

Production of resonances in a thermal model

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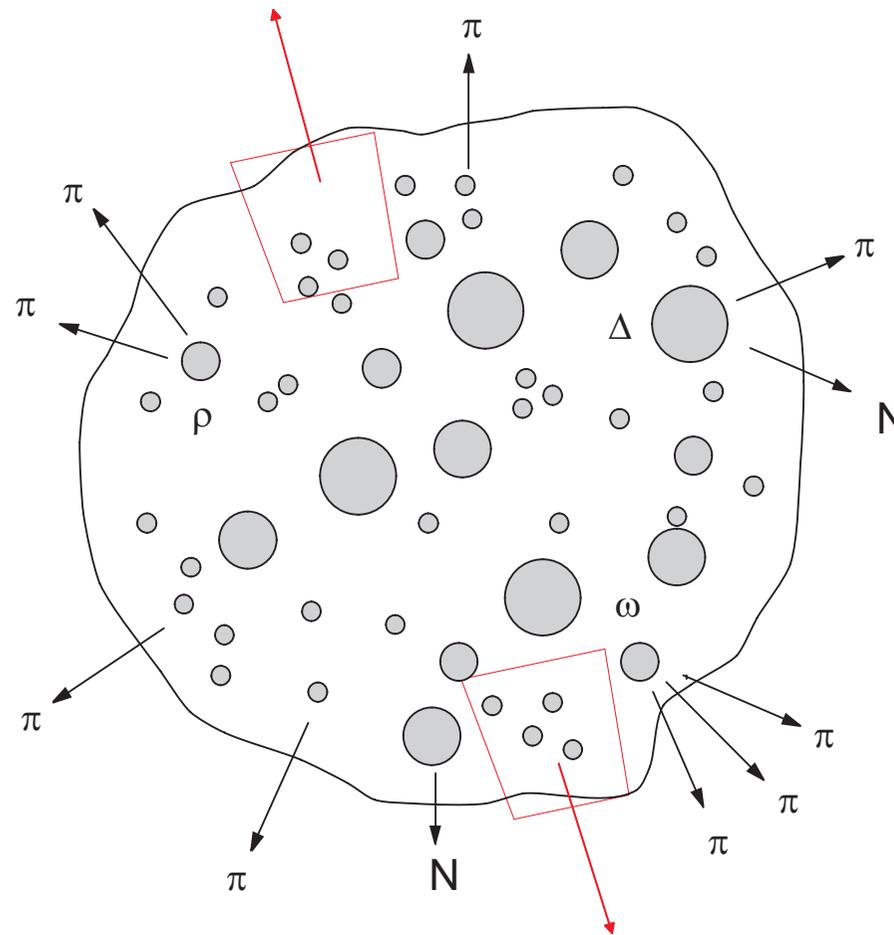
WB + Wojciech Florkowski, PRL 87 (2001) 272302; PRC 65 (2002) 064905

WB+ Anna Baran + WF, Acta Phys. Polon. B33 (2002) 4235

WB+ WF+ Brigitte Hiller (Coimbra), PRC 68 (2003) 034911

Thermal models

Koppe (1948), Fermi (1950), Landau, Hagedorn, Rafelski, Bjorken, Heinz, Sollfrank, Wiedemann, Gaździcki, Bugaev, Gorenstein, Braun-Munzinger, Stachel, Cleymans, Redlich, Buda-Lund, Akkelin, Sinyukov, Becattini, Cleymans, Letessier, (many more)



$$\sim e^{-(E-\mu)/T}$$

Specific features of our approach

1. Single freezeout approximation: $T_{\text{chem}} = T_{\text{kin}} \equiv T$, single freeze-out.

A radical simplification, supported by the RHIC HBT results:

$$R_{\text{out}}/R_{\text{side}} \sim 1,$$

$R_{\text{side}}(\phi)$ has out-of-plane elongation,

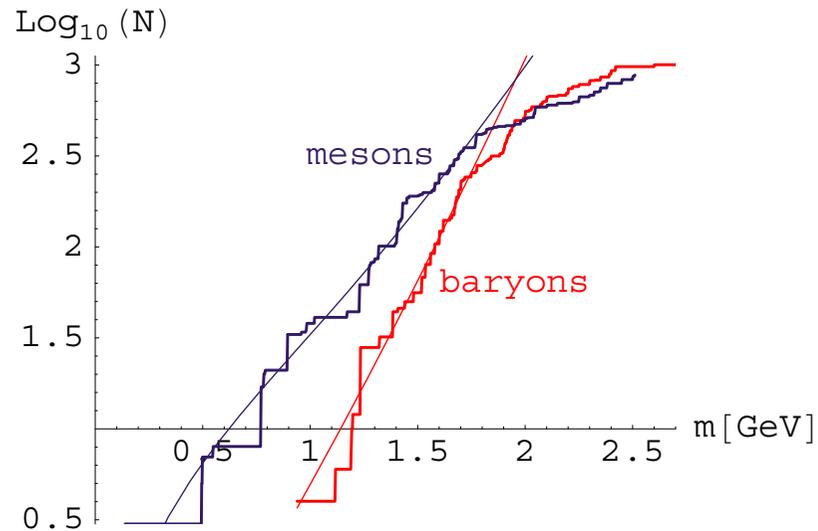
resonances seen abundantly

→ short time (< 1 fm/c) between the freeze-outs (explosive scenario).

T and μ_B are fitted from ratios of dN/dy at midrapidity

2. Ockham razor: No γ -factors for strangeness (Rafelski), excluded-volume effects (Gorenstein), canonical (Redlich) or microcanonical ensemble (Becattini), minimum number of parameters

3. Hagedorn: Complete treatment of resonances (important due to the Hagedorn-like exponential growth of the number of states)



(from WB+WF, PLB 490 (2000) 223)

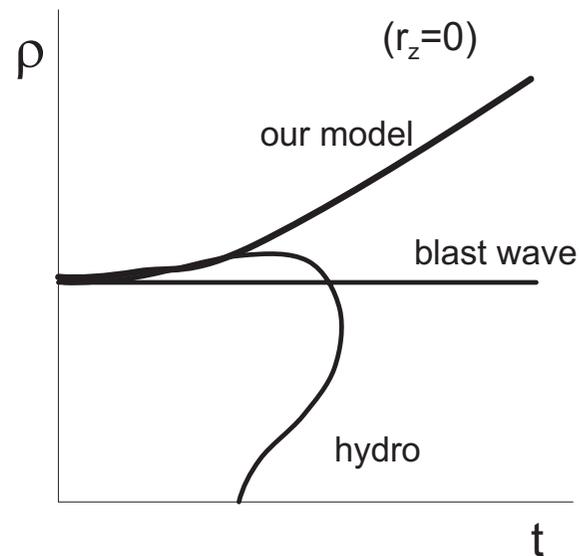
75% of pions and protons come from decays of higher states, 80% of Λ 's, 60% of Ξ 's, 30% of ρ_0 's, . . . !

Resonances mimic the interactions among hadrons

4. Geometry and flow: We **take** the hypersurface (inspired by the Bjorken and Buda-Lund models) of the form

$$\tau = \sqrt{t^2 - r_z^2 - r_x^2 - r_y^2} = \text{const}$$

and constrain the transverse size, $\rho = \sqrt{r_x^2 + r_y^2} < \rho_{\text{max}}$.



The Buda-Lund choice helps to resolve the HBT problem ($R_{\text{out}}/R_{\text{side}} \sim 1$)

The geometric parameters τ and ρ_{\max} , of the order of a few fm, are fitted to the p_{\perp} -spectra (τ^3 is the overall normalization constant, ρ_{\max} controls the slopes). The hydrodynamic four-velocity is (Hubble law)

$$u^{\mu} = \partial^{\mu} \tau = \frac{x^{\mu}}{\tau} = \frac{t}{\tau} \left(1, \frac{r_z}{t}, \frac{r_x}{t}, \frac{r_y}{t} \right)$$

Boost invariance is a good approximation for midrapidity

Other choices can be tested (Heinz+Sollfrank+Wiedemann, Torrieri+Rafelski)

Our model has altogether 4 parameters

Comments:

many fits adjust the norms

large flow in short time, stiff eq. of state

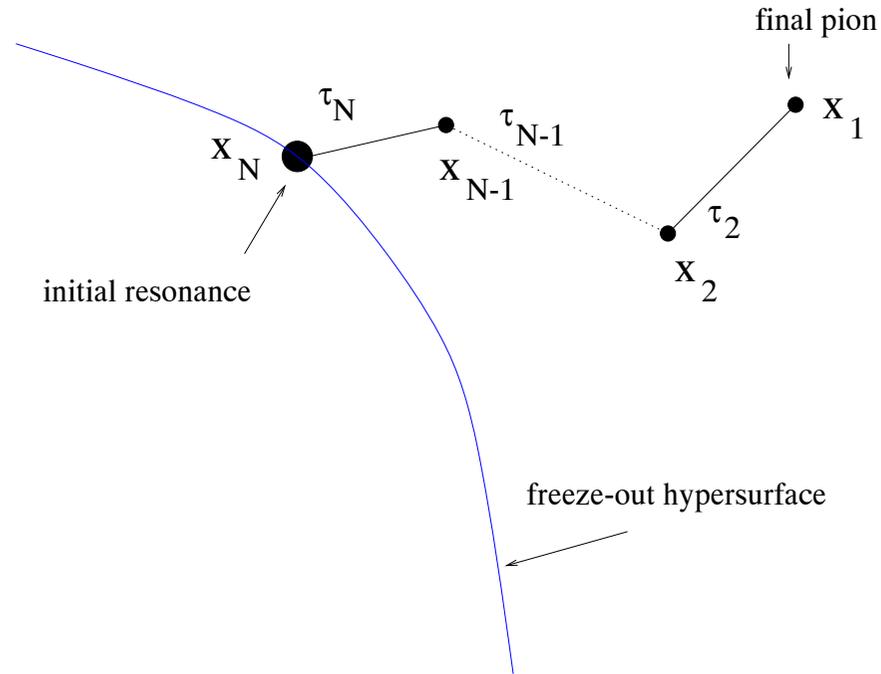
Ratios

For a boost-invariant model $\frac{dN_i/dy}{dN_j/dy} = \frac{N_i}{N_j}$ and ratios do not depend on geometry/flow.

$\sqrt{s_{NN}}$ [GeV]	130	200
T [MeV]	165 ± 7	160 ± 5
μ_B [MeV]	41 ± 5	26 ± 4
χ^2/DOF	1.0	1.5

@ 200 GeV	Model	Experiment
π^-/π^+	1.009 ± 0.003	$1.025 \pm 0.006 \pm 0.018$ $1.02 \pm 0.02 \pm 0.10$
K^-/K^+	0.939 ± 0.008	$0.95 \pm 0.03 \pm 0.03$ $0.92 \pm 0.03 \pm 0.10$
\bar{p}/p	0.74 ± 0.04	$0.73 \pm 0.02 \pm 0.03$ $0.70 \pm 0.04 \pm 0.10$ 0.78 ± 0.05
\bar{p}/π^-	0.104 ± 0.010	0.083 ± 0.015
K^-/π^-	0.174 ± 0.001	0.156 ± 0.020
$\Omega/h^- \times 10^3$	0.990 ± 0.120	$0.887 \pm 0.111 \pm 0.133$
$\bar{\Omega}/h^- \times 10^3$	0.900 ± 0.124	$0.935 \pm 0.105 \pm 0.140$

