Impact-parameter dependence of the diagonal GPD of the pion from chiral quark models

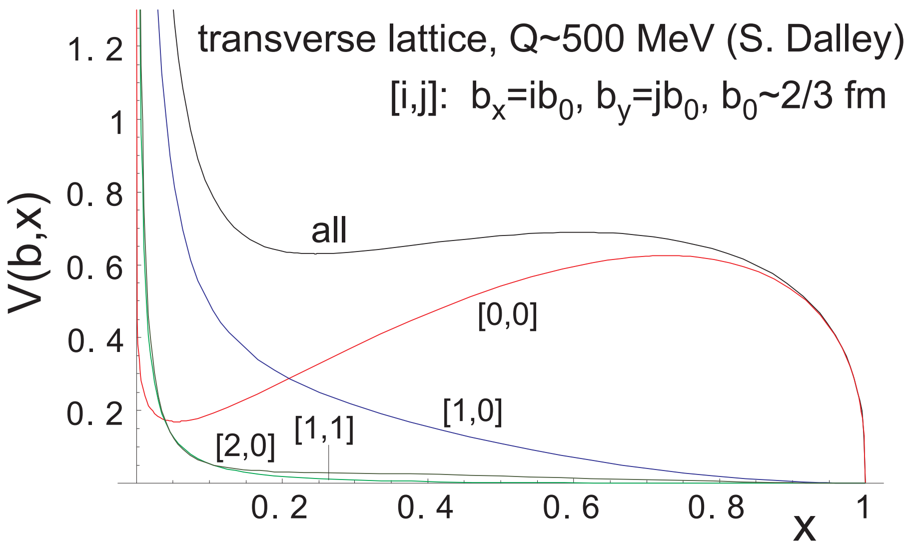
Wojciech Broniowski (INP Cracow) and Enrique Ruiz-Arriola (U. Granada)

Durham, 5-9 August 2003

- ERA+WB, Impact-parameter dependence of the generalized parton distribution of the pion in chiral quark models, hep-ph/0307198,
- ERA+WB, Spectral quark model and low-energy hadron phenomenology, Phys. Rev. **D67** (2003) 074021, hep-ph/0301202

Motivation and outline

- Transverse lattice results



- Evaluation in two chiral quark models $\to$ initial condition at the quark model scale, $Q_0 \sim 320$ MeV

- LO DGLAP evolution from Q_0 to Q $\to$ good description of the data

Definition of the impact-parameter-dependent GPD (bGPD)

The off-forward ($\mathbf{\Delta}_\perp \neq \mathbf{0}$) diagonal ($\xi = 0$) twist-2 GPD of the pion is defined as (for π^+ $H(x) \equiv H_u(x) = H_{\bar{d}}(1-x)$)

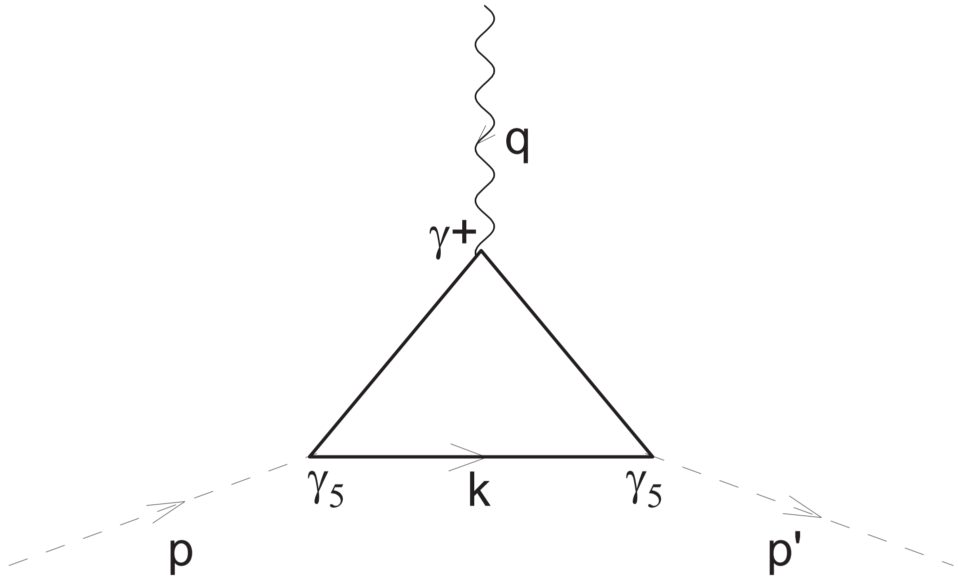
$$
\begin{aligned}
H(x, \xi = 0, -\mathbf{\Delta}_\perp^2) &= \int d^2b \int \frac{dz^-}{4\pi} e^{i(xp^+z^- + \mathbf{\Delta}_\perp \cdot \mathbf{b})} \\
&\times \quad \langle \pi^+(p') | \bar{q}(0, -\frac{z^-}{2}, \mathbf{b}) \gamma^+ q(0, \frac{z^-}{2}, \mathbf{b}) | \pi^+(p) \rangle,
\end{aligned}
$$

