Generalized parton distributions of the pion

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based on work with E. Ruiz Arriola, A. E. Dorokhov, and K. Golec-Biernat

Coimbra, 19 February 2008

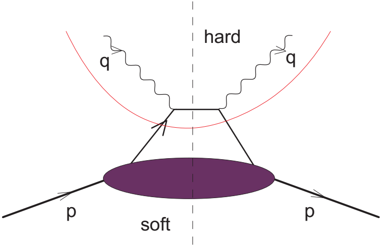
- Pion light cone wave function and pion distribution amplitude in the NJL model, Phys.Rev.D66:094016,2002, hep-ph/0207266
- Spectral quark model and low-energy hadron phenomenology, Phys.Rev.D67:074021,2003, hep-ph/0301202
- Impact parameter dependence of the GPD of the pion in chiral quark models, Phys.Lett.B574:57-64,2003, hep-ph/0307198
- Application of chiral quark models to high-energy processes, Bled 2004, 7-10, hep-ph/0410041
- Pion transition form factor and distribution amplitudes in large-N_c Regge models, Phys.Rev.D74:034008,2006, hep-ph/0605318
- Photon DA's and light-cone wave functions in chiral quark models, Phys.Rev.D74:054023,2006, hep-ph/0607171
- Pion-photon Transition Distribution Amplitudes in the Spectral Quark Model, Phys.Lett.B649:49,2007, hep-ph/0701243
- Generalized parton distributions of the pion in chiral quark models and their QCD evolution, to appear in PRD, 0712.1012 [hep-ph]

– numerous references to the field

"Low energy meets high energy"

- We want to explore the soft structure of hadrons
- Inclusive and exclusive high-energy processes provide detailed information on (soft) partonic structure of hadrons – factorization
- Chiral quark models can be used to compute the relevant low-energy hadronic matrix elements
- **Matching to QCD** at the low quark-model scale Q_0, **QCD evolution** to experimental scales
- Comparison to data allows to determine Q_0

Deep Inelastic Scattering – Parton Distribution Functions



$Q^2 = -q^2, \;\; x = \frac{Q^2}{2p\cdot q}, \;\; Q^2 \to \infty$

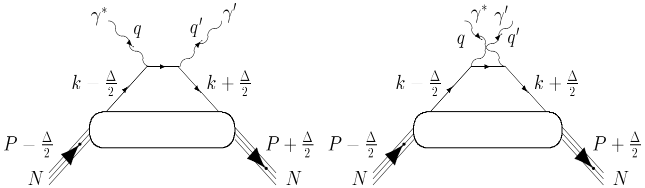
Factorization of soft and hard processes, Wilson's OPE, twist expansion

$$\langle J(q)J(-q)\rangle = \sum_i C_i(Q^2;\mu)\langle \mathcal{O}_i(\mu)\rangle, \;\; F(x,Q) = F_0(x,\alpha(Q)) + \frac{F_2(x,\alpha(Q))}{Q^2} + \ldots$$

The soft matrix element can be computed in low-energy models!

$$F_i(x,\alpha(Q_0))|_{\rm model} = F_i(x,\alpha(Q_0))|_{\rm QCD}\,, \quad Q_0 - \textbf{the matching scale}$$

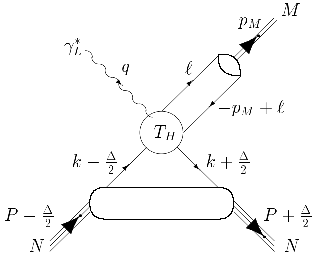
Exclusive processes in QCD



(a) (b)

Deeply Virtual Compton Scattering

non-zero momentum transfer to the target, at least one photon virtual



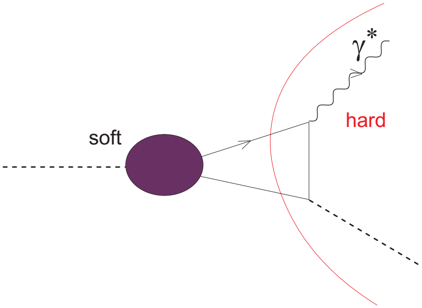
Hard Meson Production

Dictionary of matrix elements

General structure of the (soft) matrix elements: $\langle A \mid \mathcal{O} \mid B \rangle$

