

Since $\sigma_{p,\langle n\rangle}$ is not altered by the event mixing procedure, subtracting the last two equations yields

$$\sigma_{\rm dyn}^2 = \sum_n \frac{P(n)}{n^2} \sum_{i,j=1,j\neq i}^{n} \mathrm{cov}_n(p_i,p_j) \simeq \frac{1}{\langle n\rangle^2} \sum_{i,j=1,j\neq i}^{\langle n\rangle} \mathrm{cov}_{\langle n\rangle}(p_i,p_j) \qquad \textbf{(3)}$$

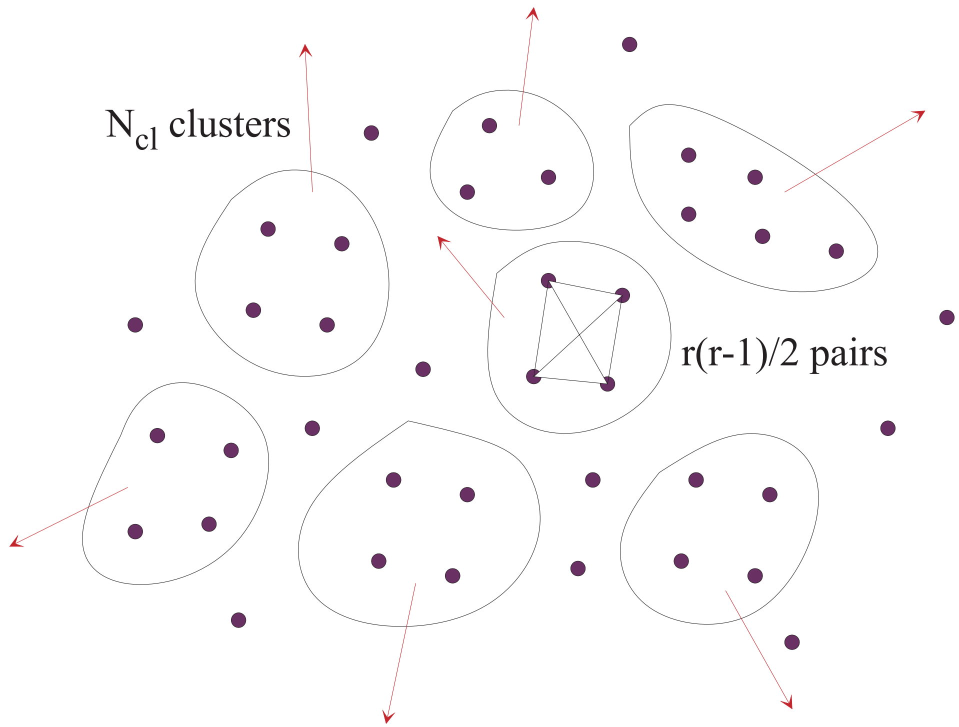
centrality	0-5%	0-10%	10-20%	20-30%
$\langle n\rangle$	59.6	53.9	36.6	25.0
σ_M	38.6	41.1	49.8	61.1
$\sigma_M^{\rm mix}$	37.8	40.3	48.8	60.0
$\sigma_{\rm dyn}\sqrt{\langle n\rangle}$	60.3 ± 1.6	59.2 ± 1.5	59.8 ± 1.2	57.7 ± 1.1

$\sigma_{\rm dyn}\sim 1/\sqrt{\langle n\rangle}$ (within 2%, round-off errors) which together with **(3)** places severe constraints on physics - not all particle can be correlated!



STAR, hep-ph/0504031 ($\frac{dN}{d\eta}\langle\Delta p_i \Delta p_j\rangle \simeq \sigma^2_{\rm dyn}\langle n\rangle/\Delta\eta$)

Multiparticle clusters



The number of correlated pairs within a cluster is $r(r-1)/2$. Some particles may be unclustered, hence $\langle N_{\rm cl}\rangle r/\langle n\rangle = \alpha$. Then

$$\sigma^2_{\rm dyn} = \frac{\alpha(r-1)}{\langle n\rangle}{\rm cov}^*,$$

which complies to the scaling of $\sigma_{\rm dyn}$. An immediate conclusion here is that $\alpha(r-1)$ cannot depend on $\langle n\rangle$ (in the fiducial centrality range) in order for the scaling to hold

Can it be jets?

