Particle spectra and correlations in a thermal model

Wojciech Broniowski
H. Niewodniczański Institute of Nuclear Physics, Cracow, Poland

Collective flow and QGP properties, RIKEN/BNL, 17 November 2003

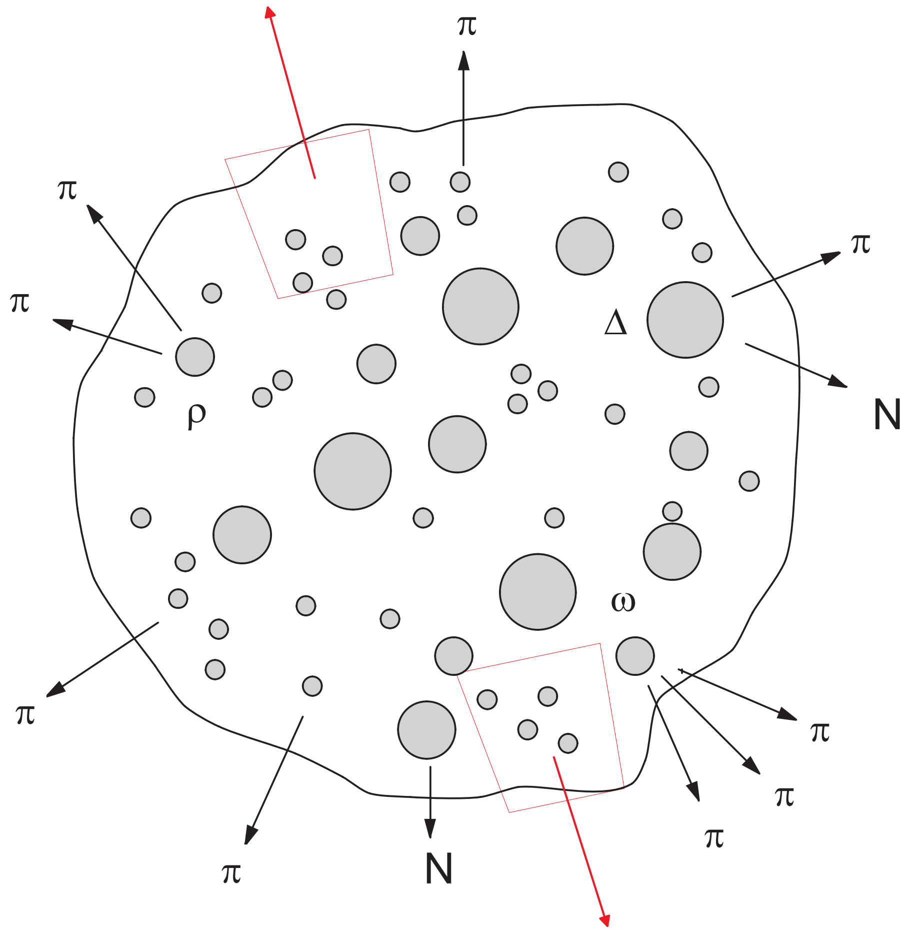
WB + Wojciech Florkowski, PRL 87 (2001) 272302; PRC 65 (2002) 064905
WB+ Anna Baran + WF, Acta Phys. Polon. B33 (2002) 4235
WB+ WF+ Brigitte Hiller (Coimbra), PRC 68 (2003) 034911
Piotr Bozek+ WB+WF, nucl-th/0310062

Outline

1. Single-freeze-out approximation
2. Importance of hadronic resonances
3. Yields
4. p_t-spectra
5. $\pi^+\pi^-$ invariant-mass correlations
6. Balance functions
7. HBT radii
8. Elliptic flow

Thermal models

Koppe (1948), Fermi (1950), Landau, Hagedorn, Rafelski ..., Heinz ..., Gaździcki, Braun-Munzinger ..., Magestro, Csörgő ..., Becattini ..., Hirano, ... (many more)



$\sim e^{-(E-\mu)/T}$

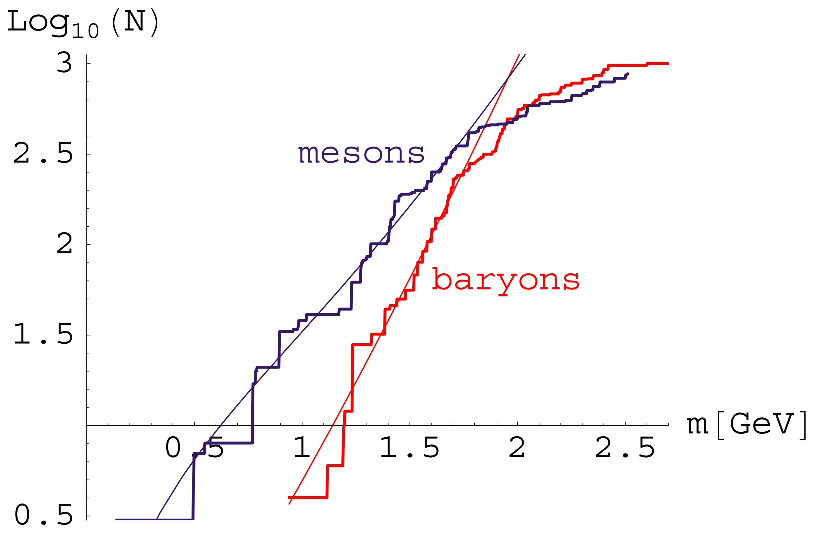
Specific features of our approach

1. Single freezout approximation: $T_{\rm chem} = T_{\rm kin} \equiv T$, single freeze-out. A radical simplification, supported by the RHIC HBT results: $R_{\rm out}/R_{\rm side} \sim 1$, $R_{\rm side}(\phi)$ has out-of-plane elongation, resonances seen abundantly $\to$ short time between the freeze-outs (explosive scenario).