- A = B = one-particle state – Parton Distribution of A (inclusive DIS)
- A = one-particle state, B = vacuum – distribution amplitude (DA) of A (hadronic form factors, HMP)
- A, B = one-particle state of different momentum – GPD (exclusive DIS, DVCS, HMP)
- A = many-particle state, B = vacuum – GDA (transition form factors)
- A $\neq$ B (A, B – different hadronic states) – Transition Distribution Amplitude ($h\bar{h} \to \gamma\gamma^*$)
- ...

Pion Distribution Amplitude



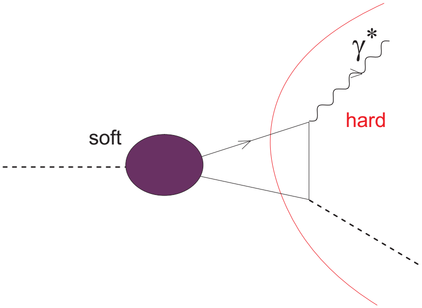
Definition (for π^+, leading twist):

$$\langle 0|\bar{d}(z)\gamma_\mu\gamma_5 u(-z)|\pi^+(q)\rangle = i\sqrt{2} f_\pi(q^2) q_\mu \int_0^1 dx e^{i(2x-1)q\cdot z}\phi(x)$$

z is along the light cone, $z^2 = 0$, $f_\pi(m_\pi^2) = 93$ MeV – pion decay constant

Normalization $\int_0^1 dx\phi(x) = 1$, since $\langle 0|A_\mu^-(0)|\pi^+(q)\rangle = i f_\pi(q^2) q_\mu$

Pion Distribution Amplitude



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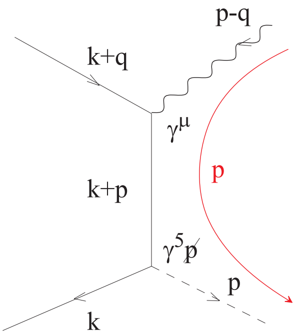
PDA is also relevant for the $\pi^0\gamma\gamma^*$ transition form factor measured by CLEO and CELLO

Leading-twist structure

A sample calculation of the leading-twist Dirac structure



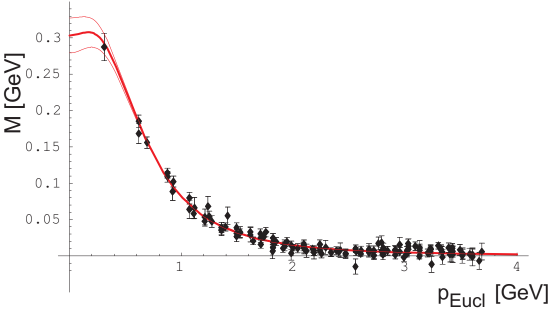
p – hard momentum

$$\gamma_5 \not{p} \frac{1}{\not{k} + \not{p} - m} \gamma^\mu \simeq \gamma_5 \gamma^\mu + \text{higher twists}$$

(crossed diagram similar)

Chiral quark models

Spontaneous chiral symmetry breaking → quark mass $M(0) \sim 300$ MeV



NJL, instanton liquid, lattices, ...$S(p) = \frac{Z(p)}{\not{p} - M(p)}$

(construction of interaction vertices subtle when $M = M(p^2)$)