Jets (minijets) which have been proposed as a possible explanation of the experimental data even at the considered soft momenta [PHENIX, PRL 93 (2004) 092301]. Jets, when fragmenting, lead to clusters in the momentum space. The full covariance from jets is then $N_{\rm cl,jet} j(j-1){\rm cov}^{\rm j}/\langle n\rangle^2$, with $N_{\rm cl,jet}$ - number of clusters originating from jets, j - number of particles in the cluster, and $2\,{\rm cov}^{\rm j}$ - covariance per pair, $N_{\rm cl,jet} j$ - total number of particles produced from jets. The commonly accepted estimate of the dependence of $N_{\rm cl,jet} j$ on centrality is accounted for by $R_{AA} \times N_{\rm bin}$

$$N_{\rm cl,jet} j \sim R_{AA} N_{\rm bin} = \frac{\langle n\rangle}{N_{\rm bin}\langle n\rangle_{pp}} N_{\rm bin} \sim \langle n\rangle,$$

which complies to the scaling of $\sigma_{\rm dyn}$. We stress that this scaling follows just from the presence of clusters, and is insensitive to the nature of their physical origin as long as one imposes $N_{\rm cl} \sim \langle n\rangle$. When the above equation is used, the explanation of the data in terms of (quenched) jets agrees with the cluster picture. However, the explanation of the centrality dependence in terms of jets based solely on the above equation is insufficient and inconclusive: any mechanism leading to clusters would do! Realistic microscopic estimates of ${\rm cov}^{\rm j}$ and j are necessary, including the interplay of jets and medium [current status: Liu and Trainor, PLB 567 (2003) 184, Mitchell, "Workshop on Correlations and Fluctuations in Relativistic Nuclear Collisions", MIT, 21-23 April 2005, http://www.mit.edu/~vaurynov/21april2005workshop]

How strong are the correlations

a - detector efficiency, number of observed particles $\sim a$, number of pairs $\sim a^2$. Thus

$$\sigma^2_{\rm dyn} = \frac{r-1}{\langle n \rangle}_{\rm full} {\rm cov}^* = \frac{a^2}{a}\frac{(r-1)}{\langle n \rangle}_{\rm obs} {\rm cov}^*$$

$${\rm cov}^* = \sigma^2_{\rm dyn}\frac{\langle n \rangle_{\rm obs}}{a(r-1)}.$$

For PHENIX $a \simeq 10\%$, which gives

$${\rm cov}^* \simeq \frac{0.035 \ {\rm GeV}^2}{(r-1)}.$$

The natural scale set by $\sigma_p^2 \simeq 0.08 \ {\rm GeV}^2$ (recall that $\mid {\rm cov}^* \mid \leq \sigma_p^2$). For $r=2$ the value of ${\rm cov}^*$ would assume 45% of the maximum possible value. This is unlikely, as argued from model estimates presented below, where ${\rm cov}^*$ at most $0.01 \ {\rm GeV}^2$. Thus a natural explanation of the above number is to take a significantly larger value of r. The higher r, the easier it is to satisfy the data even with small values of ${\rm cov}^*$. "Lumped clusters": lumps of matter move at some collective velocities, correlating the momenta of particles belonging to the same cluster

Same for STAR

Very similar quantitative conclusions from the STAR data [nucl-ex/0504031]. The measure used by STAR is just the estimator for $\sigma^2_{\rm dyn}$. It is elementary to show

$$\langle \Delta p_i \Delta p_j \rangle = \frac{N_{\rm event}-1}{N_{\rm event}} \sigma^2_M - \frac{1}{N_{\rm event}} \sum_{k=1}^{N_{\rm event}} \frac{\sigma_p^2}{N_k} = \sigma^2_{\rm dyn} \quad (1)$$

