Medium effects on meson couplings

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Research with B. Hiller (Coimbra) and W. Florkowski (INP)

This talk: WB,WF & BH, nucl-th//0103027
Earlier works: Acta. Phys. Pol. **30** (1999) 1079
Coimbra 99 talk by WF, nucl-th//9910057
Eur. Phys. J. **A** 7 (2000) 287

Introduction

- A lot of efforts have been undertaken to understand hadrons in hot/dense medium

- In heavy-ion collisions (AGS, SPS, RHIC, LHC) a proper inclusion of hadrons is necessary to describe the evolution of the hadronic soup. It is commonly accepted that hadrons are significantly modified by the medium. There are numerous calculations concerning hadron two-point functions (Walecka, BR scaling, hadronic models, QCD SR, χPT, NJL, low-density expansion). Results: masses/widths (spectral functions) can be significantly altered

- While there are a lot of studies of the meson two-point functions, there exist only very few papers devoted to meson three-point functions [1] [2] [3] [4]. Since the two-point functions are

[1]Non-relativistic, $\vec{q} = 0$: M. Herrmann, B. L. Friman, W. Nörenberg, NPA **560** (1993) 411

[2]Finite T, $\rho_B = 0$: C. Song, V. Koch, PRC **54** (1996) 3218

[3]Chiral effects: B. Krippa, NPA **672**(2000) 270

[4]Similar to our work: M. Urban, M. Buballa, R. Rapp, J. Wambach, NPA **673** (2000) 357

significantly altered by the medium, one expects that the three-point functions also change a lot.

- The $\rho\pi\pi$ vertex is especially important, since the ρ plays an essential role in hadron dynamics. The vacuum value of the coupling constant is large, $g_{\rho\pi\pi} \simeq 6$. Since the couplings of ρ and pions to nucleons and Δ isobars are large, we expect significant modifications of $g_{\rho\pi\pi}$ by the medium. This is indeed the case!.

- Our earlier work: $\omega \to \pi\pi$ in medium, which occurs when $\vec{q} \neq 0$, and has significant width

- Present work with Ph.D. students Anna Baran and Agnieszka Bieniek: $\omega\rho\pi$ vertex in medium (description of $\pi^0 \to \gamma\gamma$, $\pi^0 \to \gamma e^+ e^-$, $\omega \to \pi^0\gamma$, $\omega \to \pi^0 e^+ e^-$, *etc.*)

Basic idea and framework

Absorption and emission processes from nucleons in the Fermi-sea:

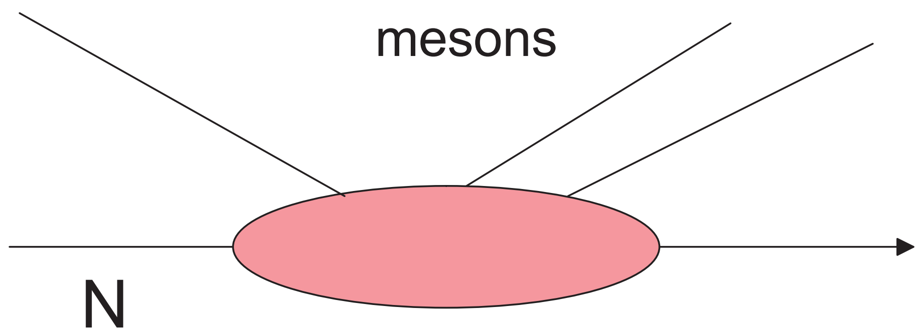

Figure 1:

- Relativistic theory, with mesons interacting with N and $\Delta(1232)$
- Vanishing temperature, $T = 0$
- Leading order in the baryon density, ρ_B

To the leading-density order only the diagrams of Fig. 2 contribute to the $\rho \to \pi\pi$ process:

