Valon Model for Double Parton Distributions

Wojciech Broniowski[1,2] **and Enrique Ruiz Arriola**[3]

[1]Institute of Nuclear Physics PAN, Cracow
[2]Jan Kochanowski U., Kielce
[3]U. of Granada

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research with Krzysztof Golec-Biernat

Motivation for multi-parton distributions

- Old story, renewed interest (LHC) [Kuti, Weiskopf 1971, Konishi, Ukwa, Veneziano 1979, Gaunt, Stirling 2010, Diehl, Ostermeier, Schafer 2012, . . . , reviews: Bartalani et al. 2011, Snigirev 2011]
- Multiple parton scattering at the LHC: will it be possible to see?
- Little explored [MIT bag: Chang, Manohar, Waalewijn 2013], valon [WB+ERA, Few Body Syst. 55 (2014) 381 (LC2013)], constituent quarks: Rinaldi, Scopetta, Vento 2013]
- Questions: factorization, interference of single- and double-parton scattering [Manohar, Waalewijn 2012, Gaunt 2013], longitudinal-transverse decoupling, positivity bounds [Diehl, Casemets 2013] (cf. the preceding talk by J. Gaunt)

Goal: construct dPDF's which satisfy the formal constraints (GS sum rules) and reproduce phenomenological sPDF's

Highlights

- "Bottom-up" construction based on educated guessing not unique
- A constructive method is based on the Fock expansion of the hadron LC wave function: $(n-1)$PDF's are recursively obtained from nPDF's
- The wave functions comes from models (physics)
- Valon model [Hwa, Zahir 1981, Hwa, Yang 2002] = model, where the only correlations come from the momentum conservation
- Pion and matching to GRV_π
- Nucleon
- Evolution and correlations

Assumed factorization

Transverse-longitudinal decoupling (?)

$$\Gamma_{ij}(x_1, x_2, b_1, b_2, Q_1^2, Q_2^2) = D_{ij}(x_1, x_2, Q_1^2, Q_2^2) f(b_1) f(b_2)$$

... or use the b-integrated dGPD's ($k_T = 0$)

$$D_{ij}(x_1, x_2, Q_1^2, Q_2^2) = \int d^2 b_1 d^2 b_2 \Gamma_{ij}(x_1, x_2, b_1, b_2, Q_1^2, Q_2^2)$$

Is phenomenology built by just assuming $D_{ij}(x_1, x_2) = D_i(x_1) D_j(x_2)$ sufficiently accurate?
(e.g., in Łuszczak, Maciuła, Szczurek 2012)

Gaunt-Stirling sum rules

[Gaunt, Stirling, JHEP (2010) 1003:005, Gaunt, PhD Thesis]

Fock-space decomposition + conservation laws $\rightarrow$

$$\sum_i \int_0^{1-y} dx\, x D_{ij}(x,y) = (1-y) D_j(y)$$

$$\int_0^{1-y} dx\, D_{i_{\rm val} j}(x,y) = \int_0^{1-y} dx\, [D_{ij}(x,y) - D_{\bar{i}j}(x,y)] = (N_{i_{\rm val}} - \delta_{ij} + \delta_{\bar{i}j}) D_j(y)$$

- Preserved by the evolution
- Non-trivial to satisfy with the (guessed) functions

Attempts of construction

- Gaunt, Stirling (2011)

$$D_{ij}(x_1,x_2) = D_i\, D_j(x_2)\, \frac{(1-x_1-x_2)^2}{(1-x_1)^{2+n_1}(1-x_2)^{2+n_2}}$$

(do not satisfy the GS sum rules)

- Lewandowska, Golec-Biernat (2014)

$$D_{ij}(x_1,x_2) = \frac{1}{1-x_2} D_i\left(\frac{x_1}{1-x_2}\right) D_j(x_2)$$

$$D_{qq}(x_1,x_2) = \frac{1}{1-x_2}\left\{D_q\left(\frac{x_1}{1-x_2}\right) - \frac{1}{2}\right\} D_q(x_2)$$