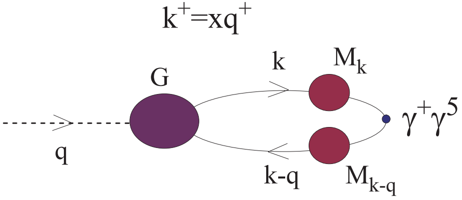
Chiral quark models 2

Model	mass	vertex $G_{\pi qq}$
Nambu-Jona-Lasinio	M=const	$i\gamma_5/F_\pi$
instanton-liquid model	$M(p^2) = M_0 r_p^2$	$i\gamma_5 r_k r_{k-q}/F_\pi$
Pagels-Stokar model	$M(p^2) = M_0 r_p^2$	$i\gamma_5(r_k^2 + r_{k-q}^2)/(2F_\pi)$

NJL needs regularization, here Pauli-Villars subtraction
All approaches satisfy chiral symmetry constraints, WT identities

QM evaluation of DA

One-loop diagram (leading $1/N_c$) with constrained integration



$$\phi(x) = -\frac{4iN_c}{f_\pi(q^2)} \int \frac{d^4k}{(2\pi)^4} \delta(k^+ - xq^+) G \frac{(M_{k-q} - M_k)k^+ + M_k q^+}{D_k D_{k-q}}$$

M_p – (momentum-dependent) *constituent* quark mass,
$D_p = p^2 - M_p^2 + i0$

Result in local model

$$\phi(x) = 1$$

(any distribution of the longitudinal momentum fraction x equally probable)

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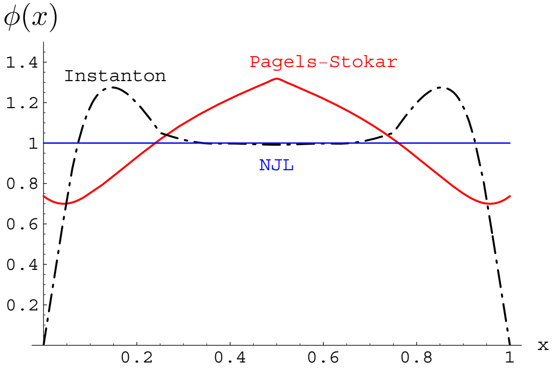
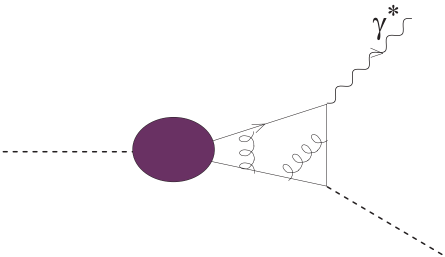
... but this is at some yet unknown QM scale Q_0!

Various models



These results are at some low quark-model scale Q_0

QCD evolution of DA



$Q_0 \to Q$

$Q_0 = 313^{+20}_{-10}$ MeV

QCD evolution - resummation of hard gluon exchanges (LO ERBL - technically simple)
(similarly for PDF - DGLAP evolution)
important: allows to determine the quark-model scale via comparison of various quantities to data
"Quark models provide initial condition for the evolution"

QCD evolution - explicit formulas

The LO evolved distribution amplitudes read (Efremov-Radyushkin, Brodsky-Lepage, Mueller 95)

$$\phi^i(x,Q^2) = \phi_{\rm as}(x) \sum_{n=0,2,4,\ldots}^{\infty} C_n^{3/2}(2x-1) a_n(Q^2),$$

$\phi_{\rm as}(x) = 6x(1-x)$, $C_n^{3/2}$ – Gegenbauer polynomials, a_n evolve with the scale:

$$a_n(Q^2) = a_n(Q_0^2) \left(\frac{\alpha(Q^2)}{\alpha(Q_0^2)} \right)^{(\gamma_n - \gamma_0)/(2\beta_0)}$$

$$a_n(Q_0^2) = \frac{2}{3} \frac{2n+3}{(n+1)(n+2)} \int_0^1 dx C_n^{3/2}(2x-1) \phi(x,Q_0^2).$$

$$\gamma_n = -\frac{8}{3} \left[3 + \frac{2}{(n+1)(n+2)} - 4 \sum_{k=1}^{n+1} \frac{1}{k} \right], \quad \beta_0 = \frac{11}{3} N_c - \frac{2}{3} N_f = 9$$

