

# Korelacje w pośpieszności na LHC

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IFJ PAN & UJK

Kawiory, 8.04.2016

Szczegóły: WB+Piotr Bożek, arXiv:1512.01945

## Motivation/new data

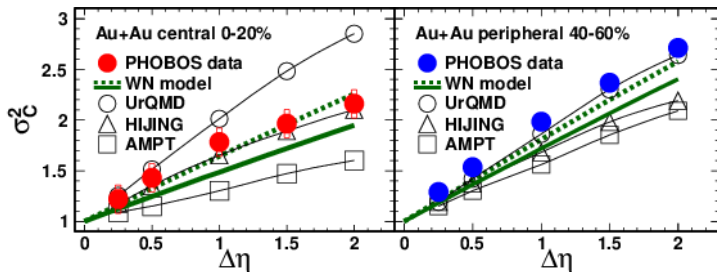
- Old story . . . , investigated at RHIC  
Białas, Zalewski, Bzdak, . . .  
(not all understood at RHIC)
- New data from the LHC, new methodology (ATLAS notes 2015)
- Interplay of long- and short-range effects
- Longitudinally-extended source model

Goal: understand key elements from an analytic model  
anatomy of the correlations

Physics issues: production mechanism in the early stage,  
degrees of freedom, . . .

# Some years ago at RHIC ...

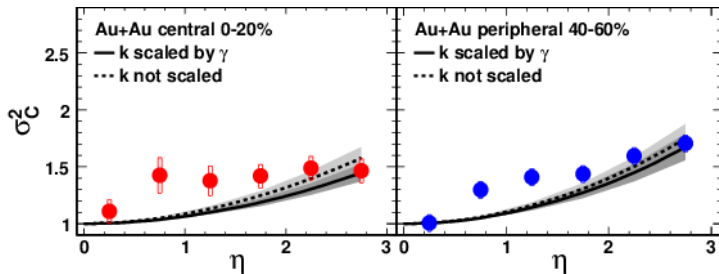
$$\text{PHOBOS: } \sigma_c^2 = \left\langle \frac{(n_F - n_B)^2}{n_F + n_B} \right\rangle$$



[WNM: Bzdak, Woźniak 2010]

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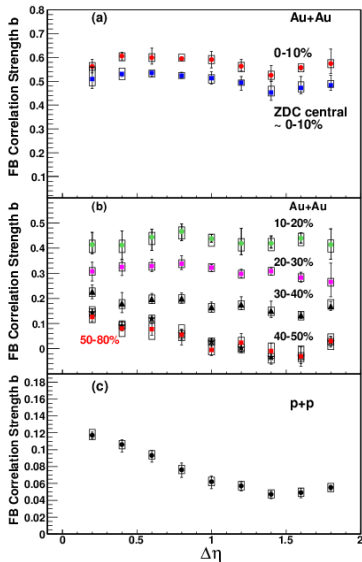
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# Some years ago at RHIC ...

STAR (for symmetric):  $b = \frac{\langle n_F n_B \rangle - \langle n_F \rangle^2}{\langle n_F n_F \rangle - \langle n_F \rangle^2}$  (same as Pearson's  $\rho$ )

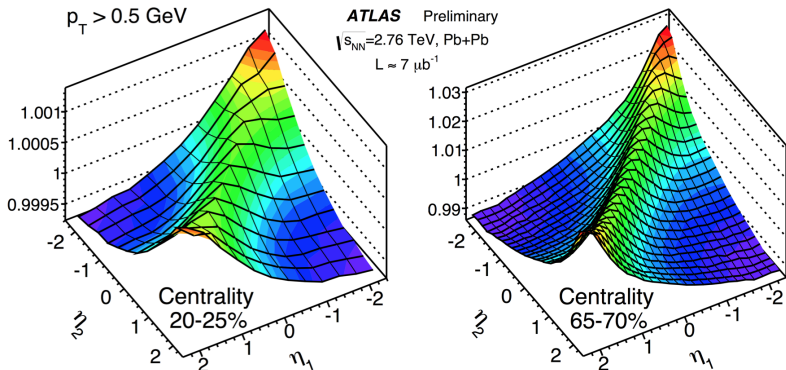


# New data (ATLAS notes CONF-2015-020, -051)

$$C(\eta_1, \eta_2) = \frac{\langle N(\eta_1, \eta_2) \rangle}{\langle N(\eta_1) \rangle \langle N(\eta_2) \rangle} = \frac{S(\eta_1, \eta_2)}{B(\eta_1, \eta_2)}$$

