Fluctuation of the initial condition from Glauber models*

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*based on WB+**P. Bożek**+**M. Rybczyński**, 0706.4266 (nucl-th)

Outline

Goals/results:

- analysis of shape fluctuations in variants of Glauber models
- understand the statistics (*e.g.*, $\Delta\varepsilon^*/\varepsilon^*(b=0) = \sqrt{\frac{4}{\pi} - 1} \simeq 0.52$)
- $\Delta v_2^*/v_2^* \simeq 0.5$ for central and peripheral collisions (* ="particle")

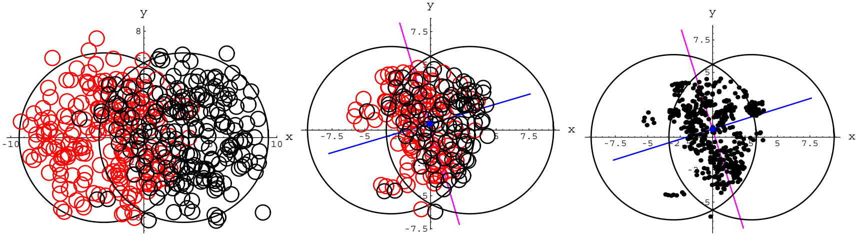
1. Fluctuations of the initial condition
   - Glauber-like models
   - Understanding the statistics
   - Monte Carlo simulations
2. Jet quenching
3. v_2, hydro, higher multipoles, *etc.*

Relations from smooth hydro:

$$\Delta v_4^*/v_4^* = 2\Delta v_2^*/v_2^*$$

$$\text{azHBT}: \; R_4 \sim R_2^2$$

A typical gold-gold event



all nucleons — wounded nucleons — binary collisions

Sizeable fluctuations of the center of mass and the quadrupole axes

Aguiar+Kodama+Osada+Hama 2001, Miller+Snellings 2003, Bhalerao+Blaizot+Borghini+Ollitrault 2005, Andrade+Grassi+Hama+Kodama+Socolowski 2006, Voloshin 2006, ...

Glauber-like models tested

- wounded nucleons, $\sigma_w = 42$ mb, $d = 0.4$ fm

one goal:

compare various Glauber-like models

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- hot spots: $\sigma_w = 42$ mb, $\sigma_{\rm bin} = 0.5$ mb. When a rare binary collision occurs it produces on the average a large amount of the transverse energy $= 14.5\% \times \sigma_{\rm w}/\sigma_{\rm bin}$
- hot spots + Γ: Sources may deposit the transverse energy with a certain probability distribution. We superimpose the Γ distribution with $\kappa = 0.5$ over the distribution of sources,

$$g(w,\kappa) = w^{\kappa-1}\kappa^{\kappa}\exp(-\kappa w)/\Gamma(\kappa),$$

where $\bar{w} = 1$ and ${\rm var}(w) = 1/\kappa$

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Notation

fixed-axes = standard

$$f(\rho,\phi) = f_0(\rho) + 2f_2(\rho)\cos(2\phi) + 2f_4(\rho)\cos(4\phi) + \dots$$

$$\varepsilon = \frac{\int \rho\, d\rho \rho^2 f_2(\rho)}{\int \rho\, d\rho \rho^2 f_0(\rho)}$$

variable-axes = participant=*

$$f^*(\rho,\phi) = f_0(\rho) + 2f_2^*(\rho)\cos(2\phi) + 2f_4^*(\rho)\cos(4\phi) + \dots$$

$$\varepsilon^* = \frac{\int \rho\, d\rho \rho^2 f_2^*(\rho)}{\int \rho\, d\rho \rho^2 f_0(\rho)}$$

(coordinates shifted to center-of-mass)

Toy model

just 2 ρ-independent terms:

$$f(\phi) = 1 + 2\epsilon\cos(2\phi)$$

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variable axes (participant)

$\varepsilon^* = \langle\langle \frac{1}{n} \sum_{k=1}^{n} \cos[2(\phi_k - \phi^*)] \rangle\rangle$

$$Y = \frac{1}{n} \sum_{k=1}^{n} \cos(2\phi_k), \quad X = \frac{1}{n} \sum_{k=1}^{n} \sin(2\phi_k)$$