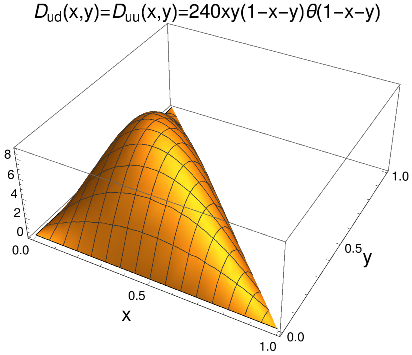
$$D_{q\bar{q}}(x_1,x_2) = \frac{1}{1-x_2}\left\{D_q\left(\frac{x_1}{1-x_2}\right) + \frac{1}{2}\right\} D_{\bar{q}}(x_2)$$

(no parton exchange symmetry, negative D_{qq} at large x)

Problems!

Non-uniqueness of the sPDF constraints

A sample function satisfying GS sum rules (valon model for the nucleon discussed later), $|p\rangle = |uud\rangle$



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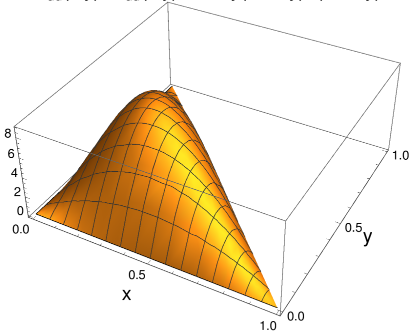
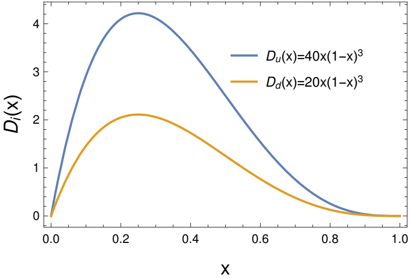
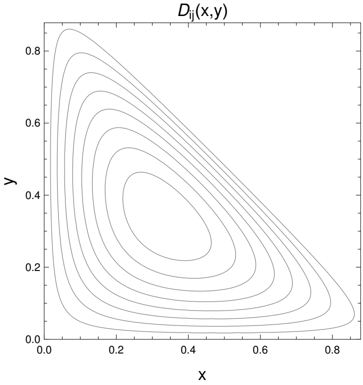




$$D_u(x) = \int dy\, D_{ud}(x,y) = \int dy\, D_{uu}(x,y),\; 2D_d(x) = \int dx\, D_{ud}(x,y)$$
$$(1-x)D_u(x) = \int dy\, y[D_{ud}(x,y) + D_{uu}(x,y),\; (1-x)D_d(x) = \int dy\, yD_{du}(x,y)$$

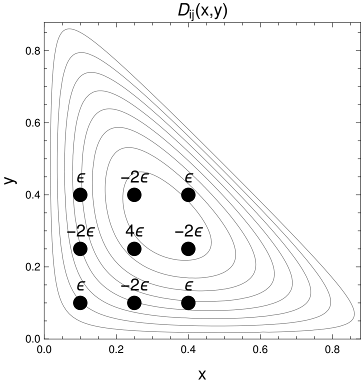
(marginal projections)

One solution $\rightarrow$ infinitely many solutions



One solution $\to$ infinitely many solutions

Explicit construction:



Integrals $\int dy\, D_{ij}(x,y)$ and $\int dy\, y D_{ij}(x,y)$ intact! (can be distributed with smeared functions of finite support). This non-uniqueness is obvious: one-particle distributions do not fix the two-particle distribution (correlations)

Top-down from LC wave function

- Light-cone Fock expansion of the proton state in partonic constituents:

$$|p\rangle = \sum_N \sum_{f_1 \ldots f_N} \int dx_1 \ldots dx_N \, \delta(1 - \sum_{k=1}^{N} x_k) \\ \times \Psi_N(x_1 \ldots x_n; f_1 \ldots f_N) \, |x_1 \ldots x_N; f_1 \ldots f_N\rangle$$