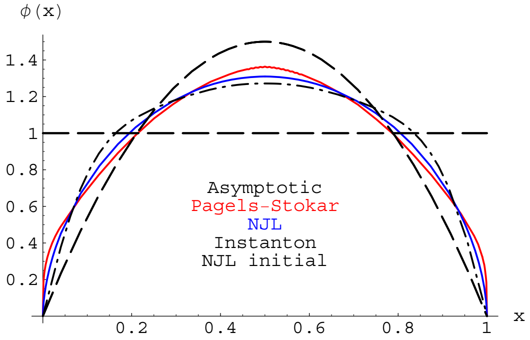
Evolved results

The analysis of Schmedding, Yakovlev, Bakulev, Mikhailov, Stefanis, of the CLEO experimental data gives $a_2(5.8\mathrm{GeV}^2) = 0.12 \pm 0.03$. Our method of determining Q_0: evolve the distribution amplitude from an arbitrary scale Q_0 to the CLEO scale $Q = 2.4$ GeV and adjust Q_0 such that $a_2 = 0.12$

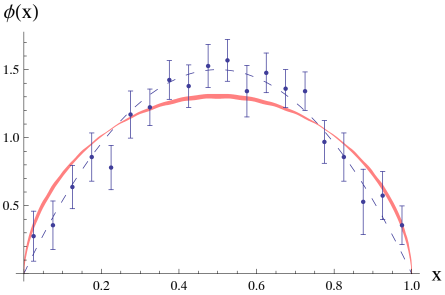
	Pagels-Stokar	Instanton	NJL/SQM
Q_0 [GeV]	0.5	0.39	0.32

Evolved DA of the pion

Pion DA evolved to the scale $Q = 2.4$ GeV from Q_0 specific to the given model

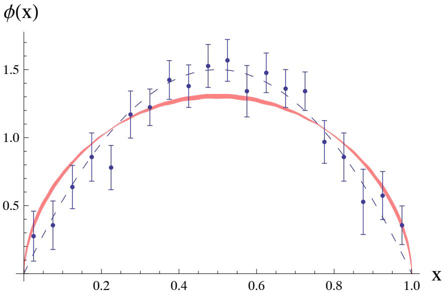


Comparison to experimental and lattice data



points: E791 data from di-jet production in $\pi + A$
band: QM at $Q = 2$ GeV
dashed line: asymptotic form ($Q \to \infty$)

Comparison to experimental and lattice data



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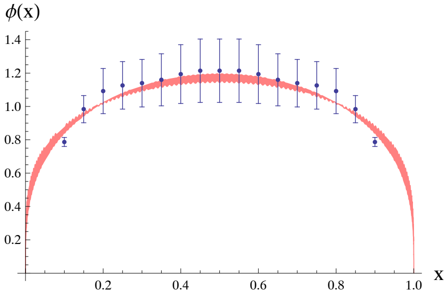


points: transverse lattice data [Dalley, van de Sande 2003]
band: QM at $Q = 0.5$ GeV

Definition of GPD

Generalized Parton Distributions

Two isospin projections of the twist-2 GPD of the pion:

$$\delta_{ab}\,\mathcal{H}^{I=0}(x,\zeta,t)=\int\frac{dz^-}{4\pi}e^{ixp^+z^-}\,\langle\pi^b(p+q)|\bar\psi(0)\gamma^+\psi(z)|\pi^a(p)\rangle\big|_{z^+=0,z^\perp=0}$$

$$i\epsilon_{3ab}\,\mathcal{H}^{I=1}(x,\zeta,t)=\int\frac{dz^-}{4\pi}e^{ixp^+z^-}\langle\pi^b(p+q)|\bar\psi(0)\gamma^+\psi(z)\,\tau_3|\pi^a(p)\rangle\big|_{z^+=0,z^\perp=0}$$

where $p^2=m_\pi^2$, $q^2=-2p\cdot q=t$, $q^+=-\zeta p^+$

$\zeta\sim$ momentum transferred along the light cone

Some background

- K. Goeke, M. V. Polyakov, and M. Vanderhaeghen, Prog. Part. Nucl. Phys. 47 (2001) 401, hep-ph/0106012
- M. Diehl, Phys. Rept. 388 (2003) 41, hep-ph/0307382
- A. V. Belitsky, A. V. Radushkin, Phys.Rept.418(2005)1, hep-ph/0504030

