Double parton distributions of the pion

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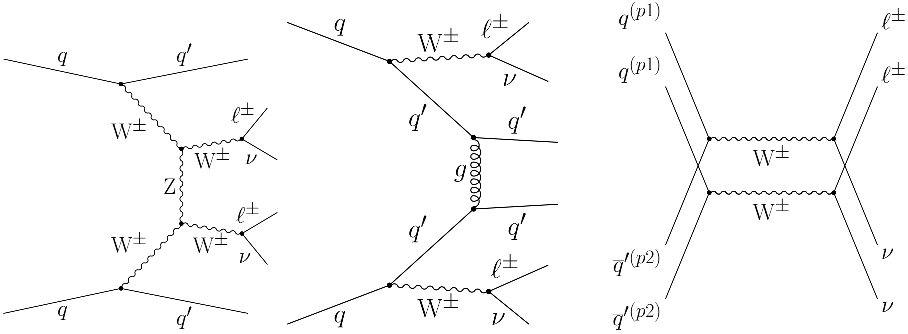
Double parton distributions

Motivation for multi-parton distributions

- Old story (Fermilab), renewed interest since ATLAS measurement for pp$\to$ W+2 jets 2013 [Kuti, Weiskopf 1971, Konishi, Ukwa, Veneziano 1979, Gaunt, Stirling 2010, Diehl, Ostermeier, Schäfer 2012, . . . , reviews: Bartalani et al. 2011, Snigirev 2011, Rinaldi,Ceccopieri 2018]
- Model exploration [MIT bag: Chang, Manohar, Waalewijn 2013], valon [WB, ERA, Golec-Biernat 2013, 2016], constituent quarks: Rinaldi, Scopetta, Traini, Vento 2013, 2018]
- Questions: factorization, interference of single- and double-parton scattering [Manohar, Waalewijn 2012, Gaunt 2013], longitudinal-transverse decoupling, positivity bounds [Diehl, Casemets 2013], transverse momentum dependence [Casemets, Gaunt 2019]
- Gaunt-Stirling sum rules [Gaunt, Stirling 2010, WB, ERA 2013, Diehl, Gaunt, Lang, Plößl, Schäfer 2019, 2020]

Example of double parton scattering (DPS)

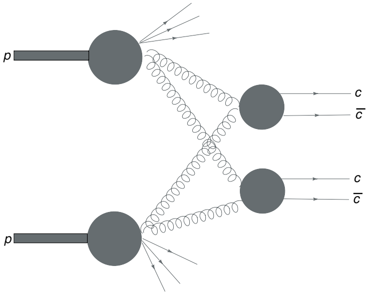
CMS 1909.06265: **Evidence** *for WW production from double-parton interactions in proton-proton collisions at* $\sqrt{s_{NN}}$*=13 TeV*

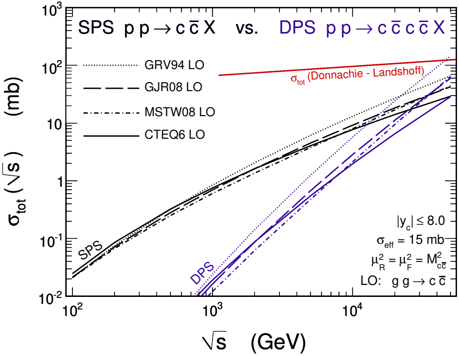


DPS → transverse distribution of partons, multi-parton correlations in the hadronic wave function, background to new physics searches

DPS searches

W+2jets, 2J/ψ, 2W, 4jets, 3jets+γ, open charm, ...
[example of predictions from Łuszczak, Maciuła, Szczurek 2011]





DPS can be comparable to SPS at the LHC

In the kinematic region of small transverse momenta of produced particles, DPS and SPS are comparable for any process [Gaunt], exploited in exp.