Assuming $a = 0.75$ we find

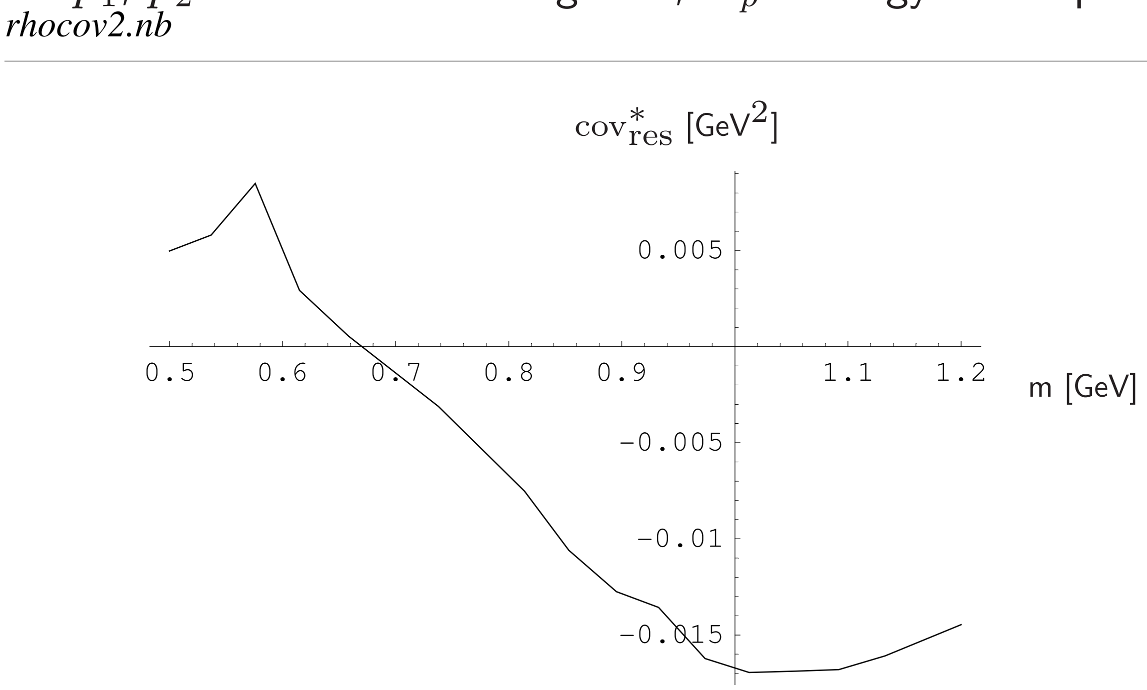
$\mathrm{cov}^*(r-1) = 0.058, 0.043, 0.035, 0.014$ GeV^2
for $\sqrt{s_{NN}} \quad = 200, \quad 130, \quad 62, \quad 20$ GeV

The value at 130 GeV is close to PHENIX. Significant beam-energy dependence! This may be due to increase of the covariance per pair with energy, and/or increase of the number of clustered particles

Covariance from decay of resonances

$$\mathrm{cov}^*_{\mathrm{res}} = \frac{\int d^3p \int \frac{d^3p_1}{E_{p_1}} \int \frac{d^3p_2}{E_{p_2}} \delta^{(4)}(p-p_1-p_2) C \frac{dN_R}{d^3p} \left(p_1^\perp - \langle p^\perp \rangle\right) \left(p_2^\perp - \langle p^\perp \rangle\right)}{\int d^3p \int \frac{d^3p_1}{E_{p_1}} \int \frac{d^3p_2}{E_{p_2}} \delta^{(4)}(p-p_1-p_2) C \frac{dN_R}{d^3p}}$$

dN_R/d^3p - resonance distribution from the Cooper-Frye formula - Cracow expansion, p_1, p_2 - momenta of daughters, E_p - energy of the particle with momentum p, C - cuts



Cancellations between contributions of various resonances are possible; `Therminator` - negligible contribution of resonances to the p_T correlations. (Of course, the "lumpy" feature of the expansion was not implemented in the calculation)

