Grawitacyjny czynnik kształtu pionu

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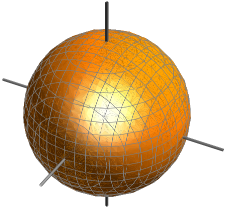
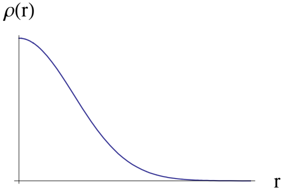
IFUJ, 2 III 08

Based on:

- *Generalized parton distributions of the pion in chiral quark models and their QCD evolution*, WB , **Enrique Ruiz Arriola**, **Krzysztof Golec-Biernat**, PRD 77 (2008) 034023
- *Gravitational and higher-order form factors of the pion in chiral quark models*, WB, ERA, PRD D78 (2008) 094011
- *A note on the QCD evolution of generalized form factors*, WB, ERA, arXiv:0901.3336 [hep-ph]

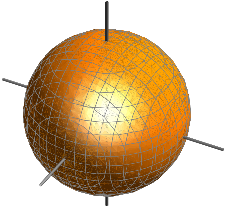
Distribution of charge

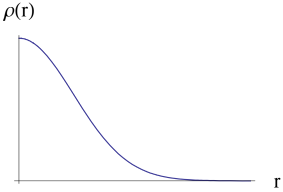




$$Q = \int d^3 r \rho(r), \qquad F(q^2) = \frac{1}{Q} \int d^3 r e^{-i\vec{q}\cdot\vec{r}} \rho(r)$$

Distribution of charge





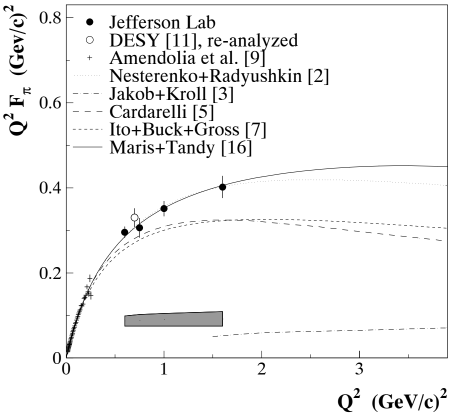
$$Q = \int d^3 r \rho(r), \qquad F(q^2) = \frac{1}{Q}\int d^3 r e^{-i\vec{q}\cdot\vec{r}} \rho(r)$$

$$= \frac{1}{Q}\int d^3 r\, \rho(r)[1 - i\vec{q}\cdot\vec{r} - \frac{1}{2}(\vec{q}\cdot\vec{r})^2 + \ldots] = 1 - \frac{q^2}{6Q}\int d^3 r\, r^2 \rho(r) + \ldots$$

$$\langle r^2 \rangle = -6 \frac{d}{dq^2} F(q^2)$$

Electromagnetic form factor of the pion from TJLAB

[Volmer et al., 2001]



$$Q^2 = \vec{q}^2 - q_0^2 = -q^2 = -t$$

Covariant definition:

$$F_\pi(q^2)\, 2P^\mu\, \delta^{ab} = \langle \pi^a(p') \mid J^\mu_{em}(0) \mid \pi^b(p) \rangle$$

where $q = p' - p$, $P = \frac{1}{2}(p' + p)$

Gravitational form factor

Electromagnetic current:

$J_V^\mu = \sum_{q=u,d,\ldots} \bar{q}(x) \frac{\tau_a}{2} \gamma^\mu q(x)$

Gravitational form factor

Electromagnetic current:

$J_V^\mu = \sum_{q=u,d,\dots} \bar{q}(x) \frac{\tau_a}{2} \gamma^\mu q(x)$

Energy-momentum tensor:

$\Theta^{\mu\nu} = \sum_{q=u,d,\dots} \bar{q}(x) \frac{\mathrm{i}}{2} \left(\gamma^\mu \partial^\nu + \gamma^\nu \partial^\mu\right) q(x) + \text{gluons}$