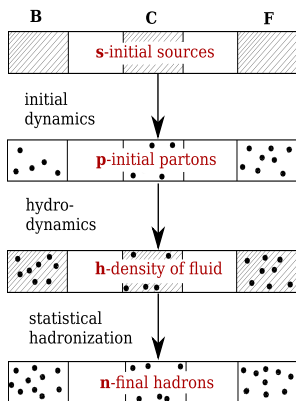
$$C_N(\eta_1, \eta_2) = \frac{C(\eta_1, \eta_2)}{C_p(\eta_1)C_p(\eta_2)}, \quad C_p(\eta_1) = \int d\eta_2 C(\eta_1, \eta_2), \quad C_p(\eta_2) = \dots$$

$\eta_1$  and  $\eta_2$  – pseudorapidities of different hadrons (no autocorrelations)



# 3-stage approach (superposition model)

Generation and propagation of e-by-e fluctuations



[Adam Olszewski+WB 2015]

Concept of **sources**: wounded nucleons, quarks, flux tubes, ...

Hydro: provides mapping  $\eta_s = \frac{1}{2} \log \frac{t+z}{t-z} \rightarrow \eta$

For long-range separations:

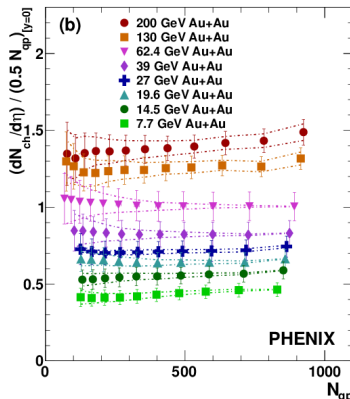
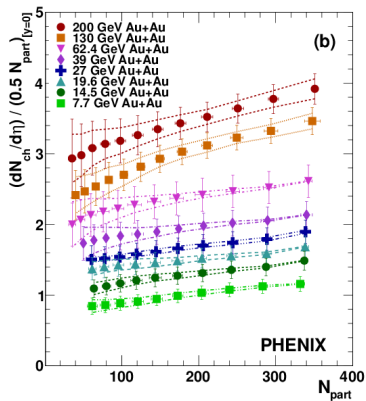
Not much mixing between the bins

$$C^s(\eta_{s,1}, \eta_{s,2}) \simeq C^n(\eta_1, \eta_2)$$

# Wounded quarks as sources

Also needed linearly of produced number of particles on the number of sources,  $n \sim s$ . This is accomplished in the wounded quark model (and not in the wounded nucleon model, which requires an admixture of a (nonlinear) binary component

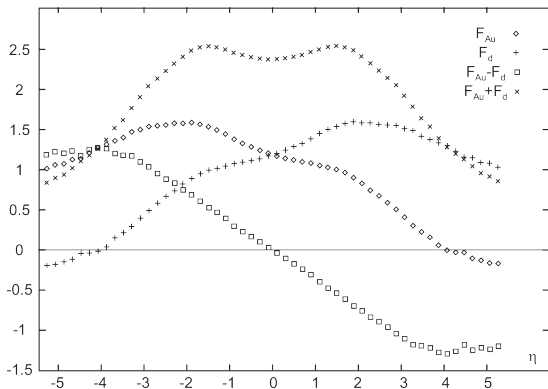
[PHENIX 2016]





# Białas-Czyż triangles in rapidity profiles

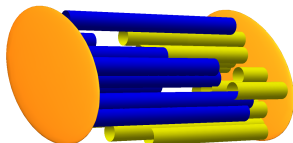
Charged hadron spectra in d+Au @ RHIC [Białas+Czyż 2004]



Asymmetric profiles used ever since ... [Bożek+Wyskiel, Bzdak]

$$\langle f_A(\eta) \rangle = h(\eta) \frac{y_b + \eta}{2y_b}, \quad \langle f_B(\eta) \rangle = h(\eta) \frac{y_b - \eta}{2y_b}$$

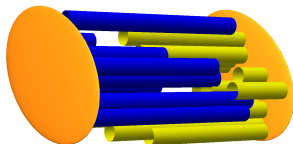
## Rapidity-extended source model



$$\begin{aligned}\langle N(\eta_1)N(\eta_2) \rangle &= \langle N_A \rangle \langle f_A(\eta_1, \eta_2) \rangle + \langle N_A(N_A - 1) \rangle \langle f_A(\eta_1) \rangle \langle f_A(\eta_2) \rangle \\ &+ \langle N_B \rangle \langle f_B(\eta_1, \eta_2) \rangle + \langle N_B(N_B - 1) \rangle \langle f_B(\eta_1) \rangle \langle f_B(\eta_2) \rangle \\ &+ \langle N_A N_B \rangle [\langle f_A(\eta_1) \rangle \langle f_B(\eta_2) \rangle + \langle f_B(\eta_1) \rangle \langle f_A(\eta_2) \rangle]\end{aligned}$$