Resonance decays in p_{\perp} -spectra



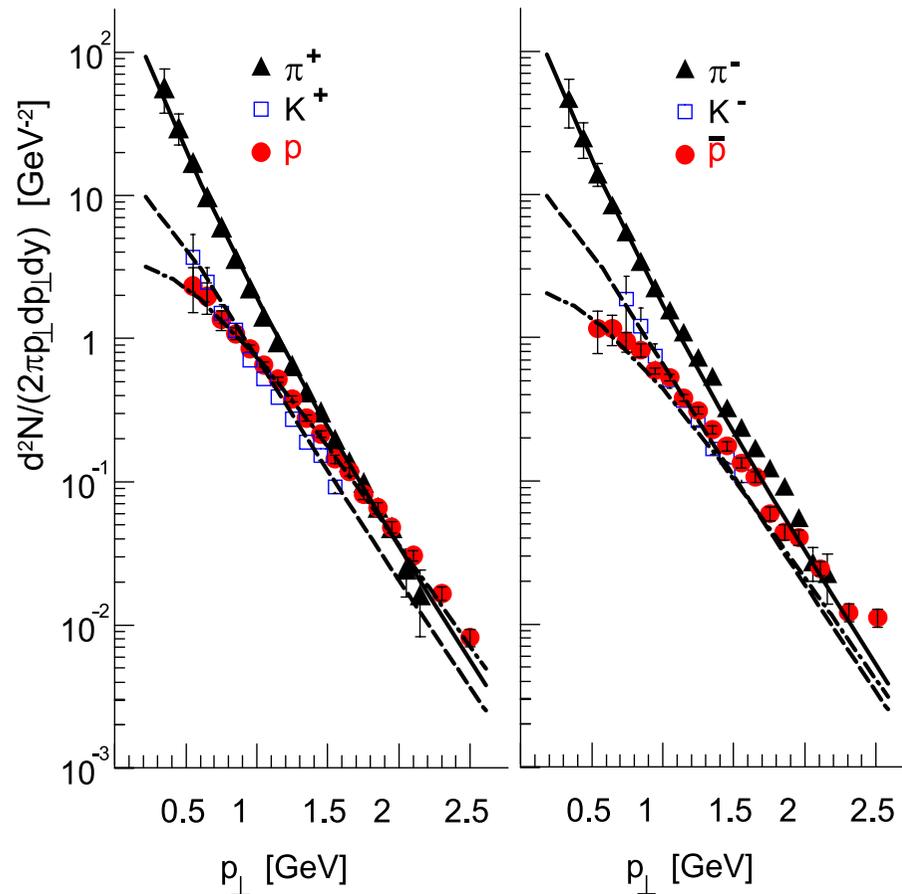
The integration over $x_{N-1} \dots x_2$ is unconstrained, while the integration over x_N is constrained to the hypersurface Σ .

$$E_{p_1} \frac{dN_1}{d^3p_1} = \int \frac{d^3p_2}{E_{p_2}} B(p_2, p_1) \dots \int \frac{d^3p_N}{E_{p_N}} B(p_N, p_{N-1}) \int d\Sigma_{\mu}(x_N) p_N^{\mu} f_N[p_N \cdot u(x_N)]$$

$$B(p_i, p_{i-1}) = \frac{b}{4\pi p_{i-1}^*} \delta\left(\frac{p_i \cdot p_{i-1}}{m_i} - E_{i-1}^*\right)$$

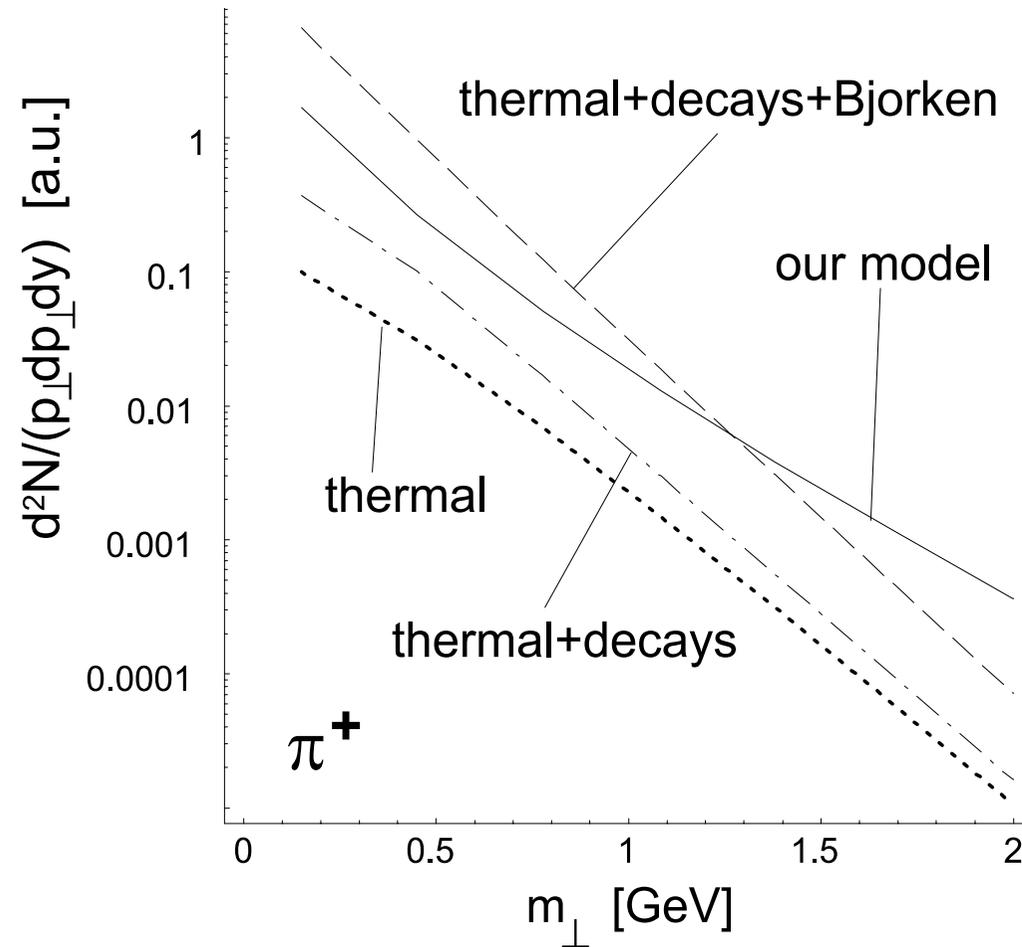
f is any distribution, not necessarily thermal, e.g. from coalescence, ...

Results for the transverse-momentum spectra

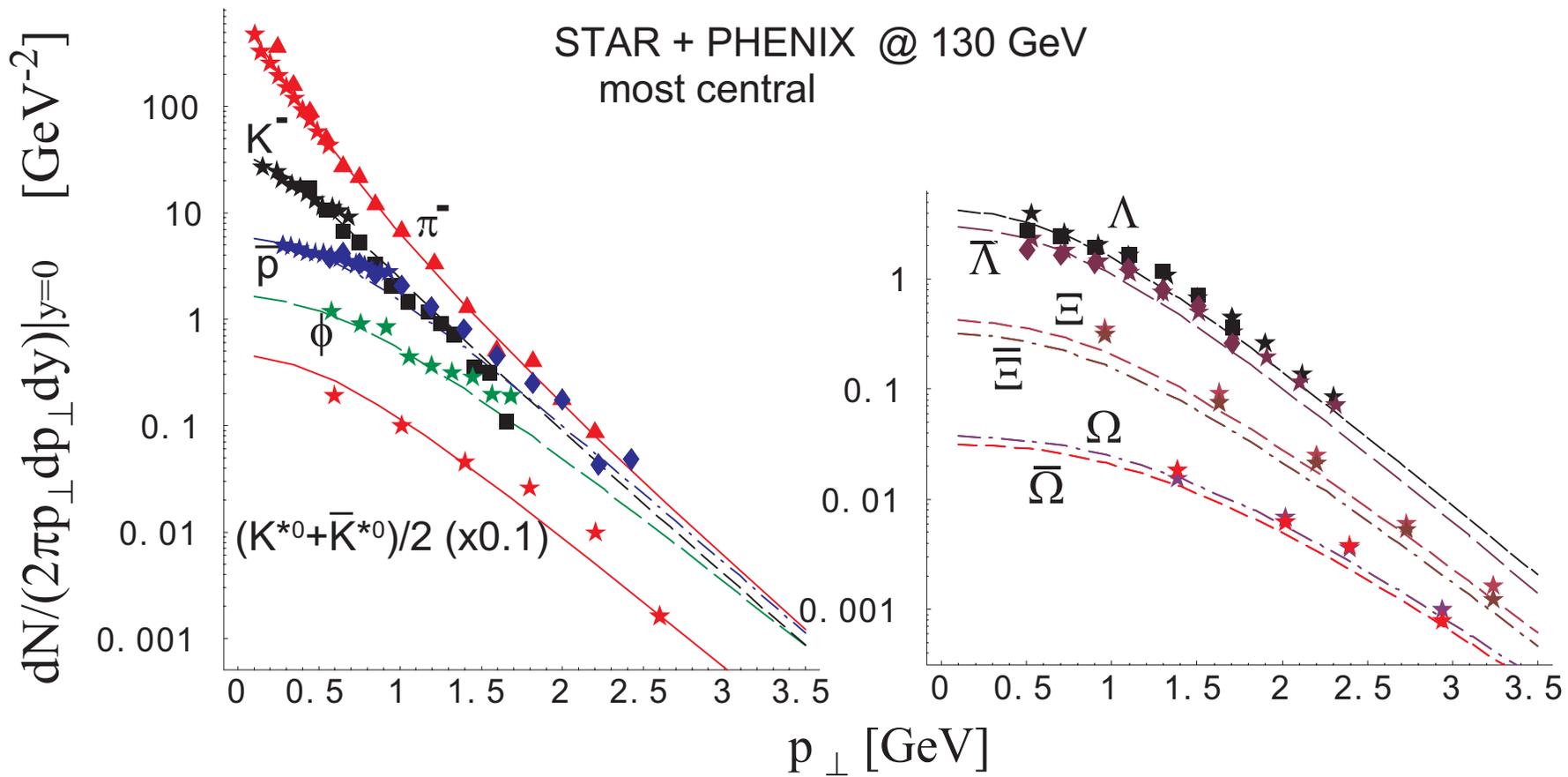


Min. bias p_{\perp} -spectra of pions, kaons, protons and antiprotons as evaluated from our model with $\tau = 6$ fm, $\rho_{\text{max}}/\tau = 0.76$, compared to the earliest PHENIX data (Velkovska, nucl-ex/0105012). Very good agreement up to $p_{\perp} \sim 2$ GeV. At larger values, where hard processes enter, the model falls below the data

“Cooling” via decays



Resonance decays lower the inverse slope by about 30 MeV



$(T = 165 \text{ MeV})$

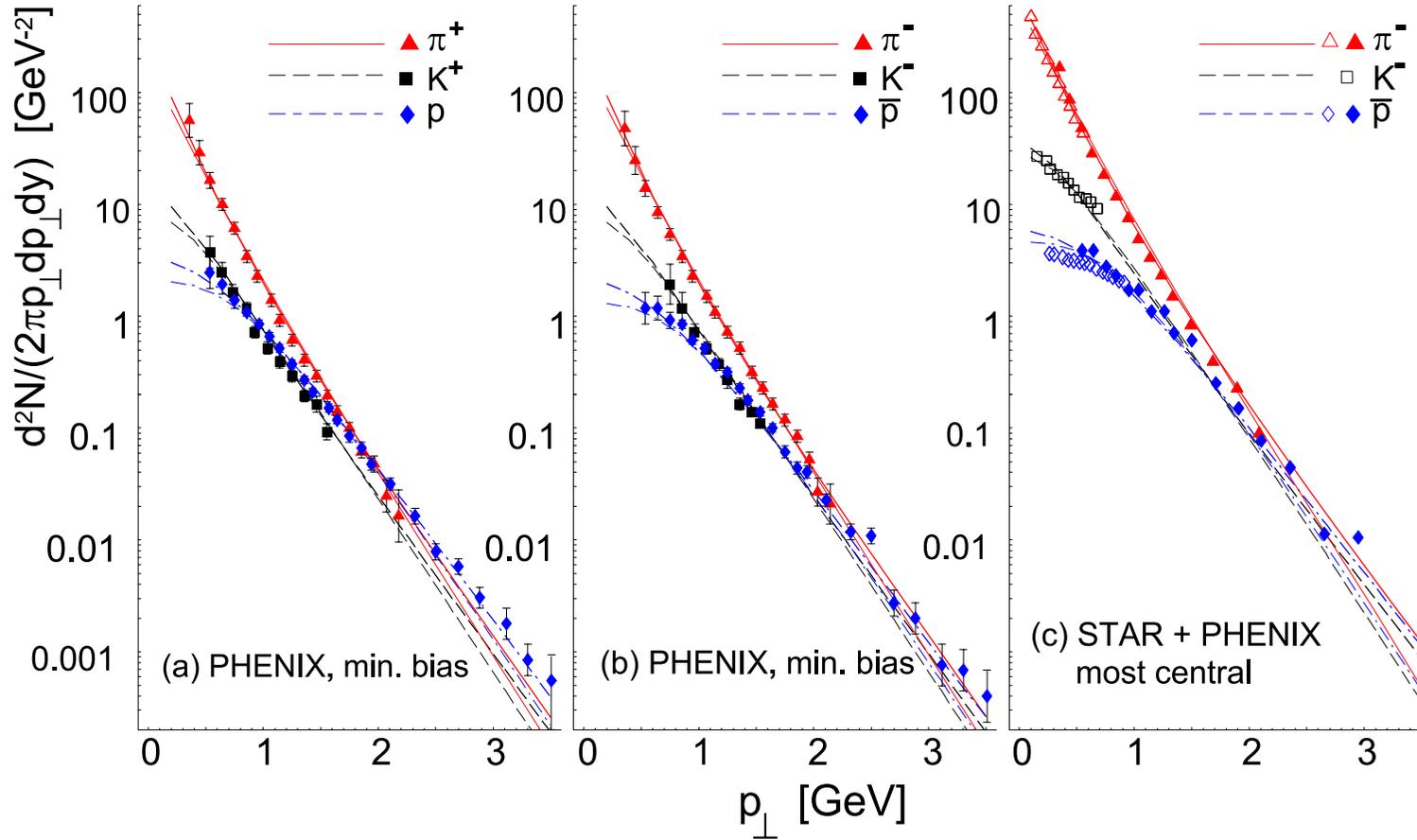
ϕ – very weak interactions, serves as a thermometer

