Fluctuations of v_2 in relativistic heavy-ion collisions*

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*based on WB+**P. Bożek+M. Rybczyński**, 0706.4266 and 0709.0123 [nucl-th] (CPOD 2007)

Introduction

- Elliptic flow [movie], measure:

$$v_2 = \frac{\langle p_x^2 \rangle - \langle p_y^2 \rangle}{\langle p_x^2 \rangle + \langle p_y^2 \rangle}$$

- Initial shape asymmetry:

$$\epsilon = \frac{\langle y^2 \rangle - \langle x^2 \rangle}{\langle x^2 \rangle + \langle y^2 \rangle}$$

- Hydro:

$$v_2 \sim \epsilon$$

  (linearity of perturbation)
- Event-by-event fluctuations of v_2 measured (PHOBOS, STAR)

$$\Delta v_2 / v_2$$

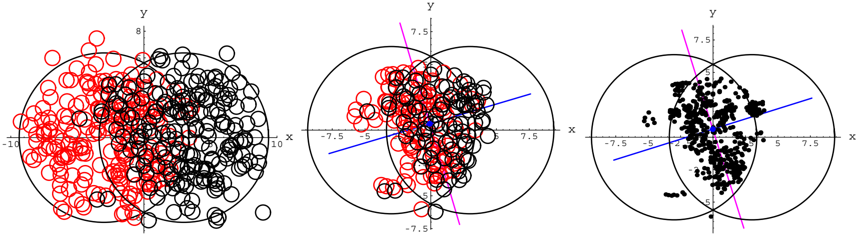
  [exp. results]

Outline

Goals/results:

- analysis of shape fluctuations in variants of Glauber models
- understand the statistics (*e.g.*, $\Delta\varepsilon^*/\varepsilon^*(b=0)=\sqrt{\frac{4}{\pi}-1}\simeq 0.52$)
- $\Delta v_2^*/v_2^* \simeq 0.5$ for central and peripheral collisions
  (* ="participant", see later)

A typical gold-gold event



all nucleons | wounded nucleons | binary collisions

Sizeable fluctuations of the center of mass and the quadrupole axes

Aguiar+Kodama+Osada+Hama 2001, Miller+Snellings 2003, Bhalerao+Blaizot+Borghini+Ollitrault 2005, Andrade+Grassi+Hama+Kodama+Socolowski 2006, Voloshin 2006, ...

Notation

fixed-axes = standard

$$f(\rho,\phi) = f_0(\rho) + 2f_2(\rho)\cos(2\phi) + 2f_4(\rho)\cos(4\phi) + \dots$$

$$\varepsilon = \frac{\int \rho\, d\rho \rho^2 f_2(\rho)}{\int \rho\, d\rho \rho^2 f_0(\rho)}$$

variable-axes = participant=* – more elongated!

$$f^*(\rho,\phi) = f_0(\rho) + 2f_2^*(\rho)\cos(2\phi) + 2f_4^*(\rho)\cos(4\phi) + \dots$$

$$\varepsilon^* = \frac{\int \rho\, d\rho \rho^2 f_2^*(\rho)}{\int \rho\, d\rho \rho^2 f_0(\rho)}$$

(coordinates shifted to center-of-mass and rotated)

RHIC measures the v_2^*, as it cannot determine accurately the reaction plane!

One-dimensional toy model

just 2 ρ-independent terms:

$$f(\phi) = 1 + 2\epsilon \cos(2\phi)$$

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variable axes (participant)

$\varepsilon^* = \langle\langle \frac{1}{n}\sum_{k=1}^{n}\cos[2(\phi_k - \phi^*)]\rangle\rangle$

$$Y = \frac{1}{n}\sum_{k=1}^{n}\cos(2\phi_k), \quad X = \frac{1}{n}\sum_{k=1}^{n}\sin(2\phi_k)$$

ϕ^* : quantity $\frac{1}{n}\sum_{k=1}^{n}\cos[2(\phi_k - \phi^*)]$ maximized in each event

$\Rightarrow \quad \cos(2\phi^*) = Y/\sqrt{Y^2+X^2}, \quad \sin(2\phi^*) = X/\sqrt{Y^2+X^2}$

$$\varepsilon^* = \langle\langle\sqrt{\left(\frac{1}{n}\sum_{k=1}^{n}\cos(2\phi_k)\right)^2 + \left(\frac{1}{n}\sum_{k=1}^{n}\sin(2\phi_k)\right)^2}\rangle\rangle$$