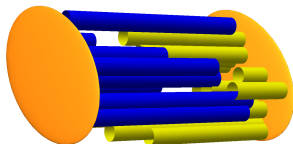
$f_{A,B}(\eta_i)$  and  $f_{A,B}(\eta_1, \eta_2)$  – probabilities of emission from a single source

# Rapidity-extended source model



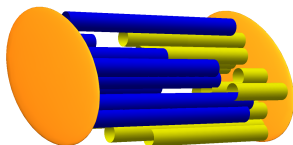
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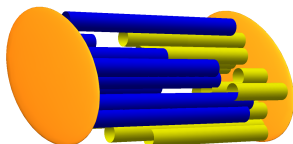
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## Rapidity-extended source model



$$\begin{aligned}\langle N(\eta_1)N(\eta_2) \rangle &= \langle N_A \rangle \text{cov}_A(\eta_1, \eta_2) + \langle N_A^2 \rangle \langle f_A(\eta_1) \rangle \langle f_A(\eta_2) \rangle \\ &+ \langle N_B \rangle \text{cov}_B(\eta_1, \eta_2) + \langle N_B^2 \rangle \langle f_B(\eta_1) \rangle \langle f_B(\eta_2) \rangle \\ &+ \langle N_A N_B \rangle [\langle f_A(\eta_1) \rangle \langle f_B(\eta_2) \rangle + \langle f_B(\eta_1) \rangle \langle f_A(\eta_2) \rangle]\end{aligned}$$

Production is independent/uncorrelated, unless from the same source. However, even if  $\text{cov}_{A,B}(\eta_1, \eta_2) = 0$  we have nontrivial  $C(\eta_1, \eta_2)$  from the fluctuation of  $N_A$  and  $N_B$  [Bzdak+Teaney 2013]

## Correlations in rapidity-extended source model

Average number of particles:  $\langle N(\eta) \rangle = \langle N_A \rangle \langle f_A(\eta) \rangle + \langle N_B \rangle \langle f_B(\eta) \rangle$  with symmetric and antisymmetric parts

$$\langle f_{A,B}(\eta) \rangle = f_s(\eta) \pm f_a(\eta)$$

After elementary transformations ( $N_+ = N_A + N_B$ ,  $N_- = N_A - N_B$ )

$$\begin{aligned} C(\eta_1, \eta_2) &= 1 + \frac{1}{\langle N(\eta_1) \rangle \langle N(\eta_2) \rangle} \times \\ &\quad \left\{ \langle N_A \rangle \text{cov}_A(\eta_1, \eta_2) + \langle N_B \rangle \text{cov}_B(\eta_1, \eta_2) \right. \\ &\quad + \text{var}(N_+) f_s(\eta_1) f_s(\eta_2) + \text{var}(N_-) f_a(\eta_1) f_a(\eta_2) \\ &\quad \left. + [\text{var}(N_A) - \text{var}(N_B)] [f_s(\eta_1) f_a(\eta_2) + f_a(\eta_1) f_s(\eta_2)] \right\} \end{aligned}$$

Correlations in elem. production + fluctuation of the number of sources

## Observation by Bzdak and Teaney

Vanishing covariance, symmetric system, profiles from the previous slides:

$$\langle N_A \rangle = \langle N_B \rangle, \quad \text{var}(N_A) = \text{var}(N_B)$$

Then

$$C(\eta_1, \eta_2) = 1 + \frac{1}{\langle N_+ \rangle^2} \left[ \text{var}(N_+) + \text{var}(N_-) \frac{\eta_1}{y_b} \frac{\eta_2}{y_b} \right]$$

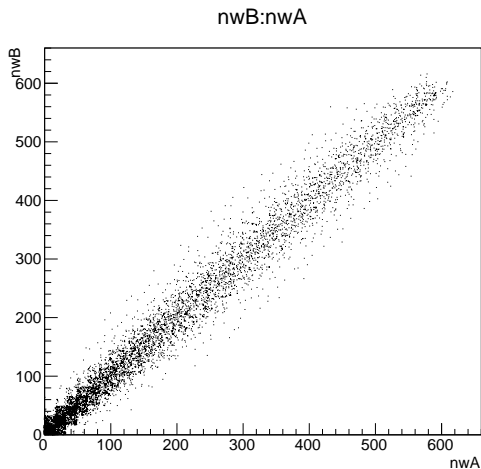
[Bzdak+Teaney 2013]

Fluctuations of  $N_A$  vs  $N_B$  yield the  $\eta_1 \eta_2$  structure in  $C(\eta_1, \eta_2)$