K^{*} – resonance, lower T would lead to much less K^{*} 's

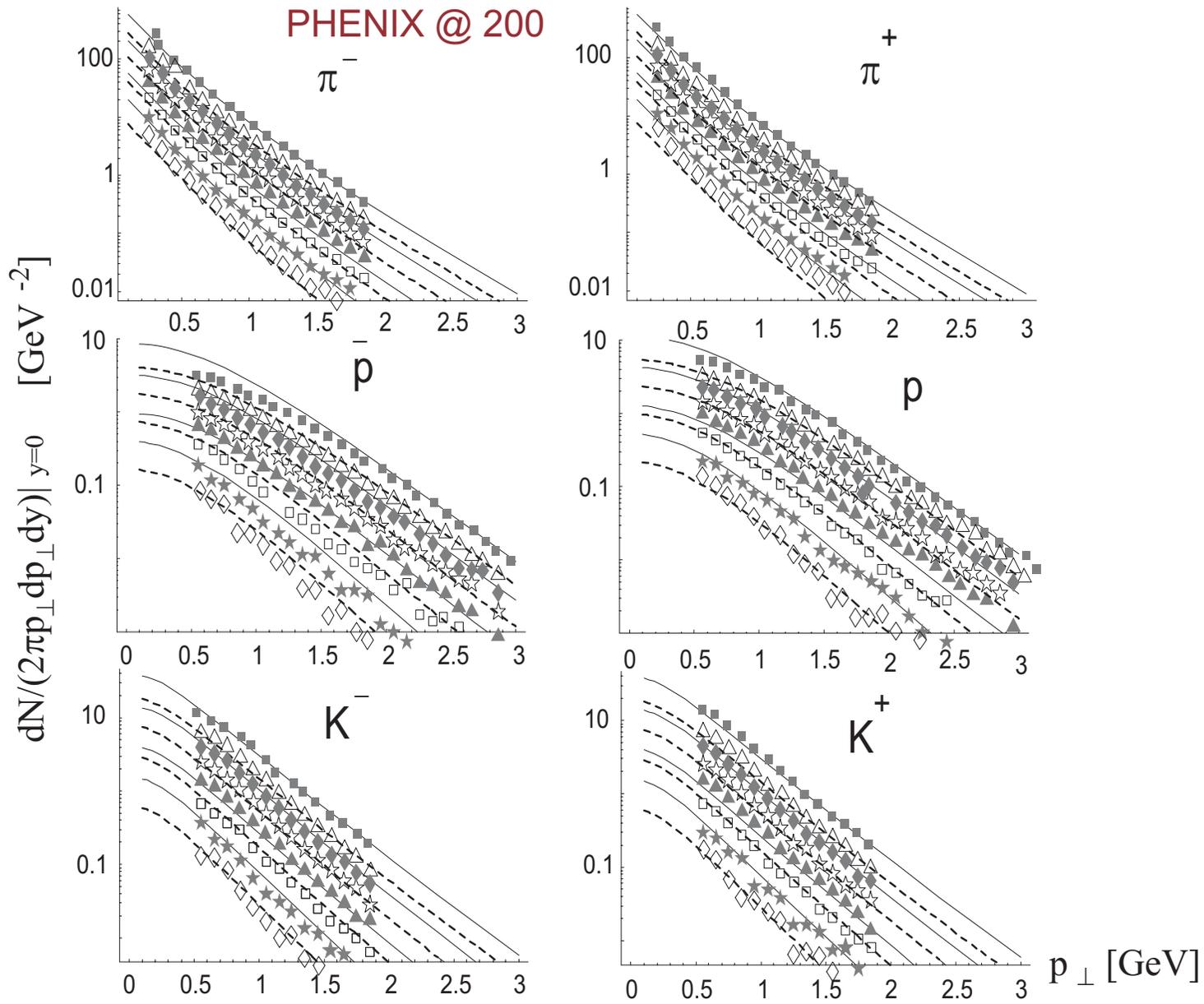
(experimental Ξ 's went down by \sim a factor of 2)

No special treatment of Ω 's

Two different expansion models



thick: present model, thin: blast-wave (from WB+WF, PRL 87 (2001) 272302)

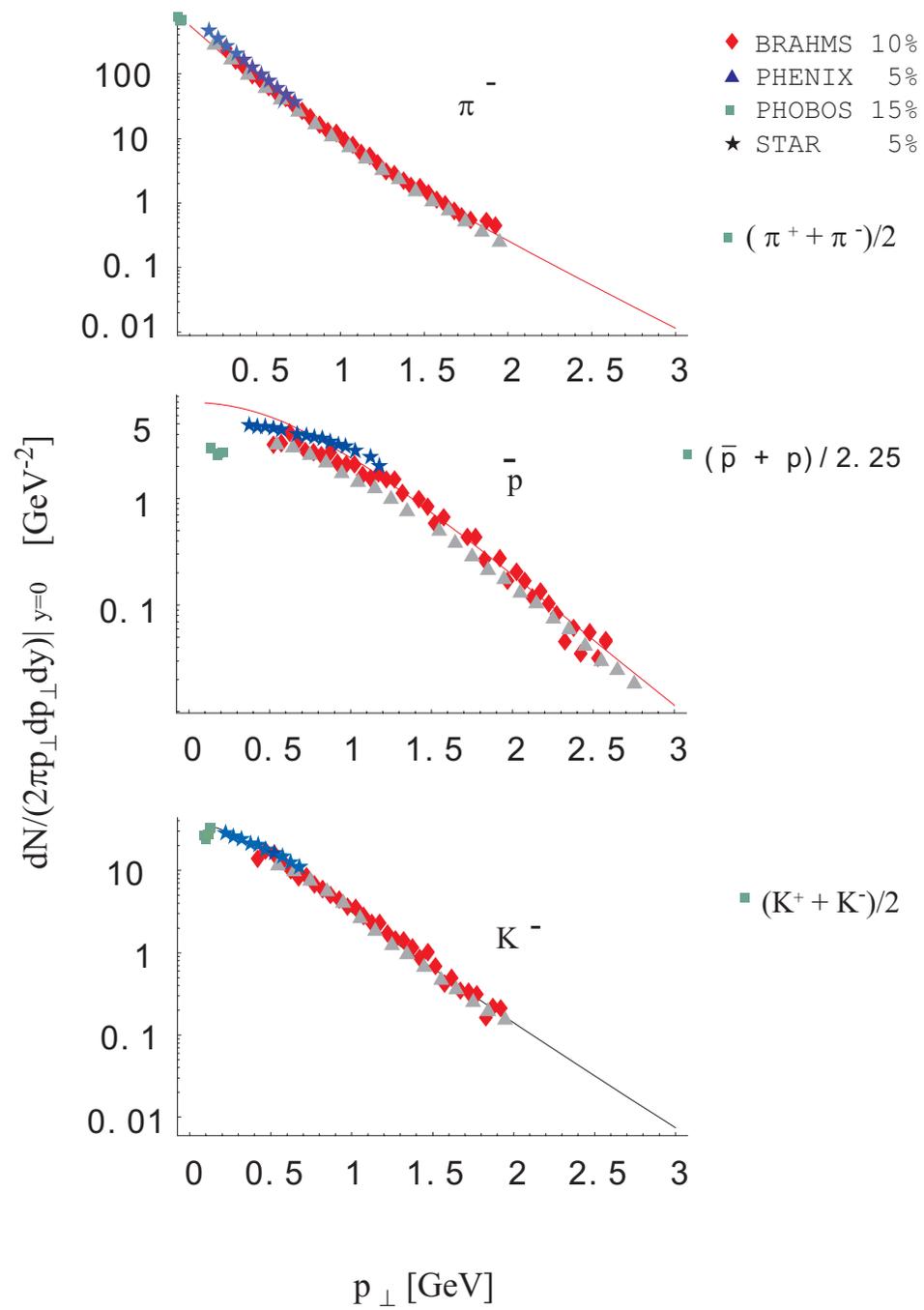


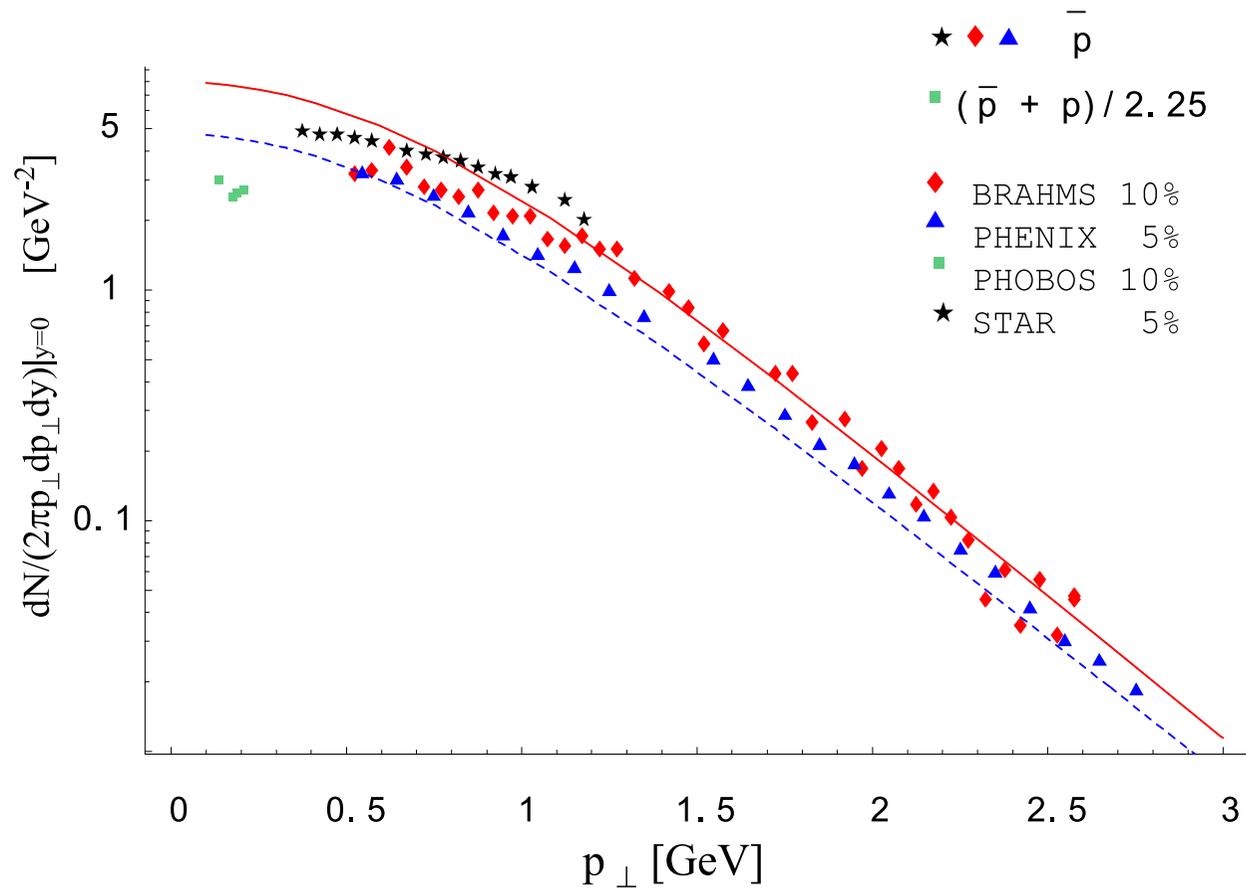
(data at different centrality, or impact parameter)