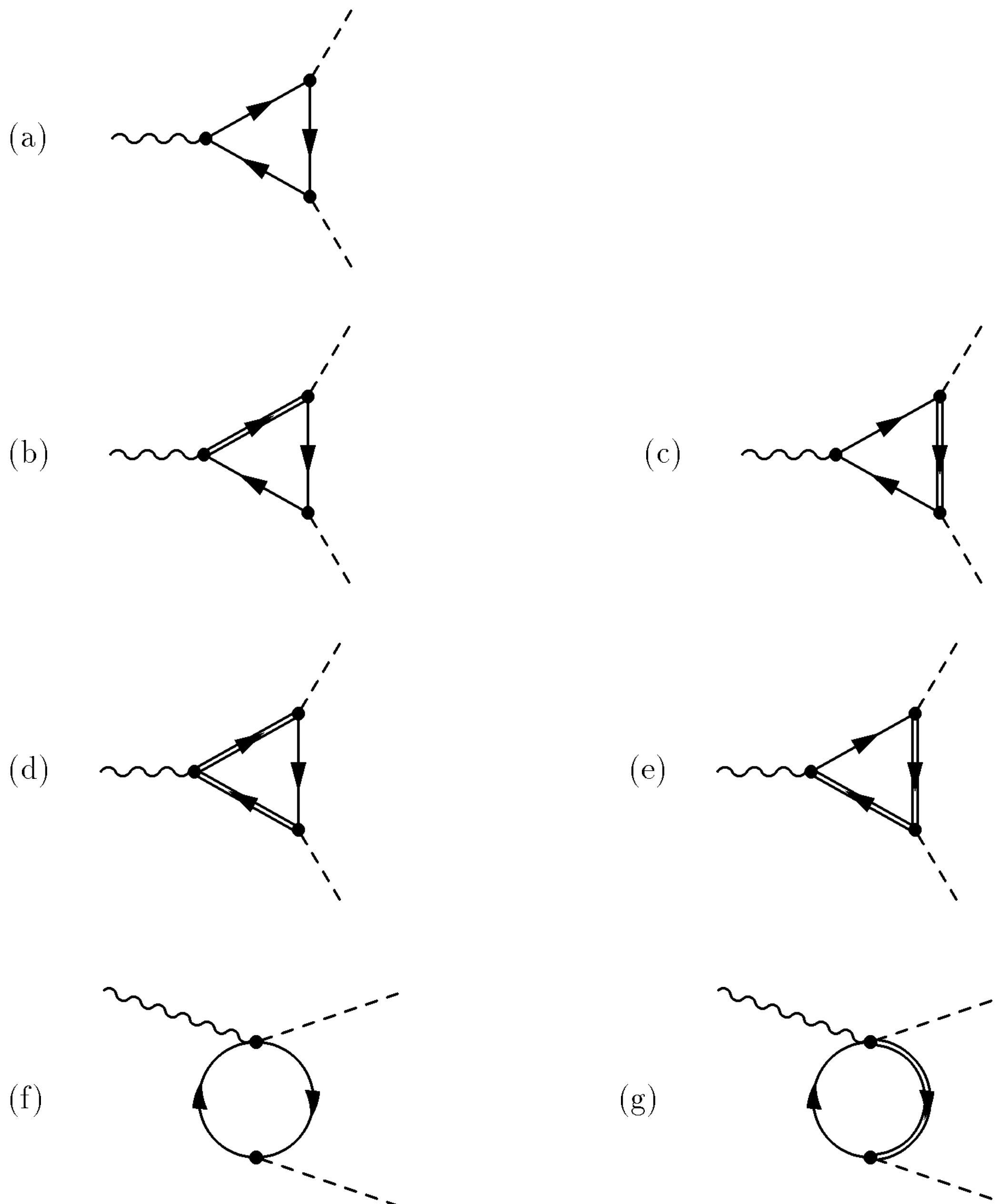

Figure 2: Diagrams included in our calculation (crossed diagrams not displayed). Wavy lines denote the ρ, dashed lines the pions, solid lines the in-medium nucleon, and double lines the Δ.

The in-medium nucleon propagator consists of the *free* and *density* parts:

$$iG(k) \equiv iG_F(k) + iG_D(k) = i(\not{k} + m_N) \times \left[\frac{1}{k^2 - m_N^2 + i\varepsilon} + \frac{i\pi}{E_k}\delta(k_0 - E_k)\theta(k_F - |\mathbf{k}|)\right],$$

where m_N is the nucleon mass, $E_k = \sqrt{m_N^2 + \mathbf{k}^2}$, and k_F is the Fermi momentum of nuclear matter. The Rarita-Schwinger Δ propagator is

$$iG_\Delta^{\alpha\beta}(k) = i\frac{\not{k} + m_\Delta}{k^2 - \left(m_\Delta - \frac{i}{2}\Gamma_\Delta\right)^2} \times \left(-g^{\alpha\beta} + \frac{1}{3}\gamma^\alpha\gamma^\beta + \frac{2k^\alpha k^\beta}{3m_\Delta^2} + \frac{\gamma^\alpha k^\beta - \gamma^\beta k^\alpha}{3m_\Delta}\right).$$

Since we are looking at density effects, one of the nucleon lines must involve G_D. For kinematic reasons, diagrams with more than one G_D vanish. The wavy lines denote the ρ, and the dashed lines the pions.

The meson-NN vertices are

$$
\begin{aligned}
-iV_{\pi^a NN} &= \frac{g_A}{2F_\pi} \not{p}\gamma_5\tau^a, \\
-iV_{\rho^b_\mu NN} &= ig_\rho(\gamma^\mu + \frac{i\kappa_\rho}{2m_N}\sigma^{\mu\nu}q_\nu)\frac{\tau^b}{2}, \\
-iV_{\rho^b_\mu \pi^a NN} &= i\frac{g_\rho g_A}{2F_\pi}\gamma^\mu\gamma_5\varepsilon^{abc}\tau_c
\end{aligned}
$$

where p is the outgoing four-momentum of the pion, q is the incoming momentum of the ρ meson, and a and b are the isospin indices of the pion and the ρ, respectively.

The meson-$N\Delta$ vertices have the form

$$
\begin{aligned}
-iV_{\pi^a N\Delta_\alpha} &= \frac{f_{\pi N\Delta}}{m_\pi}p^\alpha T^a, \\
-iV_{\rho^b_\mu N\Delta_\alpha} &= i\frac{\sqrt{2}f^*}{m_\pi}(g^{\mu\alpha} \not{q}\gamma_5 - q^\mu\gamma^\alpha\gamma_5)T^b \\
-iV_{\rho^b_\mu \pi^a N\Delta_\alpha} &= i\frac{g_\rho f_{\pi N\Delta}}{m_\pi}g^{\alpha\mu}\varepsilon^{abc}T_c.
\end{aligned}
$$

T^a are the isospin $\frac{1}{2} \to \frac{3}{2}$ transition matrices:

$$\langle \frac{3}{2}, I_3 | T^\mu | \frac{1}{2}, i_3 \rangle = \langle \frac{1}{2} 1 i_3 \mu | 1 \frac{1}{2} \frac{3}{2} I_3 \rangle.$$

For the meson-$\Delta\Delta$ couplings we have

$$\begin{aligned} iV_{\pi^a \Delta_\alpha \Delta_\beta} &= \frac{3}{2} \frac{f_\Delta}{m_\pi} g^{\alpha\beta} \not{p} \gamma_5 T^a_\Delta, \\ iV_{\rho^b_\mu \Delta_\alpha \Delta_\beta} &= i g_\rho \left(-\gamma^\mu g^{\alpha\beta} + g^{\alpha\mu} \gamma^\beta \right. \\ &\quad \left. + g^{\beta\mu} \gamma^\alpha + \gamma^\alpha \gamma^\mu \gamma^\beta \right) T^b_\Delta, \end{aligned}$$

where

$$\langle \frac{3}{2}, I'_3 | T^\mu_\Delta | \frac{3}{2}, I_3 \rangle = \frac{\sqrt{15}}{2} \langle \frac{3}{2} 1 I'_3 \mu | \frac{3}{2} \frac{1}{2} \frac{3}{2} I_3 \rangle$$