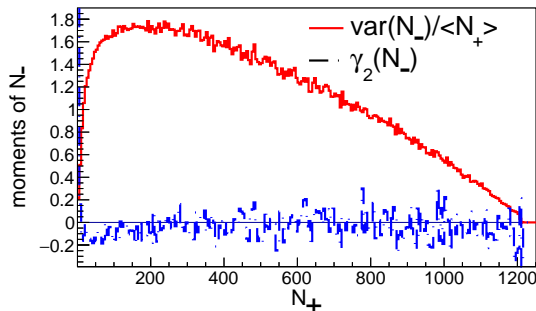
# Fluctuations of $N_A$ vs $N_B$ in the wounded quark model

Pb+Pb @ 2.76 TeV



# Fluctuations of $N_A$ vs $N_B$ in the wounded quark model

(fixed  $N_+$ )

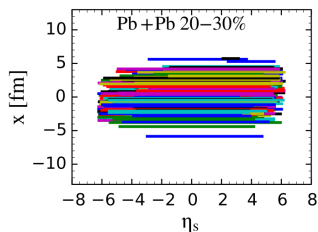


May obtain the moments of fluctuation of sources from Monte Carlo

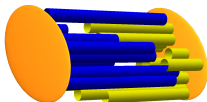
# Fluctuating length

- Idea: entropy deposition from wounded nucleons originates from string-like objects whose other end-point is randomly distributed in  $\eta$  (related to [Brodsky+Gunion+Kuhn 1977])
- *“Soft particle production ... is dominated by multiple gluon exchanges between partons from the colliding hadrons, followed by radiation of ... partons distributed uniformly in rapidity”* [Białas+Jeżabek 2004]
- Torque in p-A collisions [Bożek+WB+Moreira 2011, Bożek+WB 2015]
- Similar ideas in [Monnai+Schenke 215]
- Built-in into existing models/codes

e.g., HIJING [L.-G. Pang, QM2015]:



# What it yields?



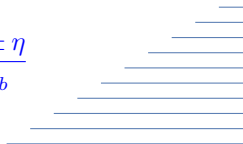
$$f_A(\eta; y) = \theta(y < \eta < y_b), \quad f_B(\eta; y) = \theta(-y_b < \eta < -y)$$

(uniform string fragmentation function)

Random end  $y$  is uniformly selected from  $[-y_b, y_b]$ , where  $y_b$  is the beam rapidity

Averaging over  $y \rightarrow$  "triangles":

$$\langle f_{A,B}(\eta) \rangle = \int_{-y_b}^{y_b} \frac{dy}{2y_b} f_{A,B}(\eta; y) = \frac{y_b \pm \eta}{2y_b}$$



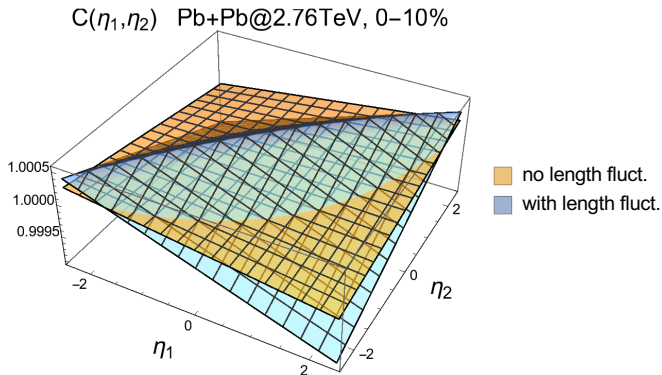
Correlations just from length fluctuations:

$$\langle f_{A,B}(\eta_1, \eta_2) \rangle = \int_{-y_b}^{y_b} dy f_{A,B}(\eta_1; y) f_{A,B}(\eta_2; y) = \frac{y_b \pm \min(\eta_1, \eta_2)}{2y_b}$$

$$\text{COV}_{A,B}(\eta_1, \eta_2) = \frac{y_b^2 - \eta_1 \eta_2 - y_b |\eta_1 - \eta_2|}{4y_b^2}$$

## Results for $C$

$$\bar{C}(\eta_1, \eta_2) = \frac{C(\eta_1, \eta_2)}{\int_{-Y}^Y \frac{d\eta_1}{2Y} \int_{-Y}^Y \frac{d\eta_2}{2Y} C(\eta_1, \eta_2)} \quad (\text{normalization to 1})$$