Centrality c is defined as a percentage of the most central events. To a **very good** accuracy

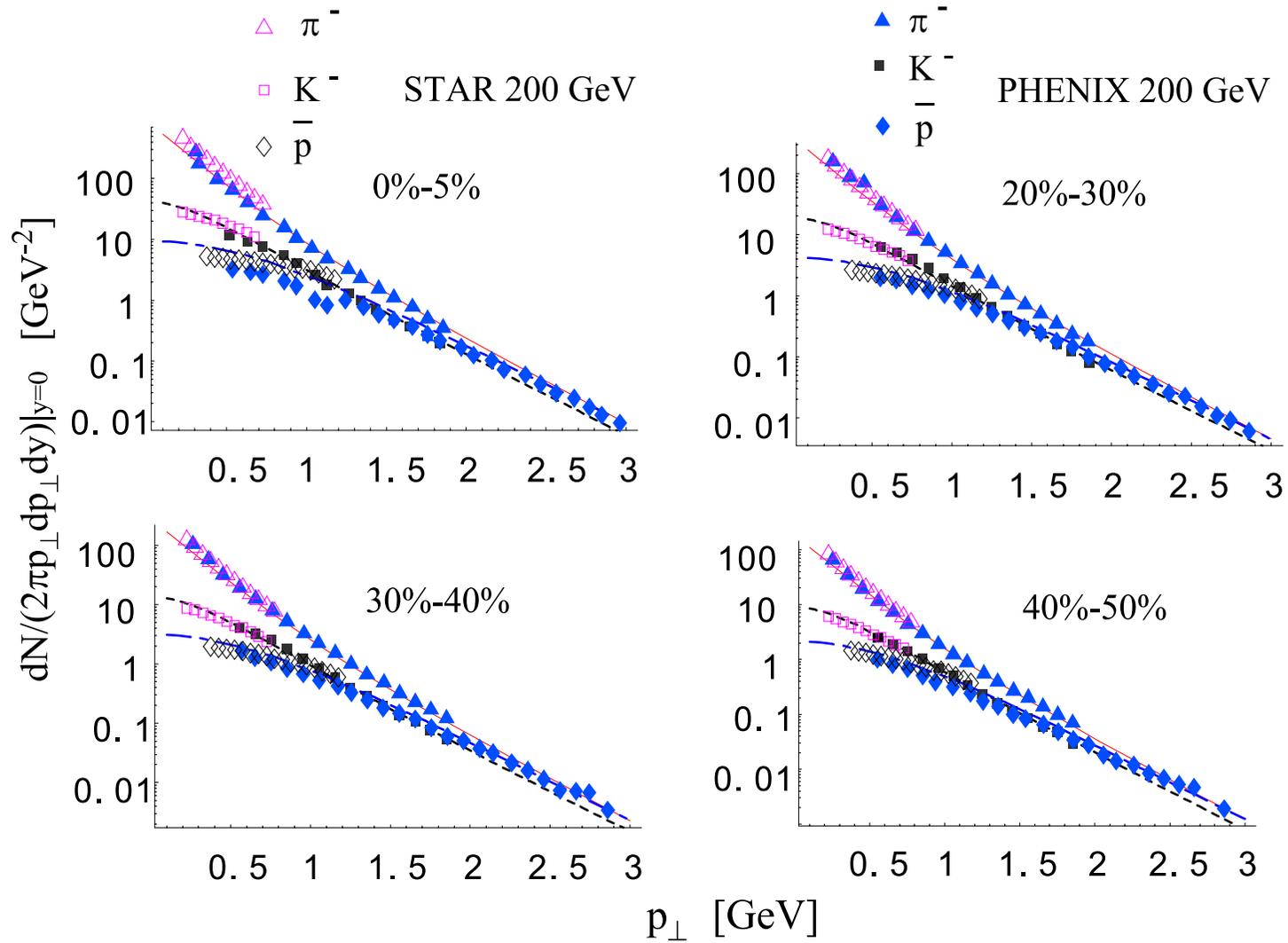
$$c \simeq \frac{\pi b^2}{\sigma_{\text{inel}}^{\text{tot}}} \simeq \frac{b^2}{4R^2}$$

(WB+WF, PRC 65 (2002) 024905)





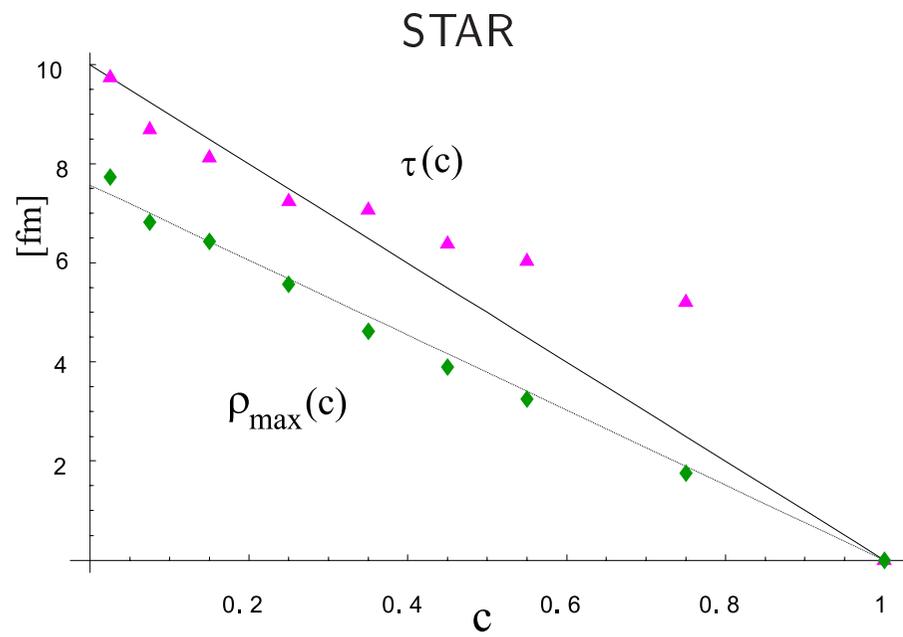
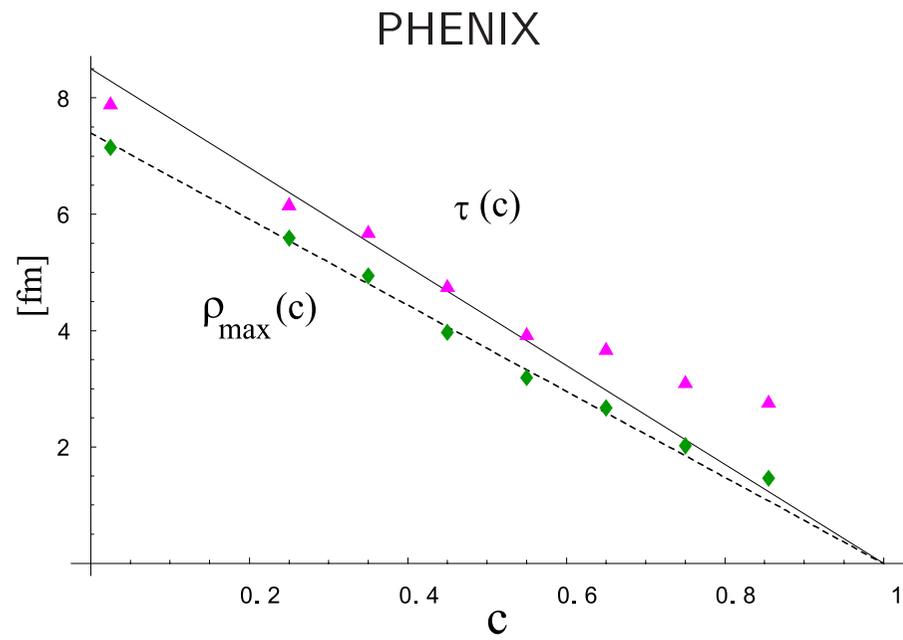
(solid – full feeding, dashed – no feeding from weak decays)



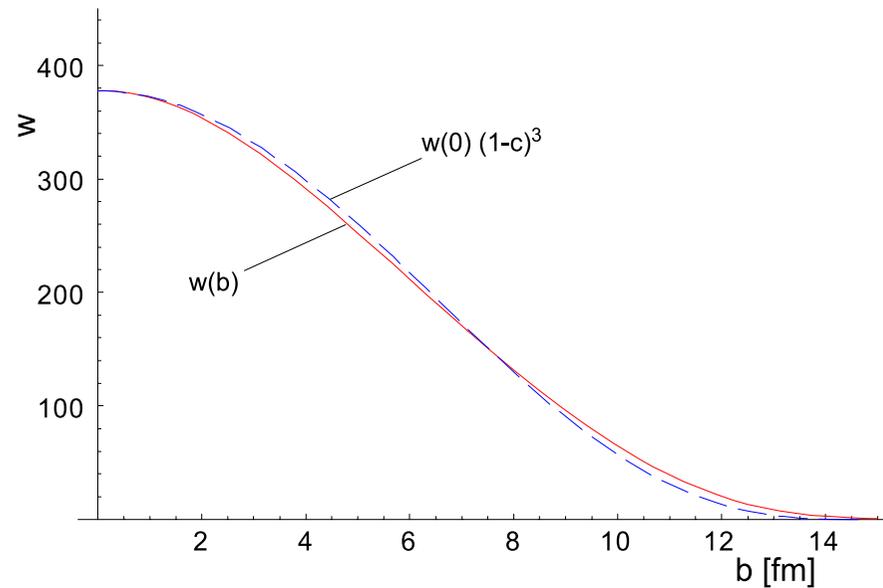
(\bar{p} from STAR more flat than from PHENIX)

Compilation of geometric parameters (by A. Baran)

	c [%]	τ [fm] (norm)	ρ_{\max} [fm]	$\langle\beta_{\perp}\rangle$ (slope)
BRAHMS	10	7.68 ± 0.19	7.46 ± 0.05	0.52 ± 0.01
STAR	0 – 5	9.74 ± 1.57	7.74 ± 0.68	0.45 ± 0.08
	5 – 10	8.69 ± 1.39	7.18 ± 0.64	0.47 ± 0.08
	10 – 20	8.12 ± 1.31	6.44 ± 0.57	0.45 ± 0.08
	20 – 30	7.24 ± 1.18	5.57 ± 0.50	0.44 ± 0.08
	30 – 40	7.07 ± 1.17	4.63 ± 0.39	0.39 ± 0.08
	40 – 50	6.38 ± 1.02	3.91 ± 0.33	0.37 ± 0.07
	50 – 60	6.19 ± 1.09	3.25 ± 0.28	0.32 ± 0.07
	70 – 80	5.48 ± 0.81	4.03 ± 0.10	0.43 ± 0.06
PHENIX	0 – 5	7.86 ± 0.38	7.15 ± 0.13	0.50 ± 0.02
	20 – 30	6.14 ± 0.32	5.62 ± 0.11	0.50 ± 0.02
	30 – 40	5.73 ± 0.16	4.95 ± 0.05	0.48 ± 0.01
	40 – 50	4.75 ± 0.28	3.96 ± 0.09	0.47 ± 0.03
	50 – 60	3.91 ± 0.23	3.12 ± 0.07	0.45 ± 0.03
	60 – 70	3.67 ± 0.12	2.67 ± 0.03	0.42 ± 0.01
	70 – 80	3.09 ± 0.11	2.02 ± 0.02	0.39 ± 0.01
	80 – 91	2.76 ± 0.20	1.43 ± 0.03	0.32 ± 0.03