(origin of GS sum rules [Gaunt, PhD thesis])

- Model Ψ_N's with the constraints from the known sPDF's,
- Compute the double distributions, $D_{f_1 f_2}(x_1, x_2)$
- Run evolution

Simplest assumption: the only correlations come from the longitudinal momentum conservation: $1 = x_1 + x_2 + \cdots + x_n$ (we call it generalized valon model):

$$\psi_N(x_1 \ldots x_n; f_1 \ldots f_N = A\psi_{f_1}(x_1) \ldots \psi_{f_N}(x_N)$$

Asymptotics

$$|\psi_N(x_1 \dots x_n; f_1 \dots f_N|^2 = A^2 \phi_{f_1}(x_1) \dots \phi_{f_N}(x_N)$$

Let the asymptotics of the single-parton functions be

$$\phi_{f_i}(x) \sim x^{\alpha_{f_i}-1} \qquad \text{at } x \to 0,$$
$$\phi_{f_i}(x) \sim (1-x)^{\beta_{f_i}} \qquad \text{at } x \to 1,$$

where for integrability $\alpha_{f_i} > 0$ and $\beta_{f_i} > -1$. Then

$$D_f(x) \sim x^{\alpha_f - 1} \qquad \text{at } x \to 0$$
$$D_f(x) \sim (1-x)^{\beta_f + \alpha_{f_{(1}} + \dots + \alpha_{f_{N-1)'}} - 1} \qquad \text{at } x \to 1$$
$$D_{fh}(x,y) \sim x^{\alpha_f + \alpha_h^N - 2} \qquad \text{at } x, y \to 0$$
$$D_{fh}(x,y) \sim \phi_f(x)\phi_h(y)(1-x-y)^{\alpha_{f_{(1}} + \dots + \alpha_{f_{N-1)'}} - 1} \quad \text{at } x + y \to 1$$

(the large-x behavior is sensitive to the low-x behavior of the other components, as in this limit the kinematics "pushes them towards 0)

Scale and evolution

- With increasing scale Q more and more partons at low x are generated, hence more and more Fock components are needed
- For practical reasons it is then favorable to use the parameterizations at lowest possible Q
- Quark-model scale $\rightarrow$ not sufficiently many gluons and sea quarks
- Cannot evolve too far down, negative distributions generated, not perturbative

Pion from GRV$_\pi$

Gluck, Reya, Vogt, Z. Phys. C 53 (1992) 651

$$xv(x) = 0.519\left(0.381\sqrt{x}+1\right)(1-x)^{0.367}x^{0.499}$$
$$xg(x) = \left(0.338\sqrt{x}+0.678\right)(1-x)^{0.39}x^{0.482}$$
$$xq_{\rm sea}(x) = 0 \qquad\qquad (Q_0 = 500\mathrm{MeV})$$

Use momentum fraction as constraints

$$2\int dx\, xv(x) = 0.584, \quad \int dx\, xg(x) = 0.416$$

Also

$$\int dx\, v(x) = 1, \quad \int dx\, g(x) = 1.46$$

hence the average number of gluons is 1.46

Pion from GRV$_\pi$ (2)

Use the simple ansatz

$$|\pi^+\rangle = A|\bar{u}d\rangle + B|\bar{u}dgg\rangle$$

The sPDF's are:

$$D_{\bar{u}}(x) = A^2|\Psi_{\bar{u}d}(x, 1-x)|^2 + B^2\int dx_3 dx_4 |\Psi_{\bar{u}dgg}(x, 1-x-x_3-x_4, x_3, x_4)|^2$$

$$D_d(x) = D_{\bar{u}}(x)$$

$$D_g(x) = B^2\int dx_1 dx_2 \left(|\Psi_{\bar{u}dgg}(x_1, x_2, x, 1-x-x_1-x_2)|^2 + |\Psi_{\bar{u}dgg}(x_1, x_2, 1-x-x_1-x_2, x)|^2\right)$$

