Uogólnione partonowe funkcje dystrybucji pionu

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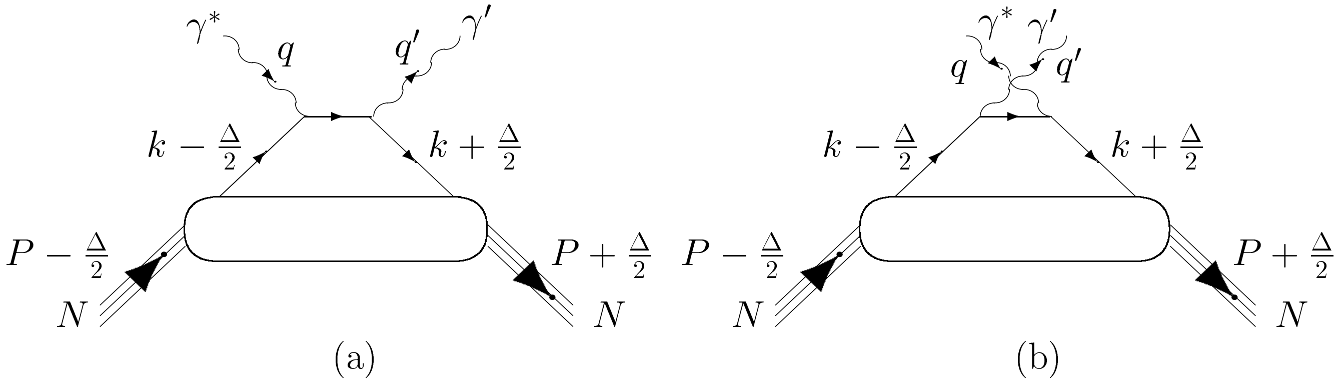
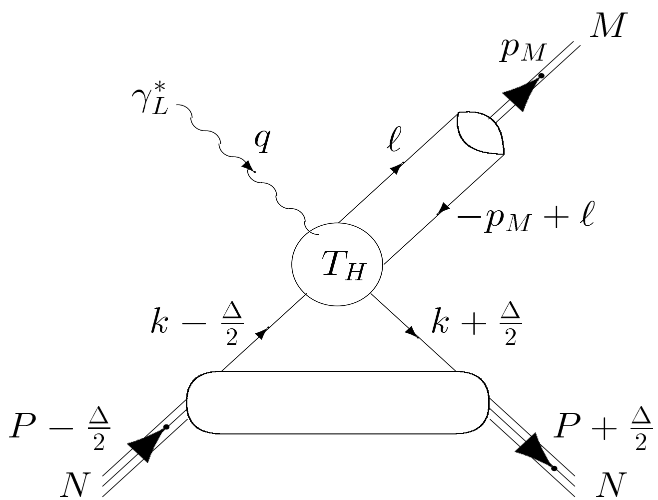
IFJ, 19 Luty 2004

- WB+ERA, Impact-parameter dependence of the generalized parton distribution of the pion in chiral quark models, Phys. Lett. **B 574** (2003) 57, hep-ph/0307198
- WB+ERA, Impact-parameter dependence of the diagonal GPD of the pion from chiral quark models, Proc. of Light Cone Physics: Hadrons and Beyond, Durham, 5-9 August 2003, hep-ph/0307198
- ERA+WB, Spectral quark model and low-energy hadron phenomenology, Phys. Rev. **D67** (2003) 074021, hep-ph/0301202

Exclusive processes in QCD



Deeply
Virtual
Compton
Scattering



Hard
Meson
Production

non-zero momentum transfer to the target, at least one photon virtual, factorization

Kinematics

Reviews:

K. Goeke, M. V. Polyakov, and M. Vanderhaeghen,
Prog. Part. Nucl. Phys. 47 (2001) 401-515, hep-ph/0106012

M. Diehl, Phys. Rept. 388 (2003) 41-277, hep-ph/0307382

Notation: $P = \frac{p+p'}{2}$, $\Delta = p' - p$, $t = \Delta^2$, $k^+ = xP^+$, $\Delta^+ = -2\xi P^+$

Dictionary:

$t = 0$ & $\xi = 0$	regular PD
$\Delta_\perp = 0$	forward GPD
$\Delta_\perp \neq 0$	off-forward GPD
$\xi = 0$	diagonal GPD (non-skewed GPD)
$\xi \neq 0$	non-diadonal GPD (skewed GPD)

Why interesting?

GPD's provide more detailed information of the structure of hadrons, three-dimensional picture instead of one-dimensional projection of the usual PD, enter sum rules,

Information on GPD may come from such processes as $ep \to ep\gamma$, $\gamma p \to pl^+l^-$, $ep \to epl^+l^-$, or from lattices (hold on!). Small cross sections of exclusive processes require very high accuracy experiments. First results are coming from HERMES and CLAS, also COMPASS, H1, ZEUS

Definition of the impact-parameter-dependent GPD (bGPD)

The twist-2 GPD of the pion is defined as
(for the case of π^+ $H(x) \equiv H_u(x) = H_{\bar{d}}(1-x)$)

$$H(x,\xi,-\mathbf{\Delta}_\perp^2) = \int \frac{dz^-}{4\pi} e^{ixp^+z^-} \langle \pi^+(p')|\bar{q}(0,-\frac{z^-}{2},0)\gamma^+ q(0,\frac{z^-}{2},0)|\pi^+(p)\rangle,$$

(Notation: $q(z^+,z^-,z_\perp)$, $z^2=0$)

Link operators $P\exp(ig\int_0^z dx^\mu A_\mu$ are implicitly present to ensure gauge invariance

Similar definition for the gluon distribution

Dictionary continued

General structure of the soft matrix element:

$$\langle A \mid \mathcal{O} \mid B \rangle$$

- A = B = one-particle state – PD of A (inclusive DIS)
- A = one-particle state, B = vacuum – distribution amplitude (DA) of A (hadronic form factors, HMP)
- A, B = one-particle state of different momentum – GPD (exclusive DIS, DVCS, HMP)
- A = many-particle state, B = vacuum – GDA (transition form factors)
- ...