For the pion two structures (form factors):

$$\langle \pi^b(p') \mid \Theta^{\mu\nu}(0) \mid \pi^a(p) \rangle = \frac{1}{2} \delta^{ab} \left[(g^{\mu\nu} q^2 - q^\mu q^\nu) \Theta_1(q^2) + 4 P^\mu P^\nu \Theta_2(q^2) \right]$$

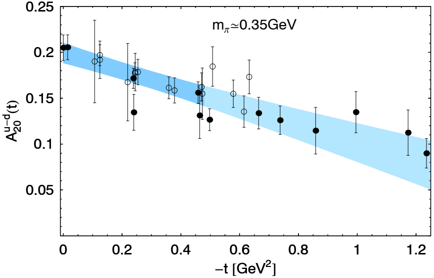
(Θ_1 - spin-2, Θ_2 - spin-0)

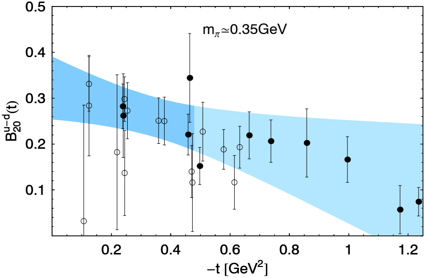
How to determine Θ_1 and Θ_2?

Lattices, exclusive high-energy processes – no need to scatter gravitons!

Nucleon

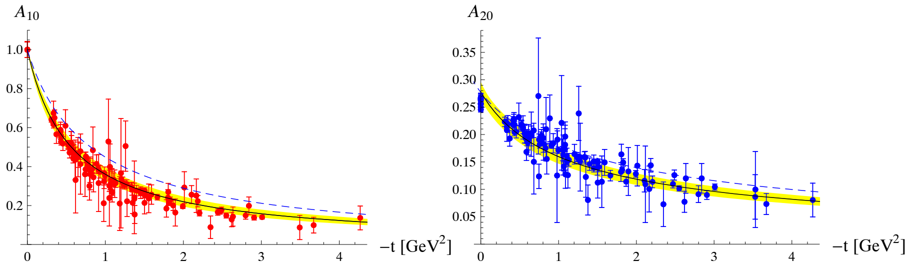
$$\langle N(p') \mid \Theta_{\mu\nu}(0) \mid N(p)\rangle = \bar{u}(p') \left[\frac{\gamma_\mu P_\nu + \gamma_\nu P_\mu}{2} A_{20}(t) + \frac{i(P_\mu \sigma_{\nu\rho} + P_\nu \sigma_{\nu\rho})q^\rho}{4M_N} B_{20}(t) + \frac{q_\mu q_\nu - q^2 g_{\mu\nu}}{M_N} C_{20}(t) \right] u(p)$$





[Hagler et al., 2007] (can compute in QM, Bochum group)

Full-QCD lattice results, pion



The electromagnetic form factor (left) and quark part of the spin-2 gravitational form factor (right) in SQM (solid line) and NJL model (dashed line) compared to the lattice data from [Brömmel 2005/7].

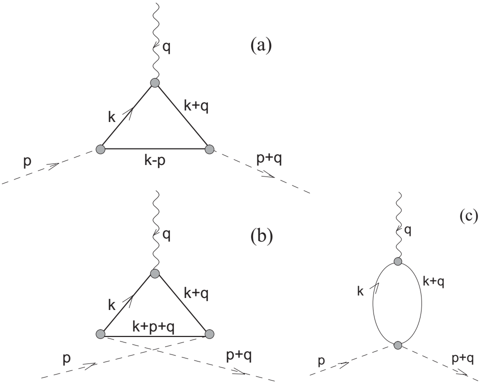
Chiral quark models at one loop (large N_c)

Nambu–Jona-Lasinio
model and followers

Vertex:

