Ewolucja Kwiecińskiego

nieprzecałkowanych partonowych funkcji rozkładu

Wojciech Broniowski
IFJ PAN

IF UJ, 25/05/04

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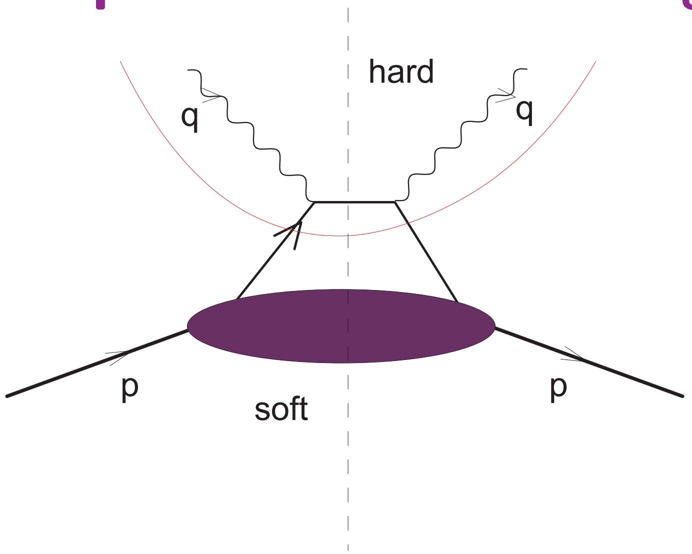
- M. A. Kimber, JK, A. D. Martin + A. M. Stasto, "The unintegrated gluon distribution from the CCFM equation," Phys. Rev. D **62**, 094006 (2000)
- JK, "Unintegrated gluon distributions from the transverse coordinate representation of the CCFM equation in the single loop approximation," Acta Phys. Polon. B **33** (2002) 1809
- Agnieszka Gawron + JK, "Unintegrated gluon distributions in a photon from the CCFM equation in the single loop approximation," Acta Phys. Polon. B **34** (2003) 133
- AG + JK + WB, "Unintegrated parton distributions of pions and nucleons from the CCFM equations in the single-loop approximation," Phys. Rev. D **68** (2003) 054001
- AG + JK, "Resummation effects in Higgs boson transverse momentum distribution within the framework of unintegrated parton distributions", arXiv:hep-ph/0309303
- JK + A. Szczurek, "Unintegrated CCFM parton distributions and transverse momentum of gauge bosons", Nucl. Phys. B **680**, 164 (2004)
- Enrique Ruiz Arriola + WB, "Solution of the Kwieciński evolution equations for unintegrated parton distributions using the Mellin transform", `hep-ph/0404008`

Deep inelastic scattering



$$Q^2 = -q^2, \quad W^2 = (p+q)^2, \quad x = \frac{Q^2}{2p \cdot q} = \frac{Q^2}{Q^2 + W^2}, \quad Q^2 \to \infty$$

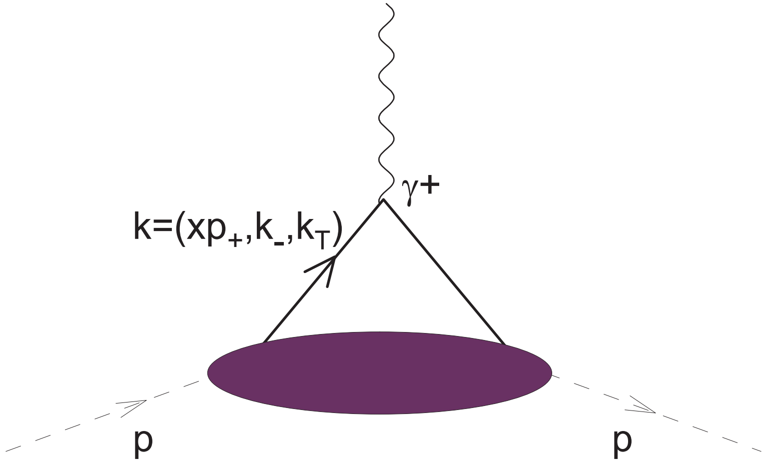
Factorization of soft and hard processess, Wilson expansion (OPE), twist expansion

$$\langle J(q) J(-q) \rangle = \sum_i C_i(Q^2; \mu) \langle \mathcal{O}_i(\mu) \rangle$$

Practical meaning for inclusive processes: $\frac{d\sigma}{dx dQ^2} = \int dx\, f(x) \frac{d\bar{\sigma}(x)}{dx dQ^2}$

Unintegrated Parton Distributions

Leading-twist (=2) UPD



No integration over $k_\perp$! Around since the dawn of QCD (Dokshitzer, Dyakonov, Troyan, 1979), formal definition (Collins, 2003):

$$
\begin{aligned}
f(x, k_\perp) &= \int \frac{dy^- d^2 y_\perp}{16\pi^3} e^{-ixp^+ y^- + i k_\perp \cdot y_\perp} \langle p \mid \bar{\psi}(0, y^-, y_\perp) W[y, 0] \gamma^+ \psi(0) \mid p \rangle \\
&\sim \langle p \mid a^\dagger(xp^+, k_\perp) a(xp^+, k_\perp) \rangle
\end{aligned}
$$

Integrated PD:

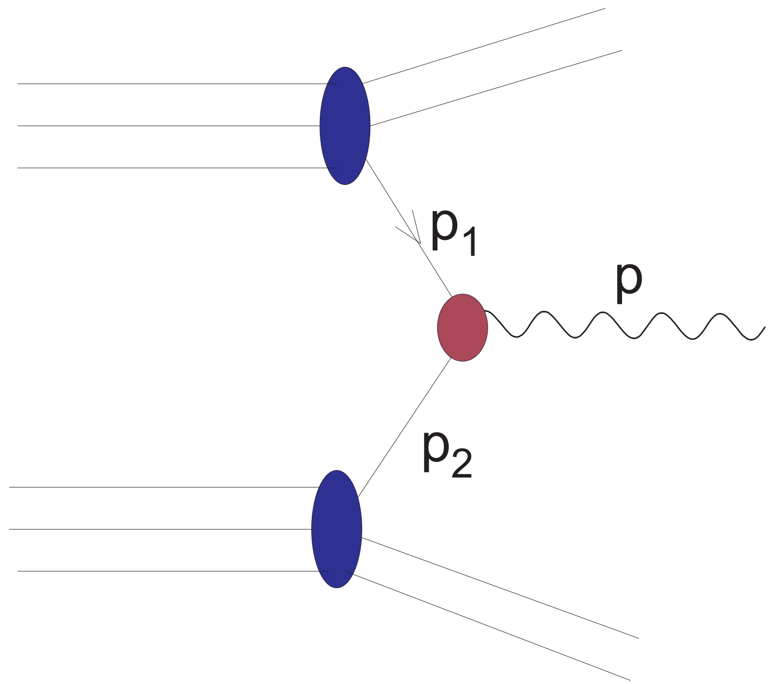
$$
f(x) = \int d^2 k_\perp f(x, k_\perp) = \int \frac{dy^-}{4\pi} e^{-ixp^+ y^-} \langle p \mid \bar{\psi}(0, y^-, 0) \gamma^+ \psi(0) \mid p \rangle
$$

where $W[y, 0] = P \exp[i \int_0^y ds_\mu A^\mu(s)]$ is 1 for the integrated PD in the light-cone gauge $(A^+ = 0)$