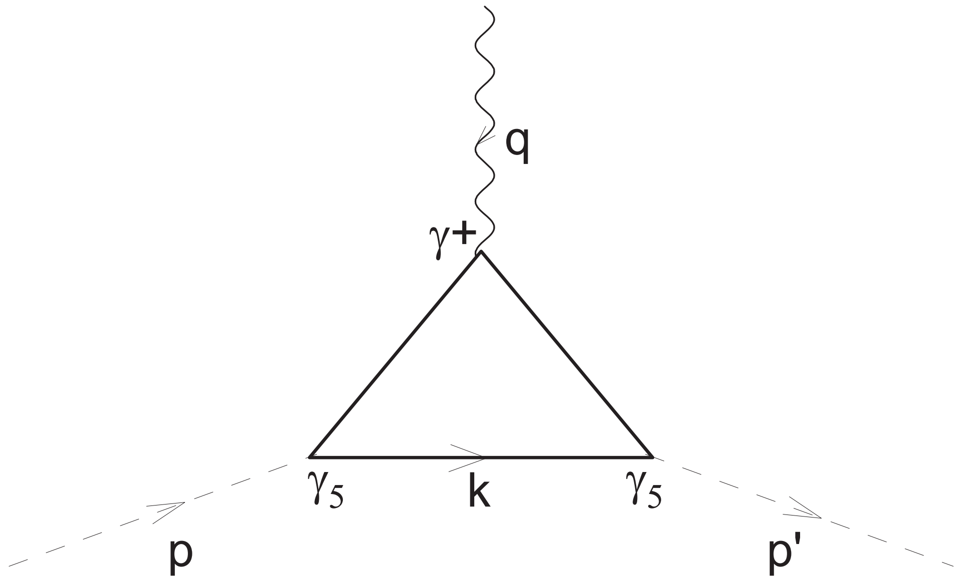
Formal properties of GPD's

$$
\begin{aligned}
H(x,\xi,-\boldsymbol{\Delta}_\perp^2) &= H(x,-\xi,-\boldsymbol{\Delta}_\perp^2) \quad \text{(time reversal)} \\
H(x,\xi,-\boldsymbol{\Delta}_\perp^2)^* &= H(x,-\xi,-\boldsymbol{\Delta}_\perp^2) \quad \text{(reality)} \\
\int_0^1 dx H(x,\xi,-\boldsymbol{\Delta}_\perp^2) &= F(-\boldsymbol{\Delta}_\perp^2) \quad \text{(form factor)} \\
H(x,0,0) &= q(x) \quad \text{(parton distribution)}
\end{aligned}
$$

GPD "links" the elastic form factor and the parton distribution (more interesting results for the case of nucleon, its spin, ...)

Evaluation in chiral quark models, $\xi = 0$ (diagonal, off-forward)

In chiral quark models the evaluation of H at the leading-N_c (one-loop) level amounts to the calculation of the diagram



where the solid line denotes the quark of mass ω.

$$H(x, 0, -\mathbf{\Delta}_\perp^2; \omega) = \frac{iN_c\omega^2}{f_\pi^2} \int \frac{d^4k}{(2\pi)^4} \mathrm{Tr}\left[\gamma^+ \frac{1}{\not k - \not p' - \omega} \gamma_5 \frac{1}{\not k - \omega} \gamma_5 \frac{1}{\not k - \not p - \omega}\right]$$
$$\times \quad \delta\left[k^+ - (1-x)P^+\right],$$

with $f_\pi = 93$ MeV. The light-cone coordinates are defined as

$$k^+ = k^0 + k^3,\ k^- = k^0 - k^3,\ \vec{k}_\perp = (k^1, k^2)$$

The calculation is done in the Breit frame, and with $\Delta^+ = 0$ and $P = (m_\pi, m_\pi, 0)$. The Cauchy theorem is applied for the k^- integration, yielding in the chiral limit

$$H(x, 0, -\mathbf{\Delta}_\perp^2; \omega) = \frac{N_c \omega^2}{\pi f_\pi^2} \int \frac{d^2\mathbf{K}_\perp}{(2\pi)^2} \frac{\left[1 + \frac{\mathbf{K}_\perp \cdot \Delta_\perp (1-x)}{\mathbf{K}_\perp^2 + \omega^2}\right]}{(\mathbf{K}_\perp + (1-x)\Delta_\perp)^2 + \omega^2},$$

where $\mathbf{K}_\perp = (1-x)\mathbf{p}_\perp - x\mathbf{k}_\perp$. The integral is $\log$-divergent, and we need regularization

Digression: unintegrated distributions

If we did not carry the integration over $d^2k_\perp$, we would be left with unintegrated distributions (Kwieciński). The most general object one can consider is therefore the unintegrated skewed off-forward parton distribution

Impact-parameter representation

We may pass to the space conjugated to $\Delta_\perp$ – the impact-parameter space

$$
\begin{aligned}
q(b,x) &= \int \frac{d^2\Delta_\perp}{(2\pi)^2} e^{-i\mathbf{b}\cdot\boldsymbol{\Delta}_\perp} H(x,0,-\boldsymbol{\Delta}_\perp^2) \\
&= \int_0^\infty \frac{\Delta_\perp d\Delta_\perp}{2\pi} J_0(b\Delta_\perp) H(x,0,-\boldsymbol{\Delta}_\perp^2).
\end{aligned}
$$

Regularization

We use two different low-energy quark models which have proven successful in describing soft physics:

1. Spectral Quark Model [SQM] (ERA + WB). Successful in describing both the low- and high-energy phenomenology of the pion (complies to the chiral symmetry, anomalies, pure twist expansion, quark propagator with no poles!).

2. Nambu–Jona-Lasinio [NJL] model with the Pauli-Villars regulator.

Spectral Quark Model

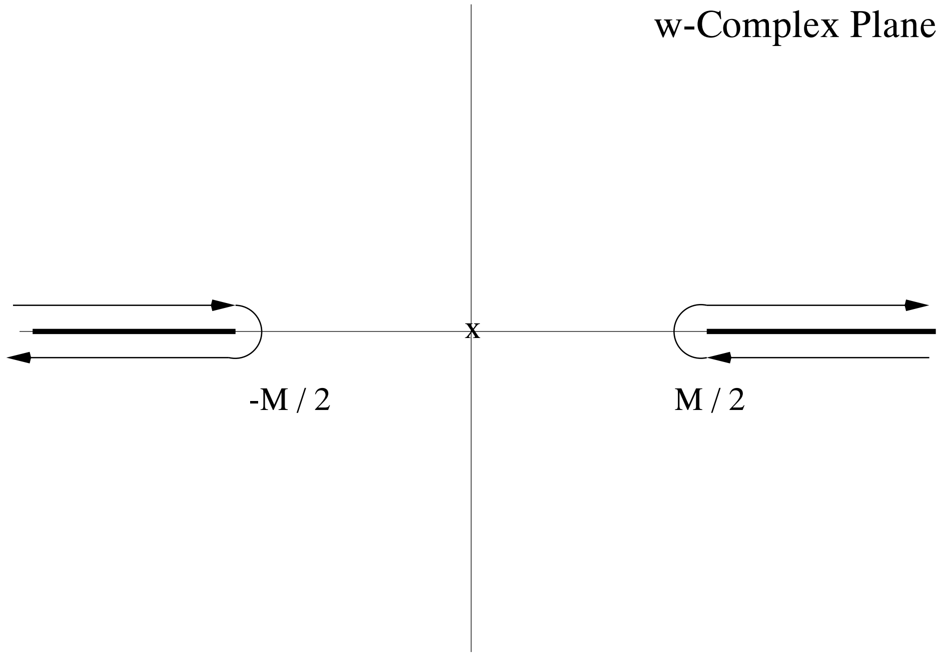
SQM amounts to supplying the quark loop with an integral over the quark mass ω weighted by a quark spectral density $\rho(\omega)$,