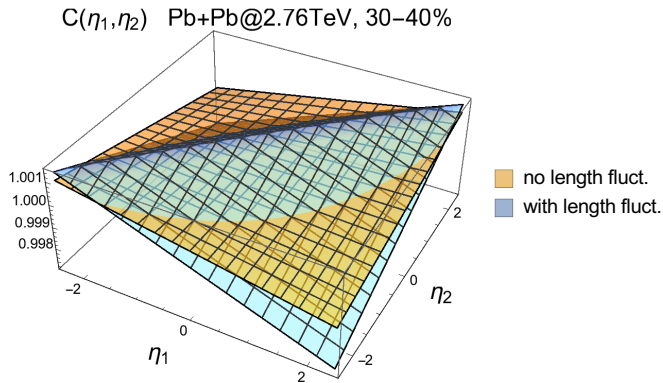


Generation of the ridge (structure from  $-|\eta_1 - \eta_2|$ )

Fluctuating length affects both short- and long-range components

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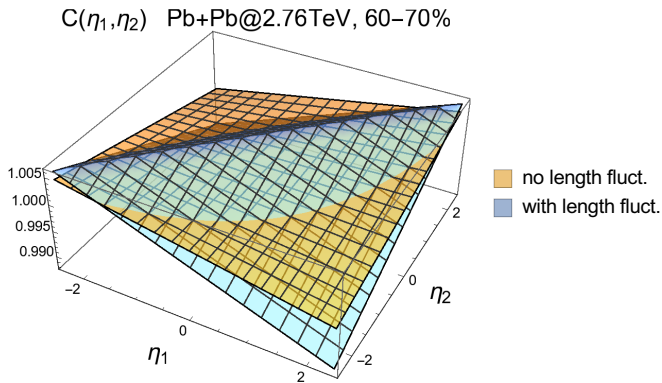


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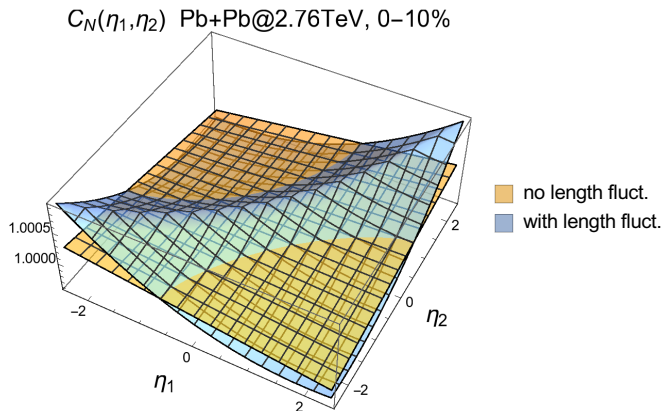


Generation of the ridge (structure from  $-|\eta_1 - \eta_2|$ )

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## Results for $C_N$

$$\bar{C}_N(\eta_1, \eta_2) = \frac{C_N(\eta_1, \eta_2)}{\int_{-Y}^Y \frac{d\eta_1}{2Y} \int_{-Y}^Y \frac{d\eta_2}{2Y} C_N(\eta_1, \eta_2)} \quad (\text{normalization to 1})$$

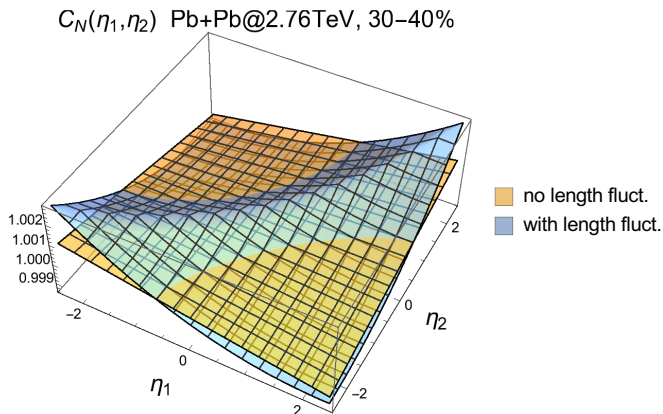


Generation of the **saddle** in the ridge (seen in experiment) is **trivial**  
Fluctuating length essential