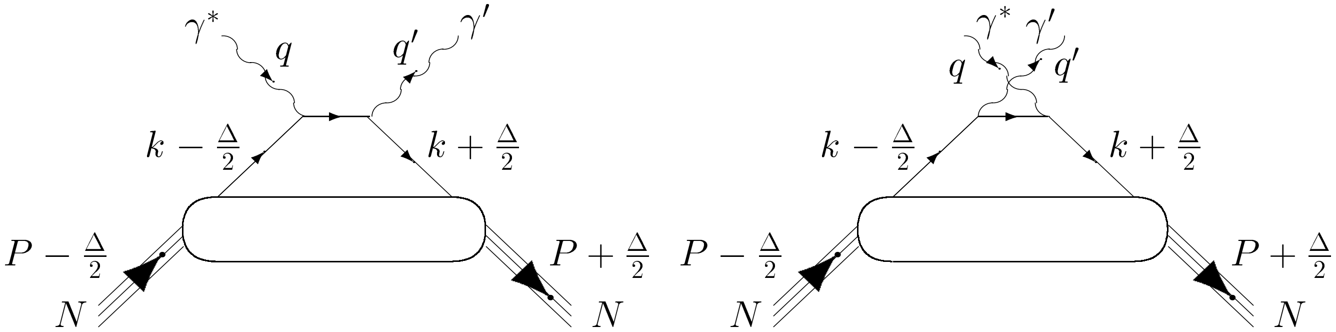
Significance of UPD's

exclusive physical processes: production of W-bosons, Higgs, heavy-flavors, jets

(H. Jung, A. Szczurek, ..., L. Motyka, A. Staśto)

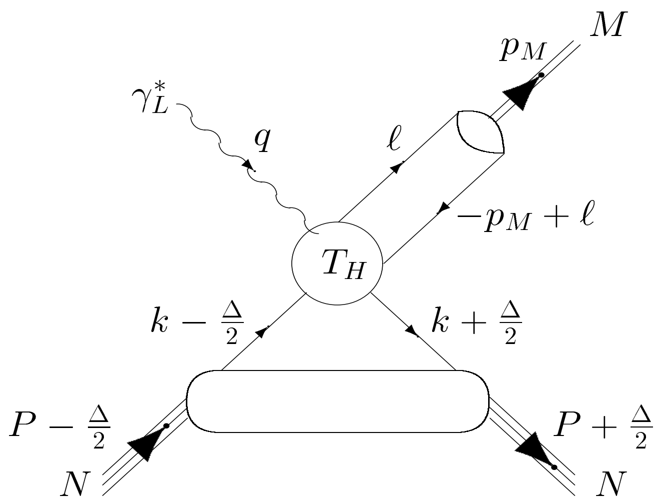


Digression: Generalized Parton Distributions / Exclusive processes in QCD



(a) (b)

Deeply
Virtual
Compton
Scattering



Hard
Meson
Production

non-zero momentum transfer to the target

Dictionary

GPD's: (ξ - transfer of q^+ parallel to the momentum of the target, $\Delta_\perp$ - perprndicular direction)

$t = 0$ & $\xi = 0$	regular PD
$\Delta_\perp = 0$	forward GPD
$\Delta_\perp \neq 0$	off-forward GPD
$\xi = 0$	diagonal GPD (non-skewed GPD)
$\xi \neq 0$	non-diadonal GPD (skewed GPD)

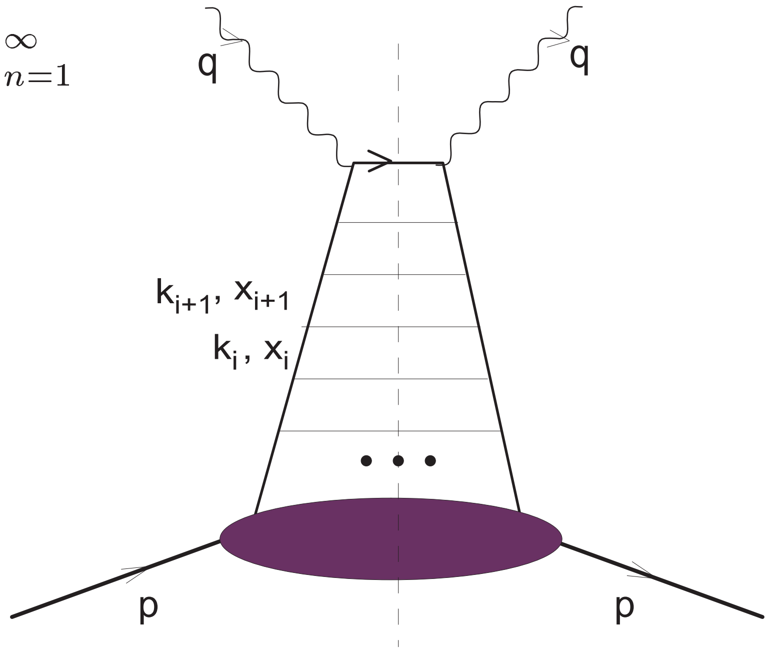
More general objects: $\langle A \mid \mathcal{O} \mid B \rangle$

- A = B = one-particle state – PD of A (inclusive DIS)
- A = one-particle state, B = vacuum – distribution amplitude (DA) of A (hadronic form factors, HMP)
- A, B = one-particle state of different momentum – GPD (exclusive DIS, DVCS, HMP)
- A = many-particle state, B = vacuum – GDA (transition form factors)
- ...

All may be unitegrated!

QCD evolution

ladder, resummation: $\sum_{n=1}^{\infty}$



phase-space restrictions:

$$\mu^2 < k_{\perp,1}^2 < k_{\perp,2}^2 < \ldots < k_{\perp,n}^2 < Q^2$$

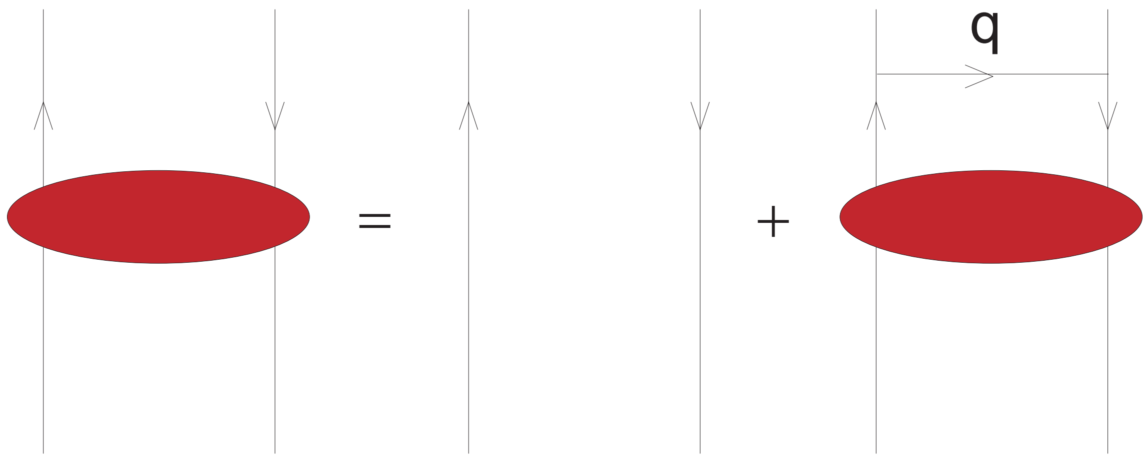
$$1 > x_1 > x_2 > \ldots > x_n = x$$

$$\alpha_s^n \int_{\mu^2}^{Q^2} \frac{dk_{\perp,n}^2}{k_{\perp,n}^2} \int_{\mu^2}^{k_{\perp,n}^2} \frac{dk_{\perp,n-1}^2}{k_{\perp,n-1}^2} \cdots \int_{\mu^2}^{k_{\perp,2}^2} \frac{dk_{\perp,1}^2}{k_{\perp,1}^2} = \frac{\alpha_s^n}{n!} \left(\log \frac{Q^2}{\mu^2} \right)^n$$