$$H_{\rm SQM}(x,0,-\mathbf{\Delta}_\perp^2) = \int_C d\omega\, \rho_V(\omega) H(x,0,-\mathbf{\Delta}_\perp^2;\omega),$$

where

$$\rho_V(\omega) = \frac{1}{2\pi i}\frac{3\pi^2 m_\rho^3 f_\pi^2}{4N_c}\frac{1}{\omega}\frac{1}{(m_\rho^2/4-\omega^2)^{5/2}},$$

and the contour C is given by



with $M = m_\rho$

Then

$$H_{\rm SQM}(x,0,-\mathbf{\Delta}_\perp^2) = \frac{m_\rho^2(m_\rho^2-(1-x)^2\mathbf{\Delta}_\perp^2)}{(m_\rho^2+(1-x)^2\mathbf{\Delta}_\perp^2)^2}\theta(x)\theta(1-x).$$

We check that

$m_\rho^2 = \frac{24\pi^2 f_\pi^2}{N_c}$

$$F(t) = \int_0^1 dx H_{\rm SQM}(x,0,t) = \frac{m_\rho^2}{m_\rho^2+t},$$

which is the built-in vector-meson dominance principle. Clearly, $F(0)=1$, and $H_{\rm SQM}(x,0,0)=\theta(x)\theta(1-x)$ [Davidson-Arriola, 1995]. We pass to the impact-parameter space by the Fourier-Bessel transformation and get

correct norm & support

$$q_{\rm SQM}(b,x) = \frac{m_\rho^2}{2\pi(1-x)^2}\left[K_0\left(\frac{bm_\rho}{1-x}\right) - \frac{bm_\rho}{1-x}K_1\left(\frac{bm_\rho}{1-x}\right)\right].$$

Nambu–Jona-Lasinio Model

In the NJL model with the Pauli-Villars regularization we get

$$H_{\rm NJL}(x,0,-\boldsymbol{\Delta}_\perp^2) = 1+\frac{N_c M^2(1-x)|\boldsymbol{\Delta}_\perp|}{4\pi^2 f_\pi^2 s_i}\sum_i c_i \log\left(\frac{s_i+(1-x)|\boldsymbol{\Delta}_\perp|}{s_i-(1-x)|\boldsymbol{\Delta}_\perp|}\right),$$

$$s_i = \sqrt{(1-x)^2\boldsymbol{\Delta}_\perp^2+4M^2+4\Lambda_i^2},$$

where M is the constituent quark mass, Λ_i are the PV regulators, and c_i are suitable constants. For the twice-subtracted case, explored below, one has, for any regulated function F, the operational definition

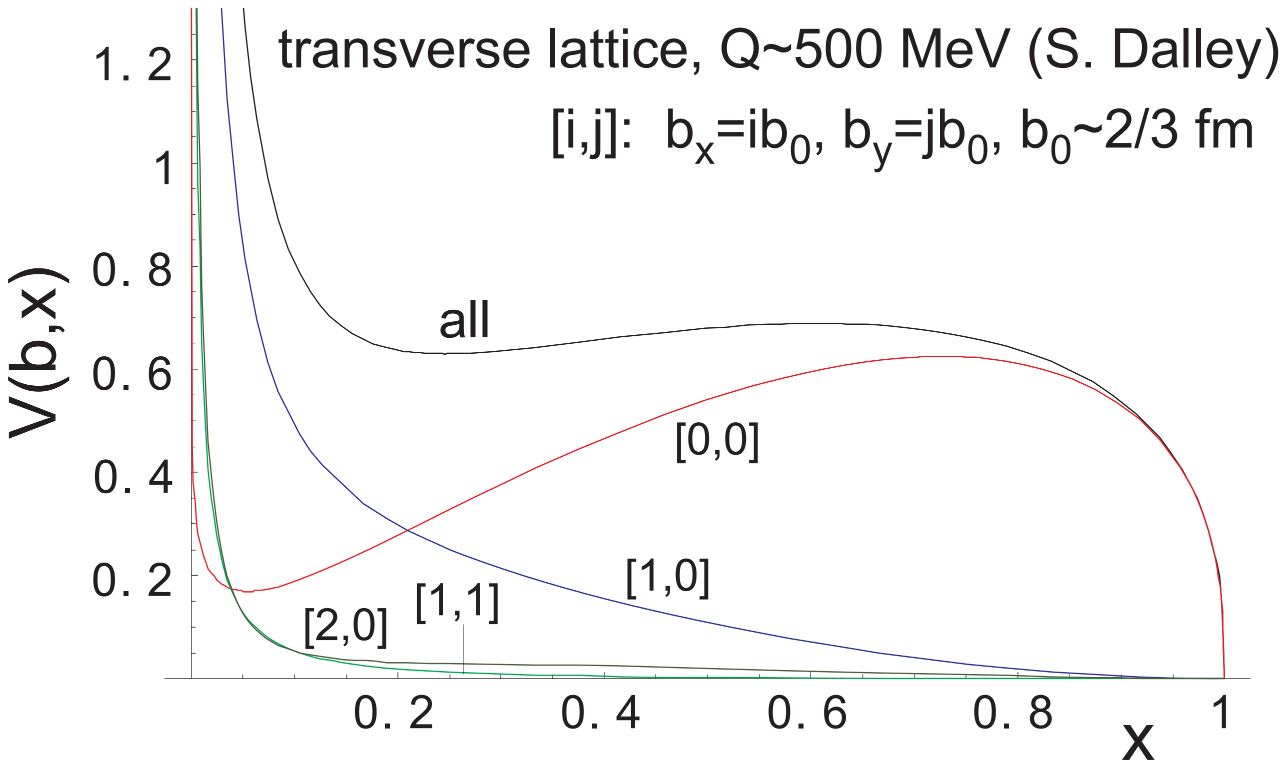
correct norm & support

$$\sum_i c_i F(\Lambda_i^2) = F(0) - F(\Lambda^2) + \Lambda^2 dF(\Lambda^2)/d\Lambda^2.$$

In what follows we use $M = 280$ MeV and $\Lambda = 871$ MeV, which yields $f_\pi = 93$ MeV.