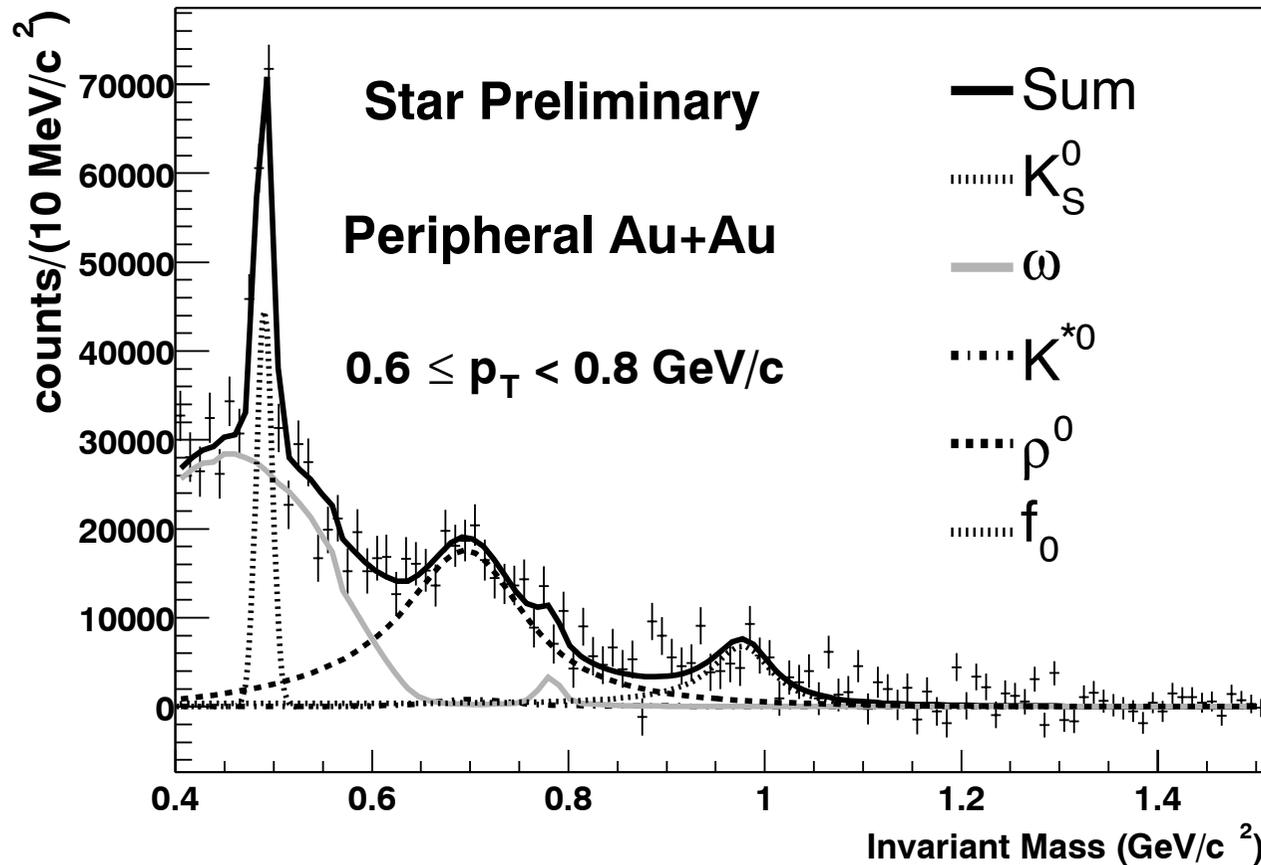


Wounded-nucleon scaling



The number of wounded nucleons, $w(b)$ (solid line) and the approximating function $w(0)(1 - c(b))^3$ (dashed line), are plotted as functions of the impact parameter b . Since the multiplicity of hadrons produced in our model is proportional to $(1 - c)^3$ at low and moderate values of c , the model conforms to the wounded-nucleon scaling

$\pi^+\pi^-$ pairs from STAR



(from J. Adams et al., nucl-ex/0307023; P. Fachini, nucl-ex/0305034)

(Brown+Shuryak, Kolb-Prakash, Rapp, Pratt+Bauer)

The phase-shift formula for the density of resonances

Resonances provide kinematic correlations

Beth,Uhlenbeck (1937); Dashen, Ma, Bernstein, Rajaraman (1974); **Weinhold (1998)**,
Friman, Nörenberg; **WB, WF, B. Hiller**, PRC **68** (2003) 034911; Pratt, Bauer,
nucl-th/0308087

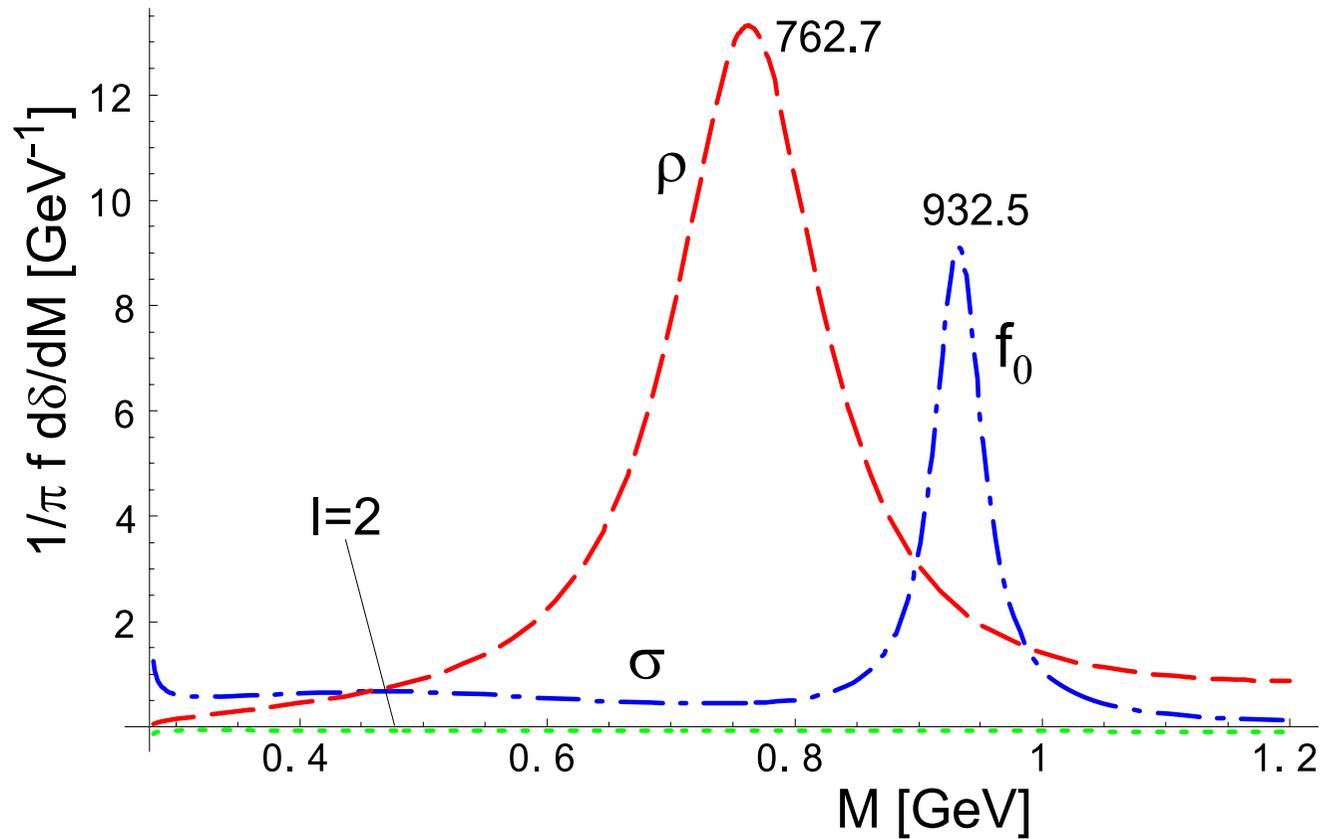
$$\frac{dn}{dM} = f \int \frac{d^3p}{(2\pi)^3} \frac{d\delta_{\pi\pi}(M)}{\pi dM} \frac{1}{\exp\left(\frac{\sqrt{M^2+p^2}}{T}\right) \pm 1}$$

For narrow resonances $d\delta(M)/dM \simeq \pi\delta(M - m_R)$, and

$$n^{\text{narrow}} = f \int \frac{d^3p}{(2\pi)^3} \frac{1}{\exp\left(\frac{\sqrt{m_R^2+p^2}}{T}\right) \pm 1}$$

In a thermal system the density of states changes \rightarrow phase shifts appear (not the spectral function) [S. Pratt, Warsaw Meeting on Particle Correlations, 2003]

$d\delta_{\pi\pi}(M)/dM$ from experiment

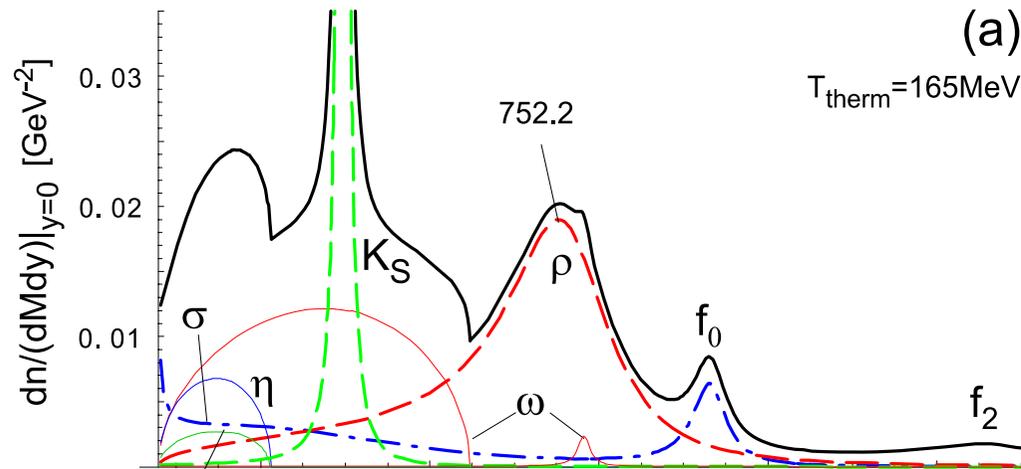


Small contribution from σ , negative and tiny contribution from $I = 2$, ρ -peak slightly shifted to lower M , $1/\sqrt{M - 4m_\pi^2}$ behavior for the σ

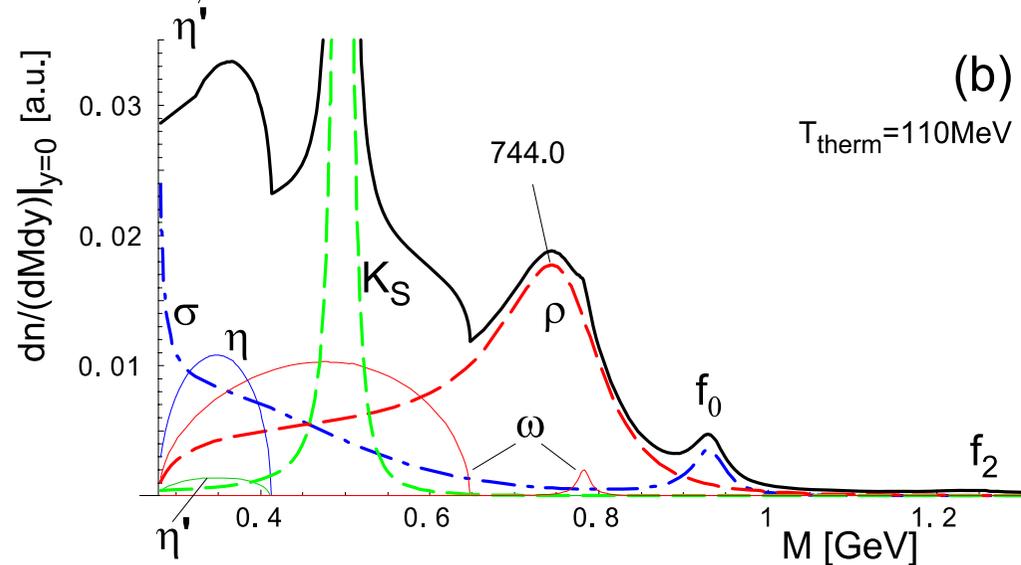
Warm-up calculation - static source

We compute the spectra at mid-rapidity, hence

$$\left. \frac{dn}{dMdy} \right|_{y=0} = \sum_i f_i \int_{0.2\text{GeV}}^{2.2\text{GeV}} \frac{p_{\perp} dp_{\perp}}{(2\pi)^2} \frac{d\delta_i(M)}{\pi dM} \frac{\sqrt{M^2 + p_{\perp}^2}}{\exp\left(\frac{\sqrt{M^2 + p_{\perp}^2}}{T}\right) - 1}$$