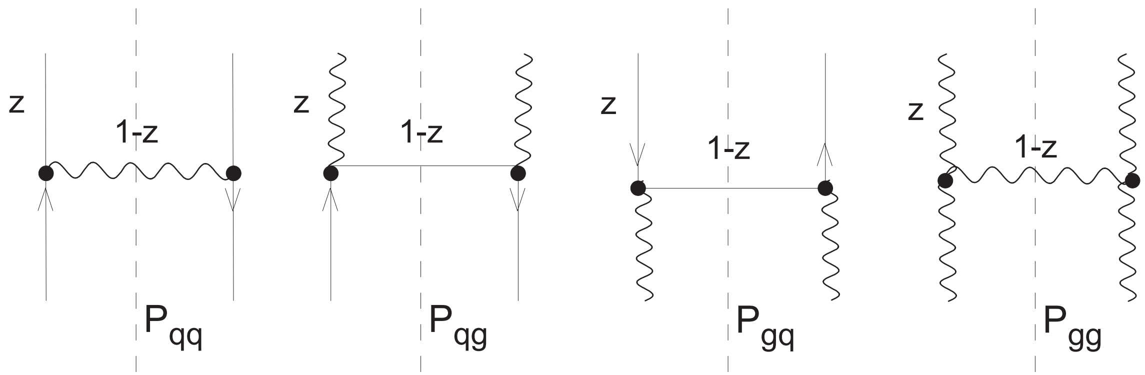
$$\alpha_s(Q^2) = \frac{4\pi}{\beta_0 \log\left(\frac{Q^2}{\Lambda_{QCD}^2}\right)}, \quad \beta_0 = 11/3N_c - 2N_f/3$$

non-planar $\sim 1/Q^2$

Bethe-Salpeter equations



Splitting functions



$$P_{qq} = C_F \frac{1+z^2}{1-z}, \; P_{qg} = N_F[z^2 + (1-z)^2], \; P_{gq} = C_F \frac{1+(1-z)^2}{z},$$
$$P_{gg} = 2N_C \left[\frac{z}{1-z} + \frac{1-z}{z} + z(1-z) \right], \quad C_F = \frac{N_C^2 - 1}{2N_C}$$

DGLAP

Dokshitzer, Gribov, Lipatov, Altarelli, Parisi (70's)

momentum ordering $\equiv$ resummation of $\alpha_s(Q^2)\log Q^2$

$$f_{NS}(x,Q) = f_{NS}(x,Q_0) + \int_0^1 dz \int_{Q_0^2}^{Q^2} \frac{dQ'^2}{Q'^2}\frac{\alpha(Q'^2)}{2\pi} P_{qq}(z) \times \left[\Theta(z-x) f_{NS}\left(\frac{x}{z}, Q'\right) - f_{NS}(x,Q')\right]$$

SFSC – "similarly for the singlet channel"

or

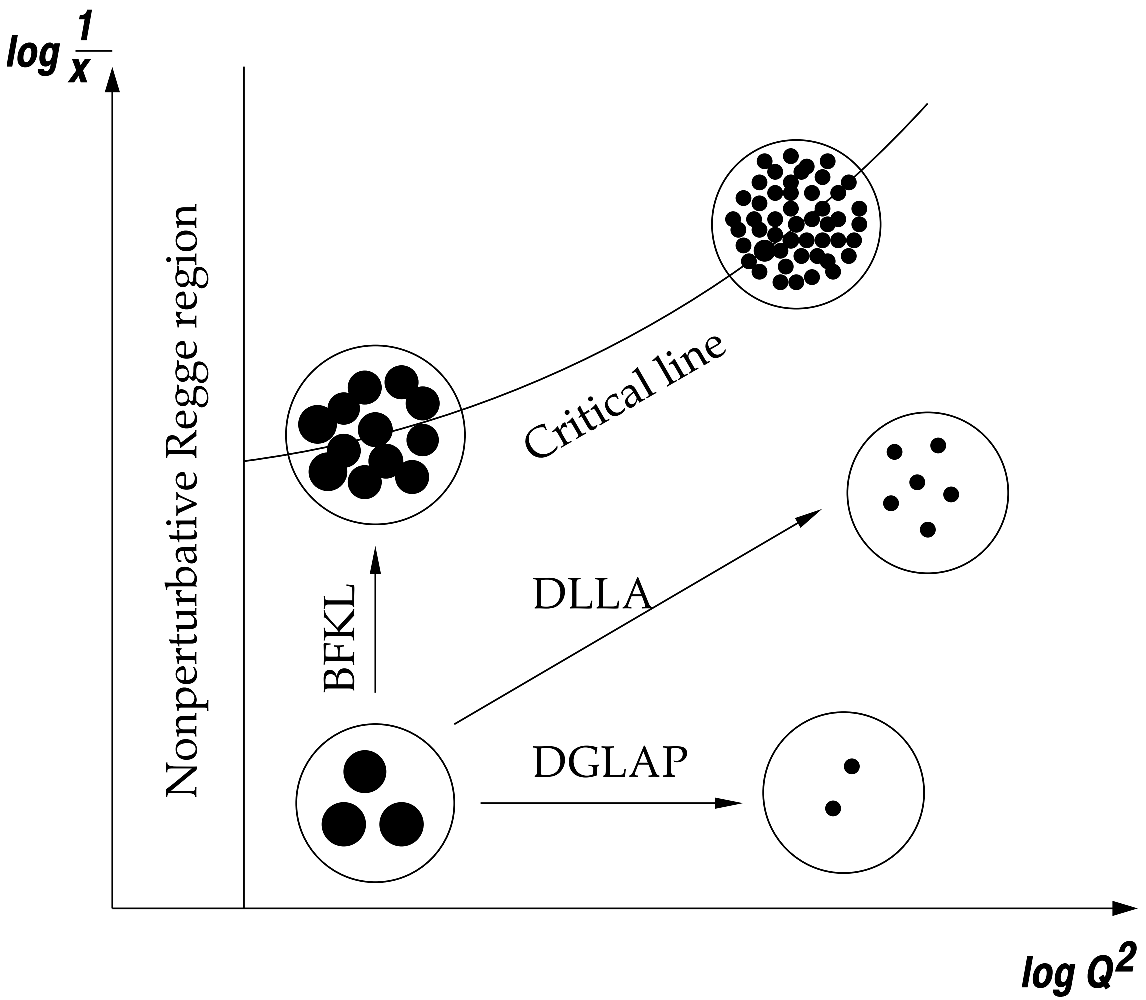
$$Q^2 \frac{d}{dQ^2} f_{NS}(x,Q) = \frac{\alpha(Q^2)}{2\pi}\int_0^1 dz P_{qq}(z)\left[\Theta(z-x) f_{NS}\left(\frac{x}{z}, Q\right) - f_{NS}(x,Q)\right]$$

$f_j = \frac{x}{2} p_j$, and *e.g.* for π^+

$$p_{\rm NS} = \bar{u} - u + d - \bar{d}, \quad p_S = \bar{u} + u + d + \bar{d} + \bar{s} + s + \ldots,$$

$$p_{\rm sea} \equiv p_S - p_{\rm NS} = 2\bar{d} + 2u + \bar{s} + s + \ldots, \quad p_G = g$$

Other schemes



(from KGB)

BFKL

Balitsky, Fadin, Kuraev, Lipatov (1976)

transverse momenta not ordered but limited . . . $\equiv$ resummation of $\alpha_s \log \frac{1}{x}$

gluons only

similarly, $\log(1-x)$ should be resummed at $x \to 1$

For instance, for π^+

$p_{\rm NS} = \bar{u} - u + d - \bar{d}$, $p_S = \bar{u} + u + d + \bar{d} + \bar{s} + s + \ldots$, $p_{\rm sea} \quad \equiv \quad p_S - p_{\rm NS} = 2\bar{d} + 2u + \bar{s} + s +$

CCFM

Catani, Ciafaloni, Fiorani, Marchesini (1988-1990)