Central limit theorem

For large n the distribution of Y and X is Gaussian:

$$f(Y,X) = \frac{n}{\pi\sqrt{1-2\epsilon^2}} \exp\left[-n\left(\frac{(Y-\epsilon)^2}{1-2\epsilon^2} + X^2\right)\right]$$

Let $Y = q\cos\alpha$, $X = q\sin\alpha$.
We need the integral of $f(Y,X) = f(q,\alpha)$ over α:

$$\int_0^{2\pi} d\alpha f(q,\alpha) = \frac{2n}{\sqrt{\pi}\sqrt{1-2\epsilon^2}} \exp\left[-n\left(\frac{q^2+\epsilon^2}{1-2\epsilon^2}\right)\right] \times \sum_{j=0}^{\infty} (2q\epsilon)^j \frac{\Gamma(j+\frac{1}{2})}{j!} I_j\left(\frac{2n\epsilon q}{1-2\epsilon^2}\right)$$

ε^* in the toy model

$$\varepsilon^* = \int q\, dq\, d\alpha\, q f(q,\alpha) = \frac{1-2\epsilon^2}{\sqrt{n\pi}} \sum_{j=0}^{\infty} (2\epsilon^2)^j \frac{\Gamma(j+\frac{1}{2})\Gamma(j+\frac{3}{2})}{j!^2} {}_1F_1\left(-\frac{1}{2}, j+1; -\frac{n\epsilon^2}{1-2\epsilon^2}\right)$$

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$$\langle \varepsilon^{*2} \rangle = \langle\langle Y^2 + X^2 \rangle\rangle = \int q\, dq\, d\alpha\, q^2 f(q,\alpha) = \frac{1+(n-1)\epsilon^2}{n}$$

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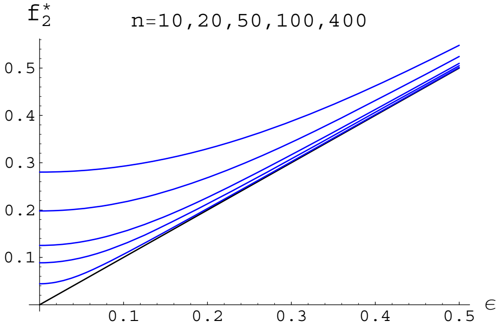
"centrality"=0

$$\varepsilon^*(\epsilon=0) = \frac{\sqrt{\pi}}{2\sqrt{n}}, \quad \frac{\Delta\varepsilon^*}{\varepsilon^*} = \sqrt{\frac{4}{\pi} - 1} \simeq 0.52$$

ε^* as a function of ε (toy model)



ε^* in the general two-dimensional case

(under the assumption of no correlations of locations of sources)

$$\varepsilon^* = \frac{\sqrt{2}\sigma_Y^2}{I_{k,0}\sqrt{\pi}\sigma_X} \sum_{j=0}^{\infty} (2\delta\sigma_Y^2)^j \frac{\Gamma\left(j+\frac{1}{2}\right)\Gamma\left(j+\frac{3}{2}\right)\, {}_1F_1\left(-\frac{1}{2}; j+1; -\frac{\bar{Y}^2}{2\sigma_Y^2}\right)}{j!^2}$$

$$\bar{Y} = I_{2,2},\ \sigma_Y^2 = \frac{1}{2n}(I_{4,0} - 2I_{2,2}^2 + I_{4,4}),\ \sigma_X^2 = \frac{1}{2n}(I_{4,0} - I_{4,4}),$$

$$\delta = \frac{1}{2\sigma_Y^2} - \frac{1}{2\sigma_X^2},\ \ I_{k,l} = \int \rho d\rho f_l(\rho)\rho^k / \int \rho d\rho f_0(\rho)$$

at $b = 0$ very simple results (independent of A, energy, model, ...)

$$\varepsilon^* = \frac{\sqrt{\pi I_{4,0}}}{2I_{2,0}\sqrt{n}}, \quad \frac{\Delta\varepsilon^*}{\varepsilon^*} = \sqrt{\frac{4}{\pi} - 1}, \quad f_2^*(\rho) = \frac{1}{2}\sqrt{\frac{\pi}{nI_{2k,0}}}\,\rho^k f_0(\rho)$$