flat

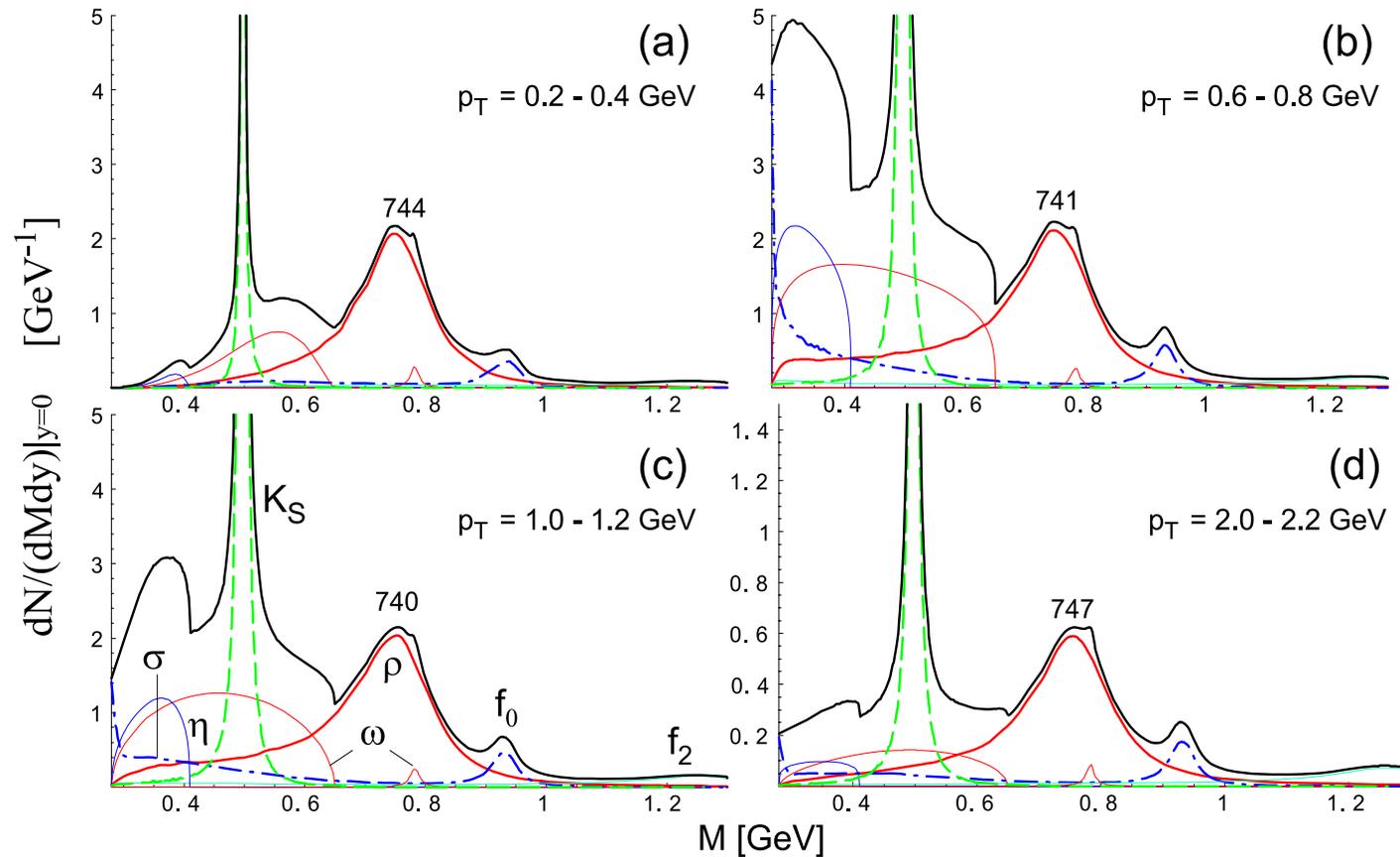


steep

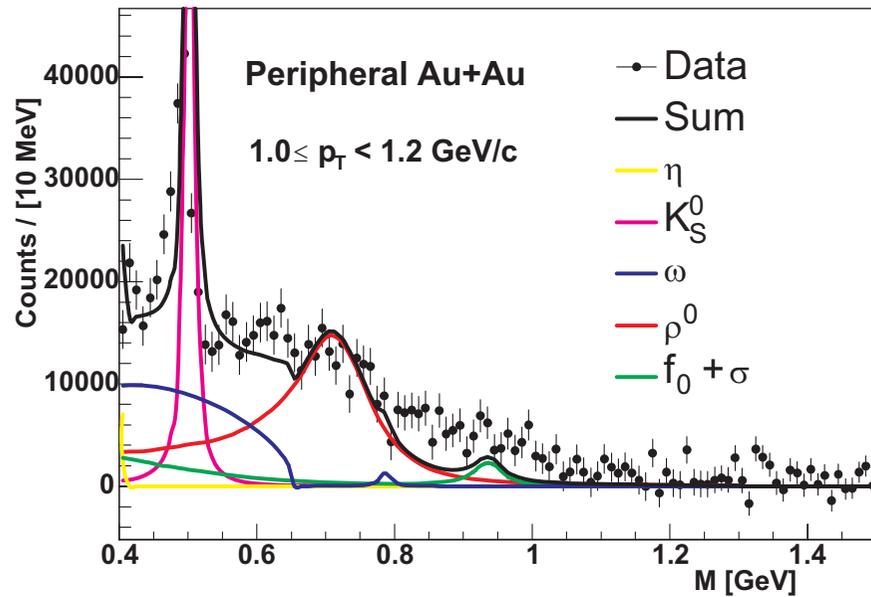
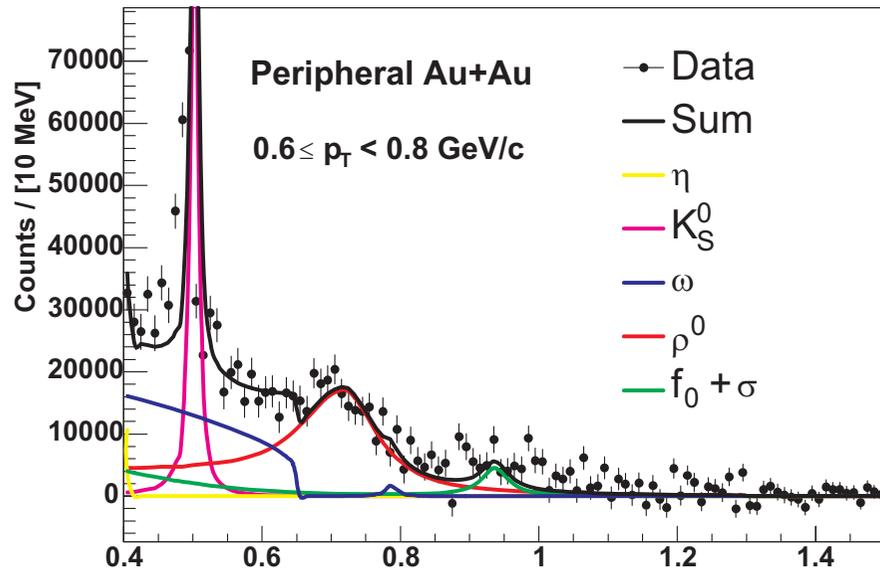
Cuts/flow + feeding from resonances

Flow has no effect on the invariant mass of a pair of particles produced in a resonance decay, since the quantity is Lorentz-invariant. Nevertheless, it affects the results since the kinematic cuts in an obvious manner break this invariance

~ 30 MeV shift of the ρ peak

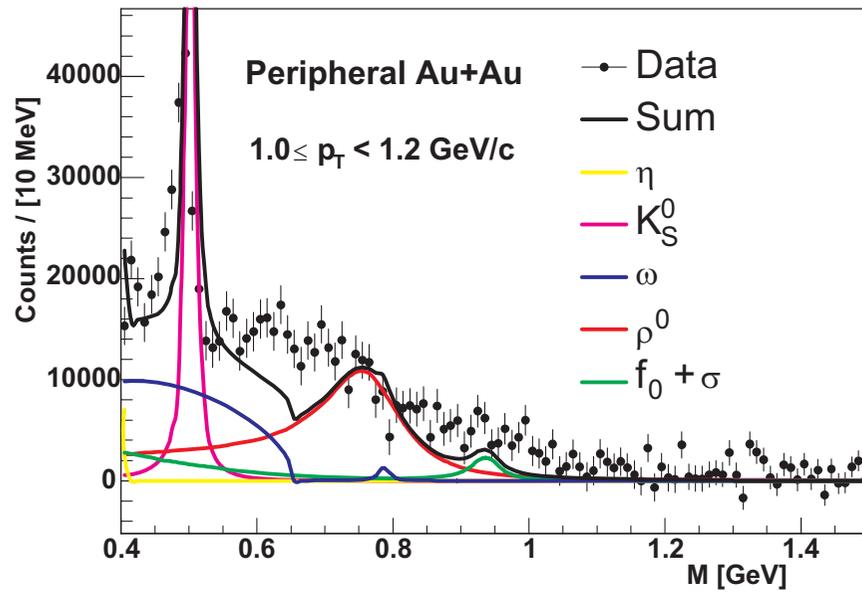
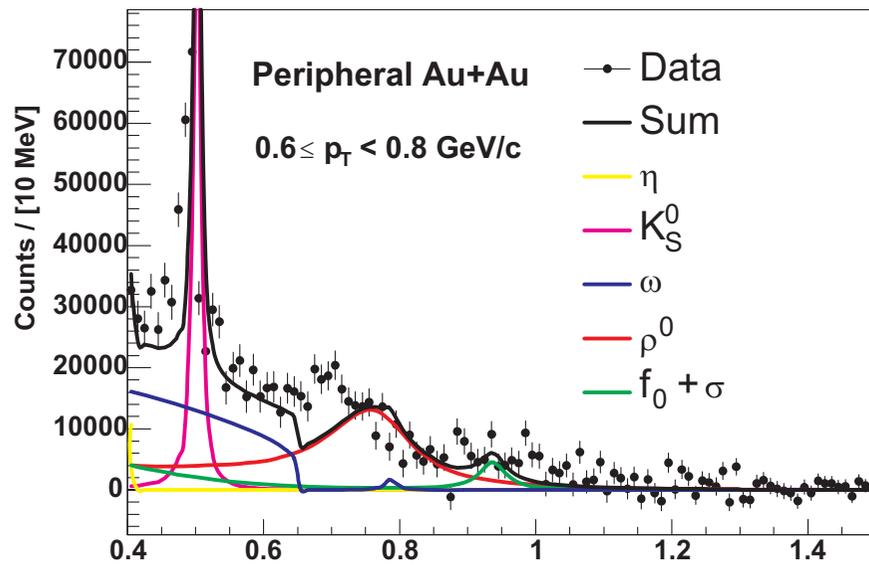