Lattice results

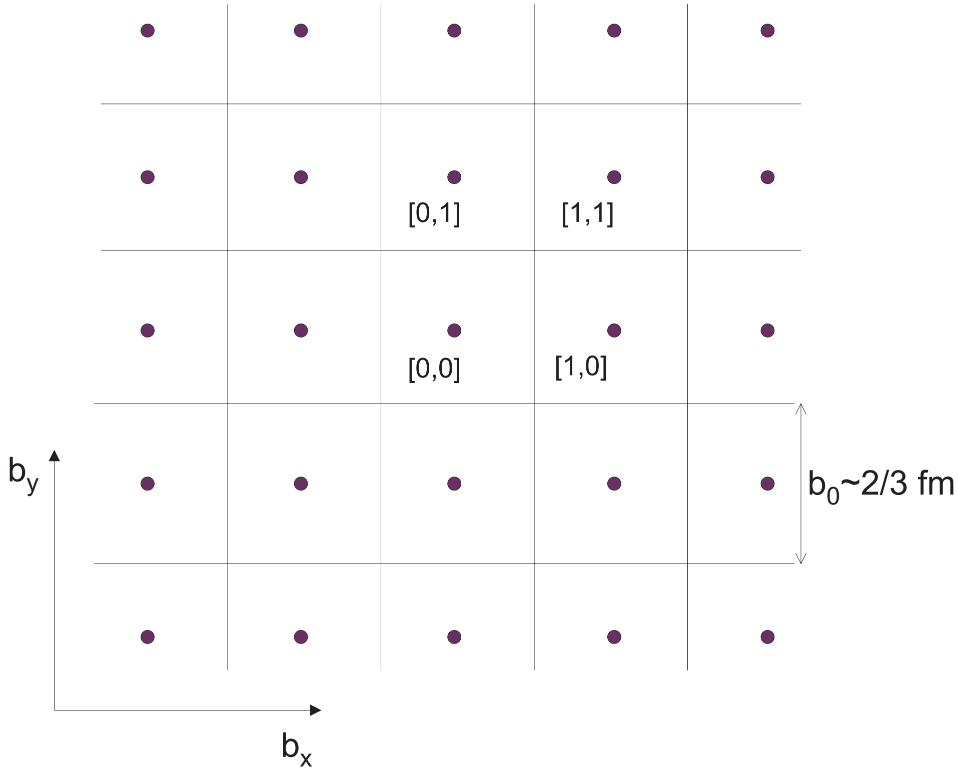
Simon Dalley



($V(b,x)$ – nonsinglet (valence) quark distribution)

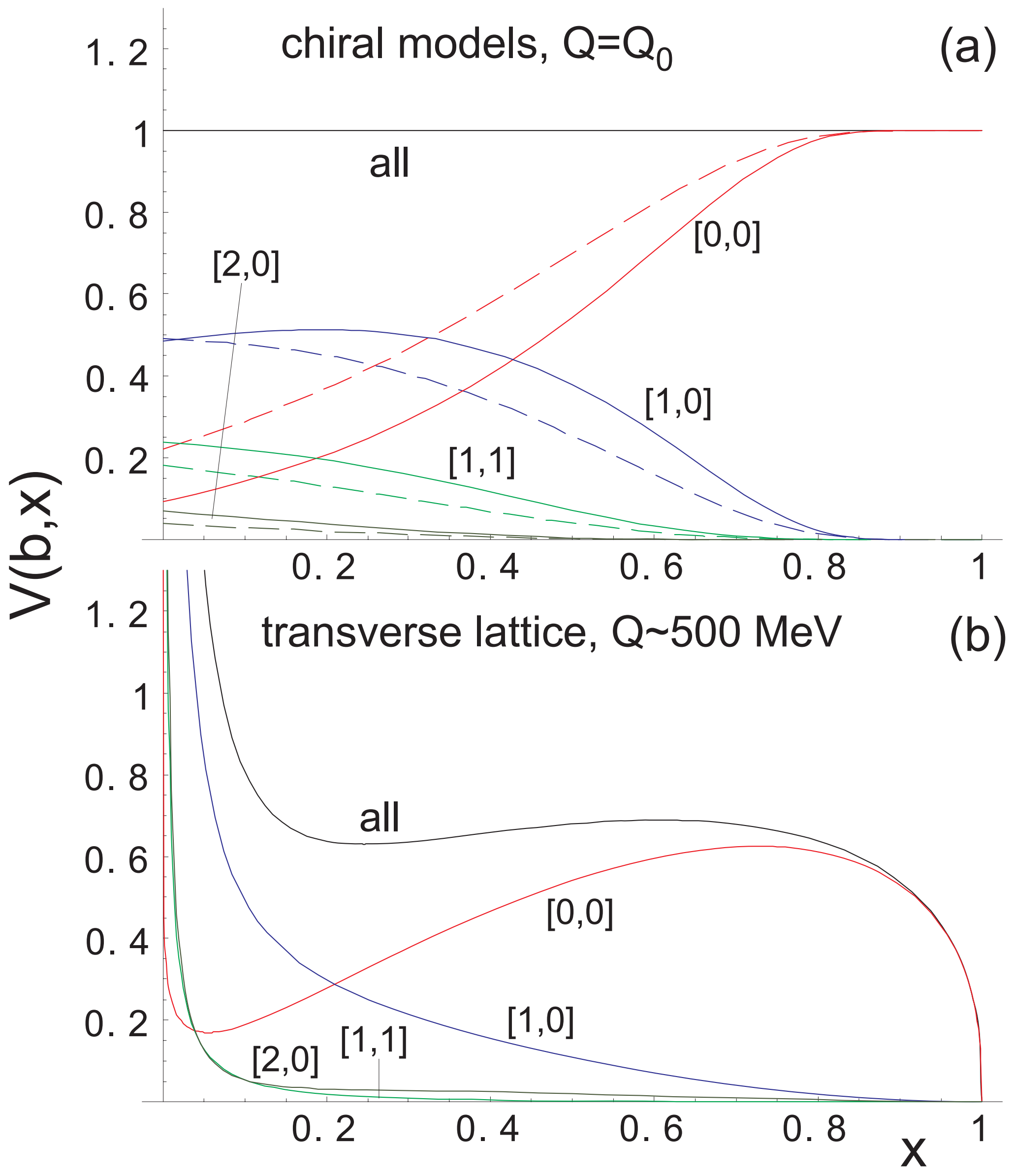
For π^+ we have $V = u - \bar{u} - d + \bar{d}$