Our choice of the physical parameters:

$$\begin{array}{ll} g_A = 1.26, & F_\pi = 93\mathrm{MeV}, \\ m_\pi = 139.6\mathrm{MeV}, & g_\rho = 5.26, \\ \kappa_\rho = 6, & f_{\pi N \Delta} = 2.12, \\ f^* = 2.12, & f_\Delta = 0.802. \end{array}$$

Low-density approximation

We evaluate the diagrams of Fig. 2 in the rest frame of the medium. The full vertex function has the form

$$A^{\mu}_{acb} = \epsilon^{acb}(A^{\mu}_{\rm vac} + A^{\mu}_{\rm med}),$$

$$A^{\mu}_{\rm vac} = g_{\rho}(2p^{\mu} - q^{\mu}),$$

$$A^{\mu}_{\rm med} = \int \frac{d^3k}{(2\pi)^3} \frac{m_N}{E_k}(Ap^{\mu} + Bq^{\mu} + Ck^{\mu})\theta(k_F - |\vec{k}|),$$

where A, B, C are certain functions of the scalar products of the four-vectors q, p, k. One can easily show that the leading-density approximation is equivalent to setting $\vec{k} = 0$ in A, B, and C. Then $\int \frac{d^3k}{(2\pi)^3}\theta(k_f - |\vec{k}|) = \frac{1}{4}\rho_B$, and

$$A^{\mu}_{\rm med} = \frac{1}{4}\rho_B(\bar{A}p^{\mu} + \bar{B}q^{\mu} + \bar{C}m_N u^{\mu}),$$

where u^{μ} is the four-velocity of the medium, and the coefficients $\bar{A}$, $\bar{B}$, $\bar{C}$ are obtained from A, B, C by setting $\vec{k} = 0$.

- No integration over $\vec{k}$ needed!

Results for ρ decaying at rest

For $\vec{q} = 0$ we find $\bar{B} = -\frac{1}{2}\bar{A}$ and $\bar{C} = 0$, hence the in-medium vertex is proportional to $2p^\mu - q^\mu$:

$$A^\mu_{\mathrm{med}}(\vec{q} = 0) = \frac{1}{8}\rho_B \bar{A}(\vec{q} = 0)(2p^\mu - q^\mu).$$

It has the same Lorentz structure as in the vacuum, hence we introduce an effective $\rho\pi\pi$ coupling constant

$$g_{\mathrm{eff}} = \left| g_\rho + \frac{1}{8}\rho_B \bar{A}(\vec{q} = 0) \right| .$$

The absolute value is taken, since with Γ_Δ the quantity $\bar{A}(\vec{q} = 0)$ is complex. In the following we treat ρ as a *virtual* particle with mass M. This is needed, *e.g.*, in the analysis of the dilepton production.

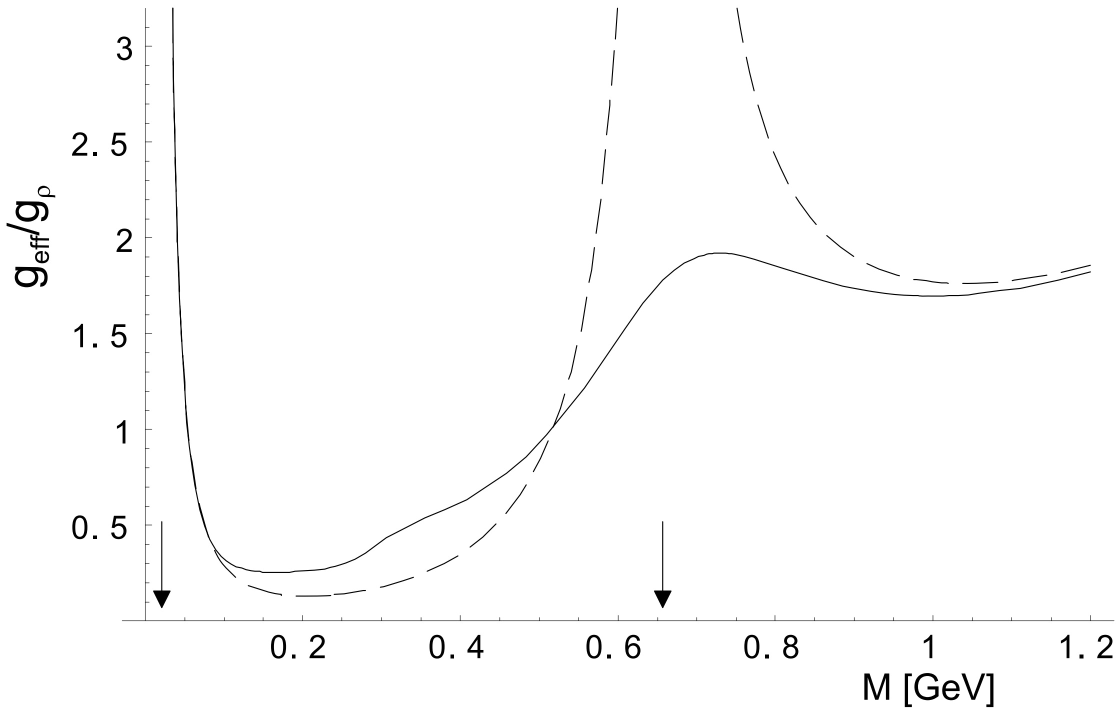

Figure 3: $g_{\rm eff}/g_\rho$ at the saturation density, as a function of M. Solid/dashed lines correspond to $\Gamma_\Delta = 120{\rm MeV}/\Gamma_\Delta = 0$. Arrows indicate positions of the singularities.