Glauber-like models tested

- wounded nucleons, $\sigma_w = 42$ mb, $d = 0.4$ fm

one goal:

compare various Glauber-like models

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- hot spots + Γ: Sources may deposit the transverse energy with a certain probability distribution. We superimpose the Γ distribution with $\kappa = 0.5$ over the distribution of sources,

$$g(w, \kappa) = w^{\kappa-1} \kappa^\kappa \exp(-\kappa w)/\Gamma(\kappa),$$

where $\bar{w} = 1$ and $\text{var}(w) = 1/\kappa$

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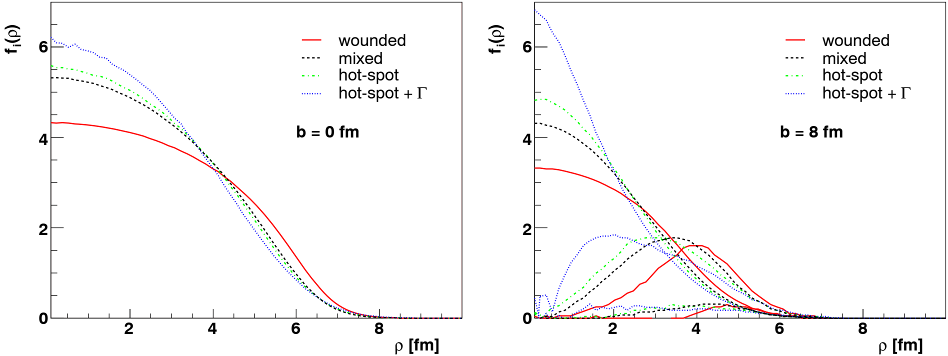
compare various Glauber-like models

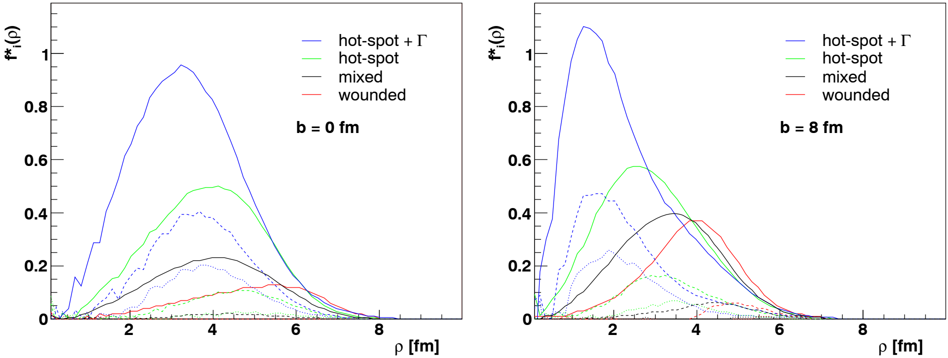
Fixed-axes (standard) profiles



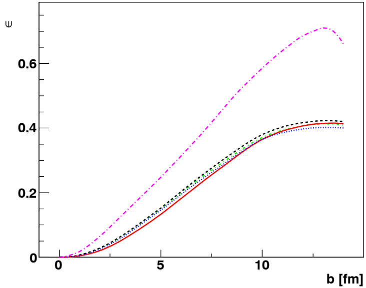
left: $f_0(\rho)$ right: $f_0(\rho)$, $f_2(\rho)$, $f_4(\rho)$

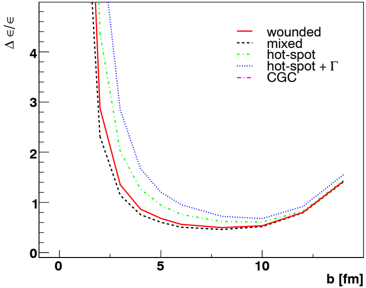
hot-spot + Γ sharpest
keeping harmonics up to $l = 4$ is sufficient