Smearing over b



$$V(x,[i,j]) \equiv \int_{(i-1/2)b_0}^{(i+1/2)b_0} db_x \int_{(j-1/2)b_0}^{(j+1/2)b_0} db_y V(x, \sqrt{b_x^2+b_y^2}).$$

The degeneracy factor for plaquettes equidistant from the origin is included, *i.e.* the $[1,0]$, $[1,1]$, and $[2,0]$ plaquettes are multiplied by 4, $[2,1]$ by 8, *etc.*



(a) SQM (solid) and NJL (dashed) at $Q = Q_0 = 313$ MeV. (b) Transverse lattice [Dalley 2003]. The initial condition of (a) needs to be evolved to a higher scale!

QCD evolution and the quark-model scale, Q_0

The models have produced GPD coresponding to a low, a priori unknown quark model scale, Q_0. A way to estimate it is to run the QCD evolution starting from various Q_0's up to a scale Q where data can be used.

QCD EVOLUTION IS OBVIOUSLY A NECESSARY STEP!

LO DGLAP

The evolution of the diagonal ($\xi = 0$) GPD's proceeds as the evolution of PD's. We use here the LO DGLAP. Note that the kernel is independent of $\mathbf{\Delta}_\perp$, or b. For the non-singlet (valence) quarks

$$
\begin{aligned}
Q^2 \frac{\partial V(x, Q, \Delta_\perp)}{\partial Q^2} &= \frac{\alpha_S(Q^2)}{2\pi} \int_0^1 dz P_{qq}(z) \left[\theta(z - x) V(\frac{x}{z}, Q, \Delta_\perp) - V(x, Q, \Delta_\perp) \right] \\
P_{qq}(z) &= \frac{4}{3} \frac{(1 + z^2)}{(1 - z)} \\
\alpha(Q) &= \left(\frac{4\pi}{\beta_0} \right) \frac{1}{\log(Q^2/\Lambda_{\mathrm{QCD}}^2)} \\
\beta_0 &= 11 - 2N_F/3, \quad N_F = 3, \quad \Lambda_{\mathrm{QCD}} = 226 \text{ MeV}
\end{aligned}
$$

The integro-differential equation can be cast in the form of differential equations in the moment space (Mellin transform).

One introduces the x-moments

$$V_n(Q,\Delta_\perp) \equiv \int_0^1 dx\, x^{n-1} V(x,Q,\Delta_\perp)$$

Then we acquire a (diagonal in n) set of equations

$$\frac{\partial V_n(Q,\Delta_\perp)}{\partial Q^2} = \frac{\alpha_S(Q^2)}{-8\pi Q^2}\gamma_n V_n(Q,\Delta_\perp)$$

where the anomalous dimensions are given by

$$\gamma_n = \int_0^1 dz(1-z^n)P_{qq} = \frac{8}{3}\left(4H_n + \frac{2}{n+1} + \frac{2}{n+2} - 3\right), \quad H_n \equiv \sum_{j=1}^{n} \frac{1}{j}$$

These equations are integrated trivially, yielding

$$V_n(Q,\Delta_\perp) = V_n(Q_0,\Delta_\perp)\exp\left(\int_{Q_0^2}^{Q^2} dQ'^2 \frac{\alpha_S(Q'^2)}{Q'^2}\right) = V_n(Q_0,\Delta_\perp)\left(\frac{\alpha_S(Q^2)}{\alpha_S(Q_0^2)}\right)^{\frac{\gamma_n}{2\beta_0}}$$

Inverse Mellin transform

We need to go back to the x-space

$$V(x,Q,\Delta_\perp) = \int_C dn\, x^{-n} V_n(Q,\Delta_\perp)$$

With $n = n_0 + it$ we have

$$V(x,Q,\Delta_\perp) = 2\int_0^\infty dt\, x^{-n_0}\left[\cos(t\log x)\mathrm{Re}V_n(Q,\Delta_\perp) + \sin(t\log x)\mathrm{Im}V_n(Q,\Delta_\perp)\right]$$

This integral can be very easily done numerically!

All takes a few lines in Mathematica

Determination of Q_0

The scale Q_0 (the quark-model scale) is defined as the scale where all momentum of the hadron is carried by the valence quarks. The valence contribution to the energy momentum tensor evolves as

$$\frac{V_1(Q)}{V_1(Q_0)} = \left(\frac{\alpha(Q)}{\alpha(Q_0)}\right)^{32/81}.$$

In [SMRS, 1992] at $Q = 2$ GeV the valence quarks carry 47% of the total momentum of the pion. Downward LO evolution requires

