Large-N_c Regge phenomenology

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Workshop on Unquenched Hadron Spectroscopy:
Non-Perturbative Models and Methods of QCD vs. Experiment
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[research with Pere Masjuan and Enrique Ruiz Arriola]

Outline

- Hagedorn spectra $\rightarrow$ string models
- Regge trajectories
- Meson dominance of the pion and nucleon form factors

Basic references:

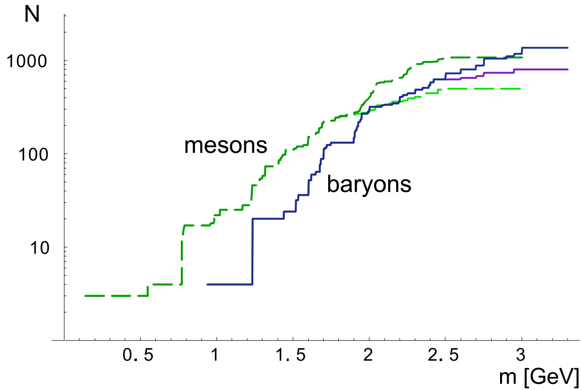
P. Masjuan, E. Ruiz Arriola, WB, PRD 85 (2012) 094006

P. Masjuan, E. Ruiz Arriola, WB, PRD 87 (2013) 014005

Hagedorn growth

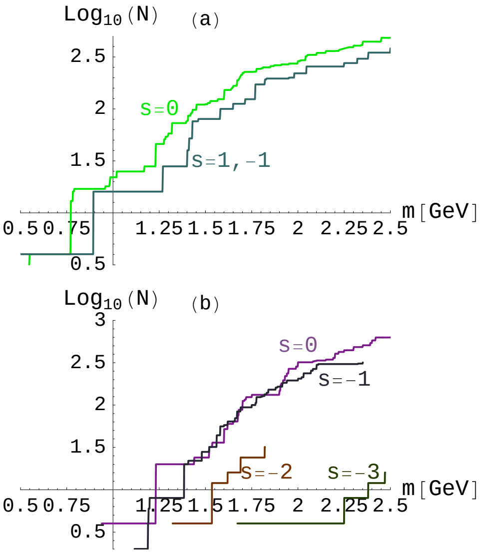
A look at PDG

Cumulative number of states



[WB, Wojciech Florkowski, Leonid Ya. Glozman, PRD 70 (2004) 117503]

Flavor independence



(a) – mesons, (b) – baryons

Exponential growth

$$\rho(m) \sim \exp\left(\frac{m}{T_H}\right) \quad \text{(Hagedorn)}$$

Consider excitation of mesonic strings, where the spectrum is generated by the harmonic-oscillator operator describing vibrations of the string:

$$N = \sum_{k=1}^{\infty} \sum_{\mu=1}^{D} k a_{k,\mu}^{\dagger} a_{k,\mu}$$

Here k labels the modes and μ labels additional degeneracy. Eigenvalues of N are composed of various modes in order to get the squared mass:

$$\alpha' m^2 = n \quad \text{(Regge formula)}$$

Partitio numerorum

Take $n = 5$. Partitions: 5, 4+1, 3+2, 3+1+1, 2+2+1, 2+1+1+1, 1+1+1+1+1. The number of partitions grows very fast with n, with the asymptotic formula of *partitio numerorum* (Ramanujan, Hardy) $\rightarrow$

$$\rho(m) \simeq \sqrt{\frac{1}{2n}} \left(\frac{D}{24n}\right)^{\frac{D+1}{4}} \exp\left(2\pi\sqrt{\frac{Dn}{6}}\right),$$

where $n = \alpha' m^2$. We can now read-off the mesonic Hagedorn temperature:

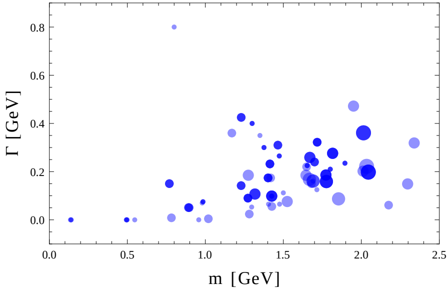
$$T_{\rm meson} = \frac{1}{2\pi}\sqrt{\frac{6}{D\alpha'}}$$