EM form factor: $Q\gamma^\mu$, Q - electric charge of the quark

gravitational form factor (quark part):



$$\Theta^{\mu\nu}(k+q,k) = \frac{1}{4}\left[(2k+q)^\mu\gamma^\nu + (2k+q)^\nu\gamma^\mu\right] - \frac{1}{2}g^{\mu\nu}\left(2\not{k} + \not{q} - M\right)$$

Spectral quark model (Vector Meson Dominance)

$$F_V^{\mathrm{SQM}}(t) = \frac{m_\rho^2}{m_\rho^2 - t}$$

$$\Theta_1^{\mathrm{SQM}}(t) = \Theta_2^{\mathrm{SQM}}(t) = \frac{m_\rho^2}{t} \log\left(\frac{m_\rho^2}{m_\rho^2 - t}\right)$$

m_ρ - mass of the ρ meson

NJL - similar, more complicated expressions

Quark-model relation:
$2\langle r^2 \rangle_\Theta = \langle r^2 \rangle_V$

(matter more concentrated than charge!)

Generalized Parton Distributions (GPD's)

[similar calculation by: Praszałowicz, Rostworowski, Bzdak, Kotko, the Valencia group]

The two isospin projections of the twist-2 GPD of the pion are defined as

$$\delta_{ab}\,\mathcal{H}^{I=0}(x,\zeta,t) = \int \frac{dz^-}{4\pi} e^{ixp^+z^-} \langle \pi^b(p+q)|\bar{\psi}(0)\gamma\cdot n\psi(z)|\pi^a(p)\rangle\Big|_{z^+=0,z^\perp=0}$$

$$i\epsilon_{3ab}\,\mathcal{H}^{I=1}(x,\zeta,t) = \int \frac{dz^-}{4\pi} e^{ixp^+z^-} \langle \pi^b(p+q)|\bar{\psi}(0)\gamma\cdot n\psi(z)\,\tau_3|\pi^a(p)\rangle\Big|_{z^+=0,z^\perp=0}$$

where $p^2 = m_\pi^2$, $q^2 = -2p\cdot q = t$, $n^2 = 0$, $p\cdot n = 1$, $q\cdot n = -\zeta$

ζ - momentum transfer along the light cone

Some reviews:

- K. Goeke, M. V. Polyakov, and M. Vanderhaeghen, Prog. Part. Nucl. Phys. 47 (2001) 401
- M. Diehl, Phys. Rept. 388 (2003) 41
- A. V. Belitsky, A. V. Radushkin, Phys. Rept. 418 (2005) 1

GPD's provide more detailed information of the structure of hadrons, three-dimensional picture instead of one-dimensional projection of the usual PD. Information on GPD's may come from such processes as $ep \to ep\gamma$, $\gamma p \to pl^+l^-$, $ep \to epl^+l^-$, or from lattices. Small cross sections of exclusive processes require very high accuracy experiments. First results are for the nucleon coming from HERMES and CLAS, also COMPASS, H1, ZEUS

Formal features

In the *symmetric* notation one introduces $\xi = \frac{\zeta}{2-\zeta}$, $X = \frac{x-\zeta/2}{1-\zeta/2}$, where $0 \leq \xi \leq 1$ and $-1 \leq X \leq 1$. Then

$$H^{I=0,1}(X,\xi,t) = \mathcal{H}^{I=0,1}\left(\frac{\xi+X}{\xi+1}, \frac{2\xi}{\xi+1}, t\right)$$

with the symmetry properties

$$H^{I=0}(X,\xi,t) = -H^{I=0}(-X,\xi,t),\ H^{I=1}(X,\xi,t) = H^{I=1}(-X,\xi,t).$$

For $X \geq 0$ we have $\mathcal{H}^{I=0,1}(X,0,0) = q^{I=0,1}(X)$, relating GPD to **PDF**

Moments of GPD's

The **polynomiality** conditions state that

$$\int_{-1}^{1} dX\, X^{2j}\, H^{I=1}(X,\xi,t) = \sum_{i=0}^{j} A_{2j+1,2i}(t)\xi^{2i},$$