T and μ_B are fitted from ratios of dN/dy at midrapidity

2. Ockham rasor: No γ-factors for strangeness (Rafelski), excluded-volume effects (Gorenstein), canonical (Redlich) or microcanonical ensemble (Becattini)

3. Hagedorn: Complete treatment of resonances (important due to the Hagedorn-like exponential growth of the number of states)



(from WB+WF, PLB 490 (2000) 223)

75% of pions and protons come from decays of higher states, 80% of Λ's, 60% of Ξ's, 30% of ρ_0's, . . . !

4. Geometry and flow: We take the hypersurface (inspired by Bjorken and Buda-Lund models) of the form

$$\tau = \sqrt{t^2 - r_z^2 - r_x^2 - r_y^2} = \text{const}$$

and constrain the transverse size, $\rho = \sqrt{r_x^2 + r_y^2} < \rho_{\max}$. The geometric parameters τ and $\rho_{\max}$, of the order of a few fm, are fitted to the $p_\perp$-spectra (τ^3 is the overall normalization constant, $\rho_{\max}$ controls the slopes). The hydrodynamic four-velocity is (Hubble law)

$$u^\mu = \partial^\mu \tau = \frac{x^\mu}{\tau} = \frac{t}{\tau}\left(1, \frac{r_z}{t}, \frac{r_x}{t}, \frac{r_y}{t}\right)$$

Boost invariance is a good approximation for midrapidity

Other choices can be tested (Heinz+Sollfrank+Wiedemann, Torrieri+Rafelski) (*e.g.* blast wave)

Altogether 4 parameters

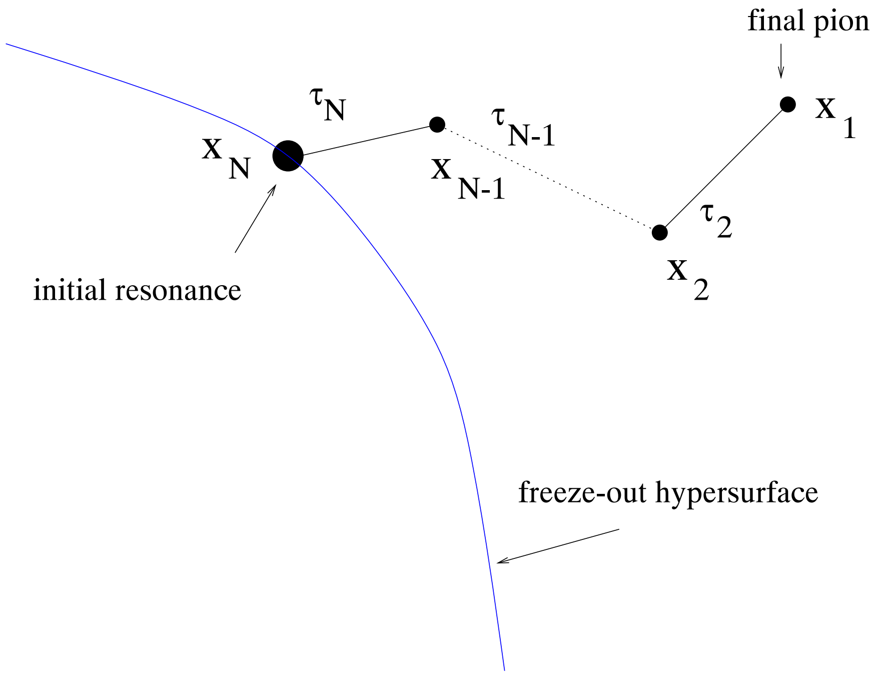
Ratios

For a boost-invariant model $\frac{dN_i/dy}{dN_j/dy} = \frac{N_i}{N_j}$ and ratios do not depend on geometry/flow.

$\sqrt{s_{NN}}$ [GeV]	130	200
T [MeV]	165 ± 7	160±5
μ_B [MeV]	41 ± 5	26±4
χ^2/DOF	1.0	1.5

@ 200 GeV	Model	Experiment
π^-/π^+	1.009 ± 0.003	$1.025 \pm 0.006 \pm 0.018$ $1.02 \pm 0.02 \pm 0.10$
K^-/K^+	0.939 ± 0.008	$0.95 \pm 0.03 \pm 0.03$ $0.92 \pm 0.03 \pm 0.10$
$\overline{p}/p$	0.74 ± 0.04	$0.73 \pm 0.02 \pm 0.03$ $0.70 \pm 0.04 \pm 0.10$ 0.78 ± 0.05
$\overline{p}/\pi^-$	0.104 ± 0.010	0.083 ± 0.015
K^-/π^-	0.174 ± 0.001	0.156 ± 0.020
$\Omega/h^- \quad \times 10^3$	0.990 ± 0.120	$0.887 \pm 0.111 \pm 0.133$
$\overline{\Omega}/h^- \quad \times 10^3$	0.900 ± 0.124	$0.935 \pm 0.105 \pm 0.140$