In the impact-parameter representation ...Burkardt...

$$
\begin{aligned}
q(b, x) &= \int \frac{d^2\Delta_\perp}{(2\pi)^2} e^{-\mathrm{i}\mathbf{b}\cdot\mathbf{\Delta}_\perp} H(x, 0, -\mathbf{\Delta}_\perp^2) \\
&= \int_0^\infty \frac{\Delta_\perp d\Delta_\perp}{2\pi} J_0(b\Delta_\perp) H(x, 0, -\mathbf{\Delta}_\perp^2).
\end{aligned}
$$

Evaluation in chiral quark models

In chiral quark models the evaluation of H at the leading-N_c (one-loop) level amounts to the calculation of the diagram



where the solid line denotes the quark of mass ω. Covariant calculation.

$$H(x, 0, -\boldsymbol{\Delta}_\perp^2; \omega) = \frac{iN_c\omega^2}{f_\pi^2}\int \frac{d^4k}{(2\pi)^4}\mathrm{Tr}\left[\gamma^+\frac{1}{\not{k}-\not{p}'-\omega}\gamma_5\frac{1}{\not{k}-\omega}\gamma_5\frac{1}{\not{k}-\not{p}-\omega}\right] \times \delta\left[k^+ - (1-x)m_\pi\right],$$

with $f_\pi = 93$ MeV. First $\int dk^-$, then $m_\pi \to 0$.

The light-cone coordinates are defined as

$$k^+ = k^0 + k^3,\ k^- = k^0 - k^3,\ \vec{k}_\perp = (k^1, k^2).$$

The calculation is done in the Breit frame, and with $\Delta^+ = 0$. The Cauchy theorem is applied for the k^- integration, yielding in the chiral limit

$$H(x, 0, -\mathbf{\Delta}_\perp^2; \omega) = \frac{N_c \omega^2}{\pi f_\pi^2} \int \frac{d^2\mathbf{K}_\perp}{(2\pi)^2} \frac{\left[1 + \frac{\mathbf{K}_\perp \cdot \mathbf{\Delta}_\perp (1-x)}{\mathbf{K}_\perp^2 + \omega^2}\right]}{(\mathbf{K}_\perp + (1-x)\mathbf{\Delta}_\perp)^2 + \omega^2},$$

where $\mathbf{K}_\perp = (1-x)\mathbf{p}_\perp - x\mathbf{k}_\perp$. The integral is lo -divergent and we need to specify the regularization:

1. Spectral Quark Model [SQM] (E. Ruiz Arriola's talk). Successful in describing both the low- and high-energy phenomenology of the pion (complies to the chiral symmetry, anomalies, pure twist expansion, quark propagator with no poles!).

2. Nambu–Jona-Lasinio [NJL] model with the Pauli-Villars regulator. Vento's talk

Spectral Quark Model

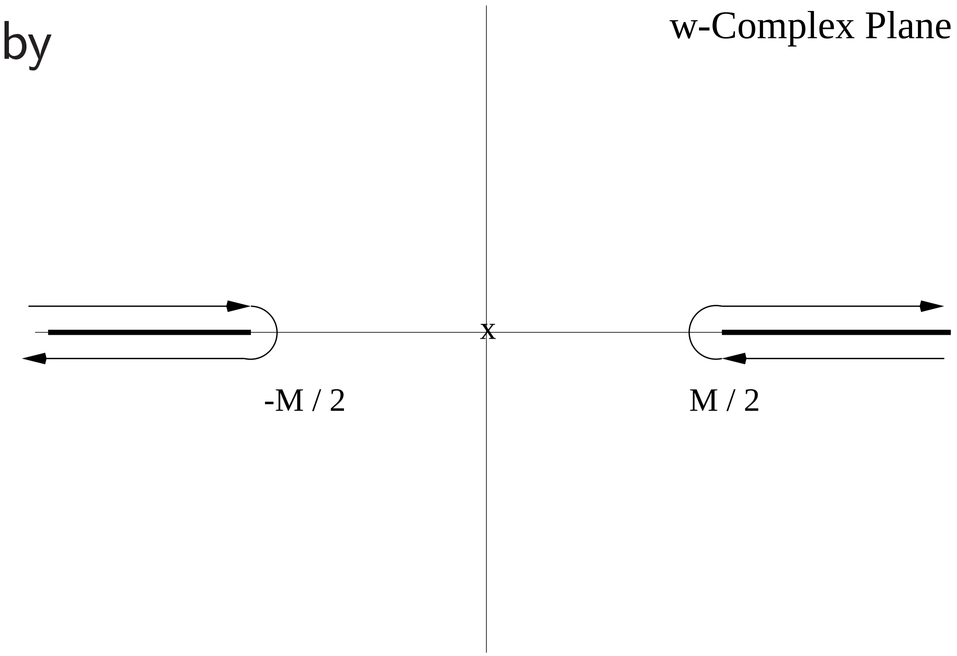
SQM amounts to supplying the quark loop with an integral over the quark mass ω weighted by a quark spectral density $\rho(\omega)$,