ϕ^* : Y maximized in each event

$\Rightarrow \qquad \cos(2\phi^*) = Y/\sqrt{Y^2 + X^2}, \qquad \sin(2\phi^*) = X/\sqrt{Y^2 + X^2}$

$$\varepsilon^* = \langle\langle \sqrt{\left(\frac{1}{n} \sum_{k=1}^{n} \cos(2\phi_k)\right)^2 + \left(\frac{1}{n} \sum_{k=1}^{n} \sin(2\phi_k)\right)^2} \rangle\rangle$$

Central limit theorem

For large n the distribution of Y and X is Gaussian:

$$f(Y,X) = \frac{n}{\pi\sqrt{1-2\epsilon^2}} \exp\left[-n\left(\frac{(Y-\epsilon)^2}{1-2\epsilon^2} + X^2\right)\right]$$

Let $Y = q\cos\alpha,\ X = q\sin\alpha$.
We need the integral of $f(Y,X) = f(q,\alpha)$ over α:

$$\begin{aligned}\int_0^{2\pi} d\alpha f(q,\alpha) &= \frac{2n}{\sqrt{\pi}\sqrt{1-2\epsilon^2}} \exp\left[-n\left(\frac{q^2+\epsilon^2}{1-2\epsilon^2}\right)\right] \\ &\times \sum_{j=0}^{\infty} (2q\epsilon)^j \frac{\Gamma(j+\frac{1}{2})}{j!} I_j\left(\frac{2n\epsilon q}{1-2\epsilon^2}\right)\end{aligned}$$

ε^* in the toy model

$$\varepsilon^* = \int q\, dq\, d\alpha\, q f(q,\alpha) =$$

$$\frac{1-2\epsilon^2}{\sqrt{n\pi}} \sum_{j=0}^{\infty} (2\epsilon^2)^j \frac{\Gamma(j+\frac{1}{2})\Gamma(j+\frac{3}{2})}{j!^2} {}_1F_1\left(-\frac{1}{2}, j+1; -\frac{n\epsilon^2}{1-2\epsilon^2}\right)$$

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$$\langle \varepsilon^{*2} \rangle = \langle\langle Y^2 + X^2 \rangle\rangle = \int q\, dq\, d\alpha\, q^2 f(q,\alpha) = \frac{1+(n-1)\epsilon^2}{n}$$

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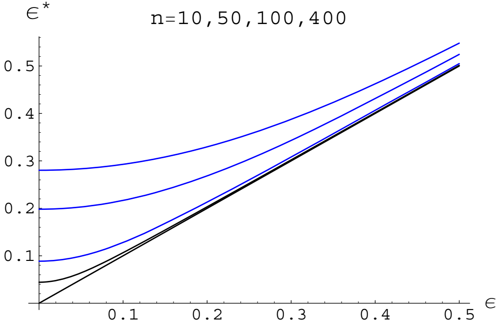
"centrality"=0

$$\varepsilon^*(\epsilon=0) = \frac{\sqrt{\pi}}{2\sqrt{n}}, \quad \frac{\Delta\varepsilon^*}{\varepsilon^*} = \sqrt{\frac{4}{\pi}-1} \simeq 0.52$$

ε^* as a function of ε (toy model)



ε^* in the general case

(under the assumption of no correlations of locations of sources)

$$\varepsilon^* = \frac{\sqrt{2}\sigma_Y^2}{I_{k,0}\sqrt{\pi}\sigma_X}\sum_{j=0}^{\infty}(2\delta\sigma_Y^2)^j\frac{\Gamma\left(j+\frac{1}{2}\right)\Gamma\left(j+\frac{3}{2}\right)\ {}_1F_1\left(-\frac{1}{2};j+1;-\frac{\bar{Y}^2}{2\sigma_Y^2}\right)}{j!^2}$$

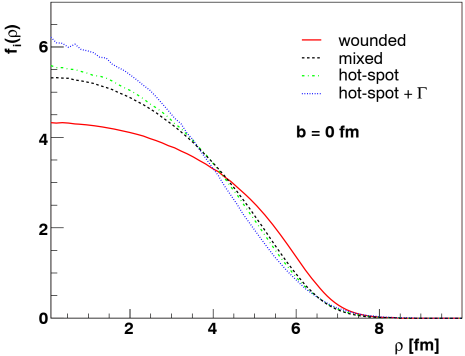
$$\bar{Y} = I_{2,2},\ \sigma_Y^2 = \frac{1}{2n}(I_{4,0}-2I_{2,2}^2+I_{4,4}),\ \sigma_X^2 = \frac{1}{2n}(I_{4,0}-I_{4,4}),$$