Master formula for DPS production

$$\sigma^{DPS}_{hh'\to AB} = \frac{m}{2}\sum_{i,j,k,l}\int dx_1 dx_2 dx'_1 dx'_2 d^2b_1 d^2b_2 d^2b \times$$

$$D^{ij}_h(x_1,x_2,b_1,b_2;\mu) D^{kl}_{h'}(x'_1,x'_2,b+b_1,b+b_2;\mu)\sigma^{ik}_A(x_1,x'_1;\mu)\sigma^{jl}_B(x_2,x'_2;\mu)$$

($m = 1$ for $A = B$ or 2 otherwise)

If factorization held:

$$\sigma^{DPS}_{hh'\to AB} = \frac{m}{2}\sum_{i,j,k,l}\int dx_1 dx_2 dx'_1 dx'_2 d^2q_\perp \times$$

$$D^i_h(x_1) D^j_h(x_2) F^{ij}_h(\boldsymbol{q}_\perp) D^k_{h'}(x'_1) D^l_{h'}(x'_2) F^{kl}_{h'}(-\boldsymbol{q}_\perp)\sigma^{ik}_A(x_1,x'_1)\sigma^{jl}_B(x_2,x'_2)$$

$$\simeq \frac{m}{2}\sigma^{SPS}_{hh'\to A}\sigma^{SPS}_{hh'\to B}/\sigma_{\text{eff}}$$

$$\sigma_{\text{eff}} = 1/\int d^2q_\perp F_h(\boldsymbol{q}_\perp) F_{h'}(-\boldsymbol{q}_\perp)$$

"effective cross section", typically of the order of the geometric size $\sim 10 - 20$ mb

Definition

Intuitive probabilistic definition:

Multi-parton distribution = probability distribution that struck partons have LC momentum fractions x_i

Field-theoretic definition of (spin-averaged) sPDF and dPDF [Diehl, Ostermeier, Schaeffer 2012] of a hadron with momentum p:

$$D_j(x) = \int \frac{\mathrm{d}z^-}{2\pi}\, e^{i\,x\,z^- p^+} \langle p|\, \mathcal{O}_j(0,z)\, |p\rangle \Big|_{z^+=0\,,\boldsymbol{z}=\boldsymbol{0}}$$

$$D_{j_1 j_2}(x_1,x_2,\boldsymbol{b}) = 2p^+ \int \mathrm{d}y^- \,\frac{\mathrm{d}z_1^-}{2\pi}\,\frac{\mathrm{d}z_2^-}{2\pi}\; e^{i\,(x_1 z_1^- + x_2 z_2^-)\,p^+} \times \langle p|\, \mathcal{O}_{j_1}(y,z_1)\, \mathcal{O}_{j_2}(0,z_2)\, |p\rangle \Big|_{z_1^+=z_2^+=y^+=0\,,\boldsymbol{z}_1=\boldsymbol{z}_2=\boldsymbol{0}}$$

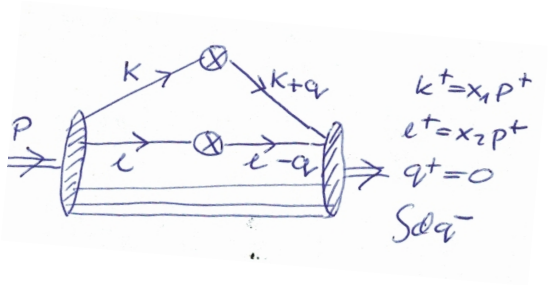
$\mathcal{O}_q(y,z) = \frac{1}{2}\,\bar{q}(y-\frac{z}{2})\gamma^+ q(y+\frac{z}{2}),\ \ldots \quad \text{(LC gauge)}$ $\qquad v^\pm = (v^0 \pm v^3)/\sqrt{2}$

$y = (y^+, y^-, \boldsymbol{b})$, ($\boldsymbol{b}$ is the transverse distance between the two quarks

dPDF in momentum space

Fourier transform in $\boldsymbol{b}$

$$D_{j_1 j_2}(x_1, x_2, \boldsymbol{b}) \to \tilde{D}_{j_1 j_2}(x_1, x_2, \boldsymbol{q})$$