$$H_{\rm SQM}(x,0,-\boldsymbol{\Delta}_\perp^2) = \int_C d\omega\, \rho_V(\omega) H(x,0,-\boldsymbol{\Delta}_\perp^2;\omega),$$

where

$$\rho_V(\omega) = \frac{1}{2\pi i}\frac{3\pi^2 m_\rho^3 f_\pi^2}{4N_c}\frac{1}{\omega}\frac{1}{(m_\rho^2/4-\omega^2)^{5/2}},$$

and C is given by



Then

$$H_{\rm SQM}(x,0,-\boldsymbol{\Delta}_\perp^2) = \frac{m_\rho^2(m_\rho^2-(1-x)^2\boldsymbol{\Delta}_\perp^2)}{(m_\rho^2+(1-x)^2\boldsymbol{\Delta}_\perp^2)^2} \quad \text{for} \;\; 0<x<1$$

We check that

$$m_\rho^2 = \frac{24\pi^2 f_\pi^2}{N_c}$$

$$F(t) = \int_0^1 dx H_{\rm SQM}(x,0,t) = m_\rho^2/(m_\rho^2+t),$$

which is the built-in vector-meson dominance principle. Clearly, $F(0)=1$, correct norm and $H_{\rm SQM}(x,0,0)=\theta(x)\theta(1-x)$ [Davidson-Arriola, 1995]. We pass to & support the impact-parameter space by the Fourier-Bessel transformation and get

$$q_{\rm SQM}(b,x) = \frac{m_\rho^2}{2\pi(1-x)^2}\left[K_0\left(\frac{bm_\rho}{1-x}\right) - \frac{bm_\rho}{1-x}K_1\left(\frac{bm_\rho}{1-x}\right)\right].$$

Nambu–Jona-Lasinio Model

In the NJL model with the Pauli-Villars regularization we get

$$
\begin{aligned}
H_{\rm NJL}(x,0,-\boldsymbol{\Delta}_\perp^2) &= 1+\frac{N_c M^2(1-x)|\boldsymbol{\Delta}_\perp|}{4\pi^2 f_\pi^2 s_i}\sum_i c_i \,\mathrm{lo}\ \left(\frac{s_i+(1-x)|\boldsymbol{\Delta}_\perp|}{s_i-(1-x)|\boldsymbol{\Delta}_\perp|}\right), \\
s_i &= \sqrt{(1-x)^2\boldsymbol{\Delta}_\perp^2+4M^2+4\Lambda_i^2},
\end{aligned}
$$

where M is the constituent quark mass, Λ_i are the PV regulators, and c_i are suitable constants. For the twice-subtracted case, explored below, one has, for any regulated function F, the operational definition

correct norm & support

$$
\sum_i c_i F(\Lambda_i^2) = F(0) - F(\Lambda^2) + \Lambda^2 dF(\Lambda^2)/d\Lambda^2.
$$

In what follows we use $M = 280$ MeV and $\Lambda = 871$ MeV, which yields $f_\pi = 93$ MeV.