Thermal clusters

Emission from local thermalized sources: each element of the fluid moves with its collective velocity and emits particles with locally thermalized spectra. The picture refelects charge conservation within the local source [Bożek, WB, Florkowski, Acta Phys. Hung. A22 (2005) 149].

$$\mathrm{cov}^*_{i,j} = \frac{\int d\Sigma_\mu u^\mu \int d^3p_1 (p_1^\perp - \langle p^\perp \rangle) f_i^u(p_1) \int d^3p_2 (p_2^\perp - \langle p^\perp \rangle) f_j^u(p_2)}{\int d\Sigma_\mu u^\mu \int d^3p_1 f_i^u(p_1) \int d^3p_2 f_j^u(p_2)}$$

$f_i^u(p) = (\exp(p \cdot u/T) \pm 1)^{-1}$ - boosted thermal distribution, $u(x)$ -expansion velocity, $d\Sigma_\mu$ - integration over the freeze-out hypersurface. Fix flow such that $\langle M \rangle = 554$ MeV

T [MeV]	10	100	120	140	165	200
$\langle \beta \rangle$	0.94	0.72	0.69	0.58	0.49	0.31
σ_p^2 [GeV^2]	0.056	0.19	0.15	0.15	0.14	0.12
$\mathrm{cov}^*_{\pi\pi}$ [GeV^2]	0.027	0.011	0.0088	0.0063	0.0034	0.0006

Results depend strongly on temperature. At realistic thermal parameters the experimental value of the covariance, $0.035\ \mathrm{GeV}^2/(r-1)$, cannot be accounted for unless the number of (charged) particles belonging to a cluster is sizeable, at least $4-10$

Conclusion

In the fiducial centrality range:

1. Constant $\langle M \rangle$ and σ_p explain the value of $\sigma_M^{\rm mix}$, which approximately scales with $1/\langle n \rangle$ (accuracy 1%)
2. The scaling of $\sigma^2_{\rm dyn}$ with $1/\langle n \rangle$ (accuracy 2%) suggest the cluster picture of the fireball
3. The magnitude of the observed $\sigma_{\rm dyn}$ can be easily achieved when several (4-10 charged) particles are present in clusters
4. Jets would just produce clusters, so it is impossible to prove or disprove their existence based solely on the centrality dependence of the correlation data at soft/medium p_T
5. The clusters may a priori originate from very different physics: jets, droplets of fluid formed in the explosive scenario of the collision, or other mechanisms leading to multiparticle correlations
6. Other authors have estimated effects of HBT correlations or elliptic flow, claiming these are small
7. Experimentalists could just compute the covariance via double sums!

Backup slides

Inclusive distributions

The commonly used *inclusive* propability distributions are related to the marginal probability distributions in the following way:

$$\rho_{\rm in}(x) \equiv \sum_n P(n) \int dp_1 \ldots dp_n \sum_{i=1}^{n} \delta(x - p_i)\rho_n(p_1, \ldots, p_n) = \sum_n n P(n)\rho_n(x),$$

$$\rho_{\rm in}(x, y) \equiv \sum_n P(n) \int dp_1 \ldots dp_n \sum_{i,j=1,j\neq i}^{n} \delta(x - p_i)\delta(y - p_j)\rho_n(p_1, \ldots, p_n)$$

$$= \sum_n n(n-1)P(n)\rho_n(x, y)$$

which are normalized to $\langle n \rangle$ and $\langle n(n-1) \rangle$, respectively.