Resonance decays in $p_\perp$-spectra



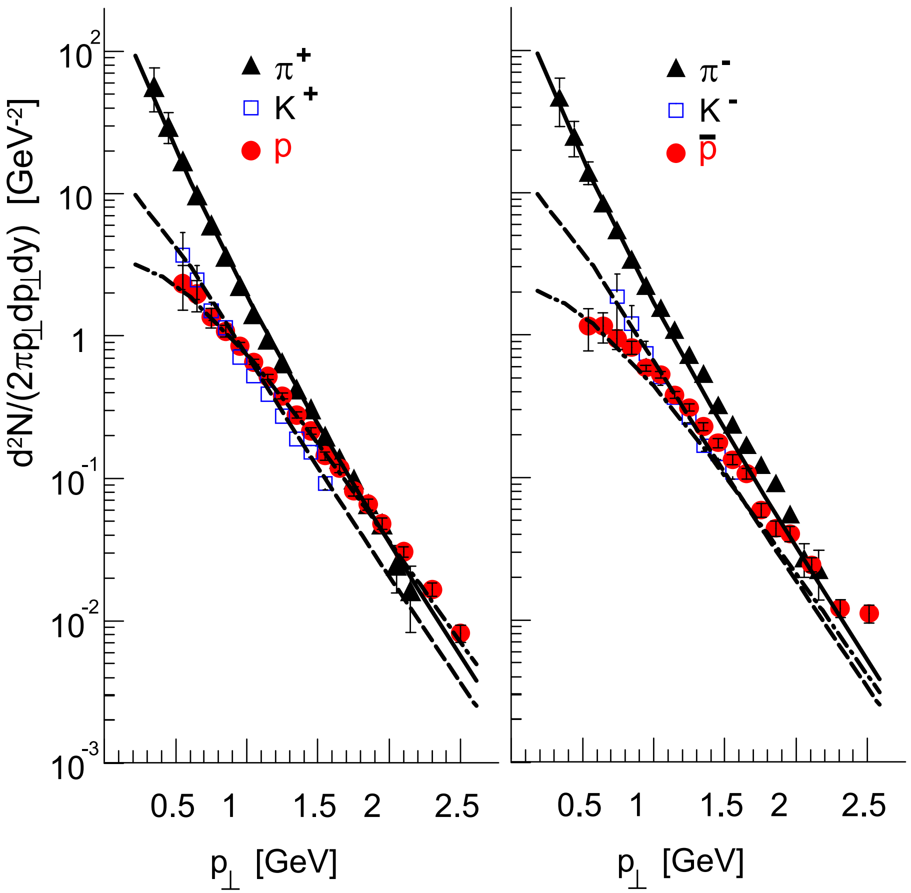
The integration over $x_{N-1} \ldots x_2$ is unconstrained, while the integration over x_N is constrained to the hypersurface Σ.

$$E_{p_1} \frac{dN_1}{d^3 p_1} = \int \frac{d^3 p_2}{E_{p_2}} B\left(p_2, p_1\right) \ldots \int \frac{d^3 p_N}{E_{p_N}} B\left(p_N, p_{N-1}\right) \int d\Sigma_\mu \left(x_N\right)\ p_N^\mu\ f_N \left[p_N \cdot u\left(x_N\right)\right]$$

$$B(p_i, p_{i-1}) = \frac{b}{4\pi p_{i-1}^*} \delta\left(\frac{p_i \cdot p_{i-1}}{m_i} - E_{i-1}^*\right)$$