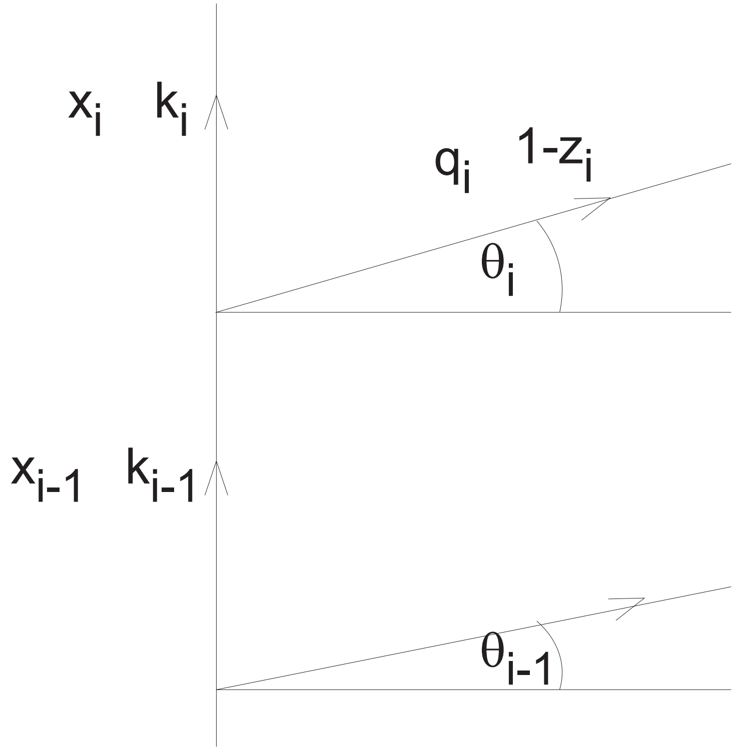
more general scheme, includes DGLAP and CCFM in limiting cases, gluons only

$z_i \equiv \frac{x_i}{x_{i-1}}$, $q'_{\perp,i} \equiv \frac{q_{\perp,i}}{1-z_i}$
$k_{\perp,i} = k_{\perp,i-1} + (1 - z_i) q'_{\perp,i}$

CCFM angular ordering:
$\theta_i > \theta_{i-1} \iff q'_{\perp,i} > z_{i-1} q'_{\perp,i-1}$

$$\iff \frac{|q_{i,\perp}|}{x_{i-1}-x_i} > \frac{|q_{i-1,\perp}|}{x_{i-2}-x_{i-1}}$$

$$\left(\sqrt{s} \tan \frac{\theta_i}{2} = \frac{|q_{i,\perp}|}{x_{i-1}-x_i}\right)$$



dressing of the vertex, CCFM splitting functions:

$$P_{gg} = \frac{\alpha_s[(1-z)q']}{1-z} + \frac{\alpha_s(k_\perp)}{z} \Delta_{NS}(z, q, k_\perp)$$

Evolution equations follow, MC codes

The Kwieciński equations

Kwieciński:

1) The so called one-loop CCFM: $q'_{\perp,i} > q'_{\perp,i-1}$ (stronger than CCFM)
2) include quarks!
3) non-Sudakov form factor set to unity
4) DGLAP splitting functions with regular terms
5) solve in transverse-coordinate space

$\longrightarrow$ simple equations (validity range: LO DGLAP)

$$f_{NS}(x, k_\perp, Q) = f_{NS}(x, k_\perp, Q_0) + \int_0^1 dz \int_{Q_0^2}^{Q^2} \frac{d^2Q'}{\pi Q'^2} \frac{\alpha(Q'^2)}{2\pi} P_{qq}(z)$$
$$\times \left[\Theta(z-x) f_{NS}\left(\frac{x}{z}, k_\perp + (1-z)Q', Q\right) - f_{NS}(x, k_\perp, Q)\right]$$

SFSC

Fourier-Bessel transformation

$$f_j(x,b,Q) \equiv \int d^2k_\perp \, e^{-ik_\perp \cdot b} f_j(x,k_\perp,Q) = \int_0^\infty 2\pi dk_\perp \, k_\perp J_0(bk_\perp) f_j(x,k_\perp,Q)$$

diagonalizes the equations in the transverse coordinate b:

$$Q^2 \frac{\partial f_{\rm NS}(x,b,Q)}{\partial Q^2} = \frac{\alpha(Q^2)}{2\pi} \int_0^1 dz \, P_{qq}(z) \left[\Theta(z-x) \, J_0((1-z)Qb) \, f_{\rm NS}\left(\frac{x}{z}, b, Q\right) - f_{\rm NS}(x,b,Q)\right]$$

SFSC

Remarks:
$b = 0 \to J_0 = 1 \longrightarrow$ equations identical to DGLAP, with the distributions f_j at $b = 0$ becoming the integrated PD's:

$$f_j(x, b=0, Q) = \frac{x}{2} p_j(x,Q)$$

"b-factorization": $f(x,b,Q)$-solution $\longrightarrow F(b) f(x,b,Q)$-solution

Kwieciński: For each b at an initial scale Q_0 the non-perturbative UPD's depending on x and b ($k_\perp$) are perturbatively evolved to a higher scale Q

DGLAP: The non-perturbative PD's depending on x are perturbatively evolved from Q_0 to a higher scale Q

Initial condition assumed, for simplicity, in a factorized form

$$f_j(x, b, Q_0) = F^{\rm NP}(b) \frac{x}{2} p_j(x, Q_0), \quad F^{\rm NP}(0) = 1$$

with the (non-perturbative) *initial profile* function $F^{\rm NP}(b)$ taken to be universal for all species of partons. Certain models do predict a factorized initial condition. The initial profile function factorizes from the evolution equations. Due to evolution, at higher scales Q we have

$$f_j(x, b, Q) = F^{\rm NP}(b) f_j^{\rm evol}(x, b, Q)$$

with $f_j^{\rm evol}(x, b, Q)$ denoting the *the evolution-generated UPD*

Initial profile

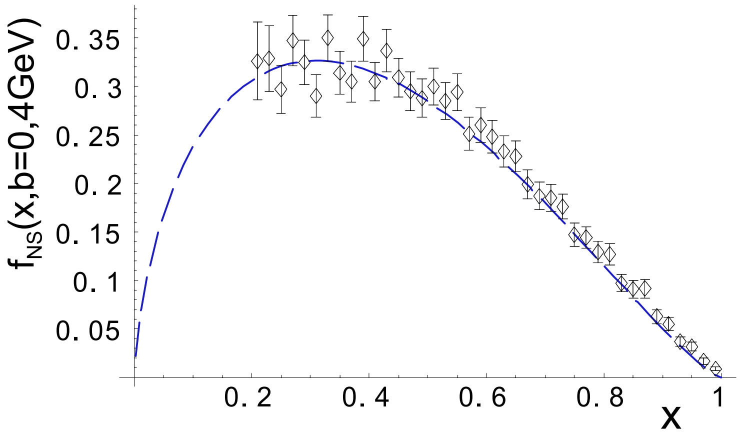
1. (Kwieciński + Gawron + WB, '03):

$$p_j(x, Q_0) = \text{GRV/GRS},\ F(b) = e^{-\frac{b^2}{b_0^2}}$$

2. (ERA+WB, '04): Chiral quark models give predictions for the pion $\rightarrow$

$$\begin{aligned} p_{NS,S}(x, b, Q_0) &= \theta(x)\theta(1-x) \\ p_G(x, b, Q_0) &= 0 \quad (\text{no gluons}) \end{aligned}$$