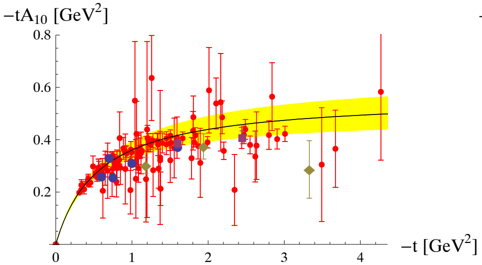
$$\int_{-1}^{1} dX\, X^{2j+1}\, H^{I=0}(X,\xi,t) = \sum_{i=0}^{j+1} A_{2j+2,2i}(t)\xi^{2i},$$

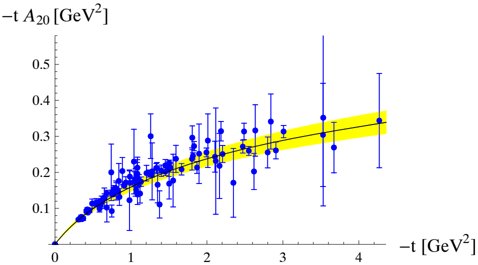
where $A_{n,k}(t)$ are the coefficient functions (form factors). The conditions follow from the Lorentz invariance, time reversal, and hermiticity. In particular

$$\forall\xi: \quad \int_{-1}^{1} dX\, H^{I=1}(X,\xi,t) = 2A_{10}(t) = 2F_V(t),$$

$$\int_{-1}^{1} dX\, X\, H^{I=0}(X,\xi,t) = A_{20}(t) + 2A_{22}(t)\xi^2 = \Theta_2(t) - \Theta_1(t)\xi^2,$$

Once again comparison to lattices





The vector (left) and spin-2 ($= \Theta_1$) gravitational (right) form factors from SQM (multiplied by $-t$) compared to the lattice data [Brömmel 2005/7]. In addition, we show the TJLAB data [Volmer 2000,Tadevosyan 2007, Horn 2006] (darker circles and squares) and the Cornell data [Bebek 1977] (diamonds)

Another way to look at the form factors

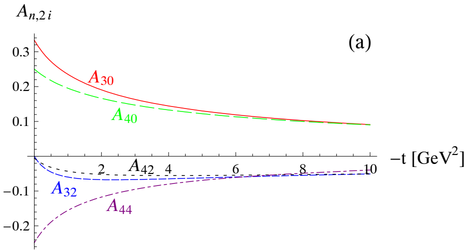
Matrix elements involving more and more covariant derivatives

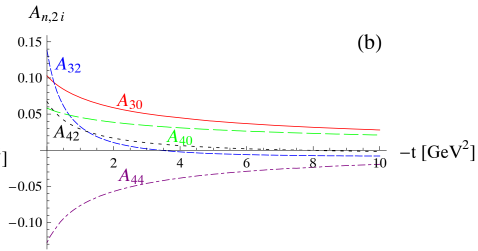
$$\left\langle \pi^+(p') \right| \overline{u}(0)\, \gamma^{\{\mu}\, i\overleftrightarrow{D}^{\mu_1} i\overleftrightarrow{D}^{\mu_2} \ldots i\overleftrightarrow{D}^{\mu_{n-1}\}}\, u(0) \left| \pi^+(p) \right\rangle =$$

$$2P^{\{\mu} P^{\mu_1} \ldots P^{\mu_{n-1}\}} A_{n0}(t) + 2 \sum_{\substack{k=2 \\ \text{even}}}^{n} q^{\{\mu} q^{\mu_1} \ldots q^{\mu_{k-1}} P^{\mu_k} \ldots P^{\mu_{n-1}\}}\, 2^{-k} A_{nk}(t)$$

$\{.\}$ - symmetrization operator

Higher-order form factors





$A_{3,2i}$ and $A_{4,2i}$ at the quark-model scale $\mu_0 \sim 320$ MeV (a), at the lattice scale $\mu = 2$ GeV (b), and the gluon form factors $A^G_{4,2i}$ at $\mu = 2$ GeV(c)