The sum variable (Białas+Koch)

Consider the variable $S_n = p_1 + \ldots + p_n$. Then

$$\begin{aligned}\langle S\rangle &= \sum_n R(n)\int dp_1\ldots dp_n S_n \rho_n(p_1,\ldots,p_n)\\ &= \sum_n R(n)n\int dp\, p\rho_n(p) = \sum_n R(n)n\langle p\rangle_n = \langle n\rangle\langle p\rangle_{\rm in},\end{aligned}$$

Similarly as for M, we find the exact results,

$$\sigma_S^2 = \langle n\rangle\sigma_{p,{\rm in}}^2 + \sum_n P(n)\left[\sum_{i,j=1,j\neq i}^{n} {\rm cov}_n(p_i,p_j)\right]$$

Note that the derived relation is completely general, true for correlated distributions, any $R(n)$, *etc.*

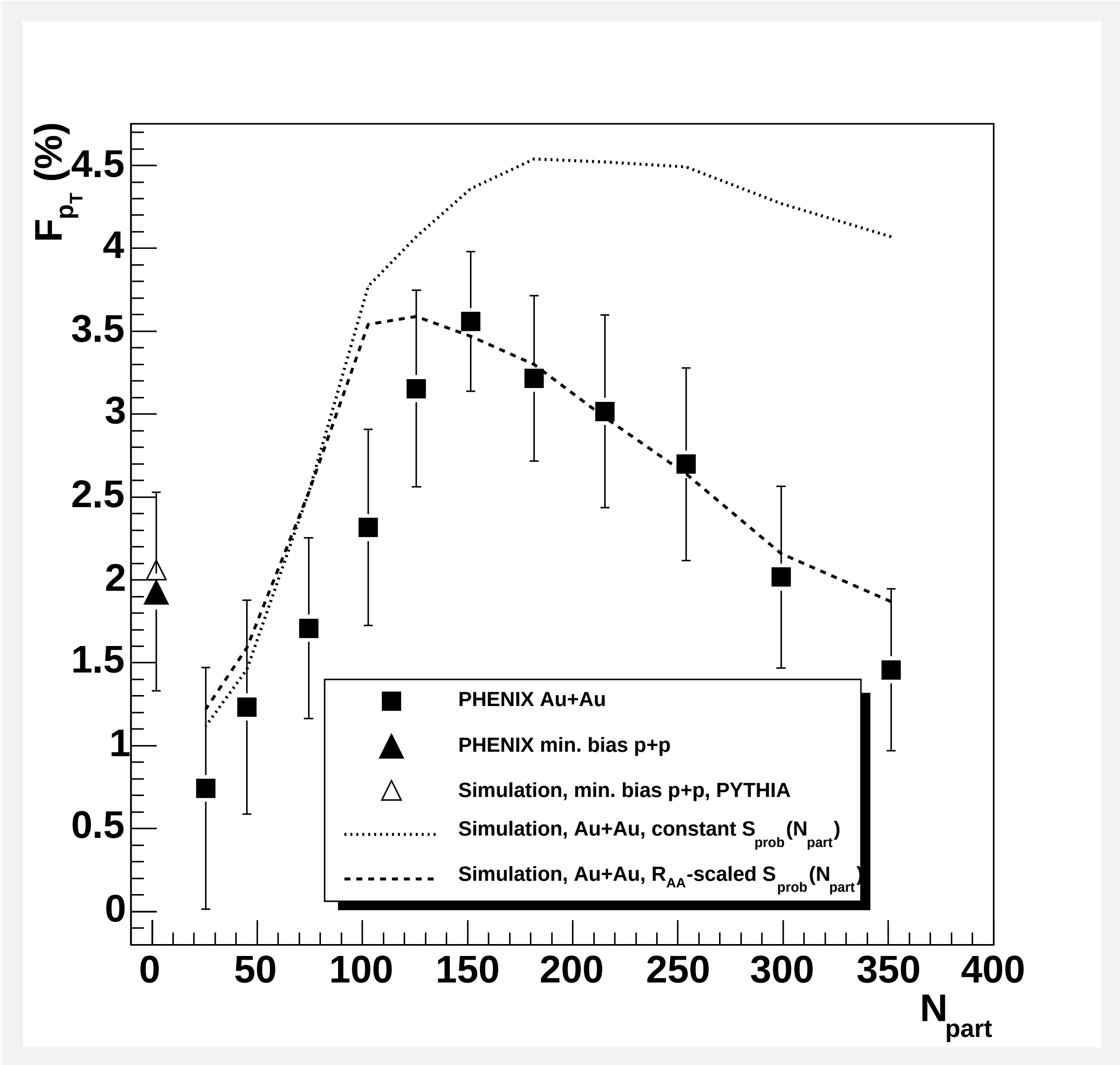
Thus, in the cluster picture

$$\sigma_S^2 - \sigma_{S,{\rm mix}}^2 = \sum_n P(n)\left[\sum_{i,j=1,j\neq i}^{n} {\rm cov}_n(p_i,p_j)\right] \sim \langle n\rangle$$