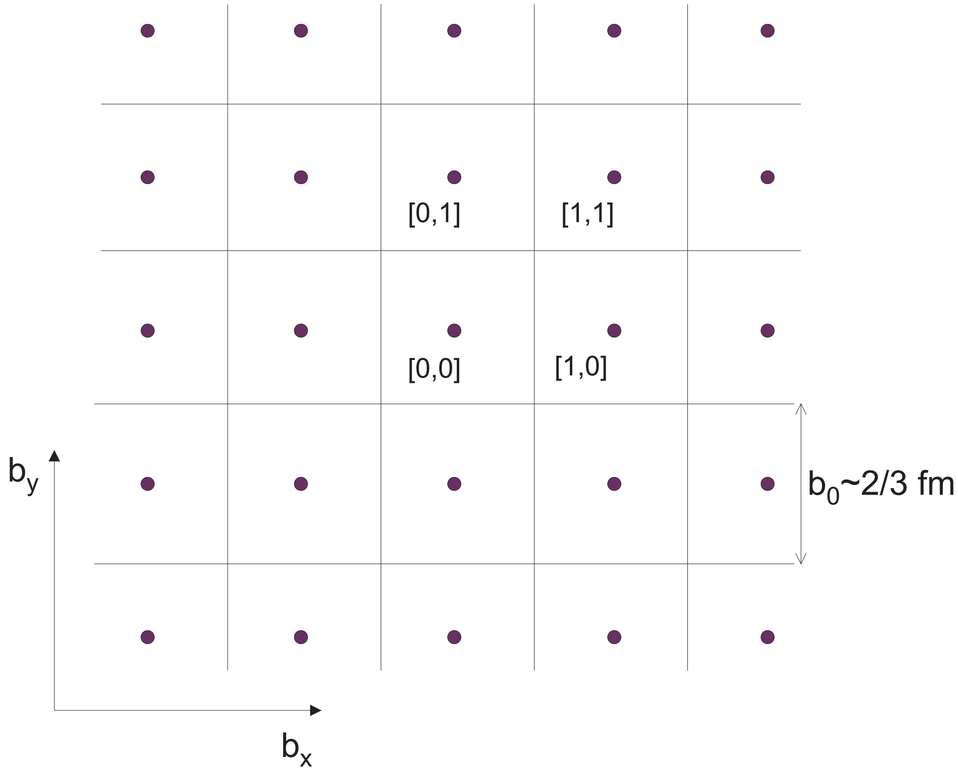
Scalin in $b/(1-x)$

Generally, the chiral quark model (one-loop) results depend on $\mathbf{\Delta}_\perp$ and x only through the combination $(1-x)^2\mathbf{\Delta}_\perp^2$. Consequently, in the b space they depend on the combination $b^2/(1-x)^2$. Due to this property

$$\frac{\int d^2b\, b^{2n} q(b,x)}{\int d^2b\, q(b,x)} \equiv \langle b^{2n}\rangle(x) = (1-x)^{2n}\langle b^{2n}\rangle(0).$$