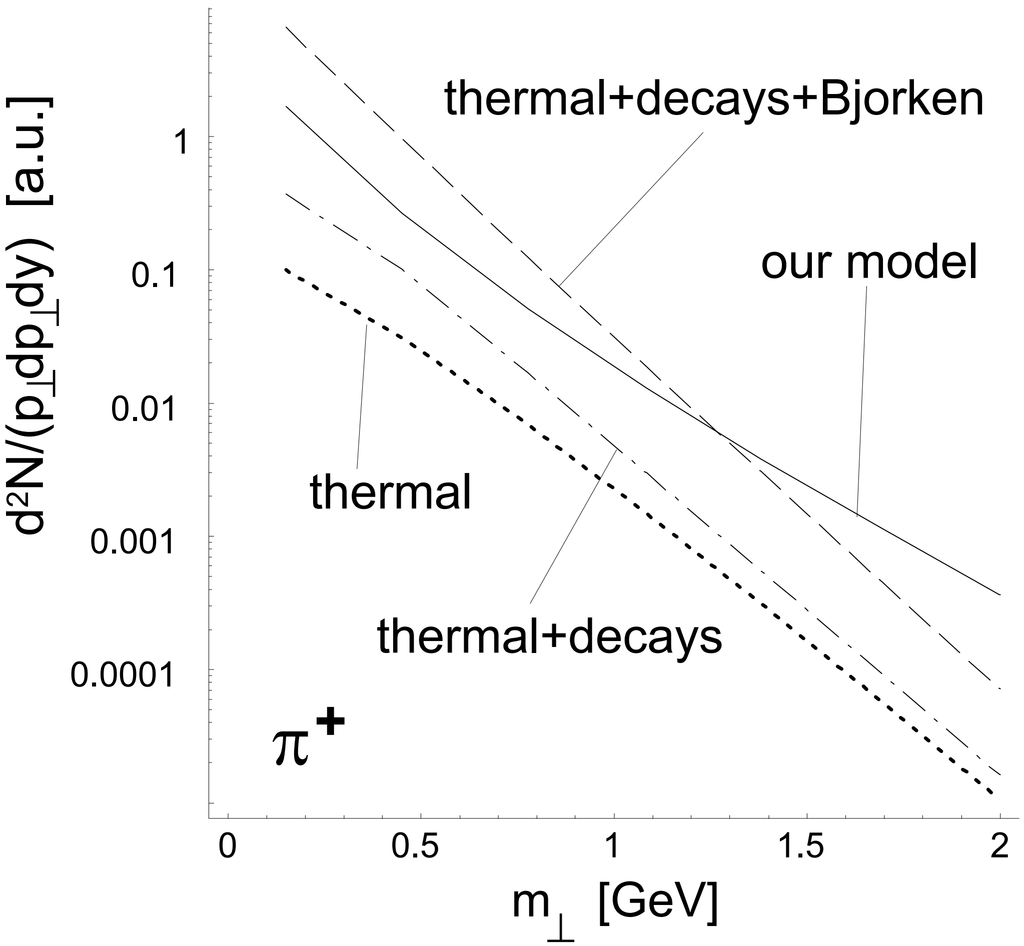
(for all details see WB+AB+WF, Acta Phys. Polon. B33 (2002) 4235)