For baryons there are 3 strings, hence

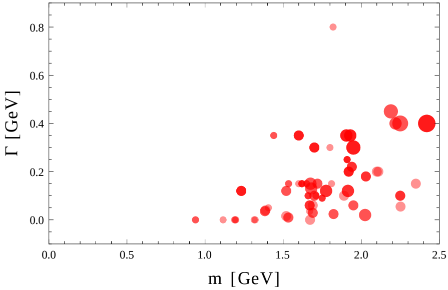
$$T_{\rm baryon} = T_{\rm meson}/\sqrt{3}$$

Widths

mesons



baryons

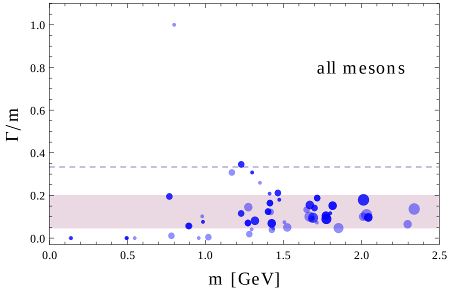


[size $\sim (2J+1)$, intensity $\sim (2I+1)$]

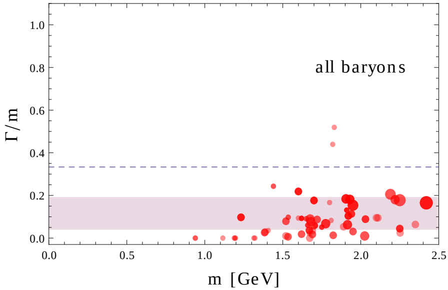
Width/mass

Mesonic strings: $l \sim m/\sigma$, the decay rate of a string per unit time, Γ, is proportional to l, which yields constant Γ/m

mesons



baryons



complies to $1/N_c$ argument

Half-width rule

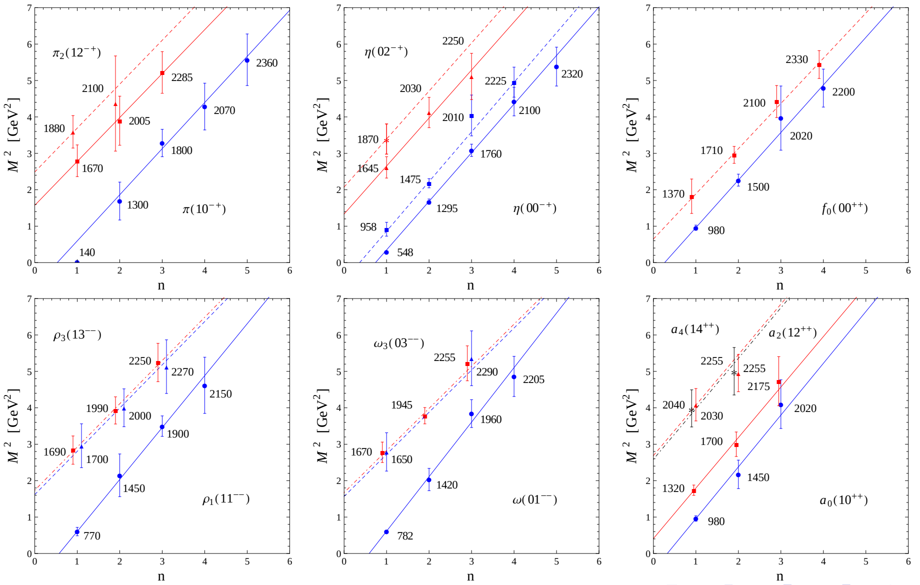
At large N_c things get simpler, as hadronic amplitudes are represented as trees of (infinitely many) mesons and glueballs. N_c counting (and data) give $\Gamma \sim m/N_c$

The BW position of the mass may move within Γ, same is expected when going from $N_c = \infty$ down to $N_c = 3$. Therefore it makes sense, in various fits, to use $\Gamma/2$ as the "error" in χ^2 – "half-width rule"