$$V_1(Q_0) = 1, \quad G_1(Q_0) + S_1(Q_0) = 0,$$

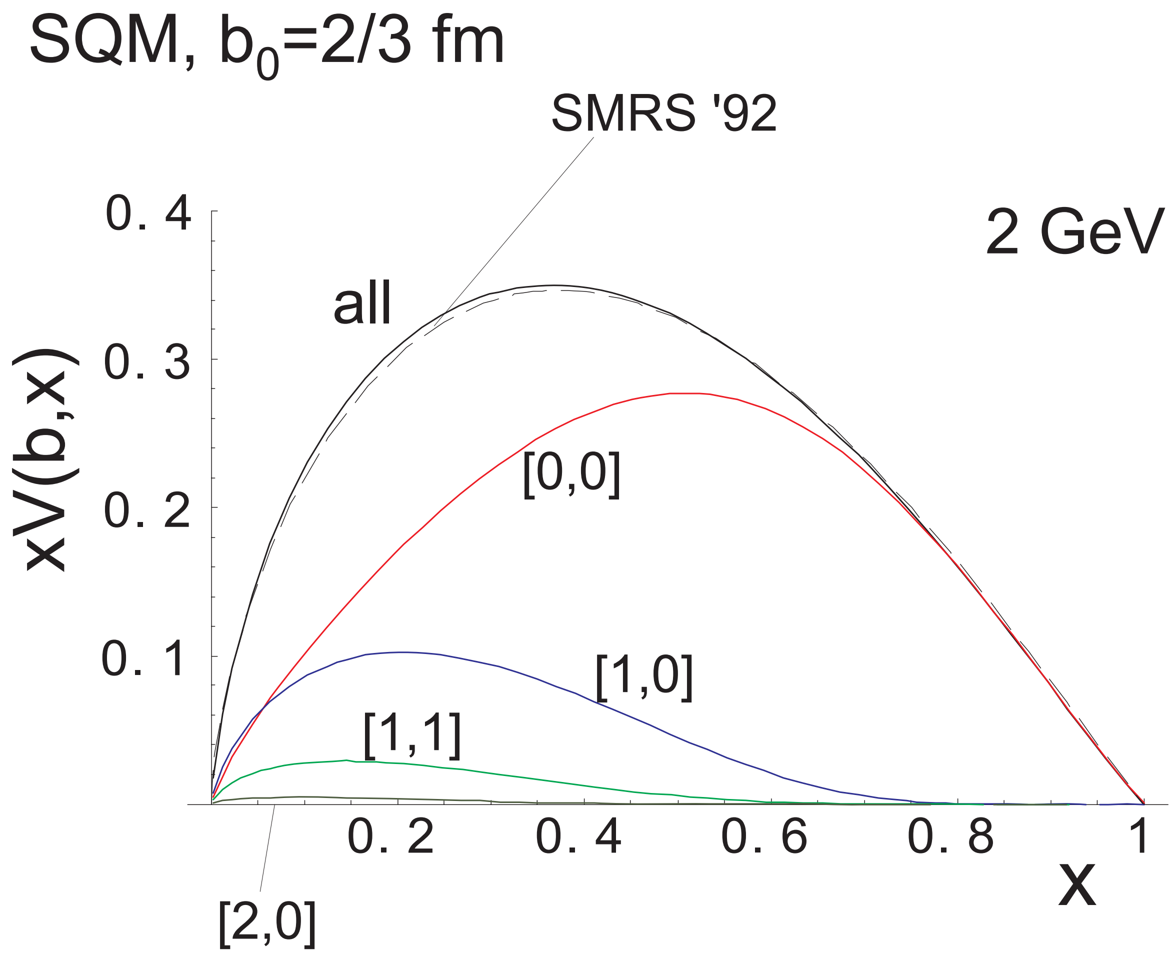
which gives

$$Q_0 = 313^{+20}_{-10} \text{ MeV}.$$

Rather low! One can hope that the typical expansion parameter $\alpha(Q_0)/(2\pi) \sim 0.34 \pm 0.04$ makes the perturbation theory still meaningful. NLO supports this assumption [Davidson + ERA, 2002]. Similar estimate for Q_0 has been obtained from an analysis of pion DA [ERA + WB,2002].

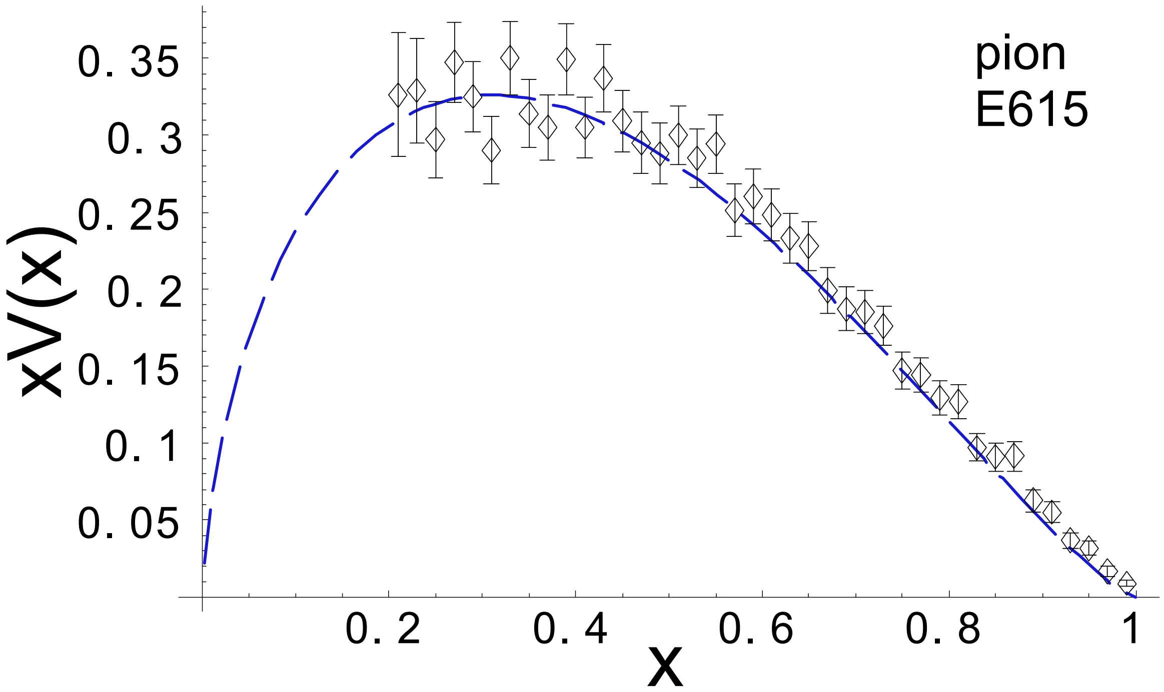
Comparison of the forward distribution to SMRS'92

$(V(x, Q_0) = \theta(x)\theta(1-x),\ Q = 2\ \text{GeV})$



Comparison of the forward distribution to Fermilab's E615

[J. S. Conway et al., PRD 39 (1989) 92], $\pi^- N \to \mu^+ \mu^- X$
$(V(x, Q_0) = \theta(x)\theta(1-x)$, $Q = 4$ GeV)