The Φ measure of Gaździcki and Mrówczyński

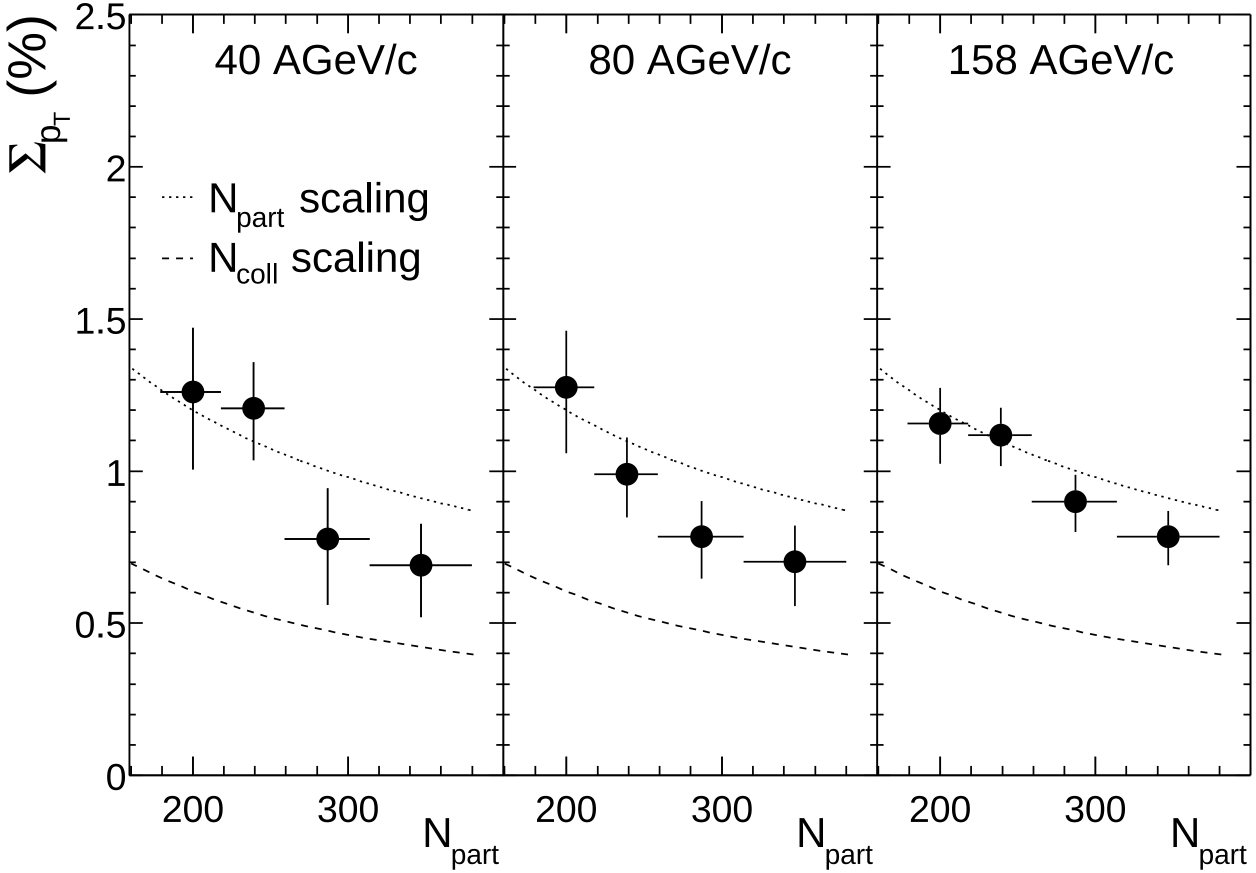
$$\Phi \equiv \sqrt{\frac{\sigma_S^2}{\langle n \rangle}} - \sigma_p \simeq \frac{a(r-1)\mathrm{cov}^*}{2\sigma_p}$$