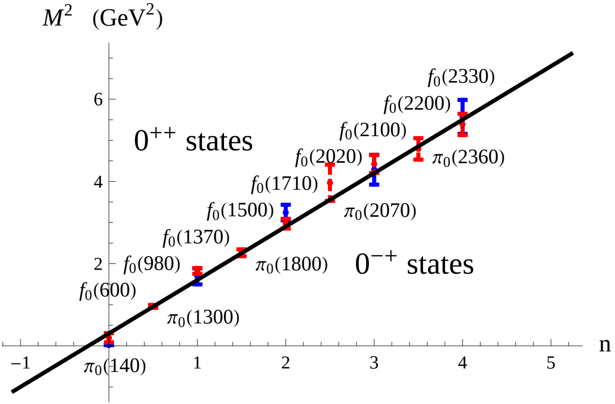
Including the width is a way of incorporating (some) $1/N_c$ corrections

Regge trajectories

Radial trajectories



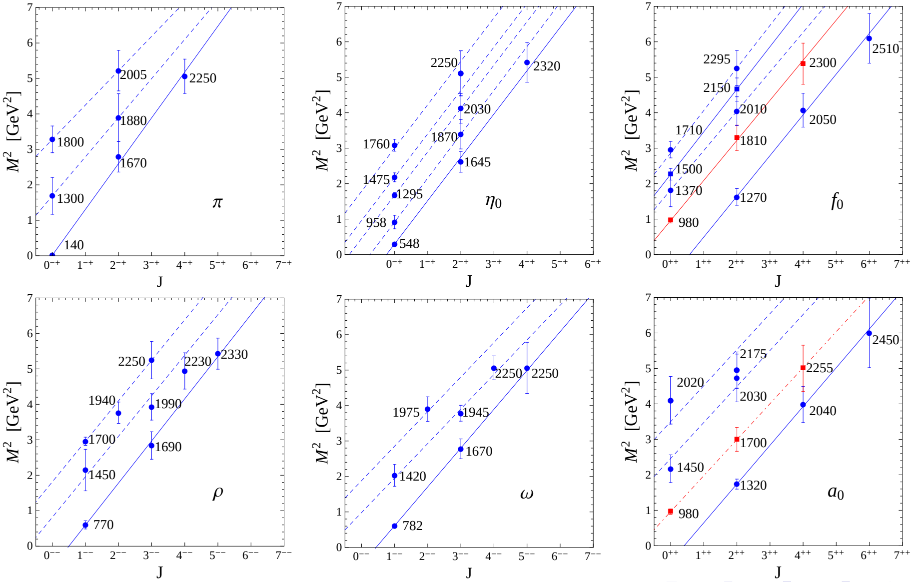
$\sigma - \pi$ trajectories



(2 σ trajectories)

[data from Anisovich, Anisovich, Sarantsev, 2000, Anisovich, 2006, last 4 σ's and last 2 π' not confirmed]

Angular-momentum trajectories



Hypothesis of universality of slopes

Above we have updated the fits of Anisovich, Anisovich, and Sarantsev, 2000
The error bars indicate the half-width

Regge formula:

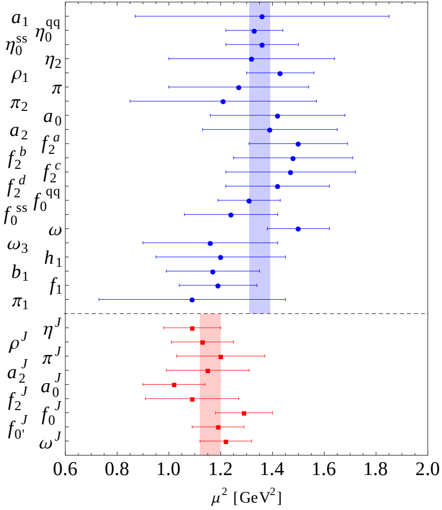
$$m^2 = an + bJ + c$$

AdS/CFT soft-wall models yield universality of slopes

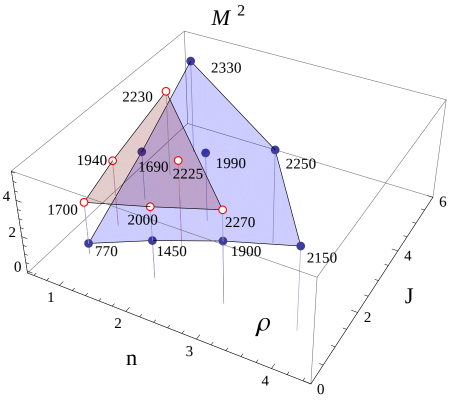
$$m^2 = a(n + J) + c$$

What do the data say?