GPD's provide more detailed information of the structure of hadrons than PDF's (structure functions). Information on GPD's may come from such processes as $ep \to ep\gamma$, $\gamma p \to pl^+l^-$, $ep \to epl^+l^-$, or from lattices. Small cross sections of exclusive processes require very high accuracy experiments. First results are for the nucleon coming from HERMES and CLAS, also COMPASS, H1, ZEUS

Formal features

Symmetric notation: $\xi = \frac{\zeta}{2-\zeta}$, $X = \frac{x-\zeta/2}{1-\zeta/2}$, with $0 \leq \xi \leq 1$, $-1 \leq X \leq 1$

$H^{I=0}(X,\xi,t) = -H^{I=0}(-X,\xi,t)$, $H^{I=1}(X,\xi,t) = H^{I=1}(-X,\xi,t)$.

For $X \geq 0$ we have $\mathcal{H}^{I=0,1}(X,0,0) = q(X)$ - the usual PDF
The following **sum rules** hold:

$$\forall\xi : \quad \int_{-1}^{1} dX\, H^{I=1}(X,\xi,t) = 2F_V(t),$$
$$\forall\xi : \quad \int_{-1}^{1} dX\, X\, H^{I=0}(X,\xi,t) = \theta_2(t) - \xi^2\theta_1(t),$$

where $F_V(t)$ is the electromagnetic form factor, while $\theta_1(t)$ and $\theta_2(t)$ are the gravitational form factors of the pion. The sum rules express the electric charge conservation and the momentum sum rule in DIS

The **polynomiality** conditions (Lorentz invariance, time reversal, and hermiticity) state that

$$\int_{-1}^{1} dX\, X^{2j}\, H^{I=1}(X,\xi,t) = \sum_{i=0}^{j} A_i^{(j)}(t)\xi^{2i},$$

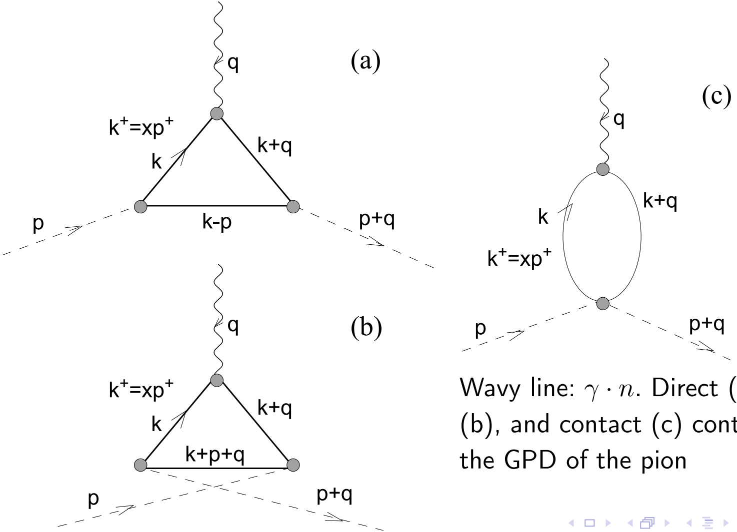
$$\int_{-1}^{1} dX\, X^{2j+1}\, H^{I=0}(X,\xi,t) = \sum_{i=0}^{j+1} B_i^{(j)}(t)\xi^{2i},$$

where $A_i^{(j)}(t)$ and $A_i^{(j)}(t)$ are form factors dependent on j and i.
The **positivity bound** requires

$$|H_q(X,\xi,t)| \le \sqrt{q(x_{\rm in})q(x_{\rm out})}, \qquad \xi \le X \le 1.$$

where $x_{\rm in} = (x+\xi)/(1+\xi)$, $x_{\rm out} = (x-\xi)/(1-\xi)$.
Finally, a **low-energy theorem** $H_{I=1}(2z-1,1,0) = \phi(z)$ holds
All above relations and bounds form severe constraints for the form of the pion GPD
All are satisfied in our QM calculation