Results for the transverse-momentum spectra

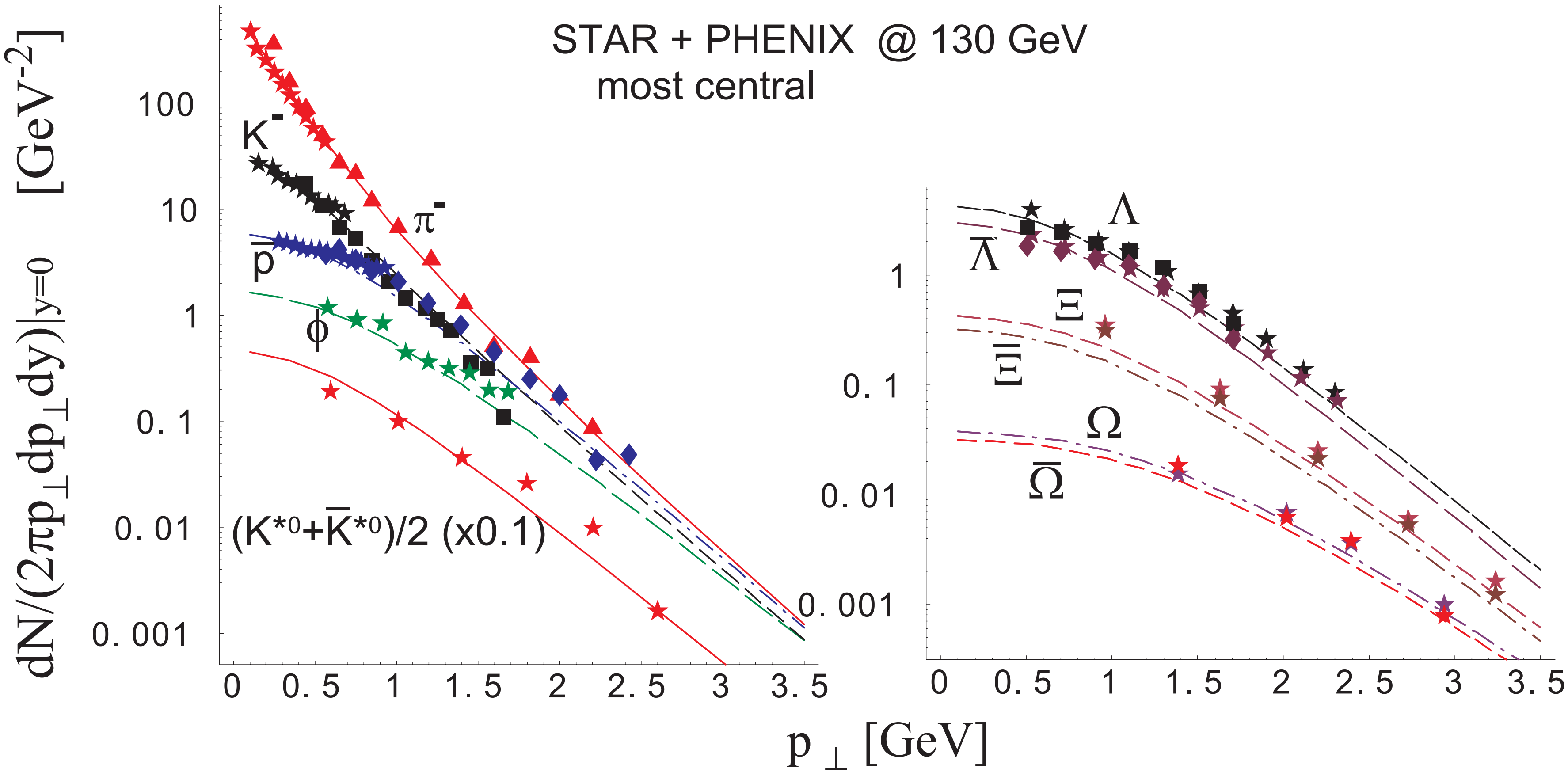


Min. bias $p_\perp$-spectra of pions, kaons, protons and antiprotons as evaluated from our model with $\tau = 6$ fm, $\rho_{\max}/\tau = 0.76$, compared to the earliest PHENIX data (Velkovska, nucl-ex/0105012). Very good agreement up to $p_\perp \sim 2$ GeV. At larger values, where hard processes enter, the model falls below the data

“Cooling” via decays



Resonance decays lower the inverse slope by about 30 MeV



($T = 165$ MeV)

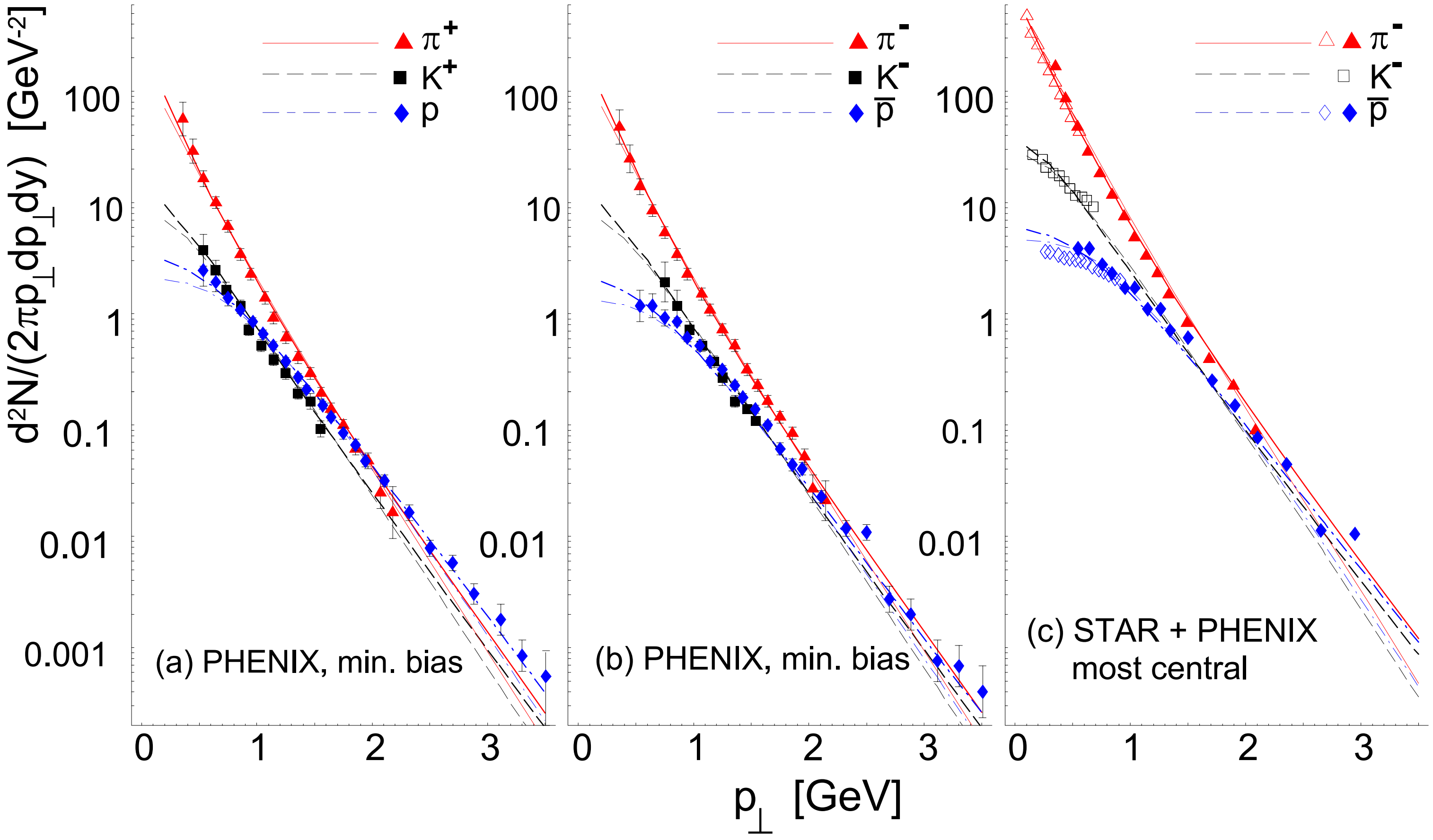
ϕ – very weak interactions, serves as a thermometer