Analyticity in M of the vertex function is nontrivial, which can be inferred from the denominators of diagrams of Fig. 2. For $\Gamma_\Delta = 0$ the poles occur at

$$M^2 = \left(\frac{m_\Delta^2 - m_N^2 - m_\pi^2}{m_N}\right)^2 = (0.657{\rm GeV})^2,$$

$$M^2 = \left(\frac{m_\pi^2}{m_N}\right)^2 = (0.021{\rm GeV})^2.$$

Triangles additionally bring in high-lying singularities at $M^2 = (2m_N)^2$ and at $M^2 = (m_\Delta + m_N)^2$, which are physically irrelevant.

Highlights:

- The considerable difference between the solid and dashed curves for M between 0.6 and 1GeV shows that the results are quite sensitive to Γ_Δ
- At M between ~ 0.07 and $\sim 0.55\mathrm{GeV}$ the effective coupling $g_{\rm eff}$ is lower than the vacuum value
- For M above $\sim 0.55\mathrm{GeV}$ the coupling is increased
- Around the physical ρ mass, $M = m_\rho$, the effective coupling is roughly two times larger than in the vacuum. For the width of the $\rho \to \pi\pi$ decay this means a factor of 4 enhancement, giving an in-medium width to the ρ of about 600MeV at the saturation density

Anatomy of g_{eff} at $M = m_\rho$, $\rho_B = \rho_0$, and $\Gamma_\Delta = 120\text{MeV}$:

$$(a) = -0.08,\ (b) = 0.22 - 0.09i,$$
$$(c) = -0.008 - 0.008i,\ (d) = 0.10 - 0.11i,$$
$$(e) = -0.11 + 0.63i,\ (f) = -0.008,$$
$$(g) = 0.62 - 1.16i,\ (\text{all}) = 0.73 - 0.75i.$$

- The $N - \Delta$ bubble diagram (g) dominates

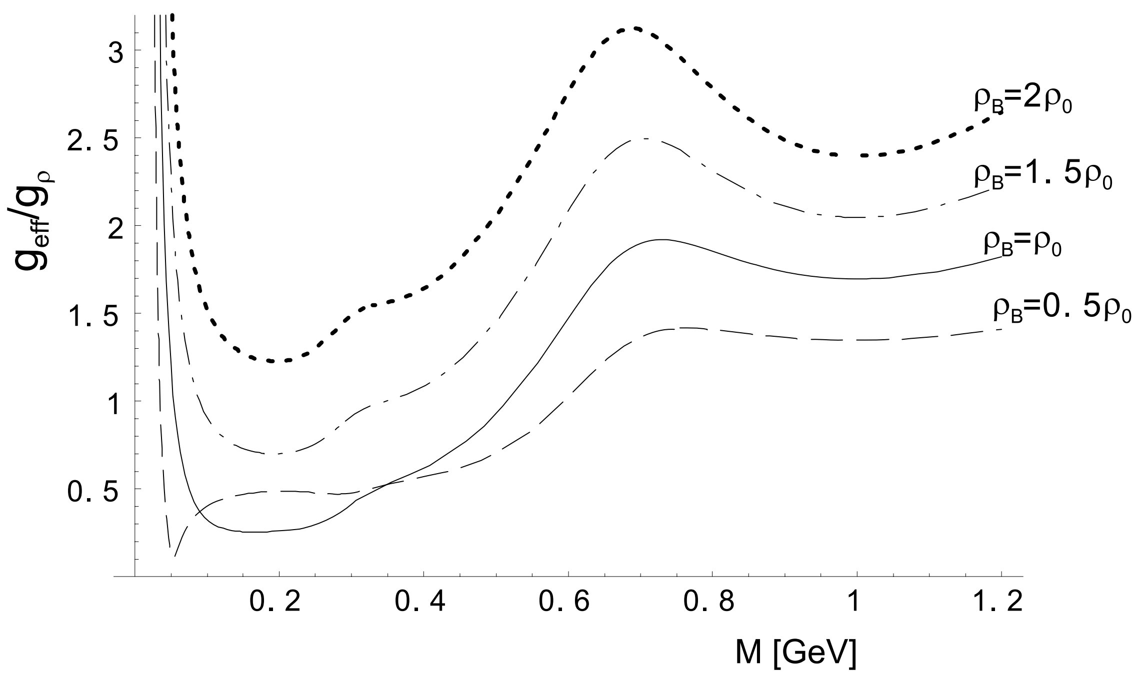

Figure 4: Same as Fig. 3 for different values of ρ_B, with $\Gamma_\Delta = 120\text{MeV}$.

Results for moving ρ, *i.e.* $\vec{q} \neq 0$

- Different behavior for transverse and longitudinal polarizations, defined by quantizing the spin along the direction of $\vec{q}$

The width for ρ with polarization P:

$$\Gamma^{P}{}_{\rho\to\pi\pi} = \frac{1}{n_s}\sum_s \frac{1}{2q_0}\int \frac{d^3p}{(2\pi)^3 2p_0}\int \frac{d^3p'}{(2\pi)^3 2p_0'} \times$$
$$|\mathcal{M}|^2 (2\pi)^4 \delta^{(4)}(q-p-p'),$$

where n_s is the number of helicity states s of the ρ meson. The transverse ρ has two helicity states $s = \pm 1$, while the longitudinal ρ has one helicity state $s = 0$.

An explicit calculation yields

$$\begin{aligned}
|\mathcal{M}_T|^2 &= -\left|2(g_\rho + \frac{1}{8}\rho_B \bar{A})\right|^2 p_\mu T^{\mu\nu} p_\nu, \\
|\mathcal{M}_L|^2 &= -(2(g_\rho + \frac{1}{8}\rho_B \bar{A})^* p_\mu + \bar{C}^* m_N u_\mu) \times \\
&\quad L^{\mu\nu}(2(g_\rho + \frac{1}{8}\rho_B \bar{A}) p_\nu + \bar{C} m_N u_\nu).
\end{aligned}$$

where the polarization tensors have the form

$$T^{\mu\nu} = g^{\mu\nu} - u^\mu u^\nu - \frac{(q^\mu - q\cdot u\ u^\mu)(q^\nu - q\cdot u\ u^\nu)}{q\cdot q - (q\cdot u)^2},$$

$$L^{\mu\nu} = -\frac{q^\mu q^\nu}{q\cdot q} + u^\mu u^\nu + \frac{(q^\mu - q\cdot u\ u^\mu)(q^\nu - q\cdot u\ u^\nu)}{q\cdot q - (q\cdot u)^2}.$$