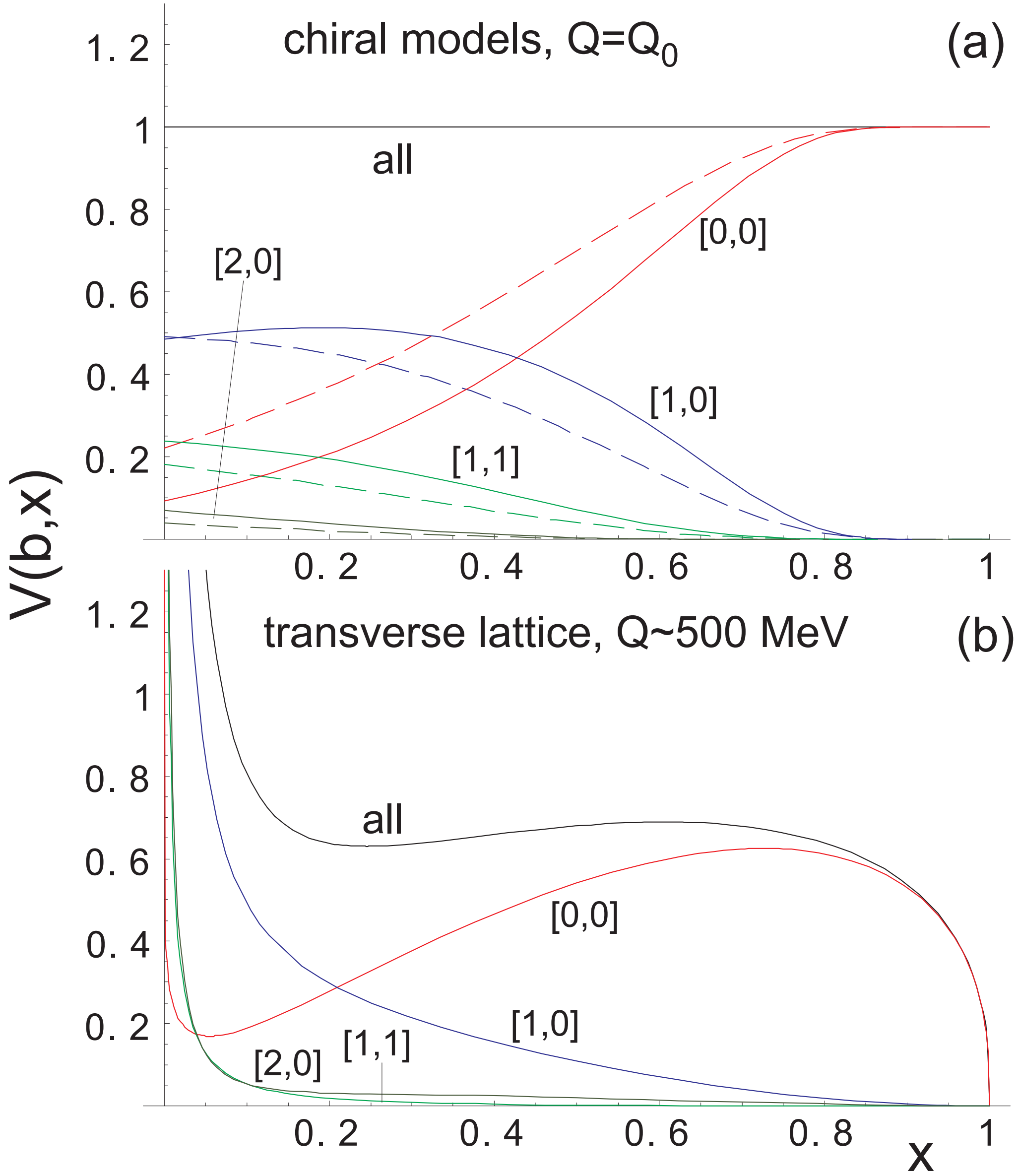
This means, that all the moments except for $n=0$ vanish as $x \to 1$, or $q(b,x)$ becomes a $\delta(b)$ function in this limit. This behavior is seen in the transverse lattice data.

Smearin over b



$$V(x,[i,j]) \equiv \int_{(i-1/2)b_0}^{(i+1/2)b_0} db_x \int_{(j-1/2)b_0}^{(j+1/2)b_0} db_y V(x, \sqrt{b_x^2+b_y^2}).$$

The degeneracy factor for plaquettes equidistant from the origin is included, *i.e.* the $[1,0]$, $[1,1]$, and $[2,0]$ plaquettes are multiplied by 4, $[2,1]$ by 8, *etc.*



(a) SQM (solid) and NJL (dashed) at $Q = Q_0 = 313$ MeV. (b) Transverse lattice [Dalley 2003]. The initial condition of (a) needs to be evolved!

QCD evolution and the quark-model scale, Q_0

The models have produced GPD corresponding to a low, a priori unknown quark model scale, Q_0. A way to estimate it is to run the QCD evolution starting from various Q_0's up to a scale Q where data can be used. Alternatively, one may use the known momentum fraction carried by the quarks at a scale Q and the downward evolution

Arriola's talk

We use the LO DGLAP evolution with $\alpha(Q) = \left(\frac{4\pi}{\beta_0}\right) \frac{1}{\log(Q^2/\Lambda^2_{\rm QCD})}$, where $\beta_0 = 11 - 2N_F/3$, $N_F = 3$, and $\Lambda_{\rm QCD} = 226 \text{ MeV}$, which gives

$$Q_0 = 313^{+20}_{-10} \text{ MeV}.$$

Rather low! One can hope that the typical expansion parameter $\alpha(Q_0)/(2\pi) \sim 0.34 \pm 0.04$ makes the perturbation theory meaningful. NLO supports this assumption [Davidson-Arriola, 2002]. Similar estimate for Q_0 obtained form an analysis of pion DA [Arriola-WB,2002].