K^* – resonance, lower T would lead to much less K^*'s

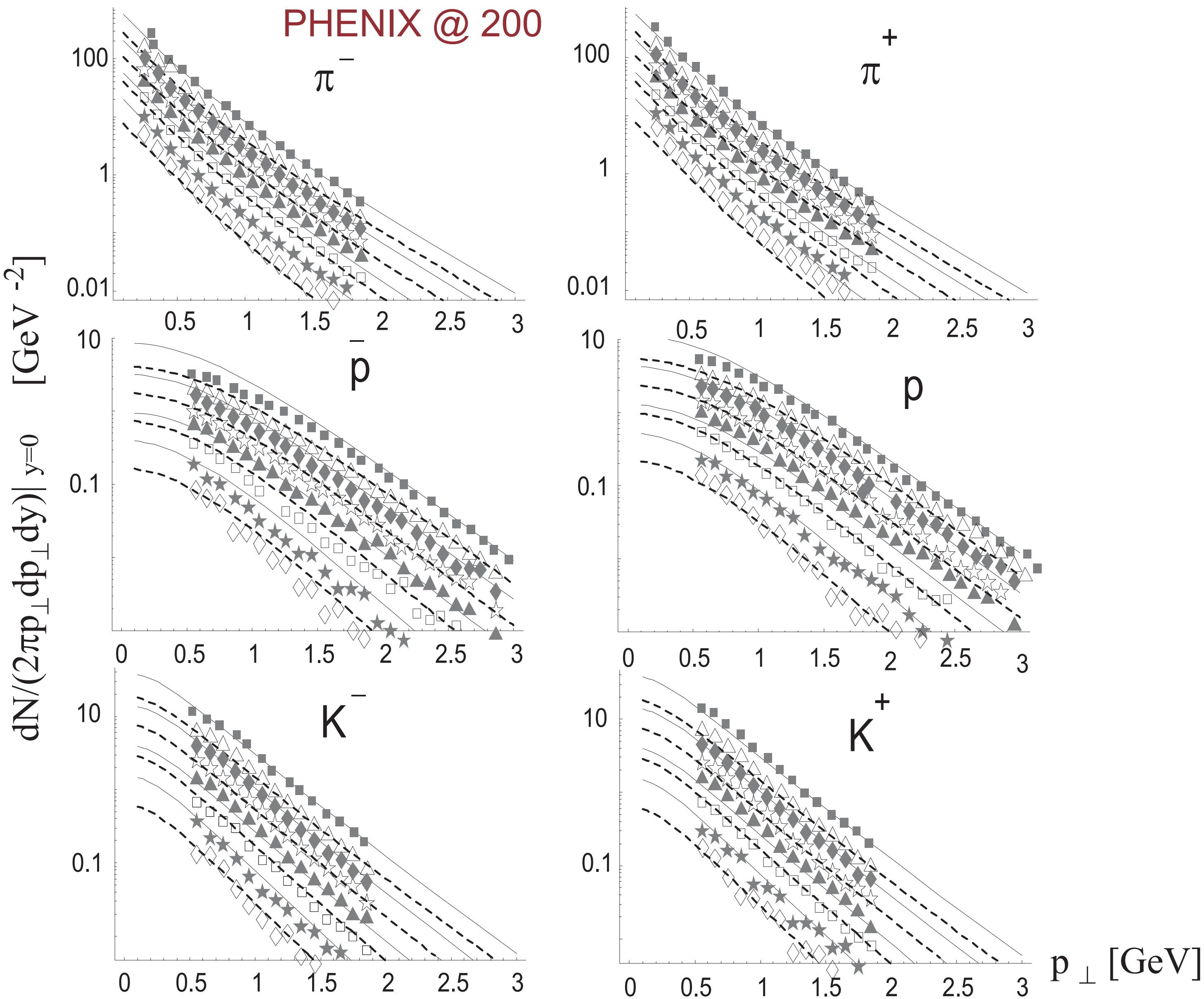
(experimental Ξ's went down by $\sim$ a factor of 2)

No special treatment of Ω's

Two different expansion models



thick: present model, thin: blast-wave (from WB+WF, PRL 87 (2001) 272302)


PHENIX @ 200
π⁻
π⁺
p̄
p
K⁻
K⁺
dN/(2πp⊥dp⊥dy)|y=0 [GeV -2]
p ⊥ [GeV]