Special case of $\boldsymbol{q} = \boldsymbol{0}$:

$$D_{j_1 j_2}(x_1, x_2) = \tilde{D}_{j_1 j_2}(x_1, x_2, \boldsymbol{q} = \boldsymbol{0})$$

Gaunt-Stirling sum rules

[Gaunt, Stirling, JHEP (2010) 1003:005, Gaunt, PhD Thesis]

Fock-space decomposition on LC + conservation laws $\rightarrow$

$$|P\rangle = \sum_N \int d[x, \boldsymbol{k}]_N \Phi(\{x_i, \boldsymbol{k}_i\}) |\{x_i, \boldsymbol{k}_i\}\rangle_N$$

$$d[x, \boldsymbol{k}]_N = \prod_{i=1}^{N} \left[\frac{dx_i d^2 k_i}{\sqrt{2(2\pi)^3 x_i}} \right] \delta\left(1 - \sum_{i=1}^{N} x_i\right) \delta^{(2)}\left(1 - \sum_{i=1}^{N} \boldsymbol{k}_i\right)$$

Gaunt-Stirling sum rules

[Gaunt, Stirling, JHEP (2010) 1003:005, Gaunt, PhD Thesis]

Fock-space decomposition on LC + conservation laws $\to$

$$\sum_i \int_0^{1-x_2} dx_1\, x_1 D_{ij}(x_1,x_2) = (1-x_2) D_j(x_2) \quad \text{(momentum)}$$

$$\int_0^{1-x_2} dx_1\, D_{i_{\mathrm{val}}j}(x_1,x_2) = (N_{i_{\mathrm{val}}} - \delta_{ij} + \delta_{\bar{i}j}) D_j(x_2) \quad \text{(quark number)}$$

$(A_{i_{\mathrm{val}}} \equiv A_i - A_{\bar{i}})$

$$N_{i_{\mathrm{val}}} = \int_0^1 dx\, D_{i_{\mathrm{val}}}(x)$$

- Preserved by dDGLAP evolution
- Non-trivial to satisfy with the (guessed) function
- Checked in light-front perturbation theory and in lowest-order covariant calculations in [Diehl, Plößl, Schäfer 2019]

Important and fundamental constraints!

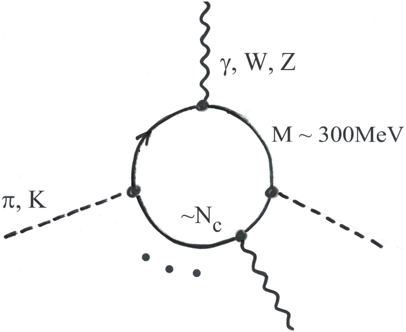
Why the pion

dPDF of the pion

- Much simpler theoretically than the proton
- Possible to obtain results form the lattice 2-current correlators in the pion state, extending the studies of [Zimmermann et al. 2017, 2018], in particular moments in x_1 and x_2 and the transverse form factor
- Nicely illustrates formalism

NJL in high-energy processes

Chiral quark models

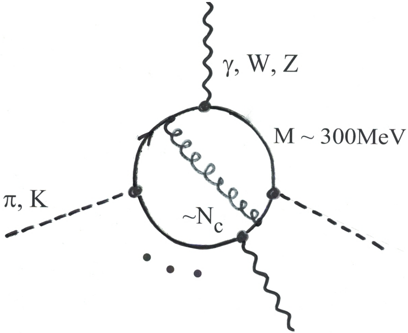


- χSB breaking $\rightarrow$ massive quarks
- Point-like interactions
- Soft matrix elements with pions (and photons, W, Z)
- Large-N_c $\rightarrow$ one-quark loop
- Regularization

pion – Goldstone boson of χSB, fully relativistic $q\bar{q}$ bound state of the Bethe-Salpeter equation

Quantities evaluated at the quark model scale
(where constituent quarks are the only degrees of freedom)