PHENIX @ 130 GeV



$$\omega = \frac{\sigma_M}{< M >}, \quad F_{p_T} = \frac{\omega_{\text{ data}} - \omega_{\text{ mix}}}{\omega_{\text{ mix}}} \simeq \frac{a(r-1)\text{cov}^*_{\text{res}}}{2\sigma_p^2}$$

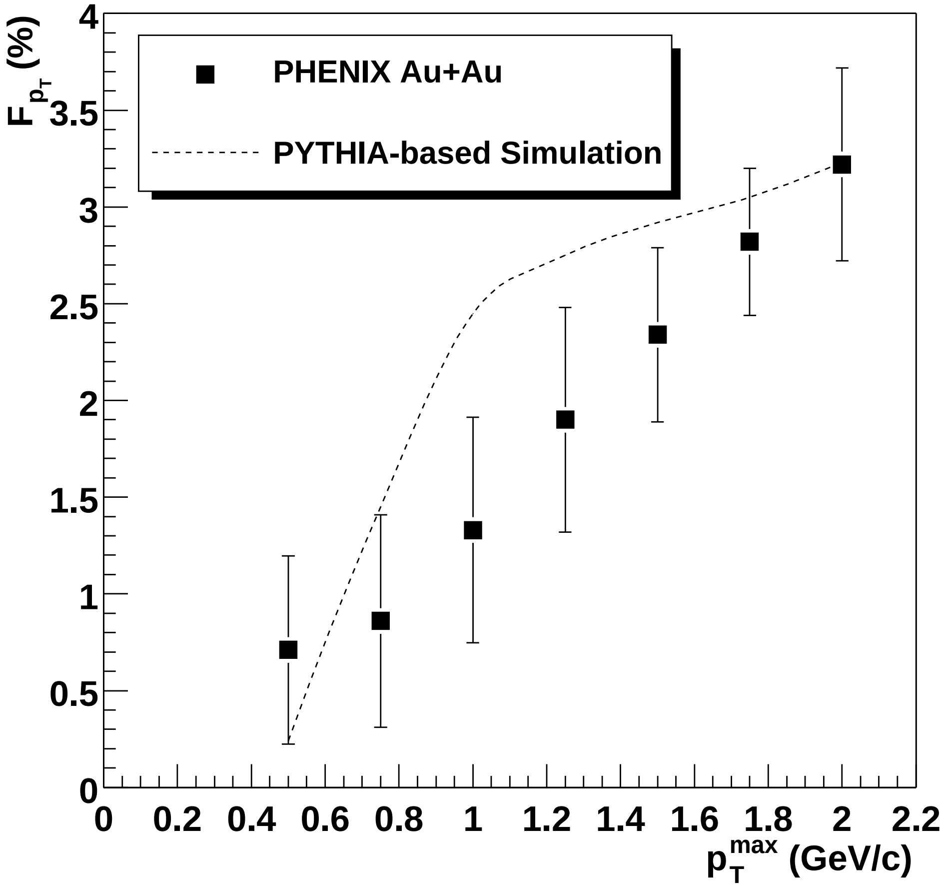
CERES



$$\Sigma_{p_T} \equiv \frac{\sigma_{\rm dyn}}{\langle p_T \rangle} \sim \frac{1}{\sqrt{\langle n \rangle}}$$