## Results for $C_N$

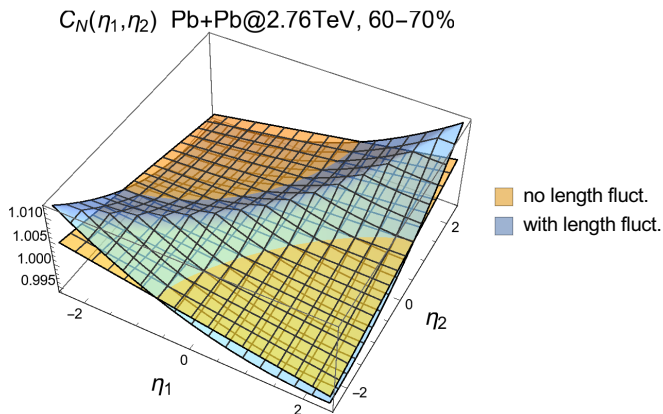
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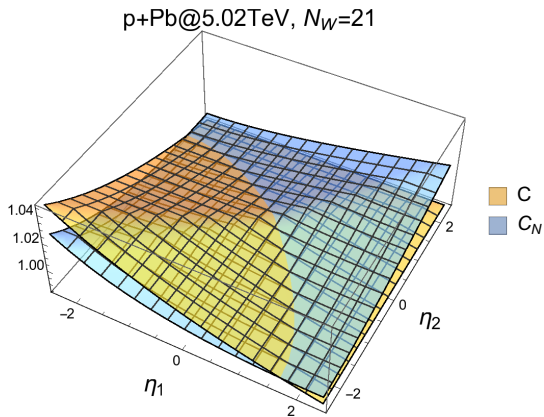
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Generation of the **saddle** in the ridge (seen in experiment) is **trivial**  
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# $C$ and $C_N$ for p-Pb collisions



## $a_{nm}$ coefficients

$$a_{nm} = \int_{-Y}^Y \frac{d\eta_1}{Y} \int_{-Y}^Y \frac{d\eta_2}{Y} C(\eta_1, \eta_2) T_n\left(\frac{\eta_1}{Y}\right) T_m\left(\frac{\eta_2}{Y}\right)$$
$$T_n(x) = \sqrt{2 + 1/2} P_n(x) \quad [\text{Bzdak+Teaney 2013, Jia 2015}]$$

(play analogous role to flow coefficients in harmonic flow)

$$a_{nn} = \frac{\frac{\text{var}(N_-)}{\langle N_+ \rangle} + \frac{\text{var}(\omega)}{\langle \omega \rangle^2}}{6\langle N_+ \rangle} \frac{Y^2}{y_p^2} \delta_{n1} - \frac{\frac{\text{var}(\omega)}{\langle \omega \rangle^2} + 1}{6\langle N_+ \rangle} \frac{Y^2}{y_p^2} \delta_{n1} + \frac{\frac{\text{var}(\omega)}{\langle \omega \rangle^2} + 1}{(2n-1)(2n+3)\langle N_+ \rangle} \frac{Y}{y_p}$$

$$a_{n,n+2} = -\frac{\frac{\text{var}(\omega)}{\langle \omega \rangle^2} + 1}{2(2n+3)\sqrt{(2n+1)(2n+5)}\langle N_+ \rangle} \frac{Y}{y_p} \quad (\text{length fluct.})$$

other = 0

## $a_{nm}$ coefficients

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$\omega$  – overlaid strength

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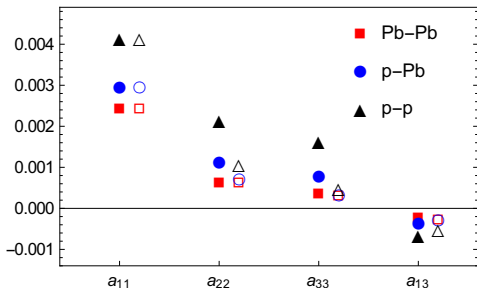
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$a_{nm} \sim 1/\langle N_+ \rangle$  – fall-off **similarly as in experiment** if  $N_+ \sim N_{\text{ch}}$

## Results for $a_{nm}$

Pb-Pb@2.76TeV



(filled – from Fig. 7 of ATLAS-CONF-2015-020, open – model)

$N_{\text{ch}}/N_+$  fitted by adjusting  $a_{11}^{\text{exp}} = c^{\text{exp}}/N_{\text{ch}} = a_{11}^{\text{mod}} = c^{\text{mod}}/N_+$

$N_{\text{ch}} = 4.7N_+$ , acceptance  $\Delta\eta = 4.8 \rightarrow dN_{\text{ch}}/d\eta \simeq 1 \times N_+$

(exp:  $dN_{\text{ch}}/d\eta \simeq (3 - 4) \times N_W$  and  $dN_{\text{ch}}/d\eta \simeq 1.4 \times Q_W \rightarrow$  wounded quarks, partons)