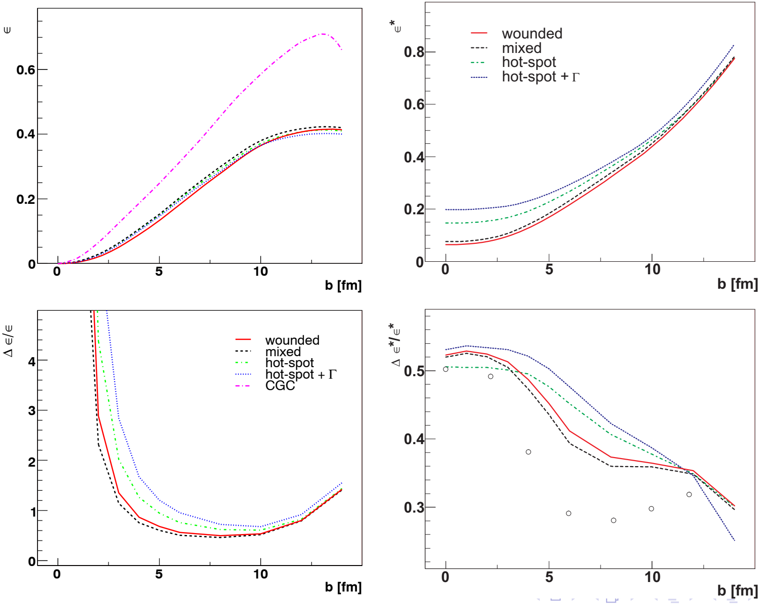
Variable-axes (participant) profiles



solid: $f_2(\rho)$, dash: $f_4(\rho)$, dots: $f_6(\rho)$

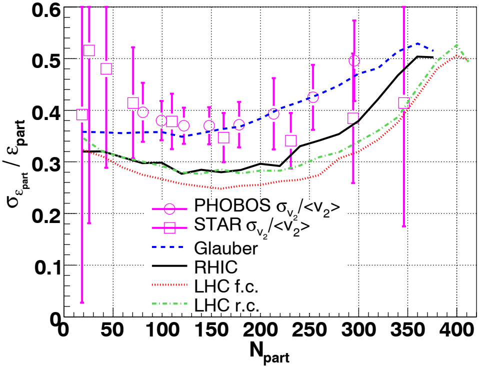

ϵ
0.6
0.4
0.2
0
0
5
10
b [fm]


Δ ϵ/ϵ
4
3
2
1
0
0
5
10
b [fm]
wounded
mixed
hot-spot
hot-spot + Γ
CGC


ε
0.6
0.4
0.2
0
0
5
10
b [fm]
ε*
wounded
mixed
hot-spot
hot-spot + Γ
0.8
0.6
0.4
0.2
0
b [fm]
Δ ε/ε
wounded
mixed
hot-spot
hot-spot + Γ
CGC
4
3
2
1
b [fm]
Δ ε*/ε*
0.5
0.4
0.3
b [fm]

Comparison to CGC

[Drescher+Nara, arXiv:0707.0249]



CGC with k_T-factorization lower than Glauber

Event-by-event fluctuations of v_2

At low azimuthal asymmetry one expects on hydrodynamical grounds

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$$\frac{\Delta v_2^*}{v_2^*} = \frac{\Delta \varepsilon^*}{\varepsilon^*}$$

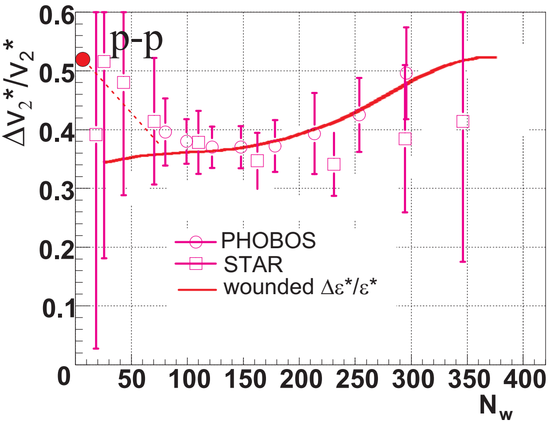
$$\frac{\Delta v_2^*}{v_2^*}(b=0) \simeq \frac{\Delta \varepsilon^*}{\varepsilon^*}(b=0) \simeq \sqrt{\frac{4}{\pi} - 1} \simeq 0.52$$

For peripheral collisions (collection of a few $p-p$ collisions) also

$$\frac{\Delta v_2^*}{v_2^*}(b \sim 2R) \simeq \sqrt{\frac{4}{\pi} - 1} \simeq 0.52$$

(replace coordinates by momenta and repeat the above analysis)

At intermediate b lower values



hydro: $\Delta\varepsilon^*/\varepsilon^* \simeq \Delta v_2^*/v_2^*$(Au+Au, central)$\simeq 0.5$

coordinates $\rightarrow p_T$: $\Delta v_2^*/v_2^*(\text{several p+p}) \simeq 0.5 =$