Conditions

$$\begin{aligned} A^2 + B^2 &= 1 \\ \int dx\, x[D_{\bar{u}}(x) + D_d(x)] &= 0.584 \\ \int dx\, x D_g(x) &= 0.416 \end{aligned}$$

provide constraints for parameters

Generalized valon ansatz

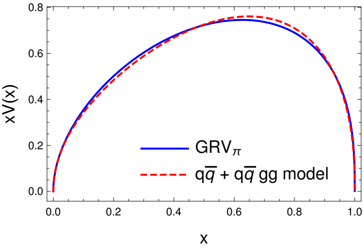
We use

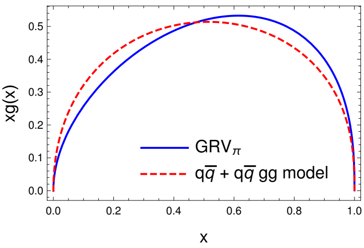
$$|\Psi_{\bar{u}d}(x_1,x_2)|^2 \sim f(x_1;a,b)f(x_2;a,b)$$
$$|\Psi_{\bar{u}dgg}(x_1,x_2,x_3,x_4)|^2 \sim f(x_1;\alpha_q,\beta_q)f(x_2;\alpha_q,\beta_q)f(x_3;\alpha_g,\beta_g)f(x_4;\alpha_g,\beta_g)$$

with

$$f(x;\alpha,\beta) = x^{\alpha-1}(1-x)^{\beta}$$

Then with $a+b=0.5$, $\alpha_q=0.5$, $\beta_q=-0.09$, $\alpha_g=0.48$, $\beta_g=-0.09$ we get

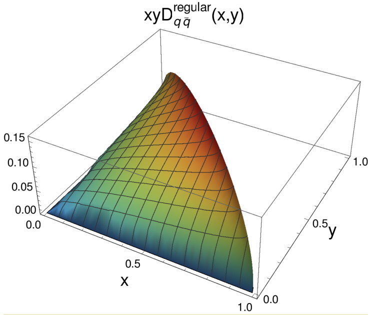


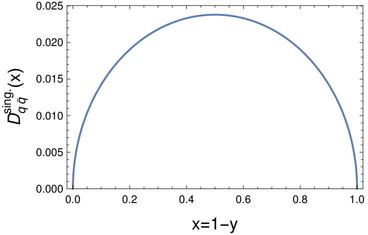


$A^2=0.15$ and $B^2=0.85$ – dominance of the component with gluons!

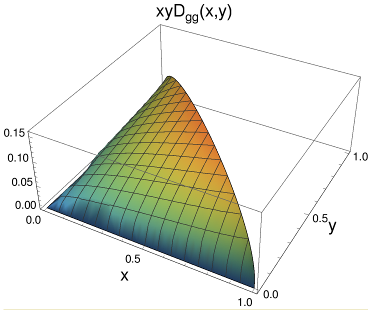
Quark dPDF's of the pion

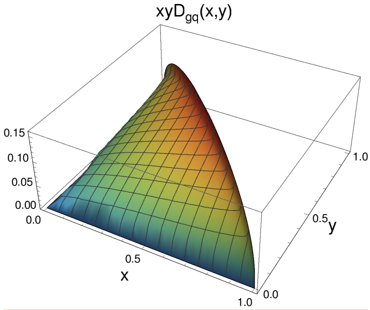
Having fixed the parameters, we may obtain the dPDF's (and nPDF's) ($\bar{u}d$ component generates a singular part, $D_{\bar{u}d} \sim f(x;a,b)f(y;a,b)\delta(1-x-y)$)