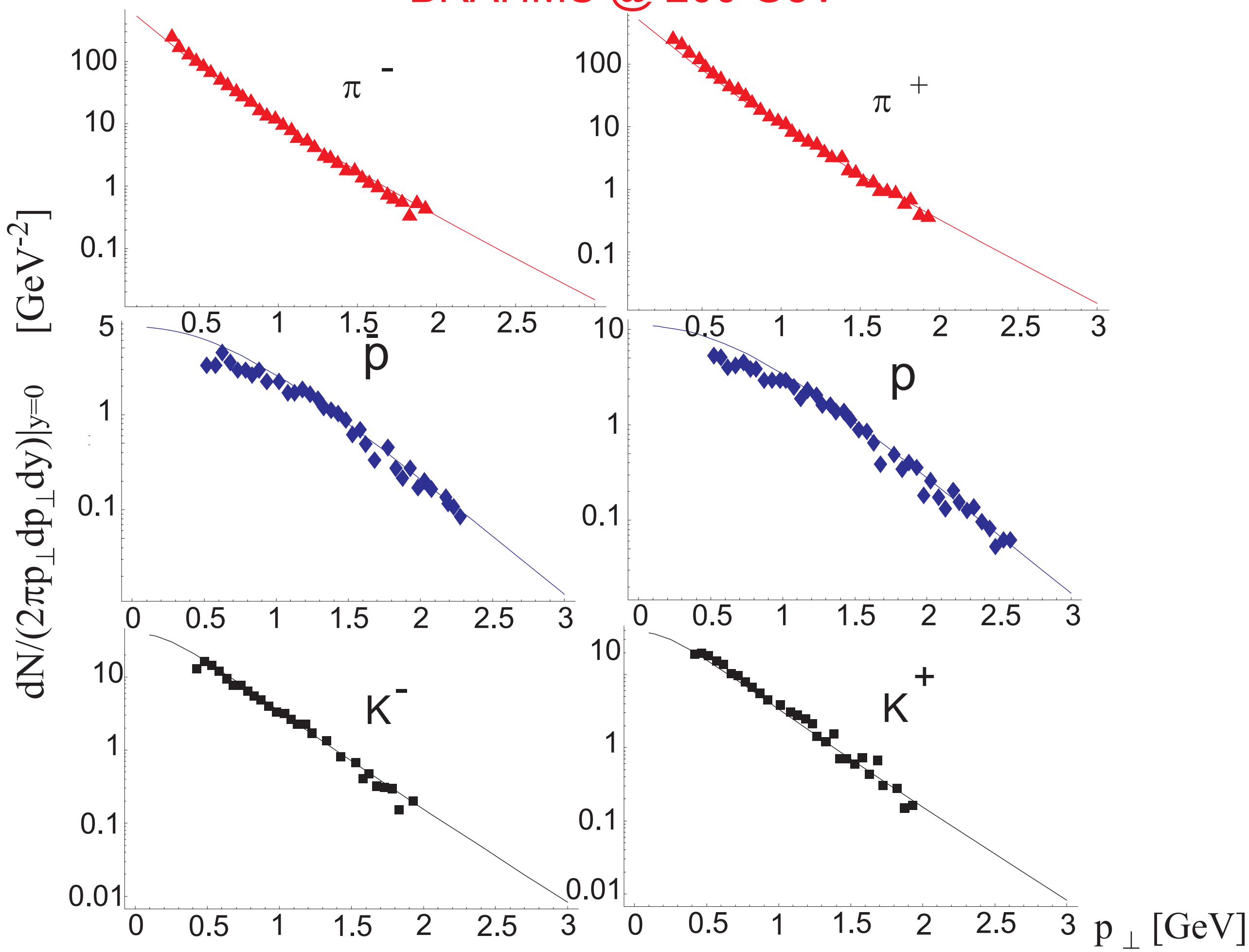
(data at different centrality, or impact parameter)

Centrality c is defined as a percentage of the most central events. To a very good accuracy

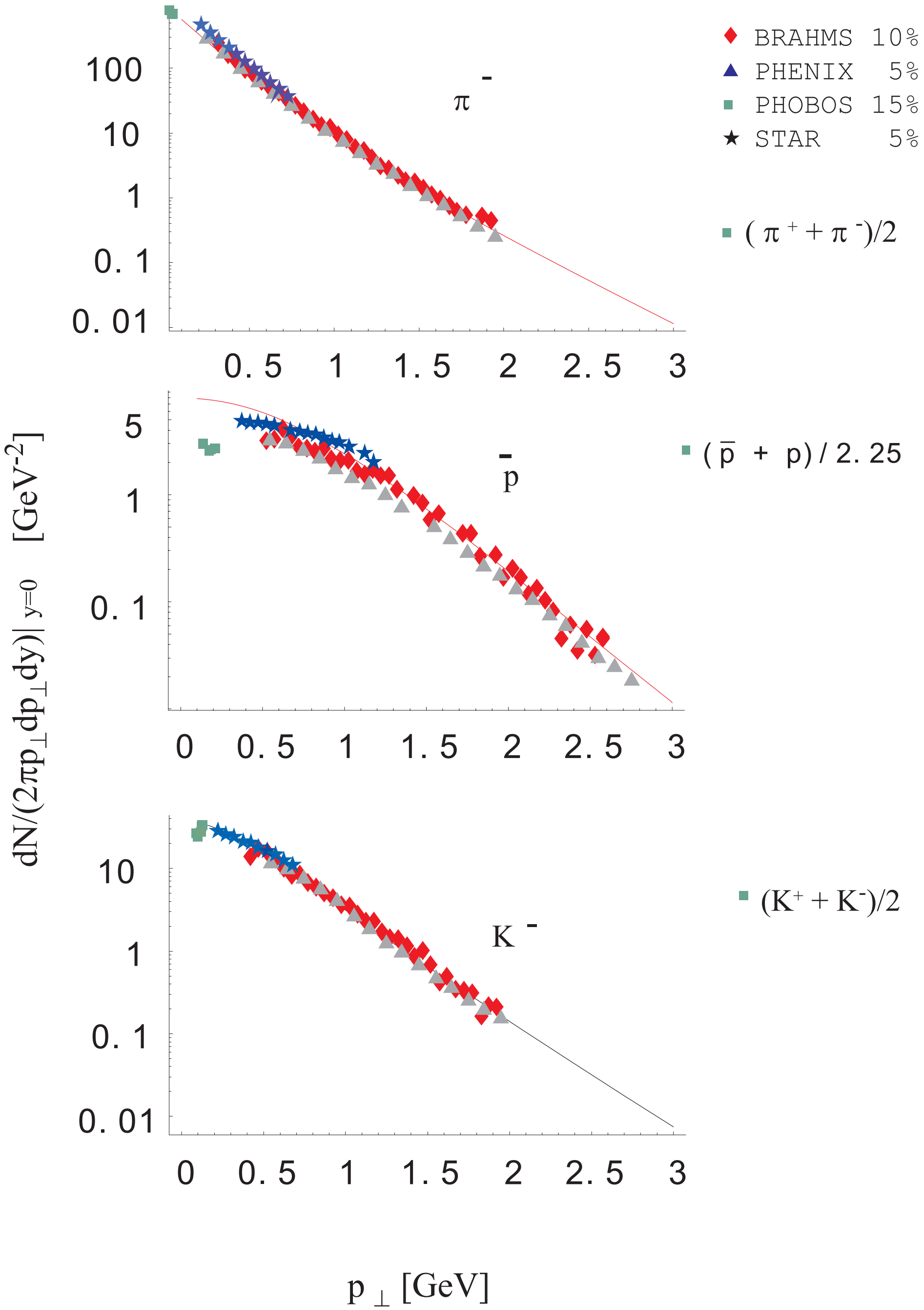
$$c \simeq \frac{\pi b^2}{\sigma_{\rm inel}^{\rm tot}} \simeq \frac{b^2}{4R^2}$$