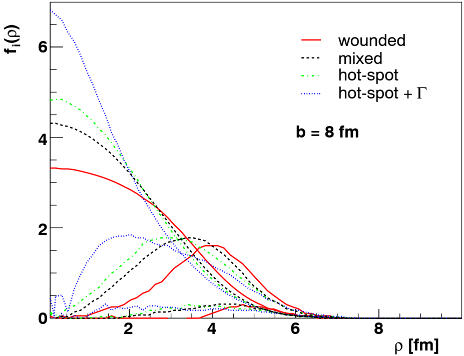
$$\delta = \frac{1}{2\sigma_Y^2}-\frac{1}{2\sigma_X^2},\quad I_{k,l} = \int \rho d\rho f_l(\rho)\rho^k \Big/ \int \rho d\rho f_0(\rho)$$

at $b=0$ very simple results

$$\varepsilon^* = \frac{\sqrt{\pi I_{4,0}}}{2I_{2,0}\sqrt{n}},\quad \frac{\Delta\varepsilon^*}{\varepsilon^*} = \sqrt{\frac{4}{\pi}-1},\quad f_2^*(\rho) = \frac{1}{2}\sqrt{\frac{\pi}{nI_{2k,0}}}\,\rho^k f_0(\rho)$$

Fixed-axes (standard) profiles

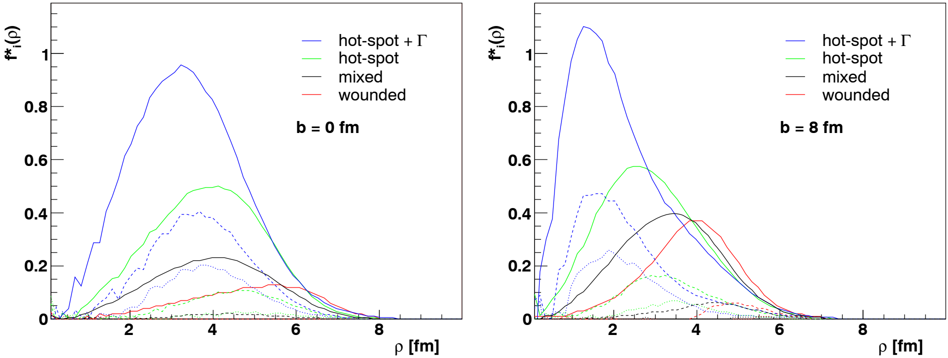




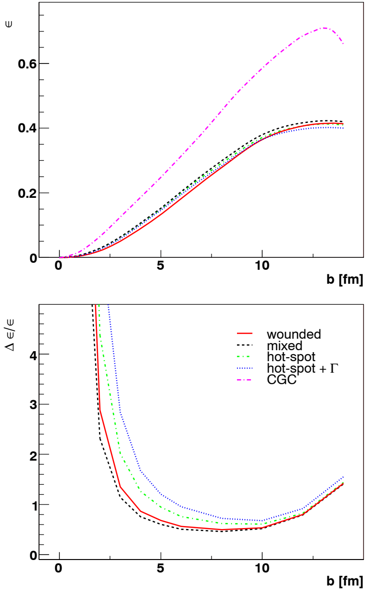
left: $f_0(\rho)$ right: $f_0(\rho)$, $f_2(\rho)$, $f_4(\rho)$

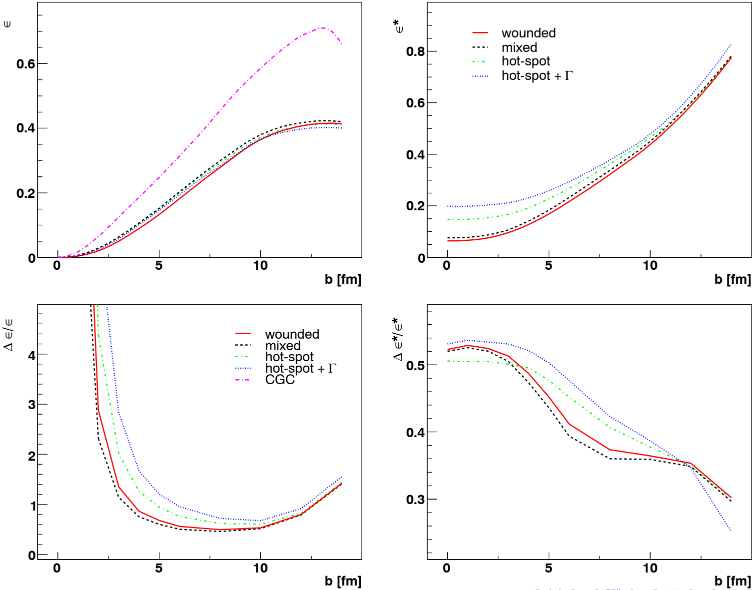
hot-spot + Γ sharpest
keeping multipoles up to $l = 4$ is sufficient