QCD evolution of the diagonal non-forward $V(x, Q, b)$

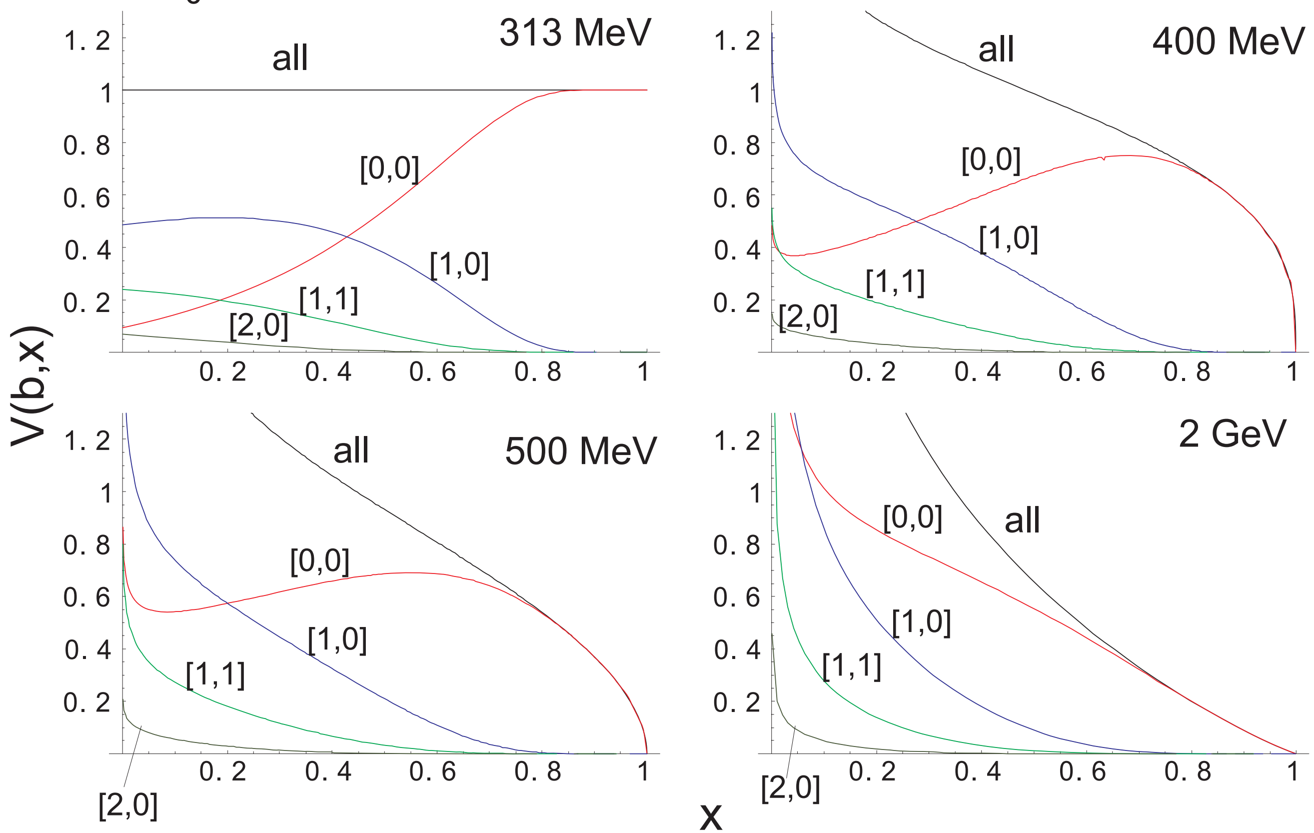
We apply the evolution to the smeared functions,

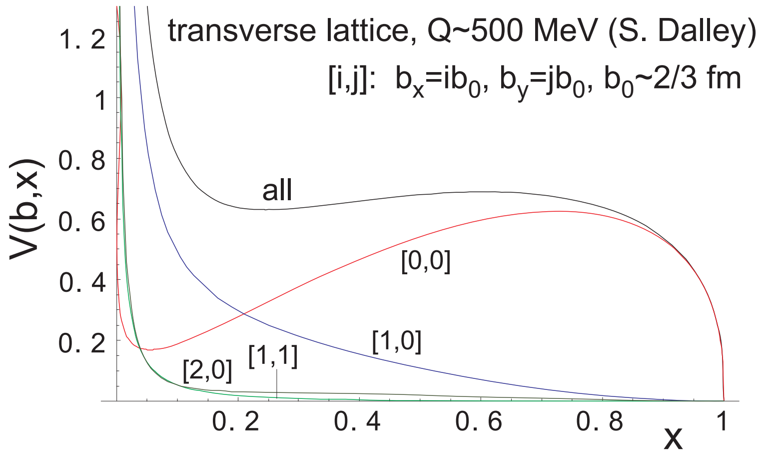
$$V(x, Q, [i,j]) = \int_{-i\infty}^{+i\infty} \frac{\mathrm{d}n}{2\pi i} x^{-n} \left(\frac{\alpha(Q)}{\alpha(Q_0)} \right)^{\gamma_n/(2\beta_0)} \int_0^1 dx'\, x'^{n-1} V(x', Q_0, [i,j])$$

where the distribution at the scale Q_0 is the prediction of either of the two considered chiral quark models

... and the results are ...