Fitted slopes - no universality



$a = 1.35(4)$, $b = 1.16(4)$ (3.4σ deviation)



Combined fit with planes containing at least 6 states:

$$m^2 = 1.38(4)n + 1.12(4)J - 1.25(4)$$

No universality at 4.5σ level

Form factors

Formulas

- Tree-level diagrams at large N_c
- Finite N_c corrections estimated from the widths Γ in the meson propagators
- Long and short distance constraints
- Logs disregarded
- Saturation with a small (minimal) number of states

Example: pion EM ff

$$F_V(-Q^2) \to \frac{16\pi f_\pi^2 \alpha(Q^2)}{Q^2}\left[1 + 6.58\frac{\alpha(Q^2)}{\pi} + \dots\right],$$

If we ignore the slowly varying logarithms, $F_V(t) = \mathcal{O}(t^{-1})$ and in the large N_c limit one has

$$F_V(t) = \sum_{V=\rho,\rho',\dots} c_V \frac{m_V^2}{m_V^2 - t}$$

with $\sum_V c_V = 1$ (long-distance constraint)

Formulas, cont.

The pion EM radius has large chiral corrections, thus we may improve the phenomenology by including ρ':

$$F_V(t) = (1-c)\frac{m_\rho^2}{m_\rho^2 - t} + c\frac{m_{\rho'}^2}{m_{\rho'}^2 - t}, \quad \frac{1}{6}\langle r^2 \rangle = (1-c)\frac{1}{m_\rho^2} + c\frac{1}{m_{\rho'}^2}$$

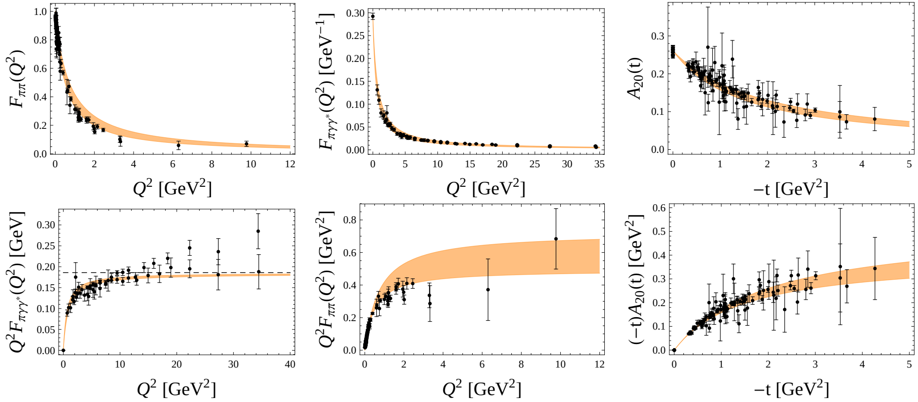
Pion gravitational ff: (relevant for Generalized Parton Distributions)

$$\langle \pi^b(p') \mid \Theta^{\mu\nu}(0) \mid \pi^a(p) \rangle = \delta^{ab}\left[(g^{\mu\nu}q^2 - q^\mu q^\nu)A_{20}(q^2) - 8P^\mu P^\nu A_{22}(q^2)\right]$$

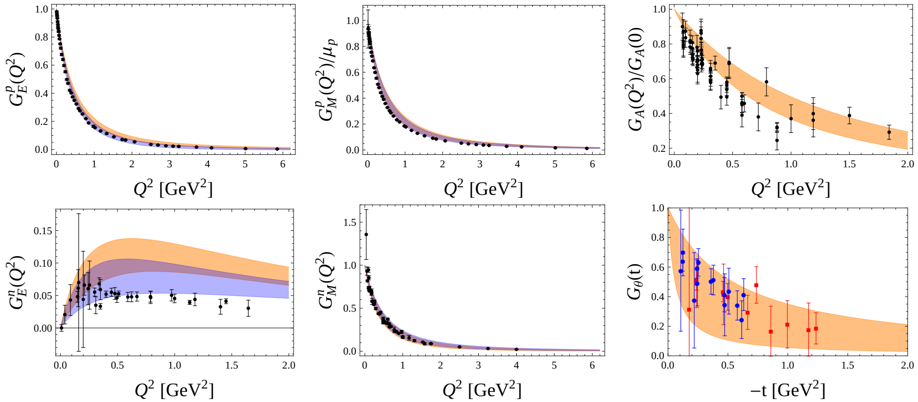
Nucleon: minimum number of resonances is 2 for the Dirac EM, 3 for the Pauli EM, 2 for axial, 1 for $G_\Theta(t)$, 2 for $F_T(t)$