QCD evolution of $V(b,x)$

We apply the LO DGLAP evolution with the kernel independent of $\mathbf{\Delta}_\perp$, or b – works well for small $\mathbf{\Delta}_\perp$, or large b. Then, in the index space,

$$V_n(Q,b) \equiv \int_0^1 dx\, x^n V(x,Q,b) = \left(\frac{\alpha(Q)}{\alpha(Q_0)}\right)^{\gamma_n^{\rm NS}/(2\beta_0)} V_n(x,Q_0,b),$$

where $\gamma_n^{\rm NS} = -8/3\left[3 + \frac{2}{(n+1)(n+2)} - 4\sum_{k=1}^{n+1}\frac{1}{k}\right]$. This equation can be inverted via the inverse Mellin transform

numerics fast & stable

$$V(x,Q,b) = \int_{-i\infty}^{+i\infty} \frac{{\rm d}n}{2\pi i} x^{-n-1} V_n(Q,b).$$

We apply the evolution to the smeared functions,

$$V(x,Q,[i,j]) = \int_{-i\infty}^{+i\infty} \frac{{\rm d}n}{2\pi i} x^{-n-1} \left(\frac{\alpha(Q)}{\alpha(Q_0)}\right)^{\gamma_n^{\rm NS}/(2\beta_0)} \int_0^1 dx'\, x'^n V(x',Q_0,[i,j]),$$

Behavior at $x \to 1$

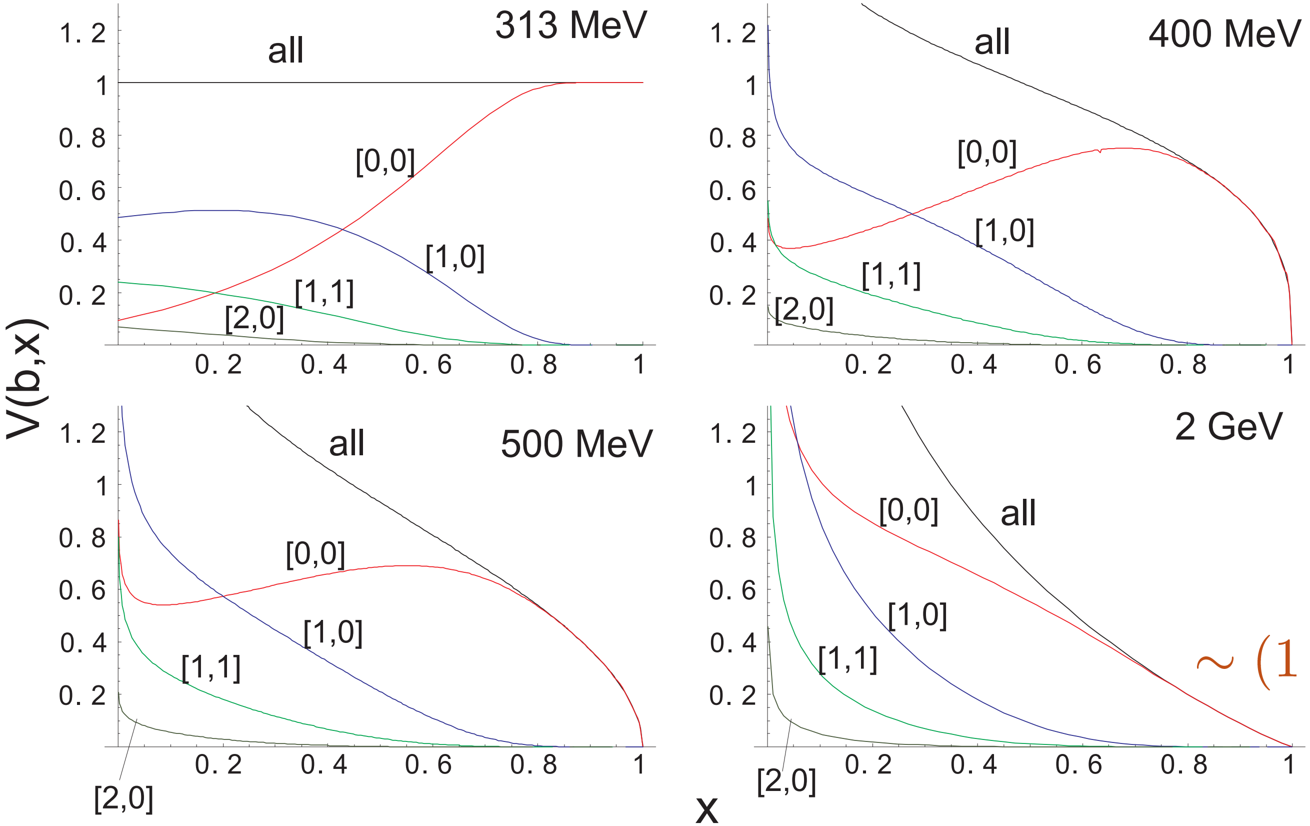
A function that initially behaves as $V(x, Q_0, b) \to C(b)(1-x)^p$ evolves into

$$V(x, Q, b) \to C(b)(1-x)^{p - \frac{4C_F}{\beta_0} \log \frac{\alpha(Q)}{\alpha(Q_0)}}, \qquad x \to 1.$$