Variable-axes (participant) profiles



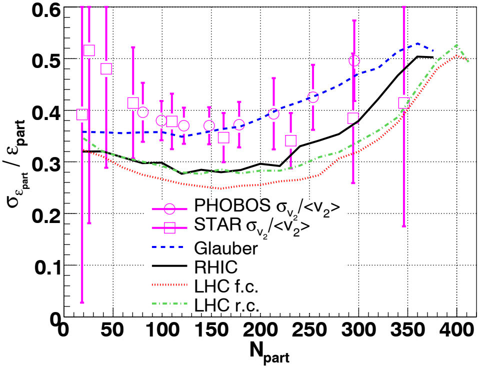
solid: $f_2(\rho)$, dash: $f_4(\rho)$, dots: $f_6(\rho)$


ϵ
0.6
0.4
0.2
0
0
5
10
b [fm]
Δ ϵ/ϵ
4
3
2
1
0
0
5
10
b [fm]
wounded
mixed
hot-spot
hot-spot + Γ
CGC


ϵ
0.6
0.4
0.2
0
0
5
10
b [fm]
ϵ*
wounded
mixed
hot-spot
hot-spot + Γ
0.8
0.6
0.4
0.2
0
0
5
10
b [fm]
Δ ϵ/ϵ
wounded
mixed
hot-spot
hot-spot + Γ
CGC
4
3
2
1
0
0
5
10
b [fm]
Δ ϵ*/ϵ*
0.5
0.4
0.3
0
5
10
b [fm]

Comparison to CGC

[Drescher+Nara, arXiv:0707.0249]



CGC with k_T-factorization lower than Glauber

Simple model of jet quenching

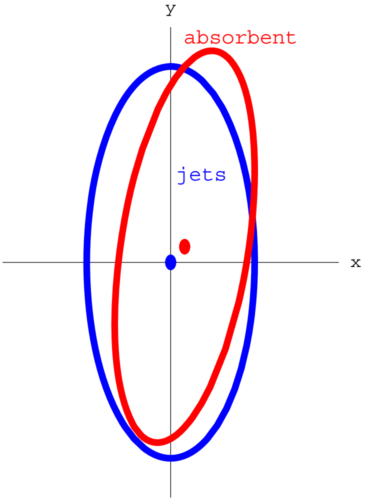
Simple model [Drees+Feng+Jia 2003, Horowitz 2005]: partons are produced in p-p collisions with the power-law spectrum $dN/dp_T^2 \propto 1/p_T^{8.1}$, the energy loss is

$$\Delta E = kE \int_0^\infty l dl \frac{\tau_0}{l+\tau_0} f^*[x_0 + v_x(l+\tau_0), y_0 + v_y(l+\tau_0)],$$

$(x_0,\ y_0)$ - jet production point generated from a binary collision in the fixed-axes frame, τ_0 - initial time, l - time measured from τ_0, $(v_x,\ v_y)$ - randomly generted transverse velocity, k - parameter fitted to reproduce the high-p_T value of R_{AA}

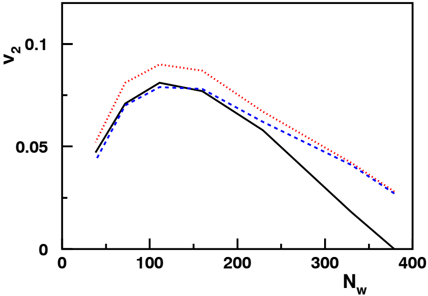
[Drescher 2006]: raise in the geometric eccentricity increases the asymmetry of the energy loss. However, the absorbing medium formed in each event is rotated and shifted!