(Figure)

- Considerable dependence on $|\vec{q}|$

- The transverse width decreases with $|\vec{q}|$, while the longitudinal does not. At lower values of M and $|\vec{q}|$ around 0.5GeV the longitudinal width develops a hill, absent in the transverse case

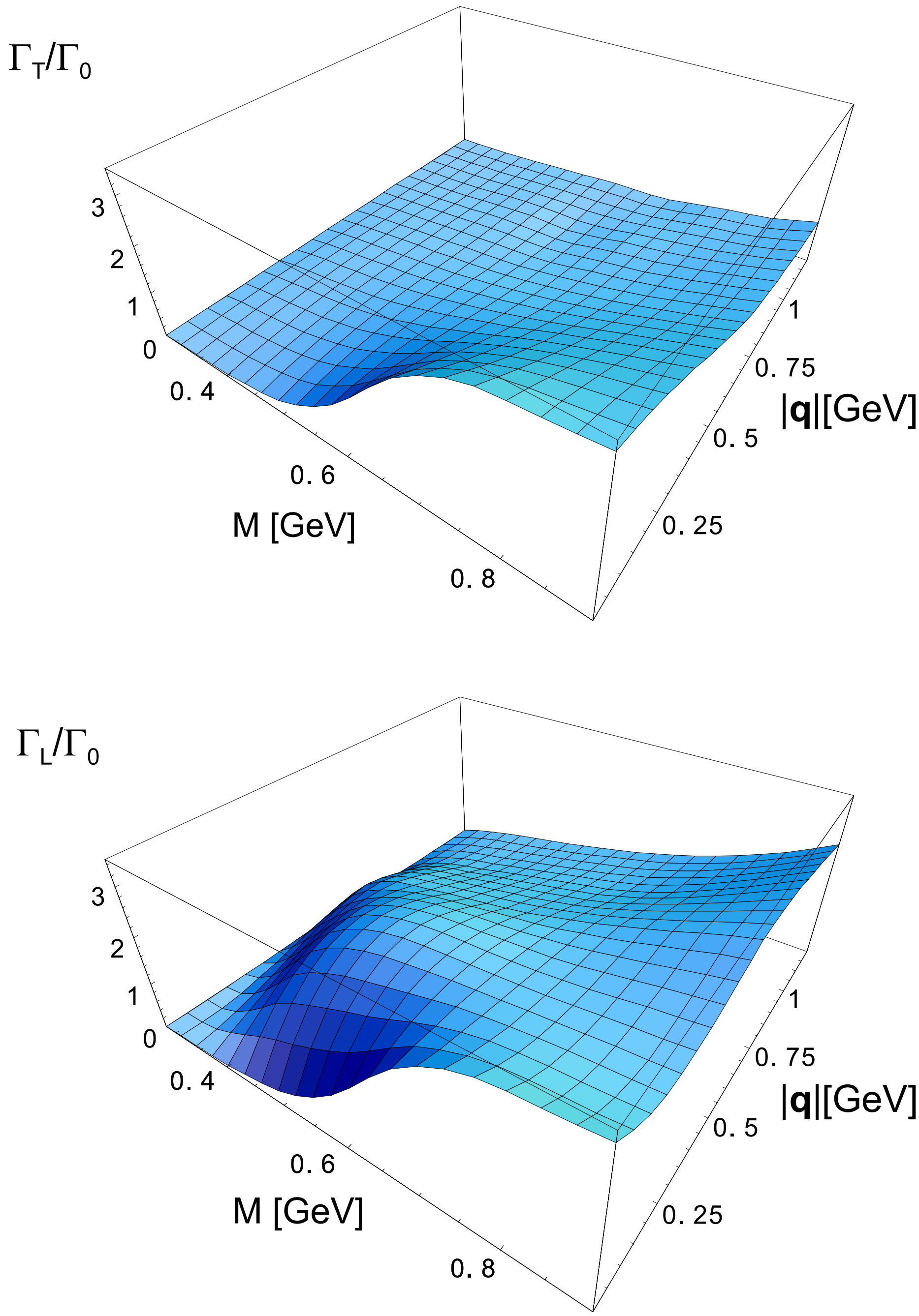

Figure 5: The ratio of the width for the $\rho \to \pi\pi$ decay at the nuclear saturation density to its vacuum value, Γ_0, plotted as a function of M and $|\vec{q}|$

The quantity which enters the formula for the dilepton production is the spectral function:

$$\mathcal{A}_P = \frac{1}{\pi}\frac{\sqrt{M^2+\vec{q}^2}\Gamma_P}{(M^2-m_\rho^{*2})^2+(M^2+\vec{q}^2)\Gamma_P^2}, \qquad P=T,L$$

where m_ρ^* is the position of the pole, with the asterisk indicating that it can be shifted from the vacuum value.

- While at $\vec{q}=0$ we obviously have $\mathcal{A}_T=\mathcal{A}_L$, at larger values of $\vec{q}$ and at M around m_ρ the transverse spectral strength becomes dominant

- The transverse spectral strength is concentrated along a ridge extending far into the large-$|\vec{q}|$ region. Thus, a proper description of propagation at finite and large values of $\vec{q}$ is needed for the description of ρ mesons in medium