QCD DGLAP-ERBL evolution, code by [KGB, Martin 1998]

Simple evolution of GFF's

For the non-singlet case one has the hierarchy

$$A_{10}(t,Q) = L_1 A_{10}(t,Q_0)$$
$$A_{32}(t,Q) = \frac{1}{5}(L_1 - L_3)A_{10}(t,Q_0) + L_3 A_{32}(t,Q_0)$$
$$A_{54}(t,Q) = \frac{1}{105}(9L_1 - 14L_3 + 5L_5)A_{10}(t,Q_0) + \frac{2}{3}(L_3 - L_5)A_{32}(t,Q_0) + L_5 A_{54}(t,Q_0)$$
$$\ldots$$
$$A_{30}(t,Q) = L_3 A_{30}(t,Q_0)$$
$$A_{52}(t,Q) = \frac{2}{3}(L_3 - L_5)A_{30}(t,Q_0) + L_5 A_{52}(t,Q_0)$$
$$\ldots$$
$$A_{50}(t,Q) = L_5 A_{50}(t,Q_0)$$
$$\ldots$$

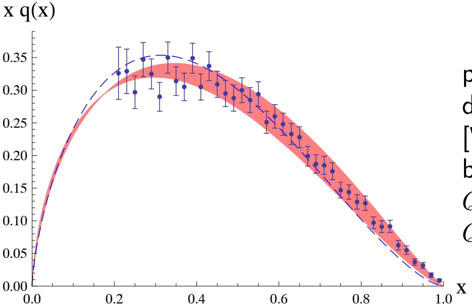
$$L_n = \left(\frac{\alpha(Q^2)}{\alpha(Q_0^2)}\right)^{\gamma_{n-1}/(2\beta_0)}, \quad L_1 = 1$$

(similarly for the singlet)

PDF, QM vs. E615

Recall: the QCD evolution is essential!

LO DGLAP QCD evolution of the non-singlet part to the scale $Q^2 = (4\ \mathrm{GeV})^2$ of the E615 Fermilab experiment:



points: Drell-Yan from E615
dashed: reanalysis of data [Wijesooriya et al., 2005]
band: valence QM PDF evolved to $Q = 4$ GeV from the QM scale $Q_0 = 313^{+20}_{-10}$ MeV

Moments of PDF's vs. lattice

With the notation for the moments at $t = 0$, $\langle x^n \rangle = A_{n+1,0}(0)$, one finds at the lattice scale of $\mu = 2$ GeV

$$\langle x \rangle = 0.271 \pm 0.016,$$
$$\langle x^2 \rangle = 0.128 \pm 0.018,$$
$$\langle x^3 \rangle = 0.074 \pm 0.027.$$
$$\text{(lattice)}$$

while in QM after the LO DGLAP evolution to the lattice scale

$$\langle x \rangle = 0.28 \pm 0.02,$$
$$\langle x^2 \rangle = 0.10 \pm 0.02,$$
$$\langle x^3 \rangle = 0.06 \pm 0.01,$$
$$\text{(chiral quark models)}$$

where the error bars come from the uncertainty of the scale μ_0

1. The spin-2 gravitational form factor of the pion from large-N_c chiral quark models agrees with the full-QCD lattice data (data for the spin-0 case have large errors).
2. In QM the mean squared EM radius is twice the gravitational one. Matter more concentrated than charge.
3. The electromagnetic and gravitational form factors do not evolve with the scale, while the higher-order GFF's do.
4. The generalized form factors at $t = 0$ from full-QCD lattices are reproduced within the error bars.
5. Our predictions can be further tested with future lattice simulations for higher-order GFF's, including the gluon GFF's. The behavior is non-trivial, with form factors having different signs, magnitude, and asymptotic fall-off.
6. LO QCD evolution of GFF's is very simple.