Geometry for jet absorption asymmetry



- (rare) jets from fixed-axes geometry, no correlation of location to the absorbing medium
- fluctuating absobing medium from variable-axes geometry, higher asymmetry
- shift of center of mass
- rotation

v_2 at high p_T



solid - fixed-axes density of the wounded nucleons

dash - variable-axes density in the hot-spot scenario (our full result)

dots - as dash but without the shift and rotation of the opaque

Result of simulation: rotation and shift practically remove the "sharpening effect" of fluctuations, except for very low b

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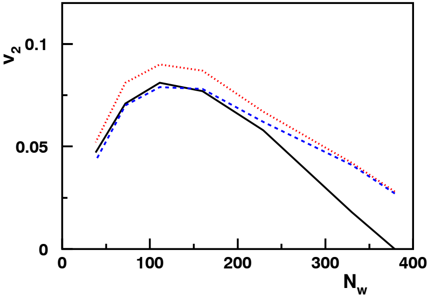

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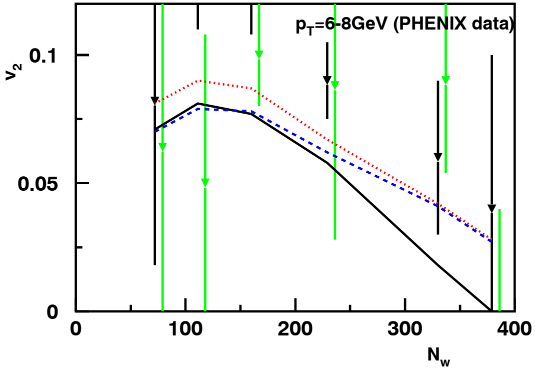
At LHC the analysis of jet tomography with respect the the event-by-event reconstructed plane must take into account the relative shift and rotation effects



Fluctuations do not help much with the jet v_2 problem

black points - $p_T = 6 - 7$ GeV, green - $p_T = 7 - 8$ GeV
blue dashed - the full model result
other Glauber models - similar, theoretical curves lower with hydro

Event-by-event fluctuations of v_2

At low azimuthal asymmetry one expects on hydrodynamical grounds

$$\frac{\Delta v_2^*}{v_2^*} = \frac{\Delta \varepsilon^*}{\varepsilon^*}$$

(see, *e.g.*, discussion in [Vogel+Torrieri+Bleicher 2007])

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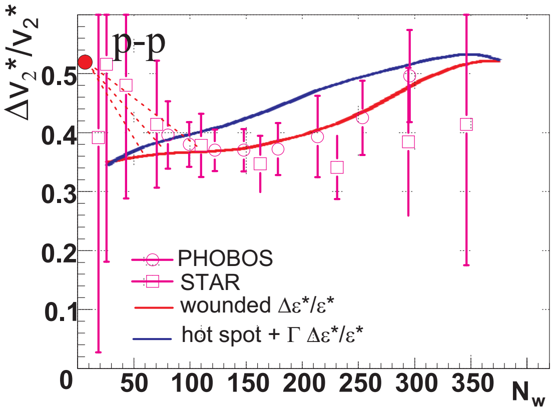
$$\frac{\Delta v_2^*}{v_2^*}(b=0) \simeq \sqrt{\frac{4}{\pi} - 1} \simeq 0.52$$

For peripheral collisions: several $p-p$, where also $\Delta v_2^*/v_2^* \simeq 0.5$
At intermediate b lower values
Agreement with PHOBOS and STAR statistics!

Simulations:

some sensitivity to the chosen Glauber-like model at intermediate centralities



endpoint values from statistics!

hydro: $\Delta\varepsilon^*/\varepsilon^* \simeq \Delta v_2^*/v_2^*$(Au+Au, central)$\simeq 0.5$

coordinates $\rightarrow p_T$: $\Delta v_2^*/v_2^*(\text{several p+p}) \simeq 0.5 =$
$\Delta v_2^*/v_2^*$(Au+Au, periph.)$\simeq 0.5$