$N_{\text{ch}} = 5.1N_A$  for p-Pb@5.02TeV

$N_{\text{ch}} = 8.1N_+$  for p-p@13TeV – requires sources at partonic level



# Conclusions

- New data coming!
- Derived analytic expressions in the simple superposition model, grasping features of more involved approaches (MC codes). Anatomy: fluctuation of sources + *intrinsic* correlations from string breakup
- The correlations from the early production mechanism contribute to both short- and long-range components in  $\eta$
- $1/N_{ch}$  scaling of  $a_{11} \rightarrow$  **linear relation**  $N_{ch} = \kappa N_{\text{sources}}$ , with the value of  $\kappa$  suggesting **wounded constituents** as degrees of freedom
- Universality of Pb-Pb, p-Pb, and (possibly) p-p should be understood if sources are partons (e.g., wounded quarks)
- Intricate procedures:  $C \rightarrow C_N$ , normalization, *removing short-range correlations* – necessary in modeling for 1-1 comparisons
- To eliminate short-range component use the **multiparticle cumulant method** [Bzdak+Bożek 2015]
- Extension to azimuthal moments ("torque" effect of event-plane decorrelation, [Bożek+WB+Moreira 2011]), CMS data
- Wounded quarks!

# BACKUP

# Fluctuating strength

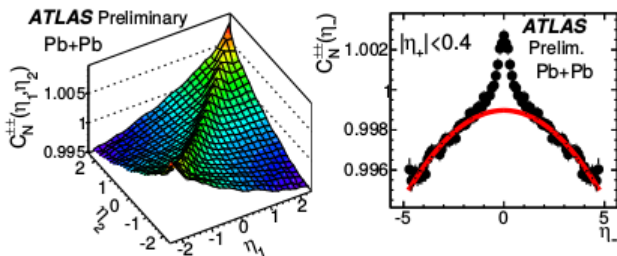
Additional ingredient:

$$f_A(\eta; y) = \omega \theta(y < \eta < y_b), \quad f_B(\eta; y) = \omega \theta(-y_b < \eta < -y)$$

$\omega$  – random strength of the source, included in models to increase multiplicity fluctuations, a.k.a. **overlaid distribution**

## Removing the short-range effects

Rather involved experimental procedure, which gets rid of correlations shorter than  $\sim 1$  unit in  $\eta_1 - \eta_2$



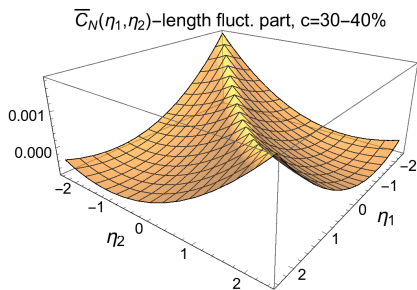
Removes resonances, jets, ..., some of the source fluctuations

After the procedure only  $a_{11}$  survives in exp., other = 0 (do not throw out the baby with the bath water!)

It should be carried out in a model simulation

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## Smearing of $\eta$

There is some “communication” between adjacent bins in  $\eta$ , so the mapping  $\eta_S \rightarrow \eta$  should be smeared

Gaussian smearing:

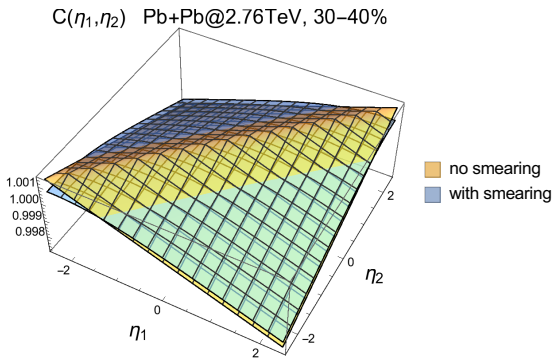
$$S_{\text{sm}}(\eta_1, \eta_2) = \int d\eta'_1 \int d\eta'_2 g(\eta_1, \eta'_1) g(\eta_2, \eta'_2) S(\eta'_1, \eta'_2)$$
$$|\eta_1 - \eta_2| \rightarrow \frac{2\sigma_\eta e^{-\frac{(\eta_1 - \eta_2)^2}{4\sigma_\eta^2}}}{\sqrt{\pi}} + (\eta_1 - \eta_2) \operatorname{erf}\left(\frac{\eta_1 - \eta_2}{2\sigma_\eta}\right)$$

with the other terms ( $\eta_1, \eta_2, \eta_1\eta_2$ ) unchanged

( $\rightarrow$  Bzdak-Teaney term not modified)