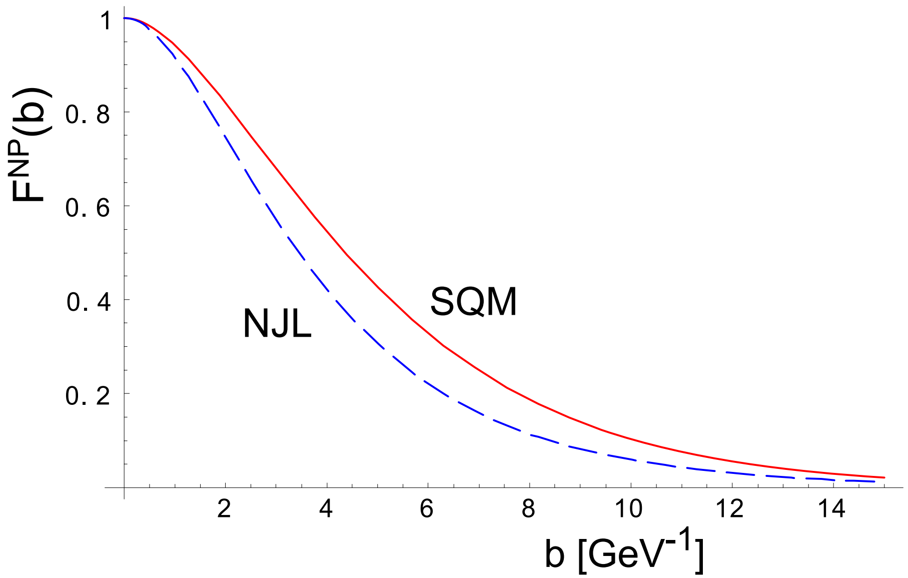
Momentum sum rule: setting $Q_0 = 313$ MeV leads to the 47% momentum fraction carried by the quarks at Q=2GeV $(\alpha(Q_0^2)/(2\pi) \sim 0.3)$, NLO analysis fine
Davidson+ERA, '95: the NS distribution evolved to 2 GeV agrees very well with the SMRS parameterization of the pion data
WB+ERA, '03: compares favorably to the E615 data at 4 GeV



NJL with PV regulator:

$$q(x, k_\perp, Q_0) = \frac{\Lambda^4 M^2 N_c}{4 f_\pi^2 \pi^3 \left(k_\perp^2 + M^2\right) \left(k_\perp^2 + \Lambda^2 + M^2\right)^2} \theta(x)\theta(1-x)$$

$$\langle k_\perp^2 \rangle_{\rm NP}^{\rm NJL} = (626 \text{ MeV})^2 \qquad (M = 280 \text{ MeV}, \Lambda = 871 \text{ MeV})$$



At large b fall off exponentially, at large $k_\perp$ fall off as a power law

. . . now we run the evolution

Kwieciński equations in the Mellin space

(ERA+WB, 2004) The Mellin moments are

$$f_j(n, b, Q) = \int_0^1 dx\, x^{n-1} f_j(x, b, Q)$$

Evolution involves the b-dependent anomalous dimensions

$b = 0 \rightarrow$ DGLAP

$$\gamma_{n,ab}(Qb) = 4 \int_0^1 dz\, \left[z^n J_0\left((1-z)Qb\right) - 1\right] P_{ab}(z)$$

Explicitly,

$$\gamma_{n,NS}(Qb) = \gamma^{(0)}_{n,NS} + \frac{4C_F}{(1+n)\,(2+n)} \left[-3 - 2\,n + 2\,(2+n)\; {}_1F_2\left(\frac{1}{2}; \frac{2+n}{2}, \frac{3+n}{2}; -\frac{Q^2 b^2}{4}\right)\right.$$
$$\left. - {}_1F_2\left(\frac{3}{2}; \frac{3+n}{2}, \frac{4+n}{2}; -\frac{Q^2b^2}{4}\right) + \frac{Q^2b^2}{2}\, {}_3F_4\left((1, 1, \frac{3}{2}; 2, 2, \frac{3+n}{2}, \frac{4+n}{2}; -\frac{Q^2b^2}{4}\right)\right]$$

where ${}_pF_q$ are the generalized hypergeometric functions and

SFSC

$$\gamma^{(0)}_{n,NS} = 2C_F\left(-3 + \frac{2}{1+n} + \frac{2}{2+n} + 4\sum_{k=1}^{n}\frac{1}{k}\right)$$

We find

$$Q^2\frac{df_{\rm NS}(n,b,Q)}{dQ^2} = -\frac{\alpha(Q^2)}{8\pi}\gamma_{n,NS}(Qb)f_{\rm NS}(n,b,Q)$$

with the formal solution

$$\frac{f_{\rm NS}(n,b,Q)}{f_{\rm NS}(n,b,Q_0)} = \exp\left[-\int_{Q_0^2}^{Q^2}\frac{dQ'^2\alpha(Q'^2)}{8\pi Q'^2}\gamma_{\rm NS}(n,b,Q')\right]$$

In the singlet channel

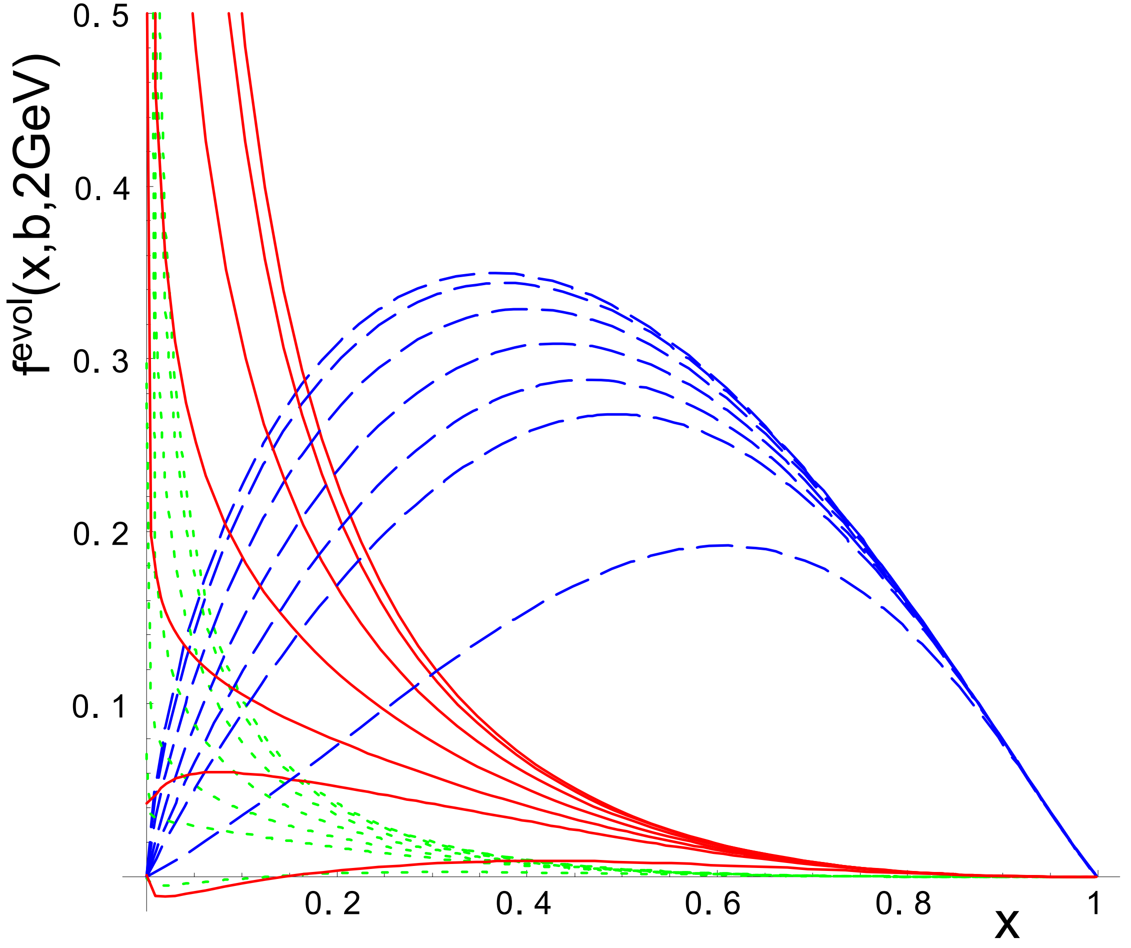
$$\begin{pmatrix} f_S(n,b,Q) \\ f_G(n,b,Q) \end{pmatrix} = \mathcal{P}\exp\left[-\int_{Q_0^2}^{Q^2}\frac{dQ'^2\alpha(Q'^2)}{8\pi Q'^2}\Gamma_n(Qb)\right]\begin{pmatrix} f_S(n,b,Q_0) \\ f_G(n,b,Q_0) \end{pmatrix},$$

$$\Gamma_n(Qb) = \begin{pmatrix} \gamma_{n,qq}(Qb) & \gamma_{n,qG}(Qb) \\ \gamma_{n,Gq}(Qb) & \gamma_{n,GG}(Qb) \end{pmatrix}$$