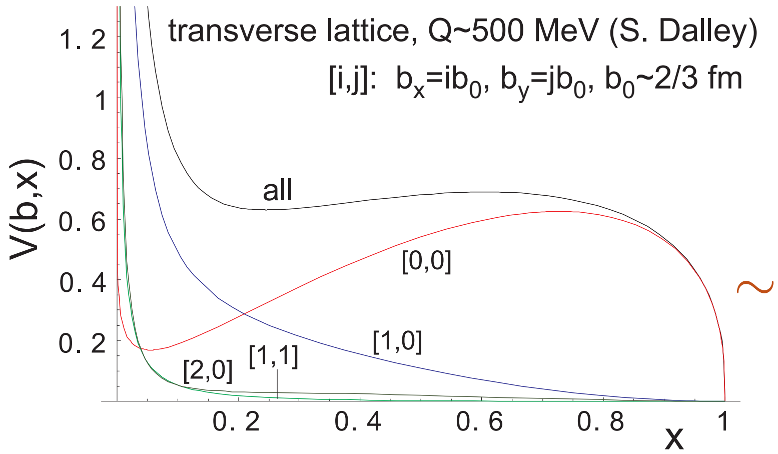
(feature of DGLAP)

No end-point problem!

SQM, b_0=2/3 fm



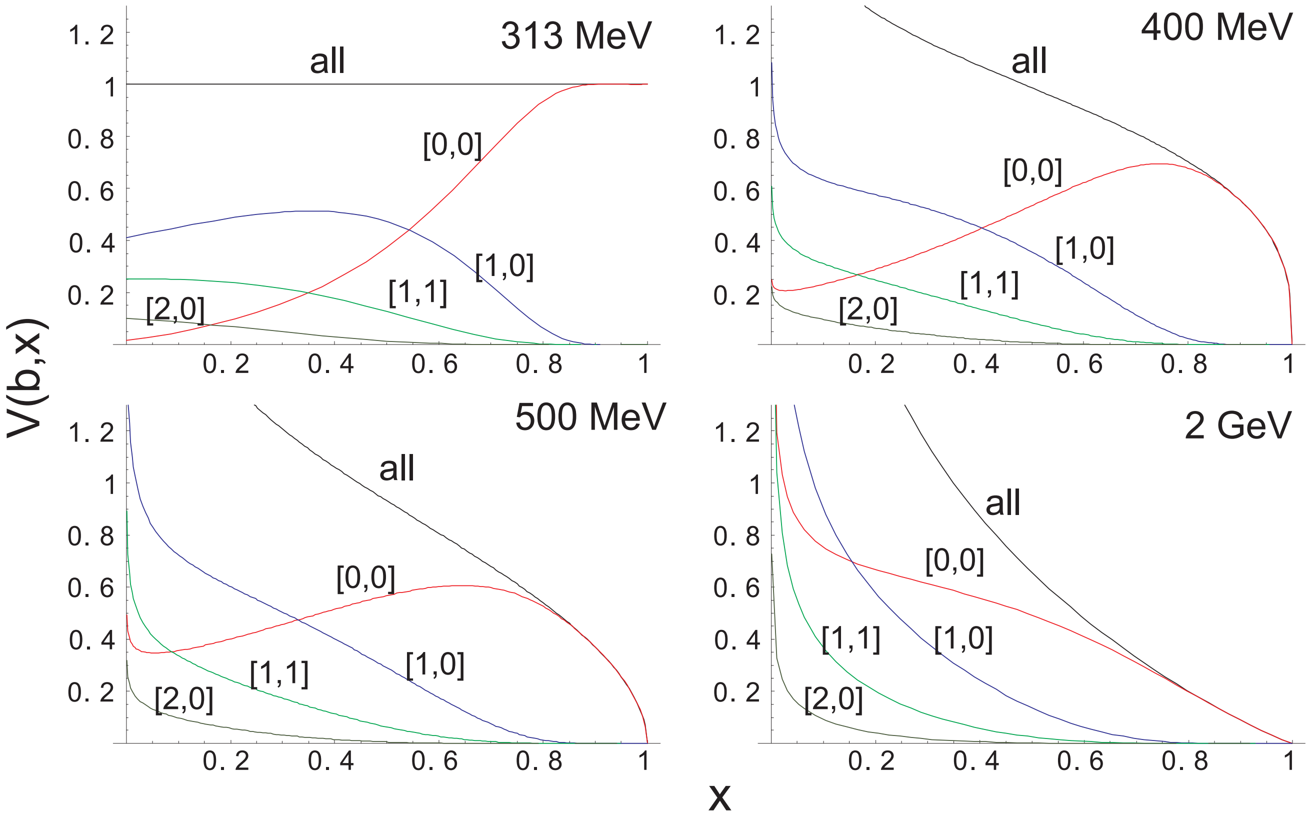
$\sim (1-x)^{1.1}$

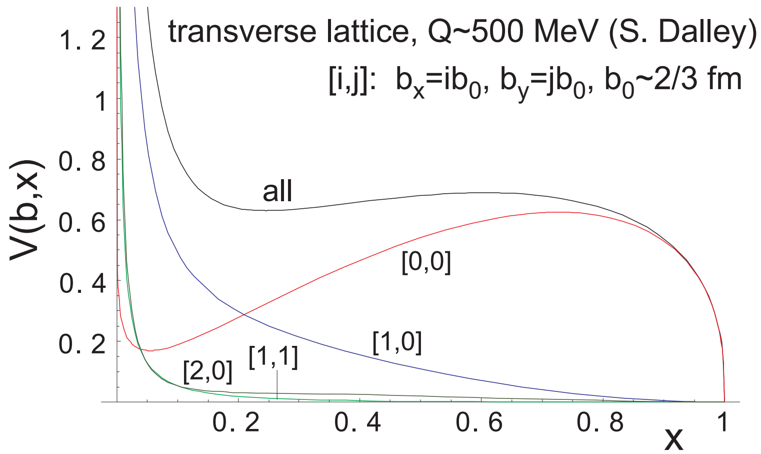


$\sim (1-x)^{0.37}$

qualitative agreement

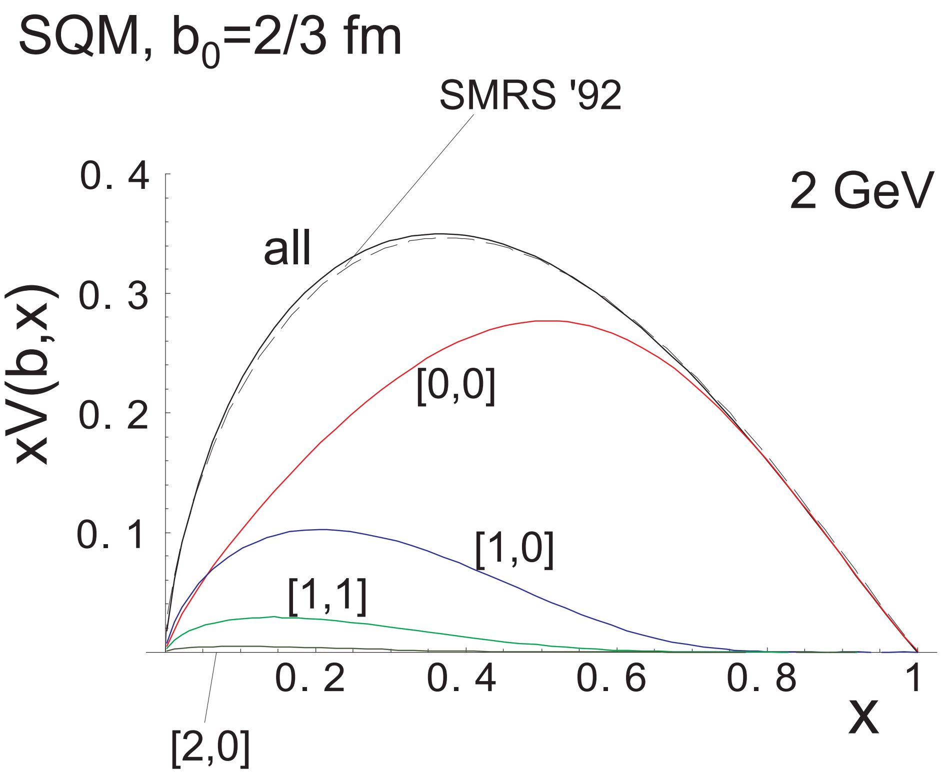
SQM, b_0=0.8 * 2/3 fm





better agreement

Comparison of the forward distribution to SMRS'92



$\sim (1-x)^{1.1}$

Chiral quark models + DGLAP → good description of the SMRS parameterization of the data! Agreement with the Durham theory and experiment !

Conclusion

Non-perturbative chiral quark models + LO DGLAP evolution $\rightarrow$ quite remarkable agreement of bGPD with the data from the transverse lattices.
The approach is in harmony with the chiral symmetry.