Quark-model evaluation of GPD



Wavy line: $\gamma \cdot n$. Direct (a), crossed (b), and contact (c) contribution to the GPD of the pion

PDF, QM vs. E615

In the special case of $\zeta = t = 0$ GPD becomes the PDF. The NJL result is (Davidson & Arriola, 1995)

$$q(x) = 1$$

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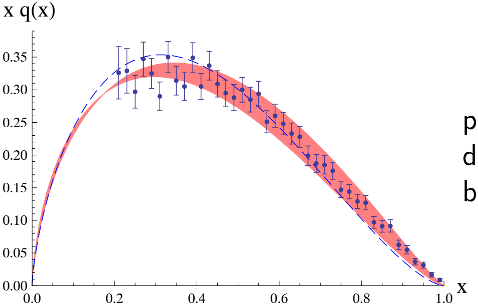
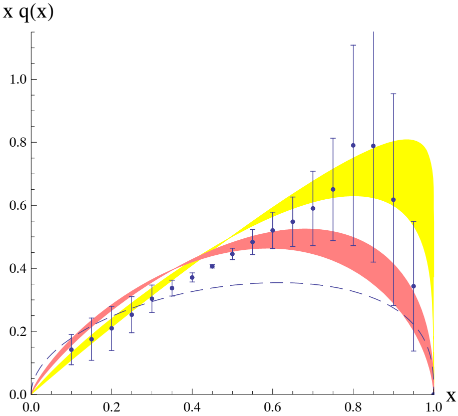
$$q(x) = 1$$

LO DGLAP QCD evolution of the non-singlet part to the scale $Q^2 = (4 \text{ GeV})^2$ of the E615 Fermilab experiment:



points: Drell-Yan from E615
dashed: 2005 reanalysis of data
band: QM evolved to $Q = 4$ GeV

PDF, QM vs. lattice



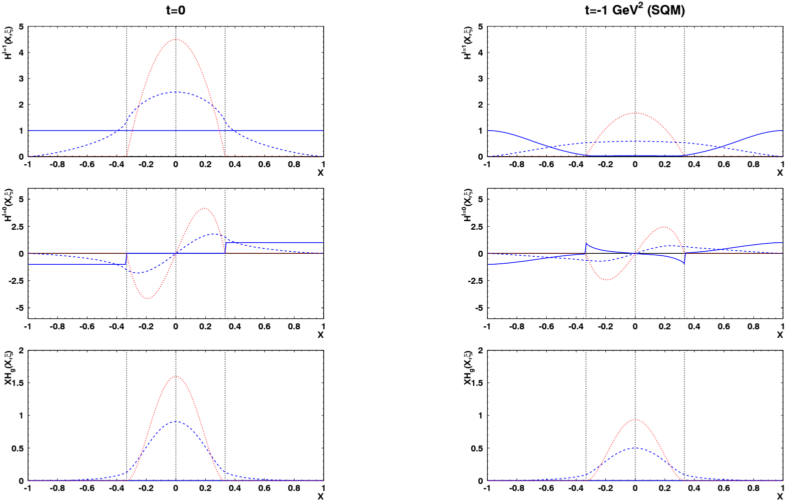
points: transverse lattice [Dalley, van de Sande 2003]
yellow: QM evolved to 0.35 GeV
pink: QM evolved to 0.5 GeV
dashed: GRS parameterization at 0.5 GeV