STAR vs. thermal model, lowered ρ



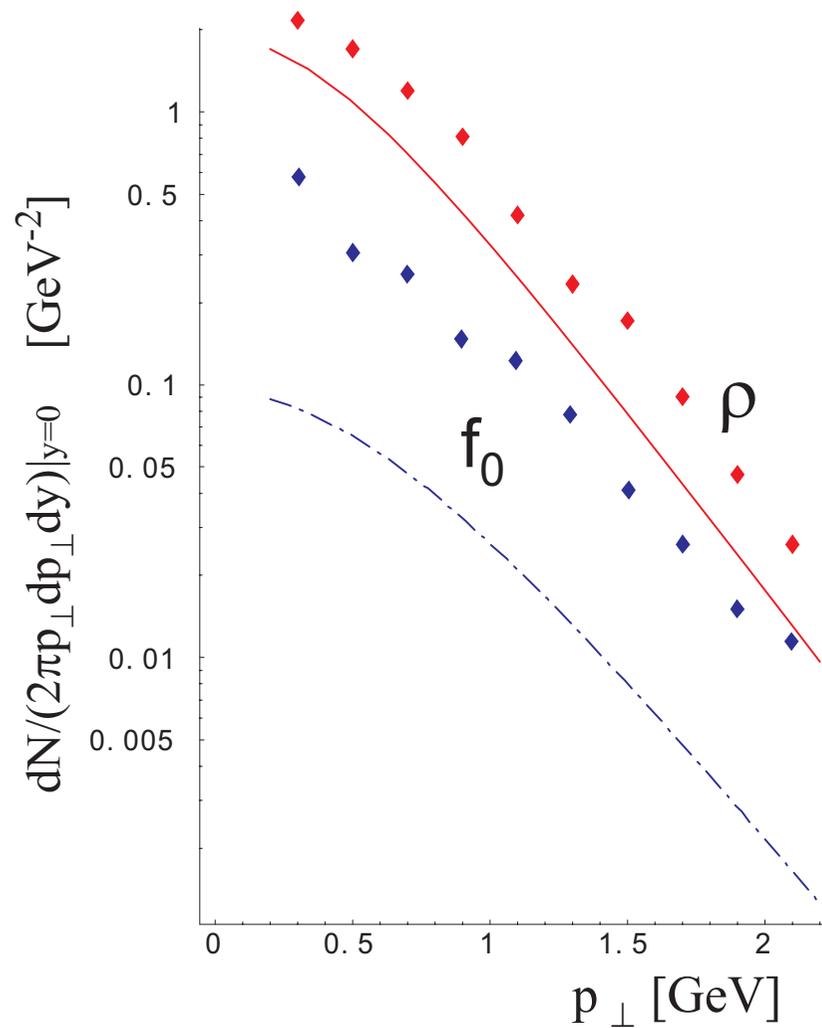
(prepared by P. Fachini)

vacuum ρ



(worse agreement)

p_{\perp} spectra of resonances



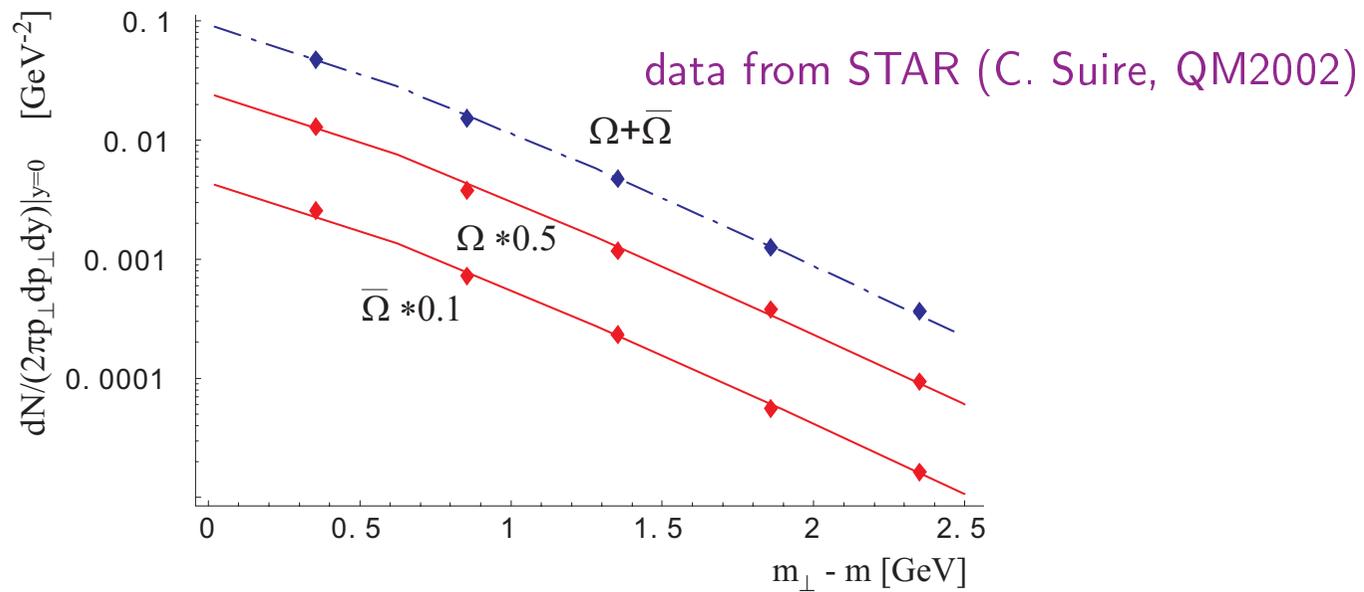
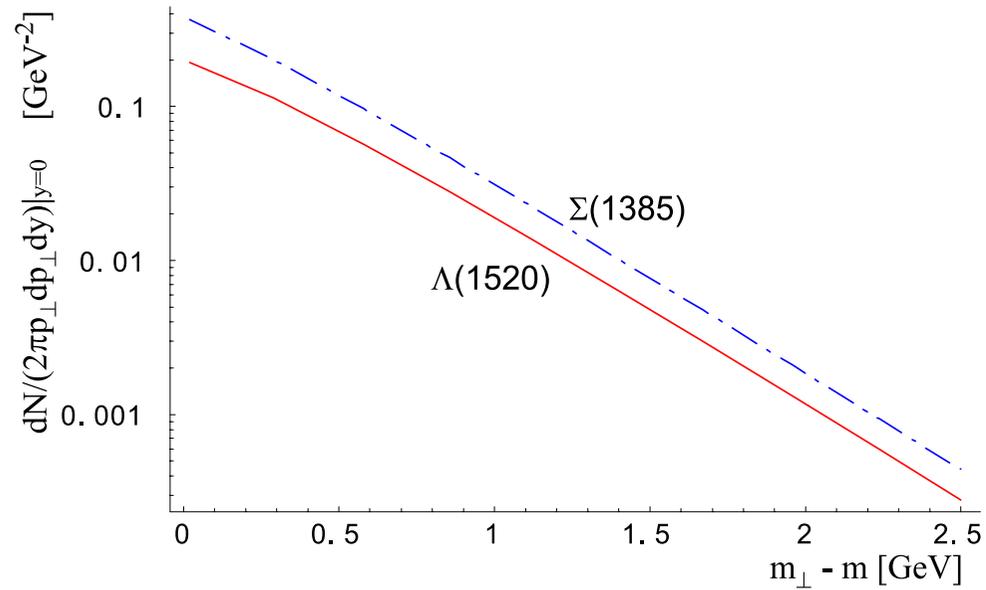
(model parameters: $\tau = 4.2$ fm and $\rho_{\max} = 3.45$ fm)

slopes OK, yields in experiment $>$ thermal model!

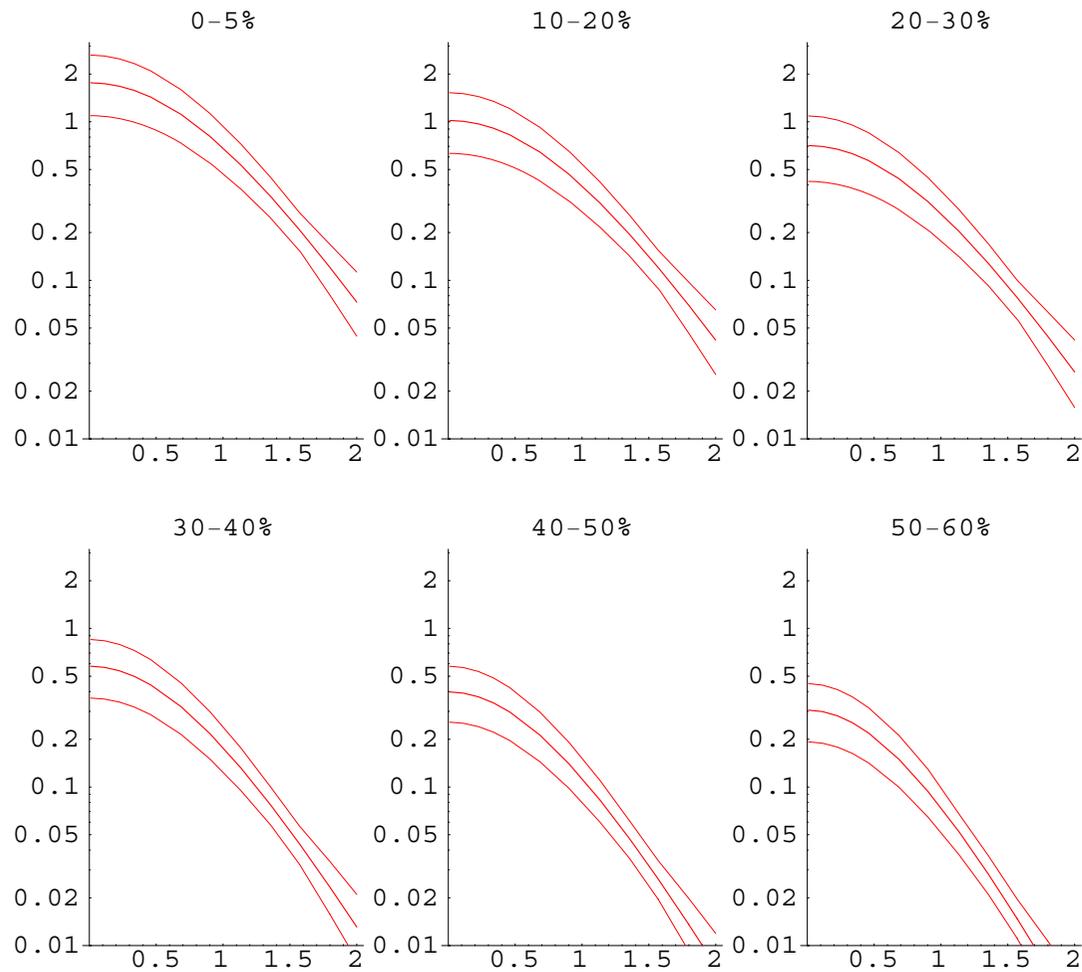
	$m_\rho^* = 770 \text{ MeV}$	$m_\rho^* = 700 \text{ MeV}$	Experiment
T [MeV]	165.6 ± 4.5	167.6 ± 4.6	
μ_B [MeV]	28.5 ± 3.7	28.9 ± 3.8	
η/π^-	0.120 ± 0.001	0.112 ± 0.001	
ρ^0/π^-	0.114 ± 0.002	0.135 ± 0.001	0.183 ± 0.028 (40-80%)
ω/π^-	0.108 ± 0.002	0.102 ± 0.002	
$K^*(892)/\pi^-$	0.057 ± 0.002	0.054 ± 0.002	
ϕ/π^-	0.025 ± 0.001	0.024 ± 0.001	
η'/π^-	0.0121 ± 0.0004	0.0115 ± 0.0003	
$f_0(980)/\pi^-$	0.0102 ± 0.0003	0.0097 ± 0.0003	0.042 ± 0.021 (40-80%)
$K^*(892)/K^-$	0.33 ± 0.01	0.33 ± 0.01	0.205 ± 0.033 (0-10%) 0.219 ± 0.040 (10-30%) 0.255 ± 0.046 (30-50%) 0.269 ± 0.047 (50-80%)
$\Lambda(1520)/\Lambda$	0.061 ± 0.002	0.062 ± 0.002	0.022 ± 0.010 (0-7%) 0.025 ± 0.021 (40-60%) 0.062 ± 0.027 (60-80%)
$\Sigma(1385)/\Sigma$	0.484 ± 0.004	0.485 ± 0.004	

The model underpredicts ρ and f_0 and overpredicts K^* and Λ . Lower T would imply less resonances!

Predictions



Prediction for Δ^{++}



p_{\perp} spectra for Δ^{++} . The bands indicate the uncertainty of τ and ρ_{\max} from the Table given above.