Chiral quark models



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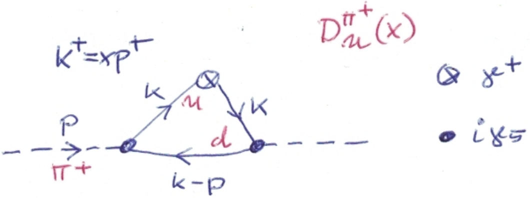
Need for QCD evolution, hard-soft factorization
Gluon dressing, renorm-group improved

sPDF in NJL

[Davidson, Arriola, 1995]



In the chiral limit of $m_\pi = 0$:

$$q_{\rm val}(x; Q_0) = 1 \times \theta[x(1-x)]$$

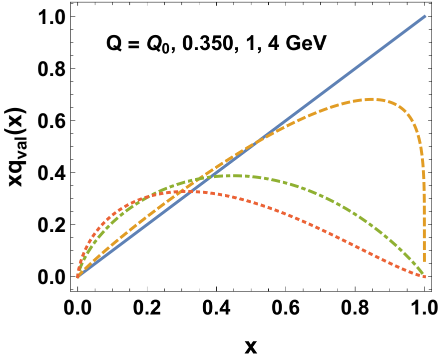
Quarks are the only degrees of freedom, hence saturate the PDF sum rules:
$\int_0^1 dx\, q_{\rm val}(x; Q_0) = 1$ (valence), $2\int_0^1 dx\, x q_{\rm val}(x; Q_0) = 1$ (momentum)

Scale and evolution

QM provide non-perturbative result at a low scale Q_0

$$A(x, Q_0)|_{\text{model}} = A(x, Q_0)|_{\text{QCD}}\,, \quad Q_0 - \text{the matching scale}$$

Quarks carry 100% of momentum at Q_0, adjusted such that when evolved to $Q = 2$ GeV, they carry the experimental value of 47% (radiative generation of gluons and sea quarks)



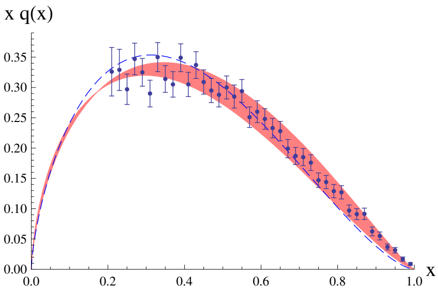
LO DGLAP evolution

$$Q_0 = 313^{+20}_{-10}\ \text{MeV}$$

NLO close to LO

$$\sim (1-x)^{p + \frac{4C_F}{\beta_0} \log \frac{\alpha(Q_0)}{\alpha(Q)}}$$

Pion valence quark sPDF, NJL vs E615



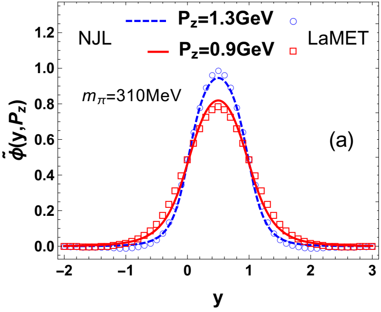
points: Fermilab E615
Drell-Yan, $\pi^{\pm}W \to \mu^{+}\mu^{-}X$

dashed line: 2005 NLO reanalysis [Wijesoorija et al.]

band: QM + LO DGLAP from $Q_0 = 313^{+20}_{-10}$ MeV to $Q = 4$ GeV

Other results for the pion

- GPD [WB, ERA, Golec-Biernat 2008]
- TDA [WB, ERA 2007]
- Equal-time wave functions [WB, ERA, Prelovsek, Šantelj 2009]
- Transversity distributions [WB, ERA, Dorokhov 2009]
- Ji's quasi parton distribution amplitude [WB, ERA, 2018]
- . . .