GPD in chiral quark models

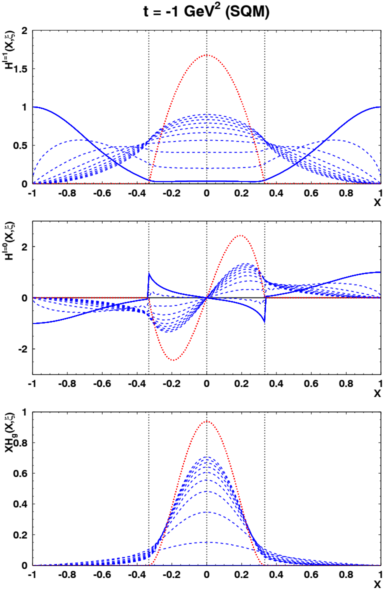
[research with K. Golec-Biernat]

Analytic formulas derived for GPD in two models: NJL and SQM (Spectral Quark Model), all formal properties satisfied, formulas fit in two long lines, **no factorization of the t-dependence** - sheds light on possible parameterizations

Similar results by [Theussl, Noguera, Vento, 2004], but no evolution
also: Praszałowicz and Rostworowski



LO QCD evolution for SQM with $\xi = 1/3$. Solid - initial condition, dashed - evolved to $Q^2 = (4\text{GeV})^2$, dotted - asymptotic form



$$Q^2 = 0.1, 1, 10, 10^2, ..., 10^8 \text{ GeV}^2$$

GPD and lattices

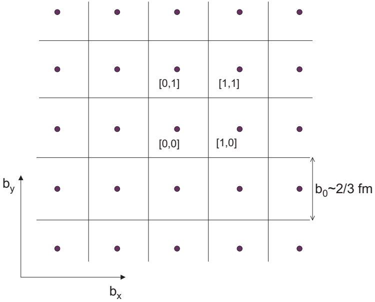
[WB+ERA'03]

$$H_{\rm SQM}(x,0,t) = \frac{m_\rho^2(m_\rho^2+(1-x)^2t)}{(m_\rho^2-(1-x)^2t)^2}\theta(x)\theta(1-x)$$

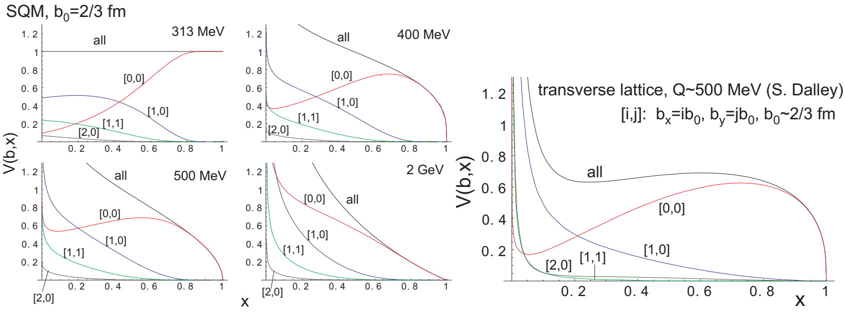
$$F(t) = \int_0^1 dx H_{\rm SQM}(x,0,t) = \frac{m_\rho^2}{m_\rho^2+t}$$

which shows the built-in vector-meson dominance in the model. We pass to the impact-parameter space by the Fourier-Bessel transformation and get

$$q_{\rm SQM}(b,x) = \frac{m_\rho^2}{2\pi(1-x)^2}\left[K_0\left(\frac{bm_\rho}{1-x}\right) - \frac{bm_\rho}{1-x}K_1\left(\frac{bm_\rho}{1-x}\right)\right]$$



labeling of lattice plaquettes



model lattice data

qualitative agreement for $Q \sim 400$ MeV

Summary

- Soft hadronic matrix elements of quark bilinears, which carry a lot of information on the quark structure of hadrons, can be evaluated for pions (and photons) in chiral quark models (large N_c, leading twist)
- The QCD evolution is necessary
- The quark-model scale Q_0 is low, ~ 320 MeV (somewhat higher in the non-local models)
- DA, GPD, PDF, GDA, TDA, light-cone wave functions (k_T-unintegrated quantities) ...
- Charge and gravitational form factors
- Overall agreement with the available data and lattice simulations very reasonable
- Link between the high- and low-energy analyses

Summary

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THANKS!