$\mathcal{P}$ indicates ordering along the integration path. The above equations are solved numerically for any value of n and b. Then the inverse Mellin transform is carried out,

$$f_j(x,b,Q) = \int_C \frac{dn}{2\pi i}\,x^{-n}f_j(n,b,Q)$$

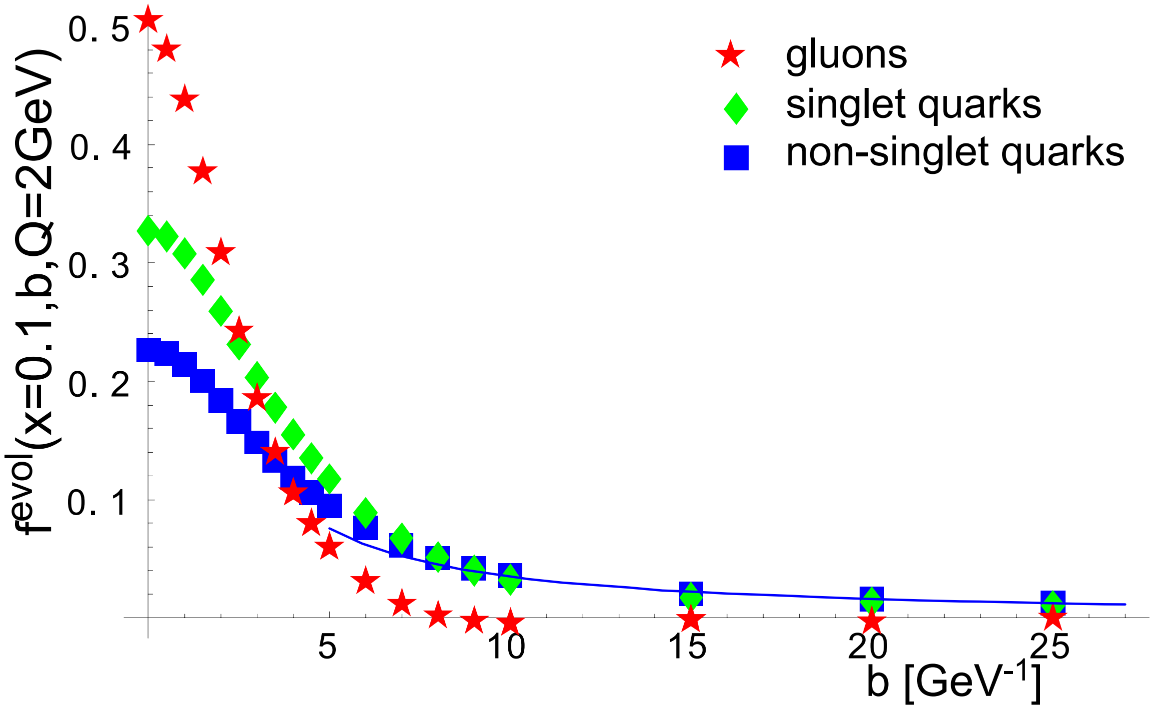
Numerical solution, $Q^2 = 4\ \text{GeV}^2$



b=0
1
2
3
4
5
10
[GeV^{-1}]

(non-singlet (valence) quarks, sea quarks ($S - NS$), and gluons)

Numerical solution, $Q^2 = 4\ \mathrm{GeV}^2$, $x = 0.1$

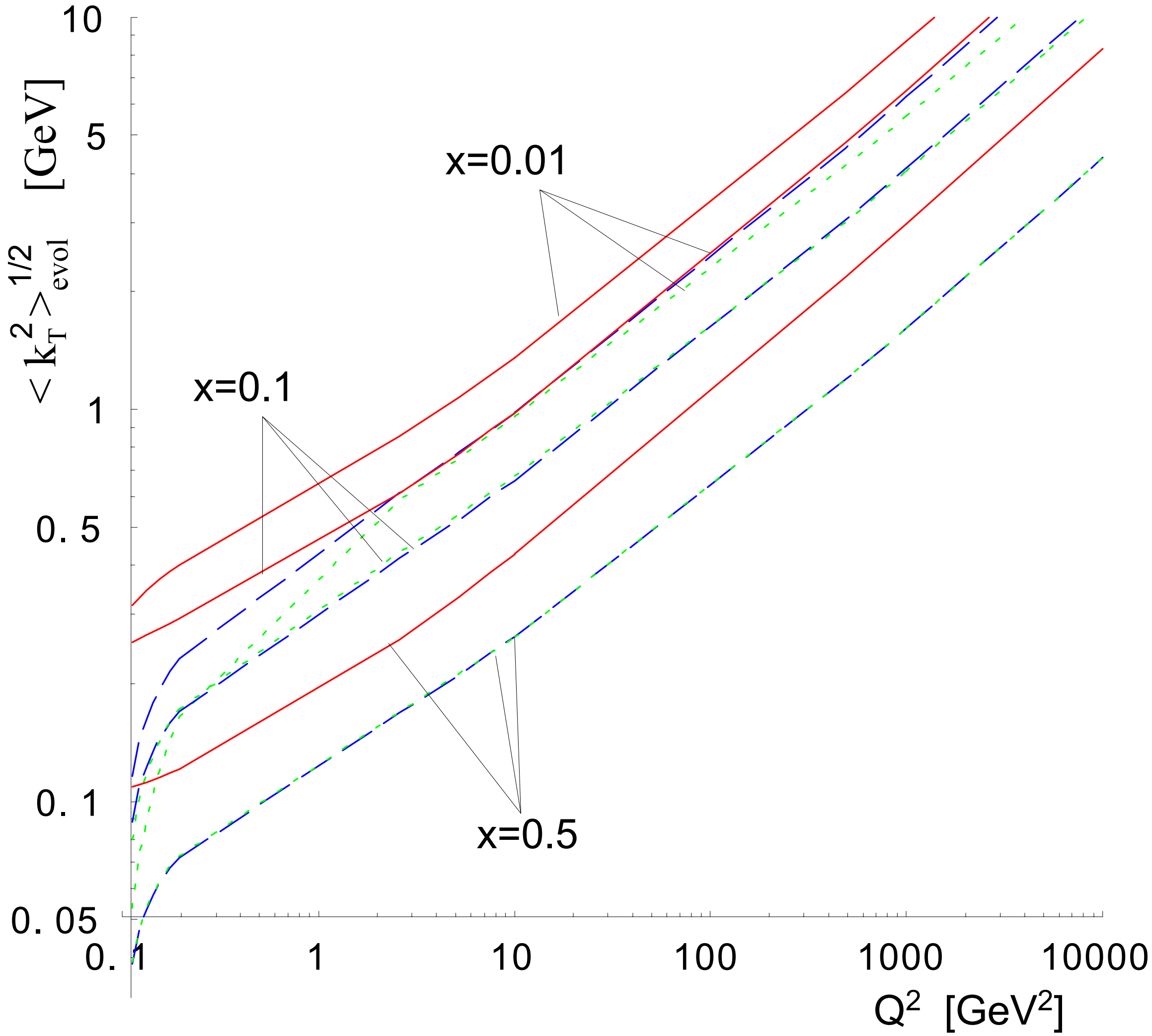


Shrinking in b (spreading in $k_\perp$) as Q grows!