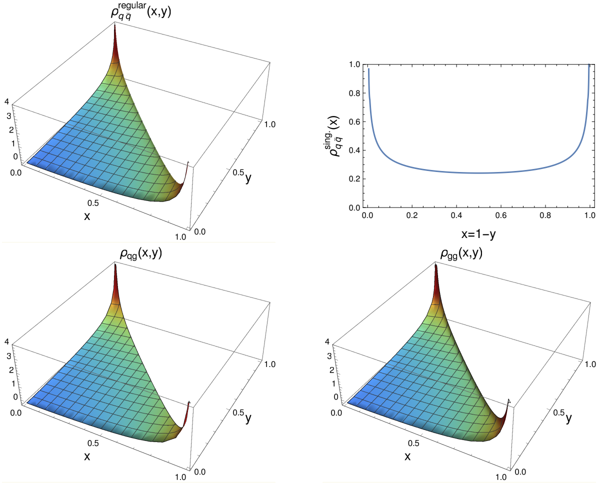
Other dPDF's of the pion





Correlation

$$\rho_{ij}(x_1, x_2) = \frac{D_{ij}(x_1,x_2)}{D_i(x_1)D_j(x_2)} - 1$$



Nucleon

Repeating the construction for the GRV parametrization of the proton ($Q_0 = 480$ MeV) would require taking at least

$$|p\rangle = A_{uud}|uud\rangle + \cdots + A_{uud(\bar{q}q)^3}|uud\bar{q})^3\rangle + \cdots + A_{uudgg}|uudgg\rangle$$

For now we take the wave function composed just of three valence quarks $|p\rangle = |uud\rangle$ [WB, ERA, LC2013 proceedings]

[Kuti, Weisskopf 1971] take the valence quark orbital in the form $\psi(x) \sim x^a$ and include correlations only from the longitudinal momentum conservation [Hwa, Zahir 1980, Hwa, Yang 2002]:

$$|\Psi_{uud}(x_1, x_2, x_3)|^2 \delta(1 - x_1 - x_2 - x_3) = A\phi_u(x_1)\phi_u(x_2)\phi_d(x_3)\delta(1 - x_1 - x_2 - x_3)$$

Nucleon in the valon model

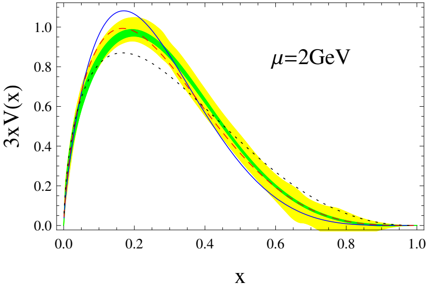
Take $\phi_a(x) = x^\alpha$. Then

$$D_{uu}(x_1,x_2) = D_{ud}(x_1,x_2) = 2\frac{\Gamma(3\alpha+3)}{\Gamma(\alpha+1)^3} x_1^\alpha x_2^\alpha (1-x_1-x_2)^\alpha$$

$$D_u(x_1) = 2D_d(x_1) = \frac{2\Gamma(3\alpha+3)}{\Gamma(\alpha+1)\Gamma(2\alpha+2)} x_1^\alpha (1-x_1)^{2\alpha+1}$$

(with $\alpha = 1$ the behavior of $D(x_1)$ at $x_1 \to 1$ conforms to the counting rules [Brodsky, Lepage 1980])