(for gravitational ff there are lattice data)

Pion form factors

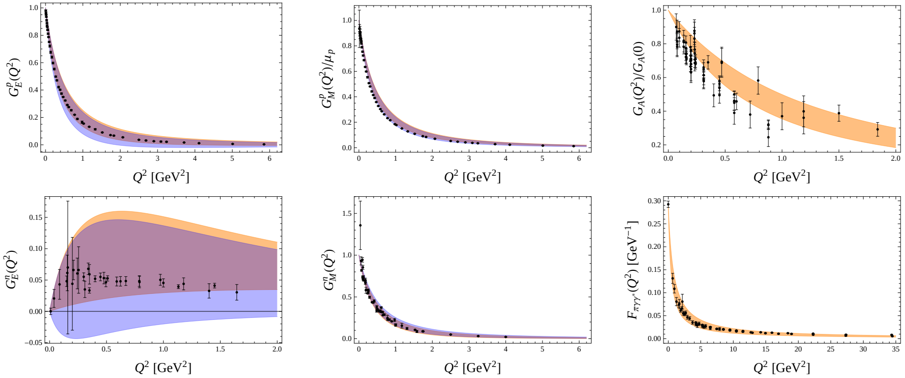


Nucleon form factors

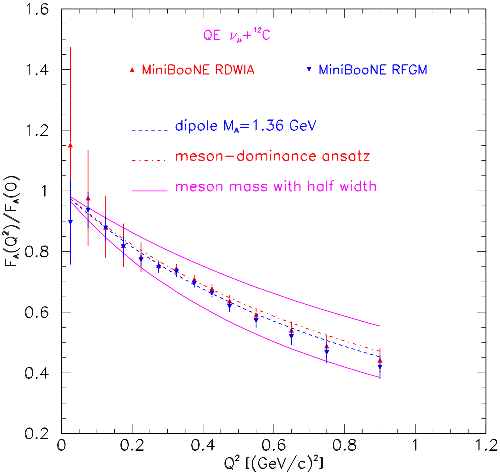


Left and middle rows: orange – $g_{\omega NN} = 9$, blue – $g_{\omega NN} = 12$

M/N_c rule

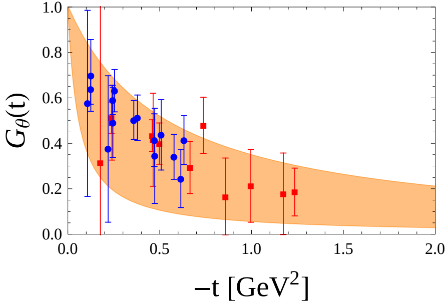

$G_E^p(Q^2)$
Q^2 [GeV^2]
$G_M^p(Q^2)/\mu_p$
Q^2 [GeV^2]
$G_A(Q^2)/G_A(0)$
Q^2 [GeV^2]
$G_E^n(Q^2)$
Q^2 [GeV^2]
$G_M^n(Q^2)$
Q^2 [GeV^2]
$F_{\pi\gamma\gamma^*}(Q^2)$ [GeV^{-1}]
Q^2 [GeV^2]

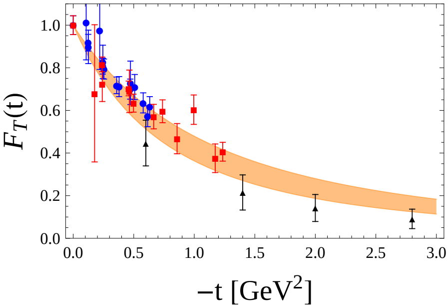
Nucleon axial form factor



[Butkevich, Perevalov, PRD 89, 053014 (2014)]

Nucleon gravitational form factors





monopole with the σ meson

(data from lattice)

Conclusions

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We live in the World of Hadrons!

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Wishes:

Long live Eef in the World of Hadrons!

Back-up

Discussion with D. Bugg