effect increases with increasing Q and dropping x, largest for gluons

Long, power-law tail in b

Spreading in $k_\perp$



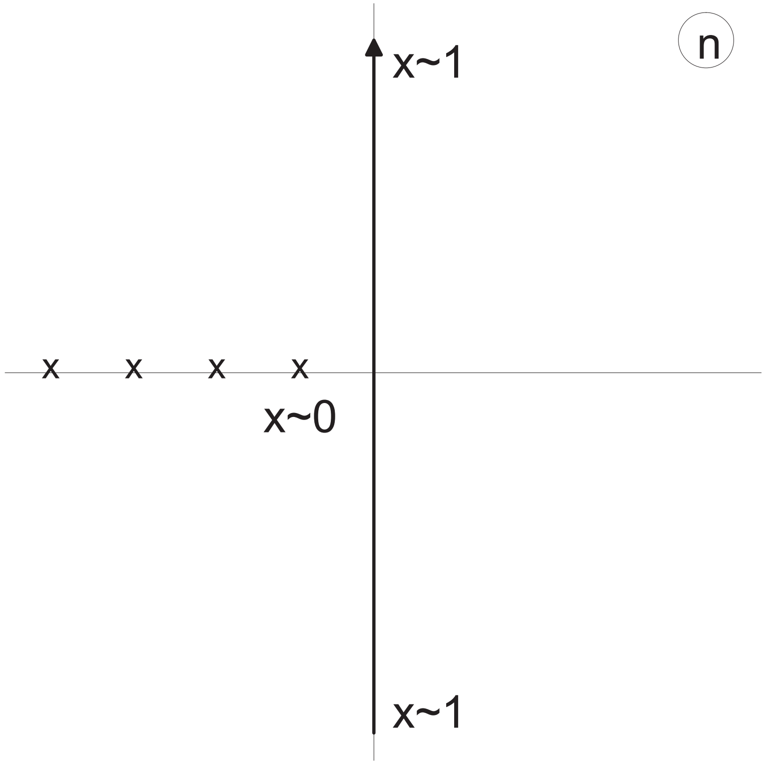
(non-singlet (valence) quarks, singlet quarks, and gluons)

Asymptotically $\langle k_\perp^2 \rangle_{\rm evol} \sim Q^2 \alpha(Q^2)$

Full width: $\langle k_\perp^2 \rangle = \langle k_\perp^2 \rangle_{\rm NP} + \langle k_\perp^2 \rangle_{\rm evol}$

Mathematical properties

. . . follow from the properties of the Mellin transform



We deal with initial condition of the form $x^{\alpha}(1-x)^{\beta} \times F(b)$

$x \sim 0$

$$f^{\rm evol}_{\rm NS,S}(x,b,Q^2) \sim x \exp\left(2\sqrt{C_F A \log\frac{1}{x}}\right)$$

$$f^{\rm evol}_{G}(n,b,Q) \sim \exp\left(2\sqrt{2N_c A \log\frac{1}{x}}\right), \quad A \geq 0$$

$$A = \int_{Q_0^2}^{Q^2} \frac{dQ^2}{2\pi Q^2}\alpha(Q^2) J_0(Qb)$$

Generalization of DLLA, since for $b=0$ $A \sim \log(Q^2)$
For $b>0$ we may have $A<0$ and then $f_j^{\rm evol}(n,b,Q)$ oscillate

$x \sim 1$ The integrated non-singlet distribution behaves as

$$f_{\rm NS}(x,0,Q^2) \sim \frac{e^{2C_F(3-4\gamma)r_0}}{\Gamma(1+8C_F r_0)}(1-x)^{\beta+8C_F r_0}$$

$$r_k = r_k(Q_0^2,Q^2) = \int_{Q_0^2}^{Q^2} \frac{dQ'^2 \alpha(Q'^2)}{8\pi Q'^2} Q'^{2k}$$

For UPD's

$$\frac{f^{\rm evol}_{\rm NS}(x,b,Q^2)}{f^{\rm evol}_{\rm NS}(x,0,Q^2)} = 1 - \frac{2C_F b^2 r_1 (1-x)^2}{(1+8C_F r_0)(2+8C_F r_0)} + \mathcal{O}((1-x)^3)$$

Large bQ From asymptotic forms of $\gamma_n(bQ)$

$$
\begin{aligned}
f_{\rm NS,S}(x,b,Q) &\sim b^{-8C_F r_0(Q_0^2,Q^2)}, \\
f_{\rm G}(x,b,Q) &\sim b^{-8N_c r_0(Q_0^2,Q^2)}
\end{aligned}
$$

Low b At $x \to 0$

$$
\langle k_\perp^2 \rangle_{\rm NS}^{\rm evol} \sim \sqrt{-\frac{C_F \log x}{r_0} r_1} \sim \sqrt{\frac{2\beta_0 C_F \log \frac{1}{x}}{\log \frac{\alpha(\Lambda^2)}{\alpha(Q^2)}} \frac{1}{8\pi}} \alpha(Q^2) Q^2
$$

At $x \to 1$

$$
\langle k_\perp^2 \rangle_{\rm NS}^{\rm evol} \to \frac{2C_F(1-x)^2 r_1}{(1+8C_F r_0)(2+8C_F r_0)} \sim \frac{\beta_0^2 (1-x)^2}{64\pi C_F \left[\log \frac{\alpha(\Lambda^2)}{\alpha(Q^2)}\right]^2} \alpha(Q^2) Q^2
$$

$\langle k_\perp^2 \rangle_{\rm NS}^{\rm evol} \to \infty$ at $x \to 0$ and $\langle k_\perp^2 \rangle_{\rm NS}^{\rm evol} \to 0$ at $x \to 1$

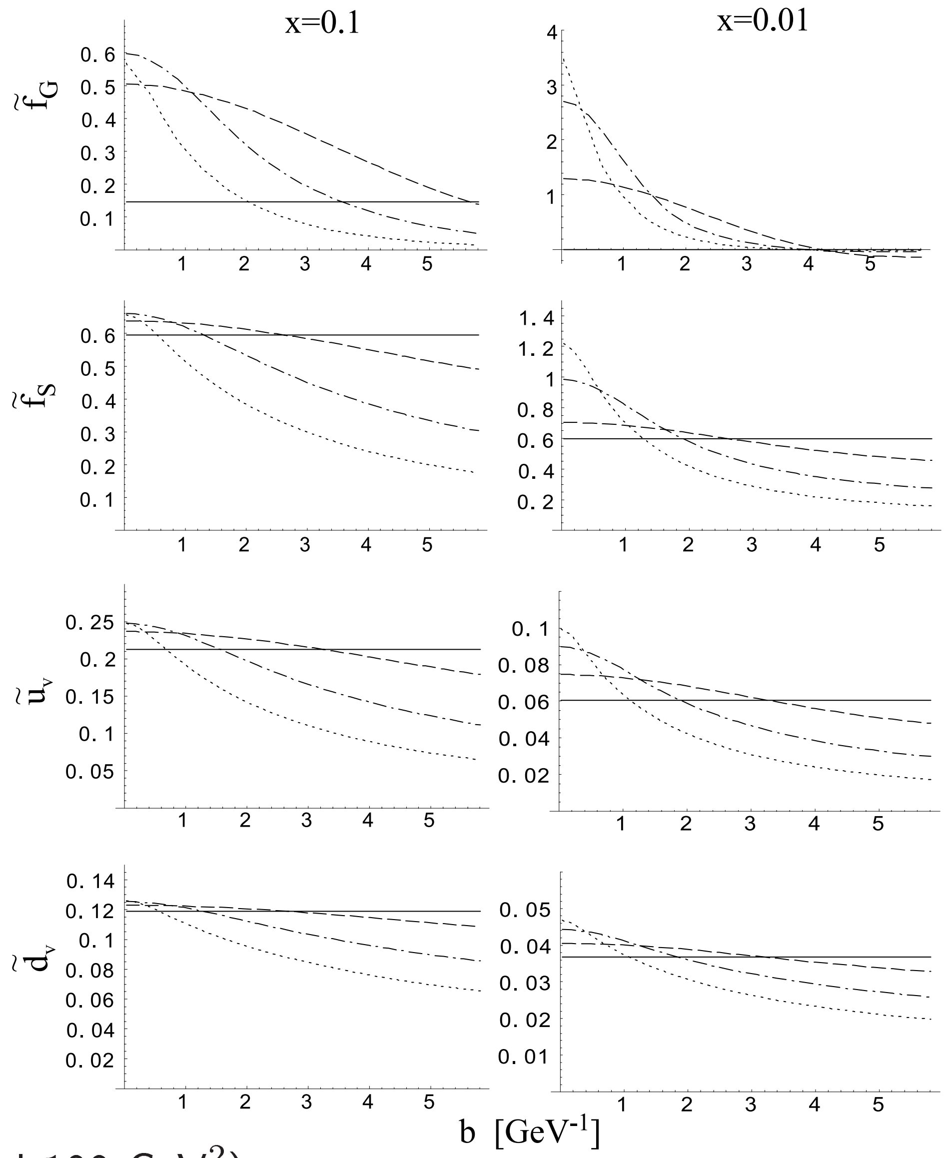
For the gluons and singlet quarks a similar asymptotic behavior of $\langle k_\perp^2 \rangle^{\rm evol}$ is found. Thus, all UPD's spread in $k_\perp$ at large Q as $Q^2 \alpha(Q^2)$