sPDF for valence vs data at $\mu = 2$ GeV



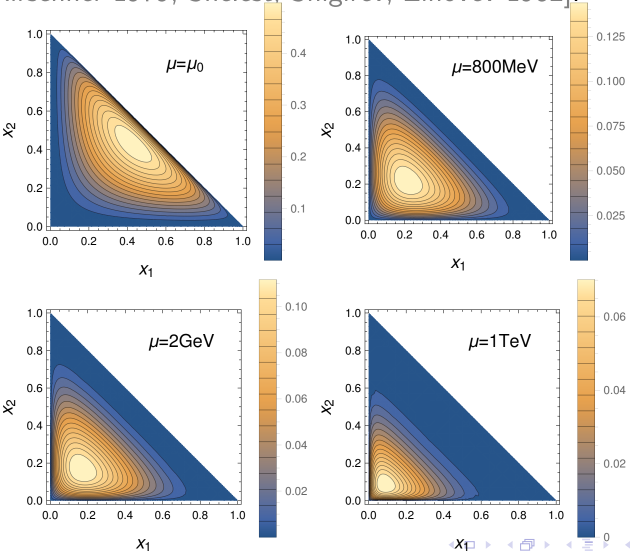
solid $\alpha = 1$, dashed $\alpha = 1/2$, dotted $\alpha = 0$
green NNPDF (no LHC), yellow NNPDF (collider)

At $\mu = 2$ GeV quarks carry 41.6% of the momentum (no gluons) $\rightarrow$ the initial scale is very low, $\mu_0 = 285$ MeV, similarly to the pion case [Davidson, Arriola 1995, WB, Arriola, Golec-Biernat 2008]

dPDF for valence

$x_1 x_2 V(x_1, x_2; \mu) \quad (\alpha = 1/2)$

Evolution [Kirschner 1979, Shelest, Snigirev, Zinovev 1982]:



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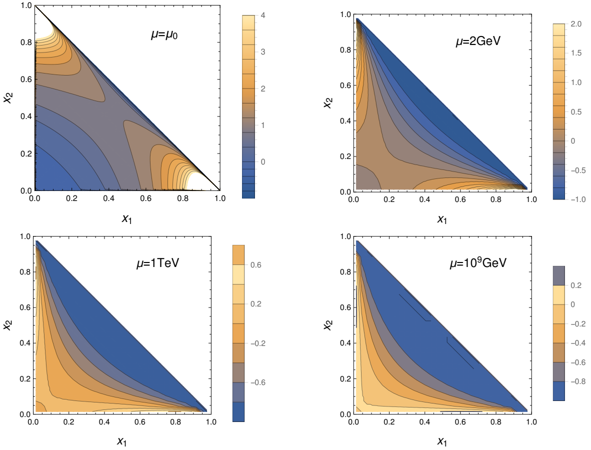


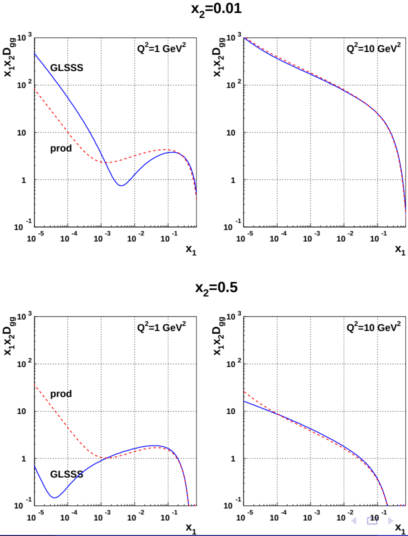
Lack of factorization [Snigirev 2003, Korotkikh, Snigirev 2004]

Evolution washes out the correlations at low x's

Gluons (only)

[Golec-Biernat, Lewandowska, Serino, Snyder, Staśto, arXiv:1507.08583]

Gluons from the MSTW parameterization, Mellin moment constraints



Summary

- Top-down strategy of constructing multi-parton distributions $\rightarrow$ formal features guaranteed
- Requires modeling the LC wave functions $\leftarrow$ physics
- Phenomenological sPDF's as constraints
- Many Fock components needed for the popular parameterizations of sPDF's, even at low scales
- The valon model offers a simple ansatz at the initial low-energy scale that grasps the essential features with just the longitudinal momentum conservation
- Evolution washes out the correlation at low x_1, x_2 , justifying the product ansatz in that limit