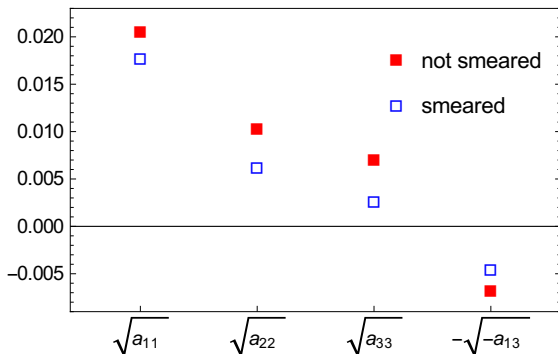
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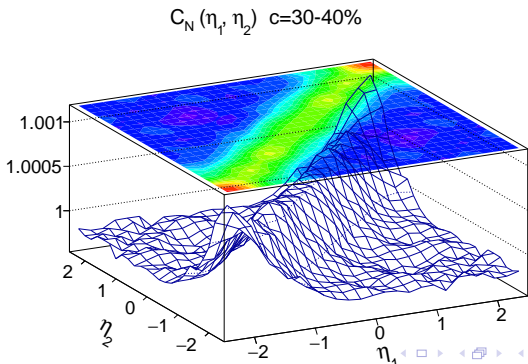
## From initial state to final hadrons

- $\eta_S \rightarrow \eta$ , some extra hydro push  $\rightarrow$  reduction of  $a_{nm}$
- resonance decays – relevant effect [PB+WB+Adam Olszewski 2015]  
 $\rightarrow$  increase
- charge conservation, other correlations – not included
- **removal of the short-range effects** (not simple in a model)

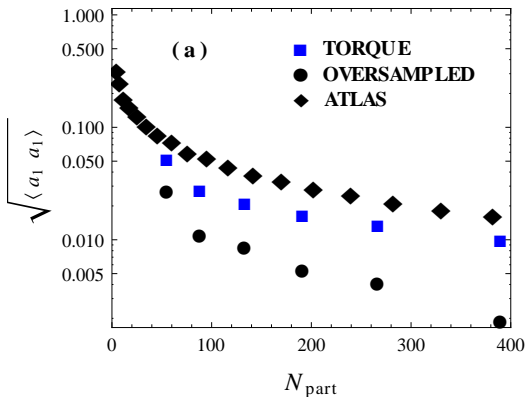
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- **removal of the short-range effects** (not simple in a model)

Hydro (no length fluctuations included here):



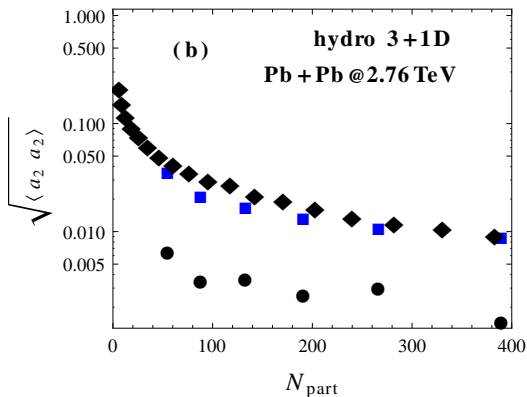
# $a_{nm}$ from hydro



$\langle a_n a_m \rangle = a_{nm}$ , “torque”=our model without length fluct.,  
“oversampled”=no resonance decays

About 50% of  $\sqrt{a_{11}}$  from resonances!

# $a_{nm}$ from hydro



$\langle a_n a_m \rangle = a_{nm}$ , “torque”=our model without length fluct.,  
“oversampled”=no resonance decays

## $b$ coefficient from $C$

The  $b$  correlation coefficient is defined for the symmetric system as follows:

$$b = \frac{\langle n_F n_B \rangle - \langle n_F \rangle \langle n_F \rangle}{\langle n_F n_F \rangle - \langle n_F \rangle \langle n_F \rangle}$$

With fixed  $N_+$  and for  $\text{cov}_{A,B}(\eta_1, \eta_2) = 0$  the one- and two-particle densities are

$$N(\eta) = N_+ f_s(\eta)$$

$$N(\eta_1, \eta_2) = N_+(N_+ - 1)f_s(\eta_1)f_s(\eta_2) + \text{var}(N_-)f_a(\eta_1)f_a(\eta_2) + \delta(\eta_1 - \eta_2)N_+f_s(\eta_1)$$

Then for  $F = [0, \infty)$  and  $B = (-\infty, 0]$

$$\begin{aligned} \langle n_F n_B \rangle - \langle n_F \rangle \langle n_F \rangle &= -N_+/4 + \text{var}(N_-) \int_0^y d\eta_1 \int_{-y}^0 d\eta_2 f_a(\eta_1) f_a(\eta_2) \\ &= -N_+/4 - \text{var}(N_-) \int_0^y d\eta_1 \int_0^y d\eta_2 f_a(\eta_1) f_a(\eta_2) = -\langle n_F n_F \rangle + \langle n_F \rangle \langle n_F \rangle \end{aligned}$$

hence  $b = -1$ , which is obvious from  $n_B = N_+ - n_F$ ,  $\langle n_F \rangle = \langle n_B \rangle = N_+/2$