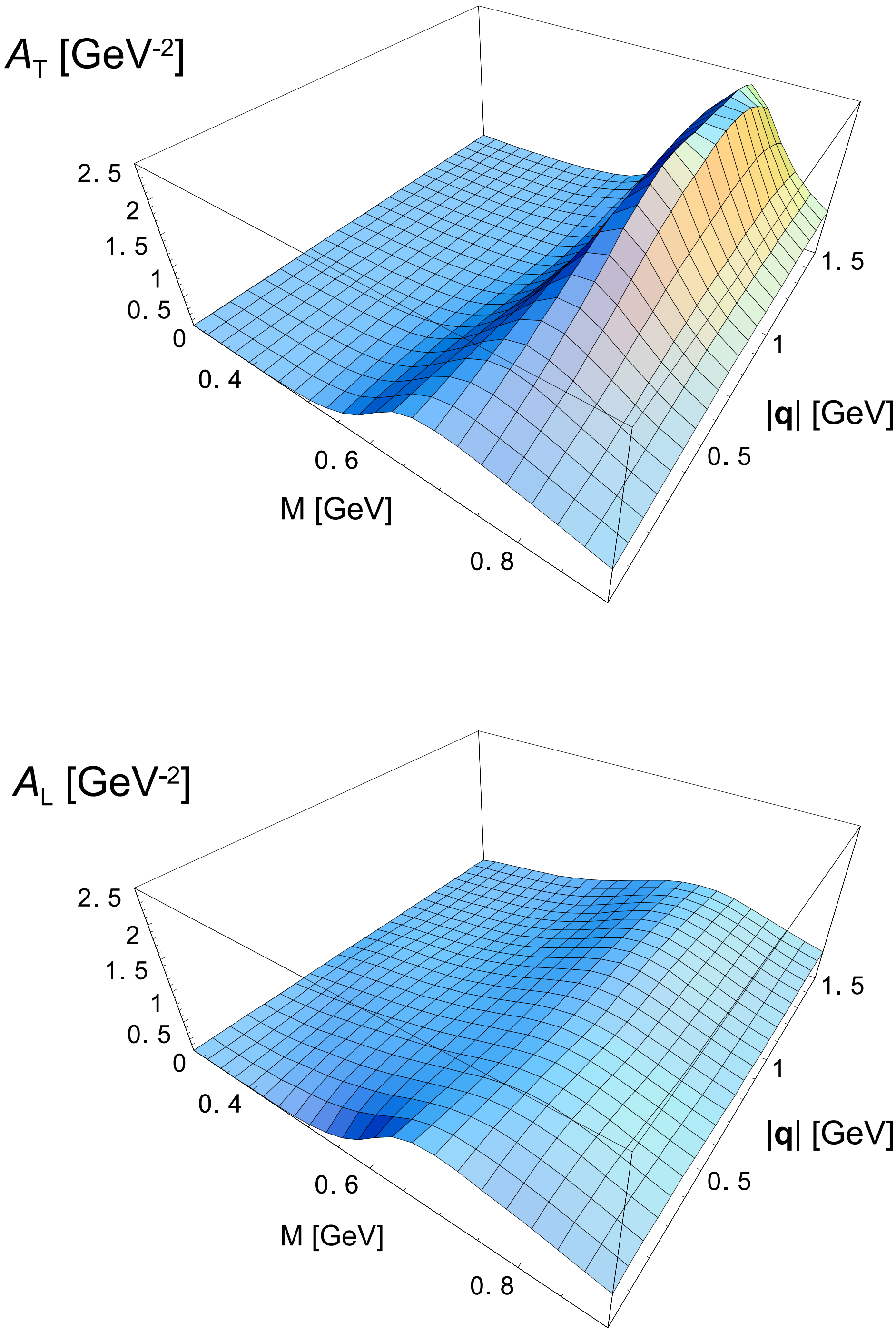

Figure 6: The spectral strengths at the nuclear saturation density, corresponding to the widths of Fig. 5

Dilepton production rate

- Measurements of the low-mass dilepton spectra (CERES, HELIOS) have shown significant excess above expected yields from the final-state hadron decays

- Properties of vector mesons, in particular the ρ, in a hadronic environment become of particular interest, since the Vector Meson Dominance Model is commonly used to make the estimates of the dilepton yields from vector-meson decays

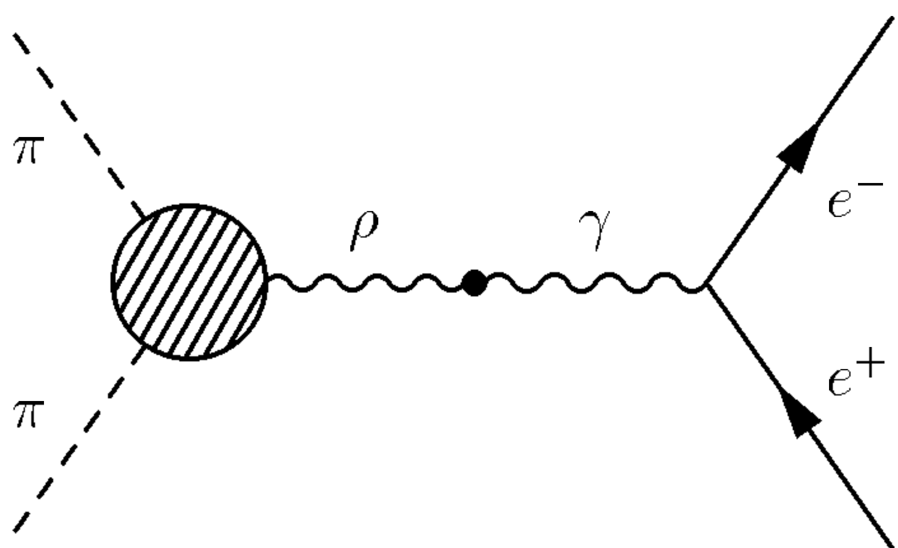

Figure 7:

The dilepton-rate formula from the ρ decays is:

$$\frac{dN}{d^4x\, dM^2} = \int \frac{d^3q}{(2\pi)^3} \frac{M}{E_q} \Gamma_{\rho \to e^+e^-} \mathcal{A} f_\rho,$$

where $\mathcal{A} = 2\mathcal{A}_T + \mathcal{A}_L$, $E_q = \sqrt{M^2 + \mathbf{q}^2}$, $\Gamma_{\rho \to e^+e^-}$ is the width for the process $\rho \to e^+e^-$, and f_ρ is the thermal Bose-Einstein distribution of ρ mesons.