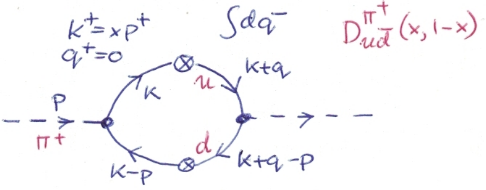


dPDF of the pion in NJL (new stuff)

- WB talk at Light Cone 2019, 16-20 Sep. 2019, Palaiseau, France
- A. Courtoy, S. Noguera, and S. Scopetta, JHEP 12, 045 (2019), arXiv:1909.09530
- WB, ERA, PRD 101 (2020) 014019, arXiv:1910.03707

dPDF of the pion in NJL model

In LC kinematics only one diagram:



In the chiral limit of $m_\pi^2 = 0$ a very simple result follows:

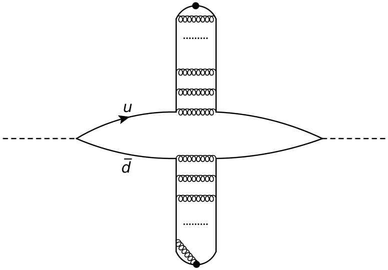
$$D_{u\bar{d}}(x_1, x_2) = \delta(1 - x_1 - x_2)\theta[x_1(1 - x_1)]\theta[x_2(1 - x_2)]F(\boldsymbol{q}_\perp^2)$$

momentum conservation support form factor

- Factorization (in the chiral limit) of the longitudinal and transverse dynamics
- GS sum rules satisfied (preserved by the evolution)
- Results at the quark-model scale → need for evolution

dDGLAP evolution in the Mellin space

[Kirschner 1979, Shelest, Snigirev, Zinovev 1982]: method of solving dDGLAP based on the Mellin moments, similarly to sPDF simplification for valence distributions



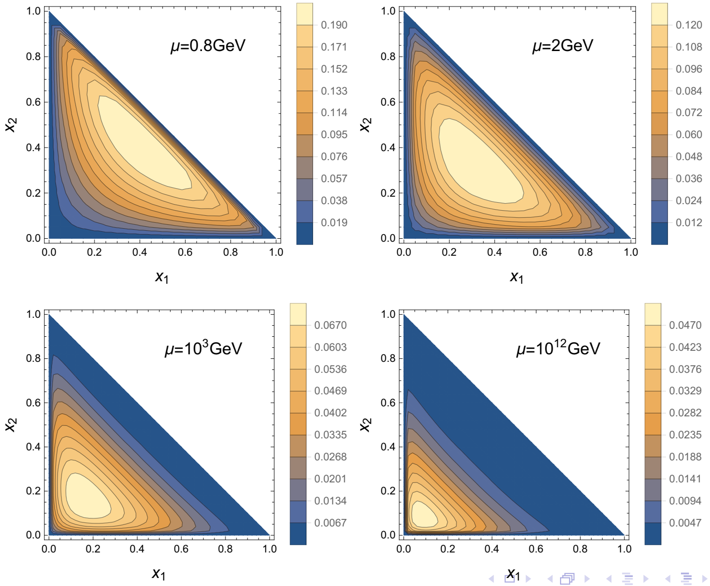
$t = \frac{1}{2\pi\beta}\log\left[1+\alpha_s(\mu)\beta\log(\Lambda_{\rm QCD}/\mu)\right]$ (single scale for simplicity), $\beta = \frac{11N_c - 2N_f}{12\pi}$

Valence:

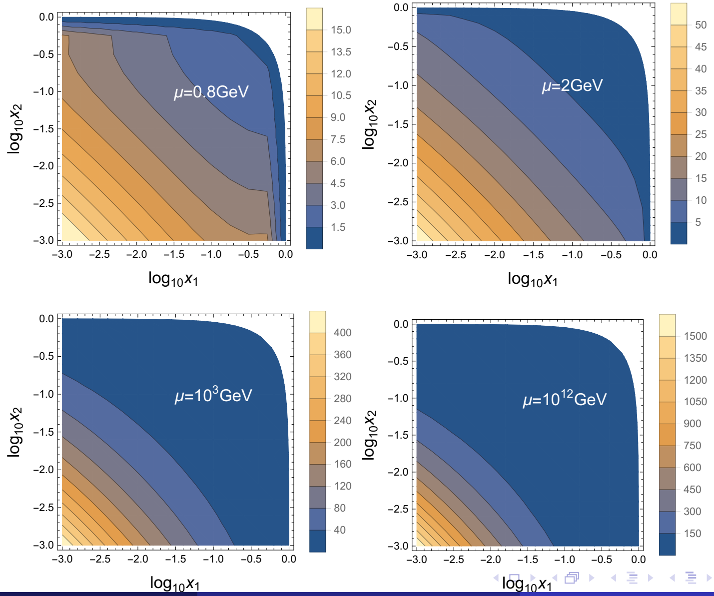
$$\mathrm{dPDF}: \quad \frac{d}{dt} M_{j_1 j_2}^{n_1 n_2}(t) = \left(P_{j_1\to j_1}^{n_1} + P_{j_2\to j_2}^{n_2}\right) M_{j_1,j_2}^{n_1 n_2}(t)$$

$$\mathrm{sPDF}: \quad \frac{d}{dt} M_j^n(t) = P_{j\to j}^n M_j^n(t)$$

$x_1 x_2 D^{\pi^+}_{u\bar{d}}(x_1, x_2)$

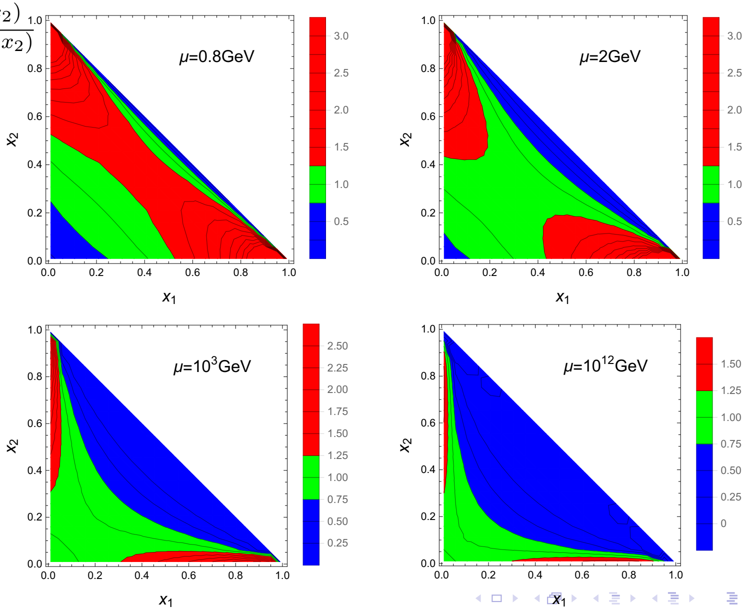


$D^{\pi^+}_{u\bar{d}}(x_1, x_2)$ – log scale



Correlation

$$\frac{D^{\pi^+}_{u\bar{d}}(x_1,x_2)}{D_u(x_1)D_{\bar{d}}(x_2)}$$

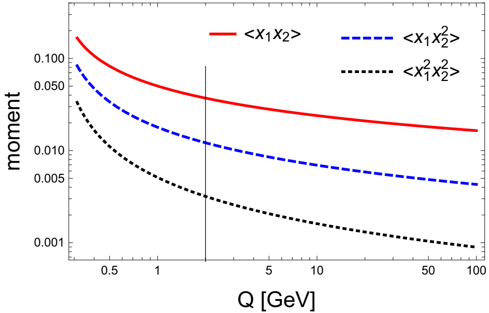


Valence moments in NJL

$$\frac{\langle x_1^n x_2^m \rangle}{\langle x_1^n \rangle \langle x_2^m \rangle} = \frac{(1+n)!(1+m)!}{(1+n+m)!} \quad \text{(NJL, any scale)}$$