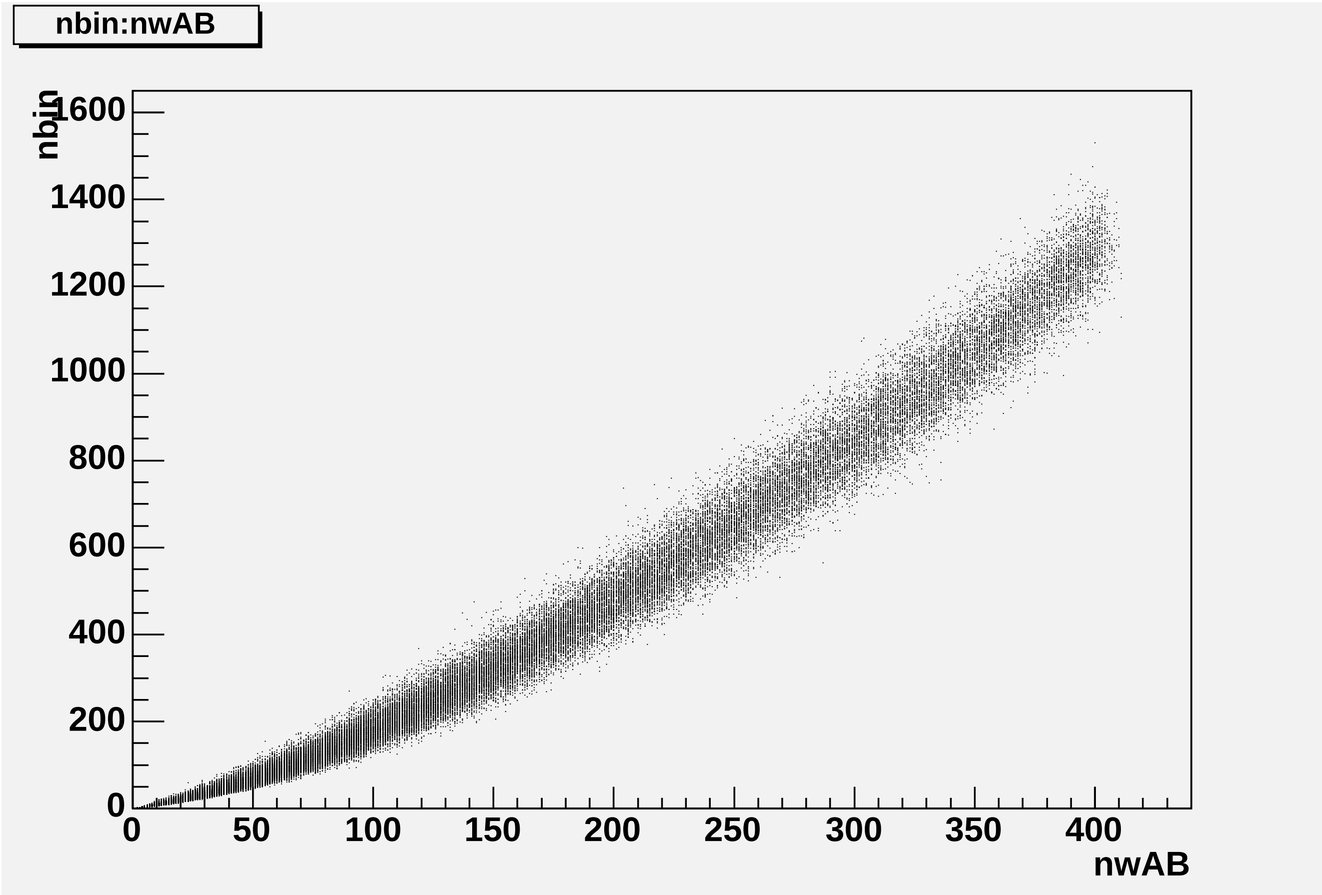
Works! (errors large)

Dependence on $p_T^{\max}$



$$\omega = \frac{\sigma_M}{< M >}, \quad F_{p_T} = \frac{\omega_{\text{ data}} - \omega_{\text{ mix}}}{\omega_{\text{ mix}}} \simeq \frac{a(r-1)\text{cov}^*_{\text{res}}}{2\sigma_p^2}$$

N_{bin} vs. N_w



Glauber Monte Carlo with $\sigma_{NN} = 41$ mb