- We include the effects of the *expansion* of the medium formed in a relativistic heavy-ion collision

- The kinematic constraint of the CERES experiment are crucial

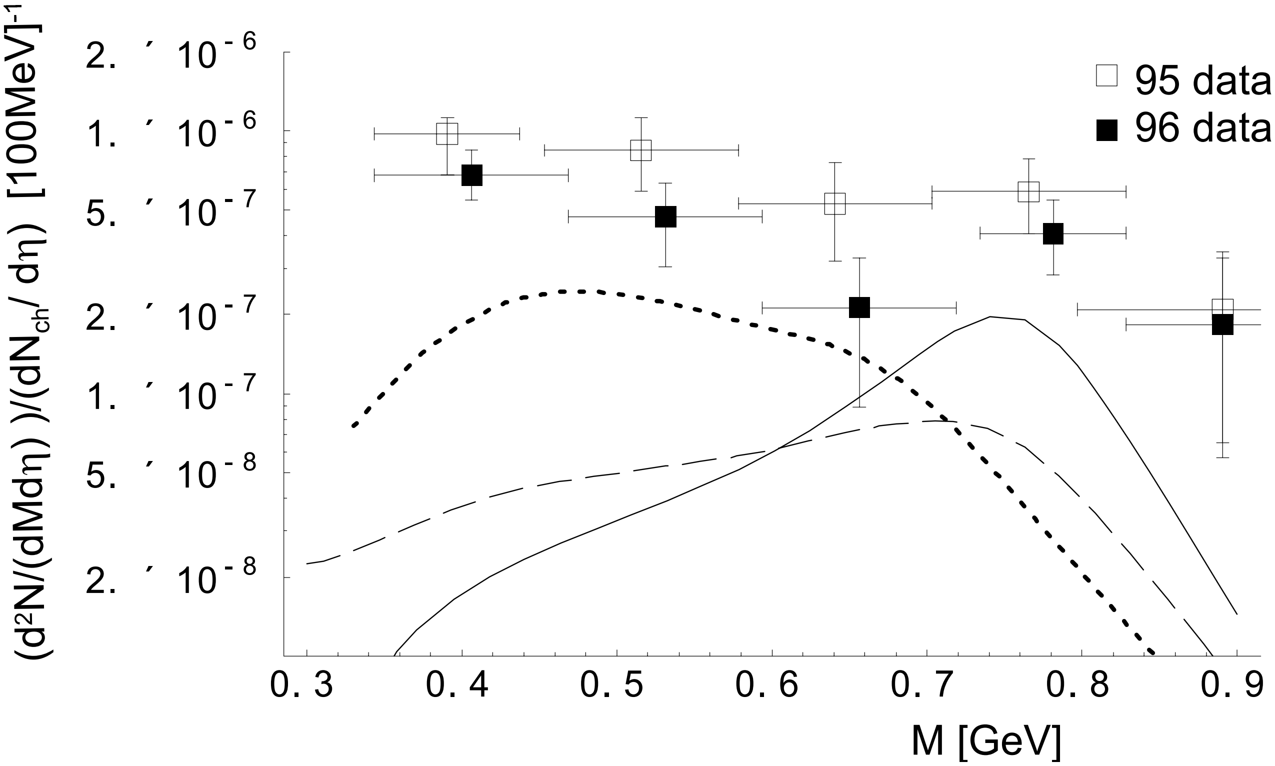

Figure 8: Dilepton yields for the 158GeV/A $Pb + Au$ CERES experiment from the ρ decays. The solid line is the result of the calculation with the vacuum ρ spectral function. The dashed line is obtained with the medium-modified spectral strength, and unchanged peak position. The dotted line is obtained with the medium-modified spectral strength, and the peak position m_ρ lowered: $m^*_\rho = (1 - 0.2\frac{\rho_B}{\rho_0})m_\rho$.

$\omega \to \pi\pi$ decay in medium

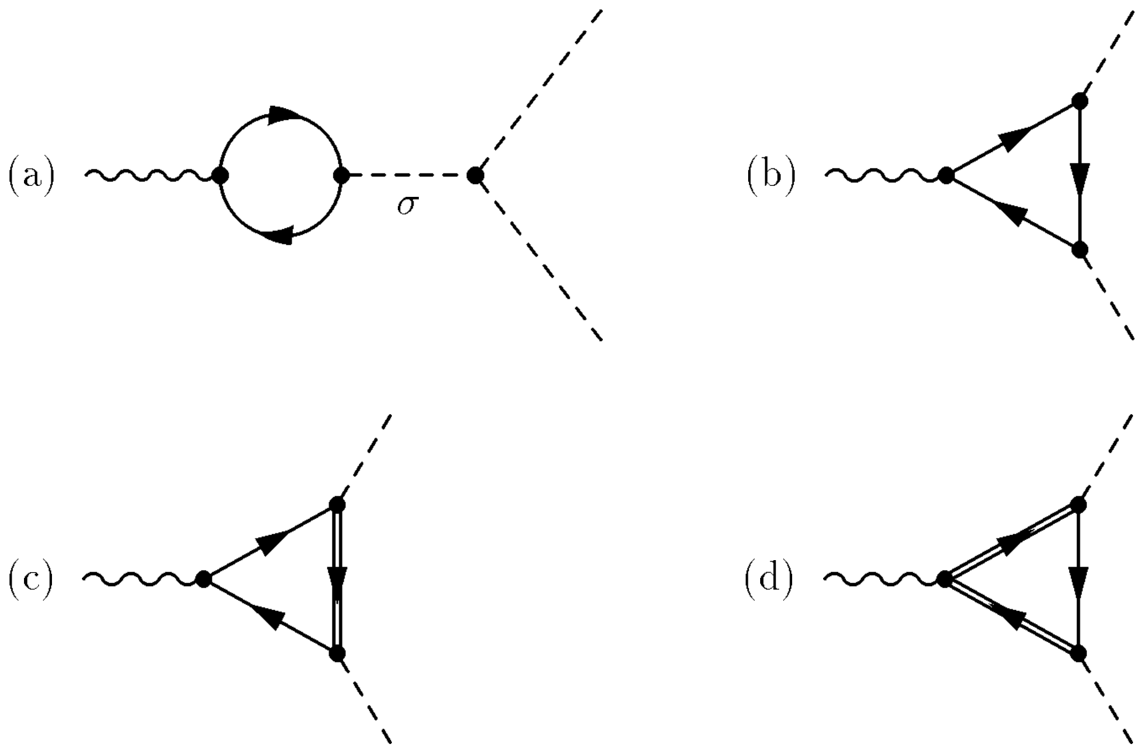

Figure 9: Diagrams contributing to the $\omega \to \pi\pi$ amplitude in nuclear medium

(Figure)

- Large longitudinal width!