(independent of the evolution scale)

	1	2	3	4
1	$\frac{2}{3}$	$\frac{1}{2}$	$\frac{2}{5}$	$\frac{1}{3}$
2	$\frac{1}{2}$	$\frac{3}{10}$	$\frac{1}{5}$	$\frac{1}{7}$
3	$\frac{2}{5}$	$\frac{1}{5}$	$\frac{4}{35}$	$\frac{1}{14}$
4	$\frac{1}{3}$	$\frac{1}{7}$	$\frac{1}{14}$	$\frac{5}{126}$



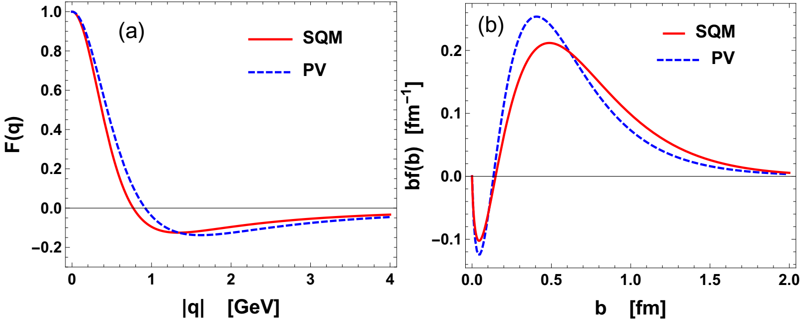
Double moments reduced compared to product of single moments
[lattice results expected, Zimmermann et al. (?)]

Transverse structure

dPDF form factor in Spectral Quark Model (regularization)

- Form factor
$$F(\boldsymbol{q}_\perp) = \frac{m_\rho^4 - \boldsymbol{q}_\perp^2 m_\rho^2}{\left(m_\rho^2 + \boldsymbol{q}_\perp^2\right)^2},$$
- In coordinate space:
$$f(b) = \int \frac{d^2\boldsymbol{q}_\perp}{(2\pi)^2} e^{i\boldsymbol{b}\cdot\boldsymbol{q}_\perp} F(\boldsymbol{q}_\perp) = \frac{m_\rho^2}{2\pi}\left[b m_\rho K_1(b m_\rho) - K_0(b m_\rho)\right]$$
- $\sigma_{\mathrm{DPS}}^{AB} = \frac{m}{2}\sigma_{\mathrm{SPS}}^{A}\sigma_{\mathrm{SPS}}^{B}/\sigma_{\mathrm{eff}}$
- Effective cross section (here coincides with geometric)
$$\sigma_{\mathrm{eff}} = \frac{1}{\int \frac{d^2\boldsymbol{q}_\perp}{(2\pi)^2} F(\boldsymbol{q}_\perp)F(-\boldsymbol{q}_\perp)} = \pi\frac{12}{m_\rho^2} = \pi\langle b^2\rangle_f = 23\ \mathrm{mb}$$

Positivity issues



- Does $F(q)$ have to be positive definite?
- It is probably an artifact of regularization (subtraction). Low-q expansion is fine, high-q is outside of the validity of the model.
- Values for large q or small b could possibly be checked on the lattice (lattice spacing $a \simeq 0.1$)
- Similar results in Courtoy et al.

Summary

- NJL: simplest field theory of the pion in the soft regime; spontaneous chiral symmetry breaking
- Covariant calculations, all symmetries preserved $\rightarrow$ good features
- dPDF in NJL = $\delta(1 - x_1 - x_2) \times F(\boldsymbol{q}_\perp^2)$ + dDGLAP evolution; factorization (in the chiral limit) of the longitudinal and transverse dynamics
- Correlations decrease with increasing evolution scale and are probably not very important ($\pm 25\%$) in the range probed by experiments (which at the moment are concerned with orders of magnitude), justifying the product ansatz in that limit
- Moments measure the $x_1 - x_2$ factorization breaking; can be verified in future lattice calculations
- The effective cross section is geometric