Conclusions

1. The Kwieciński evolution is a special case of the CCFM scheme. It is diagonal in b. It relates the UPD's at one scale to UPD's at another scale. Non-perturbative and perturbative physics is factorized
2. Equations are "semi-analytic"
3. UPD's spread in $k_\perp$ as the probing scale Q grows. Asymptotically, $\langle k_\perp^2 \rangle_{\mathrm{NS,S,G}}^{\mathrm{evol}} \sim Q^2 \alpha(Q^2)$. Spreading fastest for gluons, and at low x
4. Long, power-law tails of the evolution-generated UPD's at large b
5. Generalized DLLA at low x
6. Can be solved in the Mellin space
7. Physical predictions, cross-checks of MC codes

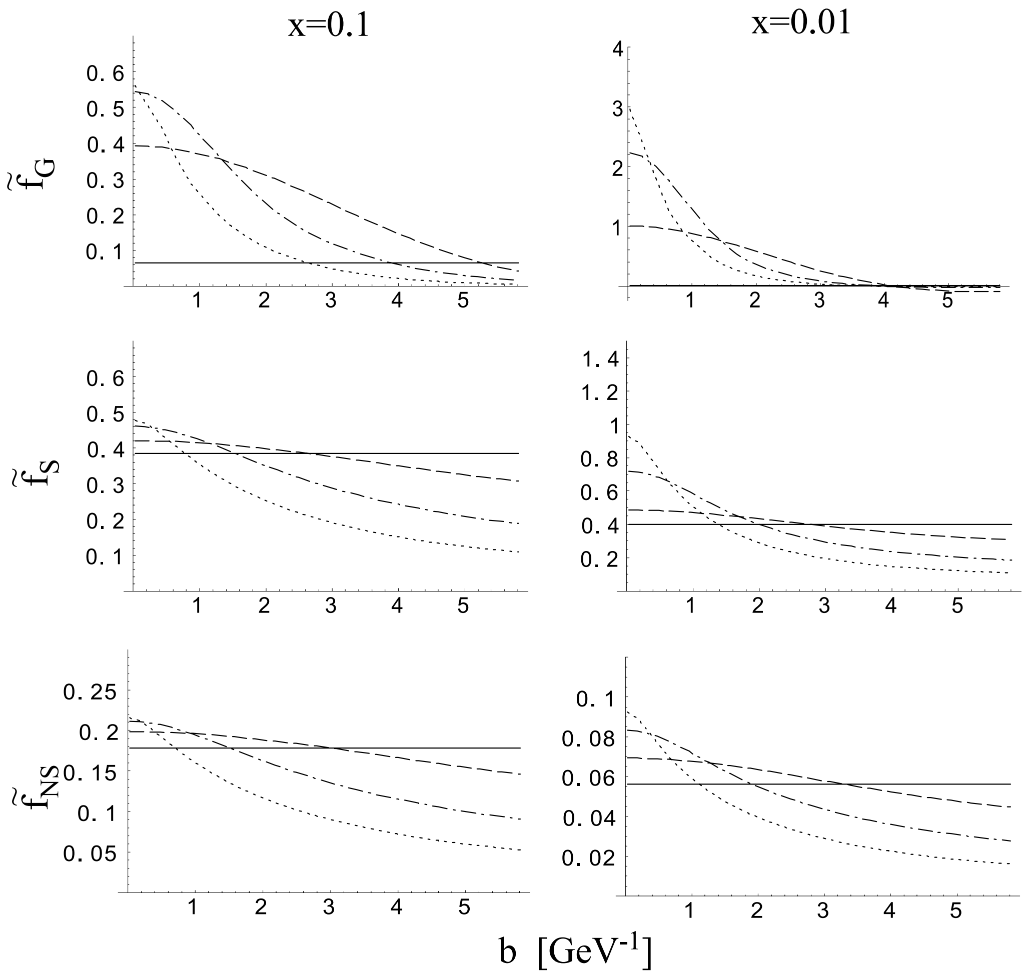
Back-up slides

Nucleon, GRV



(Q^2=0.26, 1, 10, and 100 GeV^2)

Pion, GRS



(Q^2=0.26, 1, 10, and 100 GeV^2)

Kimber + Martin + Ryskin

$$f_g(x, k_\perp) = \left. \frac{d(xg(x, Q^2))}{dQ^2} \right|_{Q^2 = k_\perp^2}$$

Generalized hypergeometric function

$$ {}_pF_q(a_1, \ldots, a_p; b_1, \ldots, b_q; z) = \sum_{k=0}^{\infty} \frac{(a_1)_k \ldots (a_p)_k}{k!\,(b_1)_k \ldots (b_q)_k} z^k $$

where

$$ (a)_k \equiv a(a+1)(a+2)\ldots(a+k-1) = \frac{\Gamma(a+k)}{\Gamma(a)} $$

Initial coditions

SQM:

$$
\begin{aligned}
q(x, k_\perp, Q_0) &= \frac{6m_\rho^3}{\pi(k_\perp^2 + m_\rho^2/4)^{5/2}}\theta(x)\theta(1-x), \\
F_{\rm SQM}^{\rm NP}(b) &= \left(1 + \frac{bm_\rho}{2}\right) \exp\left(-\frac{m_\rho b}{2}\right) \\
\langle k_\perp^2 \rangle_{\rm NP}^{\rm SQM} &= \frac{m_\rho^2}{2} = (544 \text{ MeV})^2
\end{aligned}
$$

NJL (with PV regularization):

$$
\begin{aligned}
q(x, k_\perp, Q_0) &= \frac{\Lambda^4 M^2 N_c}{4 f_\pi^2 \pi^3 \left(k_\perp^2 + M^2\right) \left(k_\perp^2 + \Lambda^2 + M^2\right)^2}\theta(x)\theta(1-x) \\
F_{\rm NJL}^{\rm NP}(b) &= \frac{M^2 N_c}{4 f_\pi^2 \pi^2}\left(2K_0(bM) - 2K_0(b\sqrt{\Lambda^2 + M^2}) - \frac{b\Lambda^2 K_1(b\sqrt{\Lambda^2 + M^2})}{\sqrt{\Lambda^2 + M^2}}\right) \\
\langle k_\perp^2 \rangle_{\rm NP}^{\rm NJL} &= (626 \text{ MeV})^2 \qquad (M = 280 \text{ MeV}, \Lambda = 871 \text{ MeV})
\end{aligned}
$$