Perturbation theory in azimuthal asymmetry

Schematically, hydro equations are $L(\psi) = 0$, where L - operator for hydrodynamics, ψ - set of hydrodynamical functions of space-time describing the state. For *smooth* evolution and small asymmetry one may expand around the azimuthally-symmetric solution ψ_0:

$$L(\psi) = L(\psi_0 + \delta\psi) \simeq L(\psi_0) + L'(\psi_0)\delta\psi$$

Since $L(\psi_0) = 0$, we have to first order

$$L'(\psi_0)\delta\psi = 0$$

Linearity $\Rightarrow ||\delta\psi(t)|| \sim ||\delta\psi(t_0)||$ for all hydrodynamic properties, in particular the shape and flow

Linearity:

$$v_2^*(t) \sim \varepsilon^*(t_0)$$

Perturbation to second order

Strong suppression of subsequent multipoles suggests the hierarchy

$$\psi = \psi_0 + \lambda\delta\psi_2 + \lambda^2\delta\psi_4 + \ldots,$$

Expansion to second order in $\lambda \sim$ a few % yields

$$L(\psi) = L(\psi_0) + \lambda L'(\psi_0)\delta\psi_2 + \lambda^2\left[L'(\psi_0)\delta\psi_4 + L''(\psi_0)(\delta\psi_2)^2/2\right]$$

The linear inhomogeneous equation for the octupole deformation:

$$L'(\psi_0)\delta\psi_4 = -\frac{1}{2}L''(\psi_0)(\delta\psi_2)^2$$

Let τ_2 and τ_4 denote the characteristic times for the operators $L'(\psi_0)$ and $L''(\psi_0)$, respectively. If $\tau_2 \gg \tau_4$ then for $t \gg t_0$

$$||\psi_4(t)|| \sim ||\delta\psi_2(t)||^2 \sim ||\delta\psi_2(t_0)||^2$$

Octupole flow:

$$v_4^*(t) \sim \varepsilon^{*2} \sim v_2^{*2}(t)$$

Simulations [Kolb 2003, Borghini+Ollitrault 2005]: v_2 saturates with time, v_4 quickly assumes the value proportional to v_2^2
Data of [Bai 2007] comply to the result, except at very low p_T

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Fluctuations of v_4:

$$\frac{\Delta v_4^*}{v_4^*} = 2\frac{\Delta v_2^*}{v_2^*} = 2\frac{\Delta \varepsilon^*}{\varepsilon^*}$$

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Fluctuations of v_4:

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At large t all deformations determined by ε^* $\Rightarrow$ other relations.
Let $R_{\rm HBT}(\phi) = R_0 + 2R_2\cos(2\phi) + 2R_4\cos(4\phi) + \ldots$. Then

$$R_4 \sim R_2^2$$

Amnesia:

In hydro (under mentioned conditions) memory of the higher multipoles is lost quickly. Only the initial quadrupole deformation matters for observables sensitive to late times

Summary

- ε, ε^*, and $\Delta\varepsilon/\varepsilon$ are sensitive to the choice of the (Glauber-like) model, while $\Delta\varepsilon^*/\varepsilon^*$ is not, changing at most by 10-15%
- Analytic formulas explain why at $b = 0$ we have (in absence of correlations) $\Delta\varepsilon^*/\varepsilon^* \simeq 0.5$, insensitive of the model used or the mass number of the colliding nuclei
- Behavior of $\Delta\varepsilon^*/\varepsilon^*$ is largely governed by the statistics
- For jet emission asymmetry, we find that the effect of the increased quadrupole eccentricity is largely canceled by the shift of the center of mass and the rotation of the axes of the absorbing medium. Only for small b the increased quadrupole moment wins over the less important shift and rotation

- Smoothing prescription for e-by-e hydro can be based on the variable-axes profiles $f_l(\rho)$
- Analysis of the variable-axes moments in the coordinate space directly carries over to the collective flow and analysis of v_2^* in the momentum space. In particular, for both central and peripheral collisions $\Delta v_2^*/v_2^* \simeq 0.5$ (statistics), lower in between
- Under assumptions of smootheness, perturbation theory made on top of azimuthally symmetric hydro leads to sensitivity of higher-multipole late-time measures, v_4, *etc.*, to the initial *quadrupole* deformation $\varepsilon(t_0)$ only. Higher multipoles of the initial shape deformation are irrelevant, as they presumably are damped fast. A number of relations follows for various measures and their e-by-e fluctuations
- Challenge to measure, *e.g.*, $\Delta v_4^*/v_4^* = 2\Delta v_2^*/v_2^*$