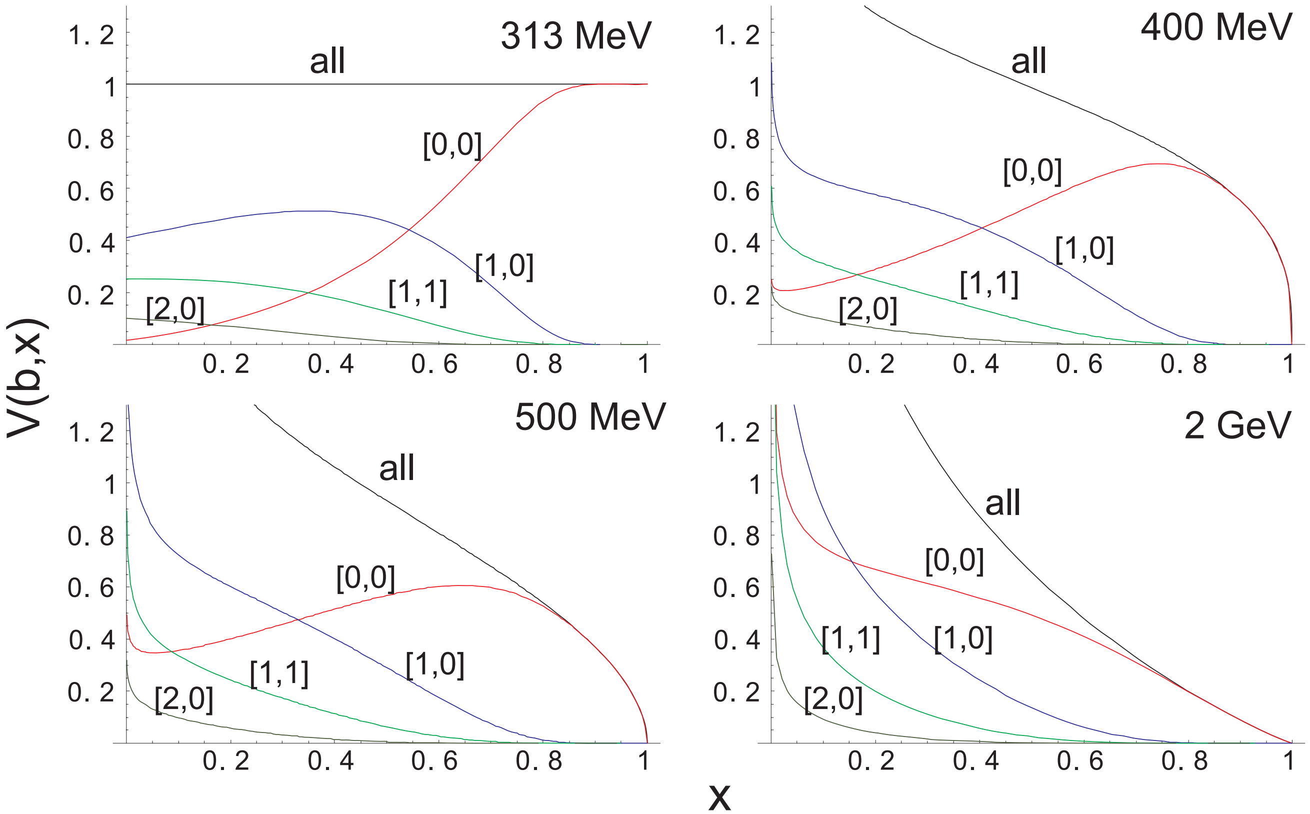
SQM, b_0=2/3 fm

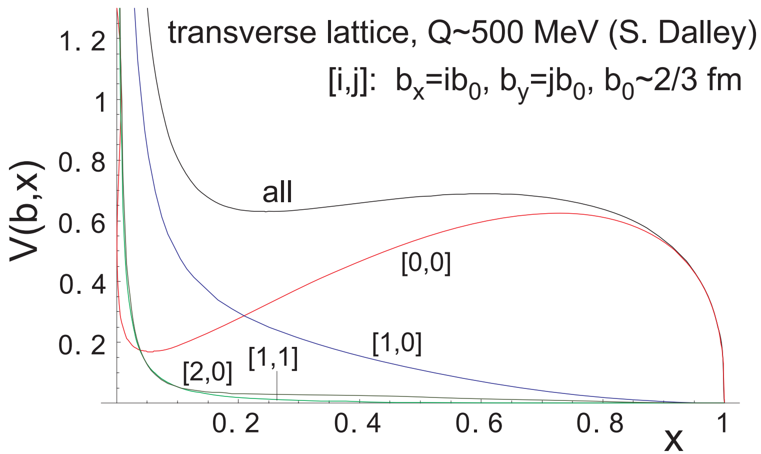




qualitative agreement

SQM, b_0=0.8 * 2/3 fm





Results and conclusions

- Chiral quark models provide GPD at a low scale, Q_0. The results are simple, especially for SQM
- The quark model scale Q_0 can be estimated with the help of the momentum fraction carried by the valence quarks. We note that the value is consistent with previous analysis based both on the forward parton distribution amplitudes as well as the light cone wave function
- Predictions of the two considered models, SQM and NJL, are qualitatively the same, with the NJL curves pushed to somewhat lower values of x
- Large effect of the DGLAP evolution on the distribution functions. Strength moved to lower values of x
- At $Q = 2$ GeV the result for the forward distribution agrees very well with the SMRS parameterization of the pion structure function [Davidson-Arriola, 1995]
- At $Q = 4$ GeV the result for the forward distribution agrees very well with the Fermilab E615 experiment!
- The results for the plaquette $[0,0]$ follow, at large x, the forward distributions. This is clear from the dependence of the initial function on the variable $b/(1-x)$. As $x \to 1$, the integration over the $[0,0]$ plaquette is the same as the integration over the whole b-space
- At $Q = 400$ MeV and 500 MeV the values of $V(x, Q, [0,0])$ reach a maximum at an intermediate value of x, and develop a dip at low x. This is in qualitative

agreement with the transverse-lattice data. We note that there the dip at low x is lower than in our model calculation, yet, in view of the simple nature of our model and approximations (chiral limit, LO evolution, evolution independent of b, uncertainties in the determination of b_0 and Q on the lattice) the similarity is quite satisfactory. We have checked that if the value the lattice-spacing parameter, b_0, were lowered, an even more quantitative agreement would follow

- The results for non-central plaquettes also qualitatively agree with the lattice measurements. In this case at $x \to 1$ the corresponding functions vanish very fast, in accordance to our model formulas
- A difference with the lattice results is that in our case the farther plaquettes naturally bring less and less, and the yield from the $[2, 0]$ plaquette is lower than for the $[1, 1]$ plaquette
- In summary, the obtained agreement of our approach, based on non perturbative chiral quark models in conjunction with perturbative LO DGLAP evolution, with the data from the transverse lattices, is quite remarkable and encouraging, baring in mind the simplicity of the models and the apparently radically different handling of chiral symmetry in both approaches
- Our analysis might be reinforced by extending our calculation to include the NLO perturbative corrections. Such a study is left for a future research

BACKUP slides

Behavior at $x \to 1$

A function that initially behaves as $V(x, Q_0, b) \to C(b)(1-x)^p$ evolves into

$$V(x, Q, b) \to C(b)(1-x)^{p - \frac{4C_F}{\beta_0} \log \frac{\alpha(Q)}{\alpha(Q_0)}}, \qquad x \to 1.$$

Moments in SQM

In the Spectral Quark Model

$$V_n(Q_0,b) = \frac{m_\rho^2\Gamma(n+1)}{\pi 2^{n+3}}\left[b m_\rho G^{4,0}_{2,4}\left(\frac{b^2 m_\rho^2}{4}\middle|\begin{matrix} \frac{n-1}{2}, \frac{n}{2} \\ -1, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}\end{matrix}\right) - G^{4,0}_{2,4}\left(\frac{b^2 m_\rho^2}{4}\middle|\begin{matrix} \frac{n}{2}, \frac{n+1}{2} \\ -\frac{1}{2}, 0, 0, 0\end{matrix}\right)\right],$$

where G denotes the Meijer G function. This form can be useful for further analytic considerations.

Scaling in $b/(1-x)$

Generally, the chiral quark model (one-loop) results depend on $\mathbf{\Delta}_\perp$ and x only through the combination $(1-x)^2\mathbf{\Delta}_\perp^2$. Consequently, in the b space they depend on the combination $b^2/(1-x)^2$. Due to this property

$$\frac{\int d^2b\, b^{2n} q(b,x)}{\int d^2b\, q(b,x)} \equiv \langle b^{2n}\rangle(x) = (1-x)^{2n}\langle b^{2n}\rangle(0).$$

This means, that all the moments except for $n=0$ vanish as $x \to 1$, or $q(b,x)$ becomes a $\delta(b)$ function in this limit. This behavior is seen in the transverse lattice data.