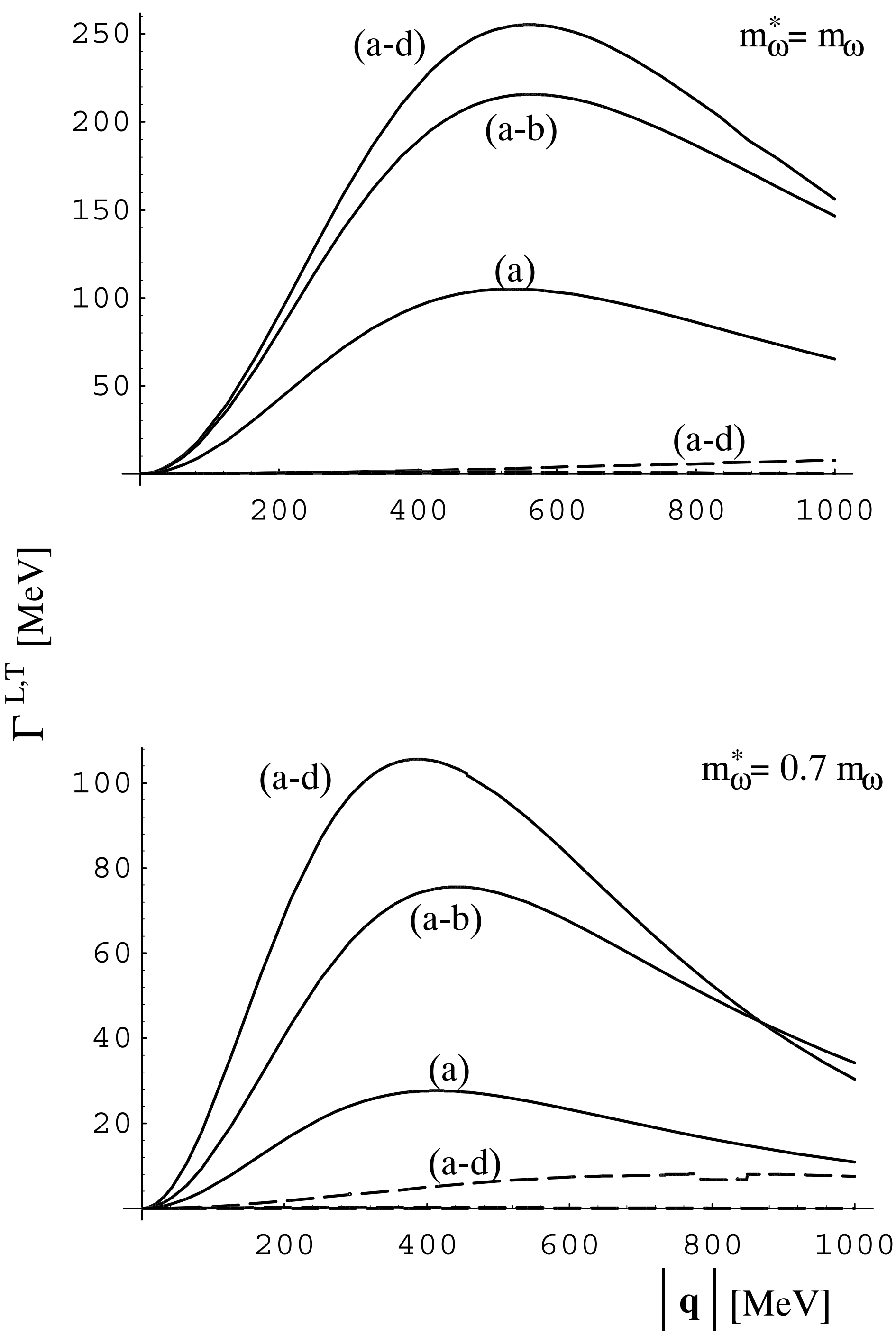

Figure 10: The in-medium width of the ω meson at the saturation density plotted as a function $|\vec{q}|$. The solid (dashed) lines correspond to the longitudinal (transverse) mode. Labels refer to Fig. 9

$\rho\omega\pi$ vertex in medium

Work with Anna Baran and Agnieszka Bieniek

- Cancellations between N and Δ diagrams in $\pi^0 \to \gamma\gamma$. Without Δ drop of $g_{\rm eff}$ to 75%, of the vacuum value, with delta to 95%
- In $\pi^0 \to \gamma e^+ e^-$ only up to 10% deviation from the vacuum
- In $\rho^0, \omega \to \pi^0 e^+ e^-$ analytic structure visible in $M_{e^+e^-}$ at

$$\sqrt{m_v m_N + m_\pi^2 + m_v^2 - \frac{m_v(m_\Delta^2 - m_\pi^2)}{m_N}} \simeq 340\text{MeV}$$

- Moderate decrease of $\omega \to \pi^0 e^+ e^-$
- Enhancement of $\rho^0 \to \pi^0 e^+ e^-$ by a factor of 2.5

Conclusions

- Large medium effects
- Δ very important
- Dependance on $\vec{q}$ - do not neglect!
- The dilepton problem remains open