Summary

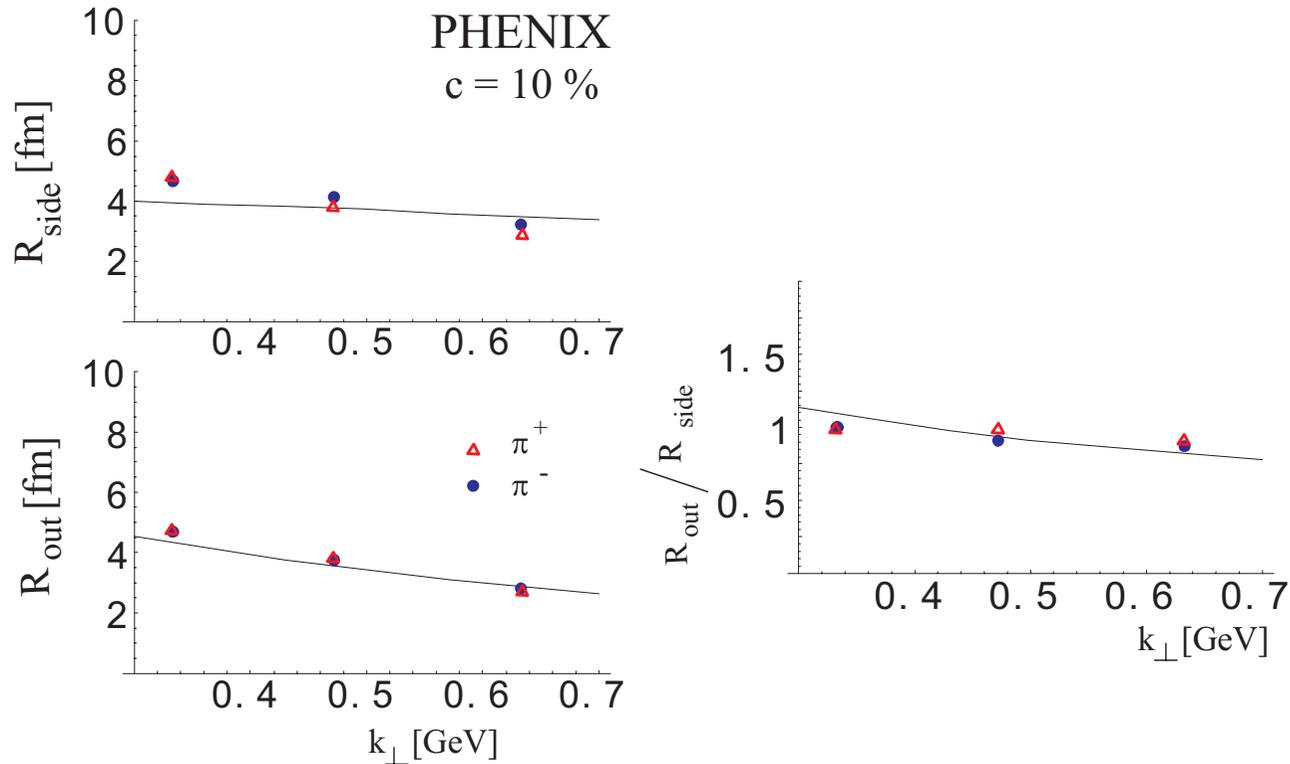
1. Works for abundances, p_{\perp} -spectra, including strange particles and resonances
2. Lower T_{kin} would lead to **much less** resonances!
3. **Resonances** are an important source of **correlations**
4. Shape of the $\pi\pi$ “spectral line” - **new thermometer**, derivative of **phase shifts** must be used, full model gives similar results at 165 MeV to the naive calculation at 110 MeV (**cooling via decays**)
5. Not possible to place the ρ peak at the experimental value. **Medium effects?**
(Brown-Rho-Shuryak)

Soft physics ($p_{\perp} < 1.5 - 2$) GeV is well described by the thermal approach with the single-freezeout approximation and resonance decays

HBT radii

$$S(x, p) = \int d\Sigma_\mu p^\mu \delta(x' - x) f(x', p)$$

$$C(\vec{q}, \vec{P}) = 1 + \frac{\left| \int d\Sigma(x) \cdot u(x) e^{iq \cdot x} S(P \cdot u(x)) \right|^2}{\int d\Sigma \cdot u S\left(\left(P + \frac{q}{2}\right) \cdot u(x)\right) \int d\Sigma \cdot u S\left(\left(P - \frac{q}{2}\right) \cdot u(x)\right)}$$



The pionic HBT radii for most-central collisions @130 GeV, and their ratio, as predicted by the model + excluded volume corrections ($\sim 30\%$ enhancement of model radii) and measured by PHENIX

Excluded-volume (Van der Waals) corrections

Such effects were found important in previous studies of the particle multiplicities in ultra-relativistic heavy-ion collisions, leading to a **significant dilution of system**. They bring in a factor (Gorenstein)

$$\frac{e^{-Pv_i/T}}{1 + \sum_j v_j e^{-Pv_j/T} n_j},$$

into phase-space integrals, where P denotes the pressure, $v_i = 4\frac{4}{3}\pi r_i^3$ is the excluded volume, and n_i is the density of particles of species i . The pressure is calculated self-consistently from the equation

$$P = \sum_i P_i^0(T, \mu_i - Pv_i/T) = \sum_i P_i^0(T, \mu_i) e^{-Pv_i/T}$$

where P_i^0 is the partial pressure of the ideal gas of hadrons of species i . If $r_i = r$, $v_i = v$, the excluded-volume correction produces a common scale factor, S^{-3} . Then

$$\frac{dN_i}{d^2p_\perp dy} = \tau^3 \int_{-\infty}^{+\infty} d\alpha_\parallel \int_0^{\rho_{\max}/\tau} \sinh\alpha_\perp d(\sinh\alpha_\perp) \int_0^{2\pi} d\xi p \cdot u S^{-3} f_i(p \cdot u)$$

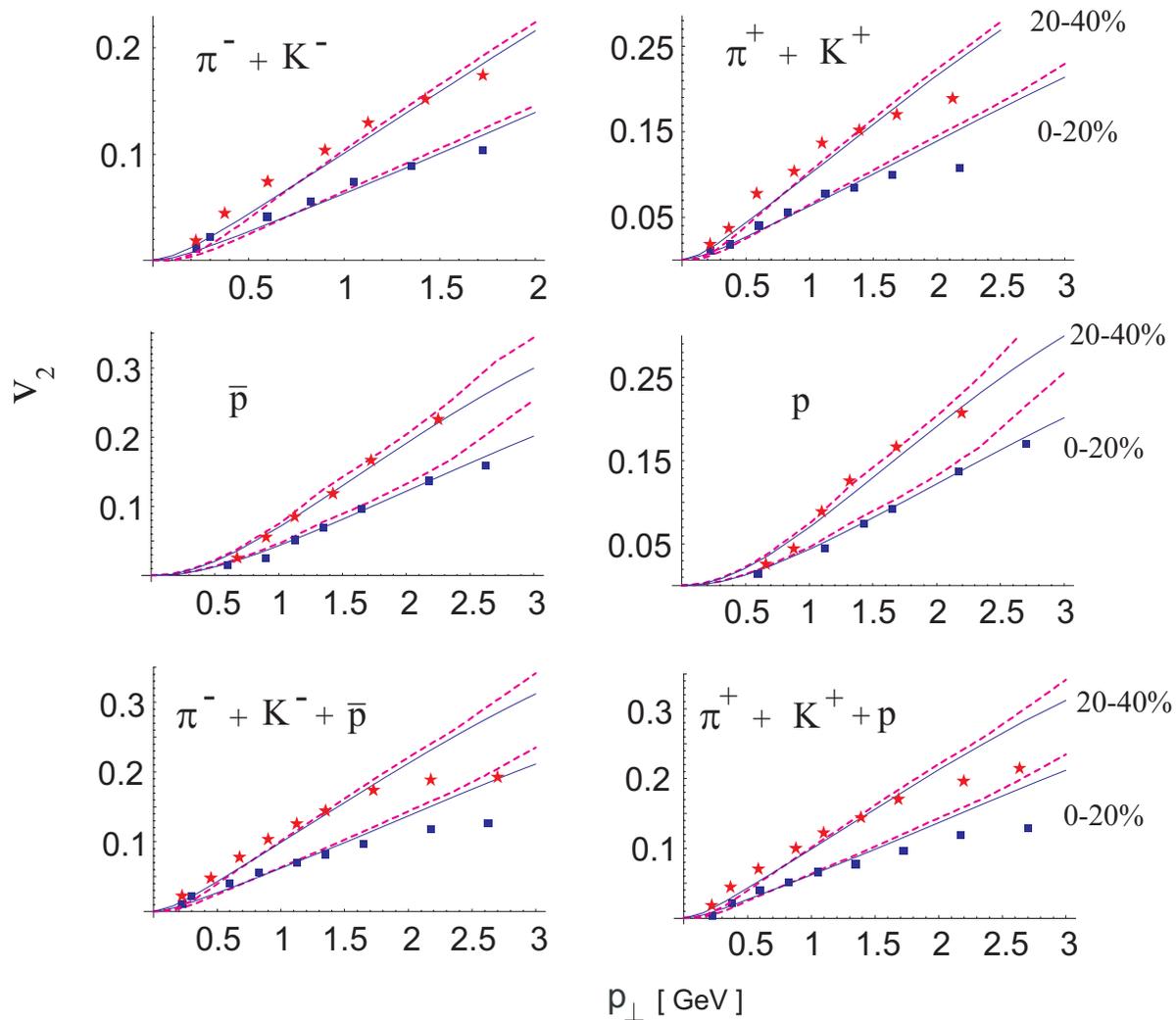
The presence of the factor S^{-3} is compensated by rescaling ρ and τ by the factor S . That way, we **retain** all the previously obtained results for the particle abundances and the momentum spectra. However, now the **system is more dilute and larger in size**.

With our values of the thermodynamic parameters we have

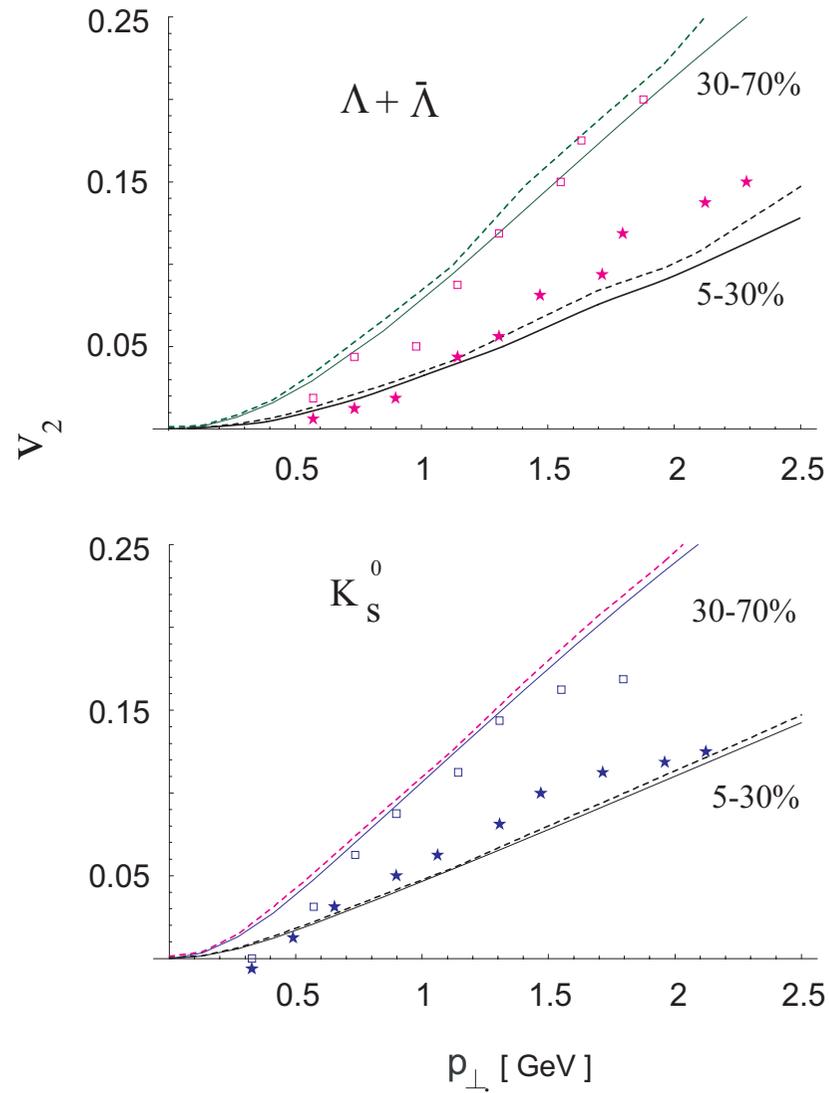
$\sum_i P_i^0(T, \mu_i) = 80 \text{ MeV}/\text{fm}^3$, which leads to $S = 1.3$ with $r = 0.6 \text{ fm}$. Values of this order have been typically obtained in other works. Thus, the excluded-volume corrections can increase the size parameters at freeze-out by about 30% and help to alleviate the problem with the HBT radii. **Hadrons have sizes!**

Elliptic flow

(Anna Baran, to be published) Idea: fix azimuthal asymmetry of shape/flow with the data on pions, kaons, ... and then make predictions for other particles. **solid (dashed): with (without) resonance decays.** (data from PHENIX @ 200 GeV)



v_2 for strange particles



(data from STAR)