$\Delta v_2^*/v_2^*$(Au+Au, periph.)$\simeq 0.5$

Perturbation theory in azimuthal asymmetry

Schematically, hydro equations are $L(\psi) = 0$, where L - operator for hydrodynamics, ψ - set of hydrodynamical functions of space-time describing the state. For *smooth* evolution and small asymmetry one may expand around the azimuthally-symmetric solution ψ_0:

$$L(\psi) = L(\psi_0 + \delta\psi) \simeq L(\psi_0) + L'(\psi_0)\delta\psi$$

Since $L(\psi_0) = 0$, we have to first order

$$L'(\psi_0)\delta\psi = 0$$

Linearity $\Rightarrow ||\delta\psi(t)|| \sim ||\delta\psi(t_0)||$ for all hydrodynamic properties, in particular the shape and flow

Linearity:

$$v_2^*(t) \sim \varepsilon^*(t_0)$$

Perturbation to second order

Strong suppression of subsequent harmonics suggests the hierarchy

$$\psi = \psi_0 + \lambda \delta\psi_2 + \lambda^2 \delta\psi_4 + \ldots,$$

Expansion to second order in $\lambda \sim$ a few % yields

$$L(\psi) = L(\psi_0) + \lambda L'(\psi_0)\delta\psi_2 + \lambda^2 \left[L'(\psi_0)\delta\psi_4 + L''(\psi_0)(\delta\psi_2)^2/2\right]$$

The linear inhomogeneous equation for the $l = 4$ deformation:

$$L'(\psi_0)\delta\psi_4 = -\frac{1}{2}L''(\psi_0)(\delta\psi_2)^2$$

Let τ_2 and τ_4 denote the characteristic times for the operators $L'(\psi_0)$ and $L''(\psi_0)$, respectively. If $\tau_2 \gg \tau_4$ then for $t \gg t_0$

$$||\psi_4(t)|| \sim ||\delta\psi_2(t)||^2 \sim ||\delta\psi_2(t_0)||^2$$

Octupole flow:

$$v_4^*(t) \sim \varepsilon^{*2} \sim v_2^{*2}(t)$$

Simulations [Kolb 2003, Borghini+Ollitrault 2005]: v_2 saturates with time, v_4 quickly assumes the value proportional to v_2^2
Data of [Bai 2007] comply to the result, except at very low p_T

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Fluctuations of v_4:

$$\frac{\Delta v_4^*}{v_4^*} = 2\frac{\Delta v_2^*}{v_2^*} = 2\frac{\Delta \varepsilon^*}{\varepsilon^*}$$

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At large t all deformations determined by $\varepsilon^* \Rightarrow$ other relations.
Let $R_{\rm HBT}(\phi) = R_0 + 2R_2\cos(2\phi) + 2R_4\cos(4\phi) + \ldots$. Then

$$R_4 \sim R_2^2$$

Amnesia:

In hydro (under mentioned conditions) memory of the higher harmonics is lost quickly. Only the initial quadrupole ($l = 2$) deformation matters for observables sensitive to late times

Summary

- ε, ε^*, and $\Delta\varepsilon/\varepsilon$ are sensitive to the choice of the (Glauber-like) model, while $\Delta\varepsilon^*/\varepsilon^*$ is not, changing at most by 10-15%. Resolving better the reaction plane would help to distinguish the models of the initial stage.
- Analytic formulas explain why at $b = 0$ we have (in absence of correlations) $\Delta\varepsilon^*/\varepsilon^* \simeq 0.5$, insensitive of the model used or the mass number of the colliding nuclei.
- (not discussed) For jet emission asymmetry, we find that the effect of the increased eccentricity is largely canceled by the shift of the center of mass and the rotation of the axes of the absorbing medium. Only at low b some effect is left.

- Smoothing prescription for e-by-e hydro can be based on the variable-axes profiles $f_l(\rho)$
- Analysis of the variable-axes moments in the coordinate space directly carries over to the collective flow and analysis of v_2^* in the momentum space. In particular, for central and peripheral collisions $\Delta v_2^*/v_2^* \simeq 0.5$ (statistics), lower in between
- Under assumptions of smootheness, perturbation theory made on top of azimuthally symmetric hydro leads to sensitivity of higher-harmonic late-time measures, v_4, *etc.*, to the initial *quadrupole* deformation $\varepsilon(t_0)$ only. Higher harmonics of the initial shape deformation are irrelevant, as they presumably are damped fast. A number of relations follows for various measures and their e-by-e fluctuations
- Challenge to measure, *e.g.*, $\Delta v_4^*/v_4^* = 2\Delta v_2^*/v_2^*$