The quark-model scale from momentum fraction

From experiment, the momentum fraction carried by the valence quarks is

$$\langle x \rangle_v = 0.47(2)$$

at $Q^2 = 4 \text{ GeV}^2$.
The QM condition $q(x) = 1$ and the leading-order DGLAP evolution with $\Lambda_{\rm QCD} = 226$ MeV yields the quark-model scale for NJL [Davidson, Ruiz Arriola 1995]

$$Q_0 = 313^{+20}_{-10} \text{ MeV}$$

At this scale $\alpha(Q_0^2)/(2\pi) = 0.34$, which makes the evolution very fast for the scales close to the initial value
Other quark models (non-local) have different value of Q_0.

Pion-photon transition form factor

Pion-photon transition form factor

$$\Gamma^{\mu\nu}_{\pi^0\gamma^*\gamma^*}(q_1,q_2) = \epsilon_{\mu\nu\alpha\beta} e_1^\mu e_2^\nu q_1^\alpha q_2^\beta F_{\pi\gamma^*\gamma^*}(Q^2,A),$$

wrere

$$Q^2 = -(q_1^2+q_2^2),\ A = \frac{q_1^2-q_2^2}{q_1^2+q_2^2},\ -1 \le A \le 1.$$

For large virtualities one finds the standard twist decomposition of the pion transition form factor (Brodsky & Lepage, 1980),

$$F_{\pi^0\gamma^*\gamma^*}(Q^2,A) = J^{(2)}(A)\frac{1}{Q^2} + J^{(4)}(A)\frac{1}{Q^4} + \dots,$$

with

$$J^{(2)}(A) = \frac{4f_\pi}{N_c}\int_0^1 dx \frac{\phi(x)}{1-(2x-1)^2A^2}$$

Pion light-cone wave function

At the quark-model scale Q_0 (in the chiral limit) we find, leaving k_T unintegrated,

NJL:

$$\Psi(x, k_T) = \frac{4N_c M^2}{f_\pi^2} \sum_j c_j \frac{1}{k_T^2 + \Lambda_j^2 + M^2} \sim (\text{two subtractions}) \sim \frac{1}{k_T^6}$$

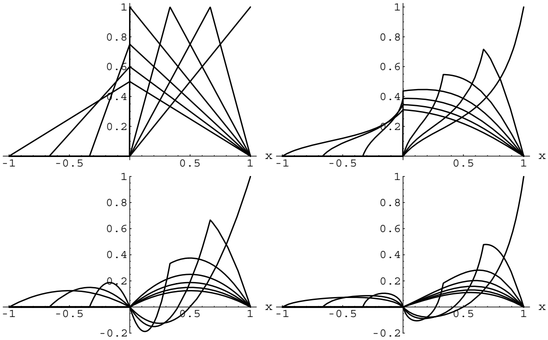
$$\langle k_T^2 \rangle = -\frac{M \langle \bar{q}q \rangle}{f_\pi^2} \sim (600 \text{ MeV})^2$$

SQM:

$$\Psi(x, k_T) = \frac{3m_\rho^3}{16\pi(k_T^2 + m_\rho^2)^{5/2}}, \quad \langle k_T^2 \rangle = \frac{m_\rho^2}{2} = (540 \text{ MeV})^2$$

Pion-photon TDA

[Pire and Szymanowski](as GPD, but between the π and γ states)



Top: vector TDA for $t = 0$ (left) and $t = -0.4$ GeV (right) several values of ζ: -1, $-2/3$, $-1/3$, 0, $1/3$, $2/3$, and 1. Bottom: the same for the axial TDA, SQM atthe scale Q_0