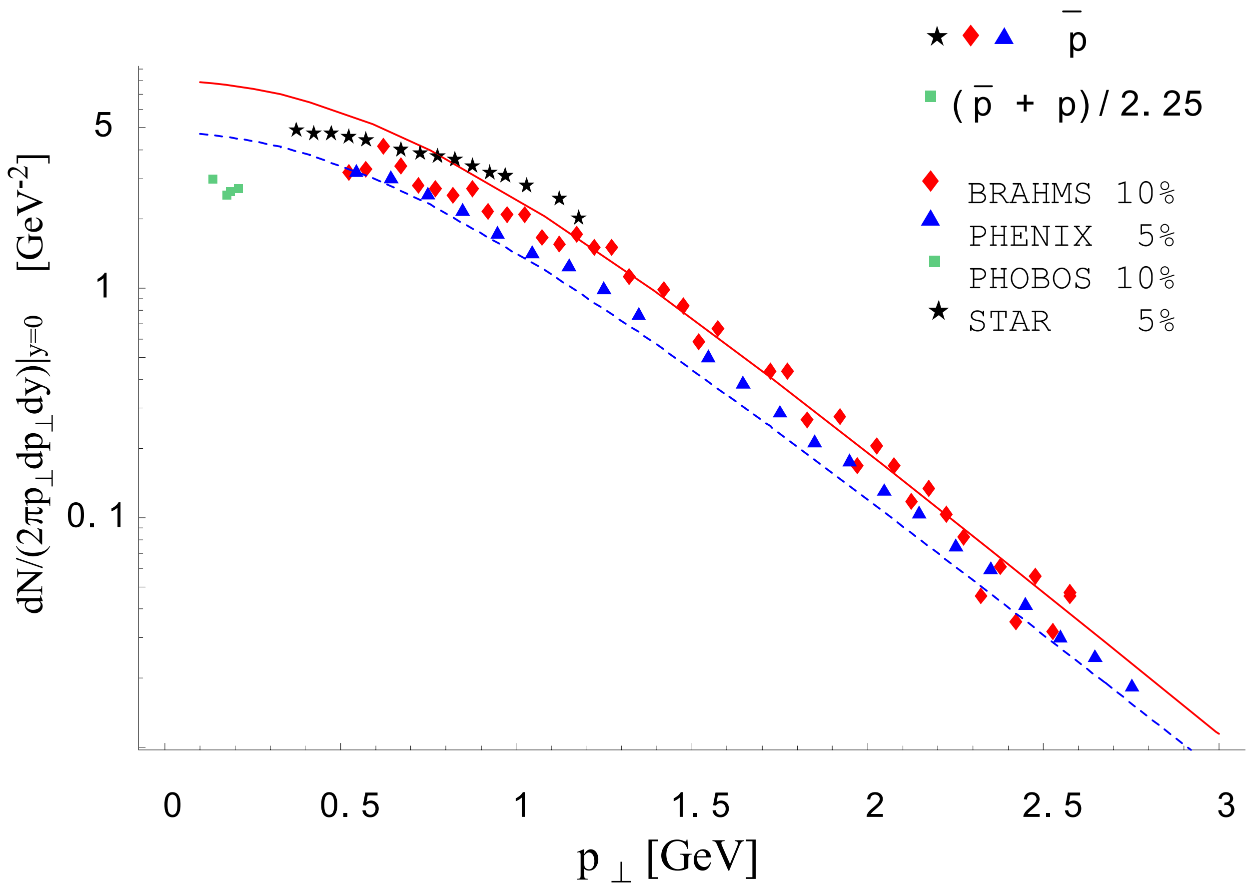
(WB+WF, PRC 65 (2002) 024905)

BRAHMS @ 200 GeV

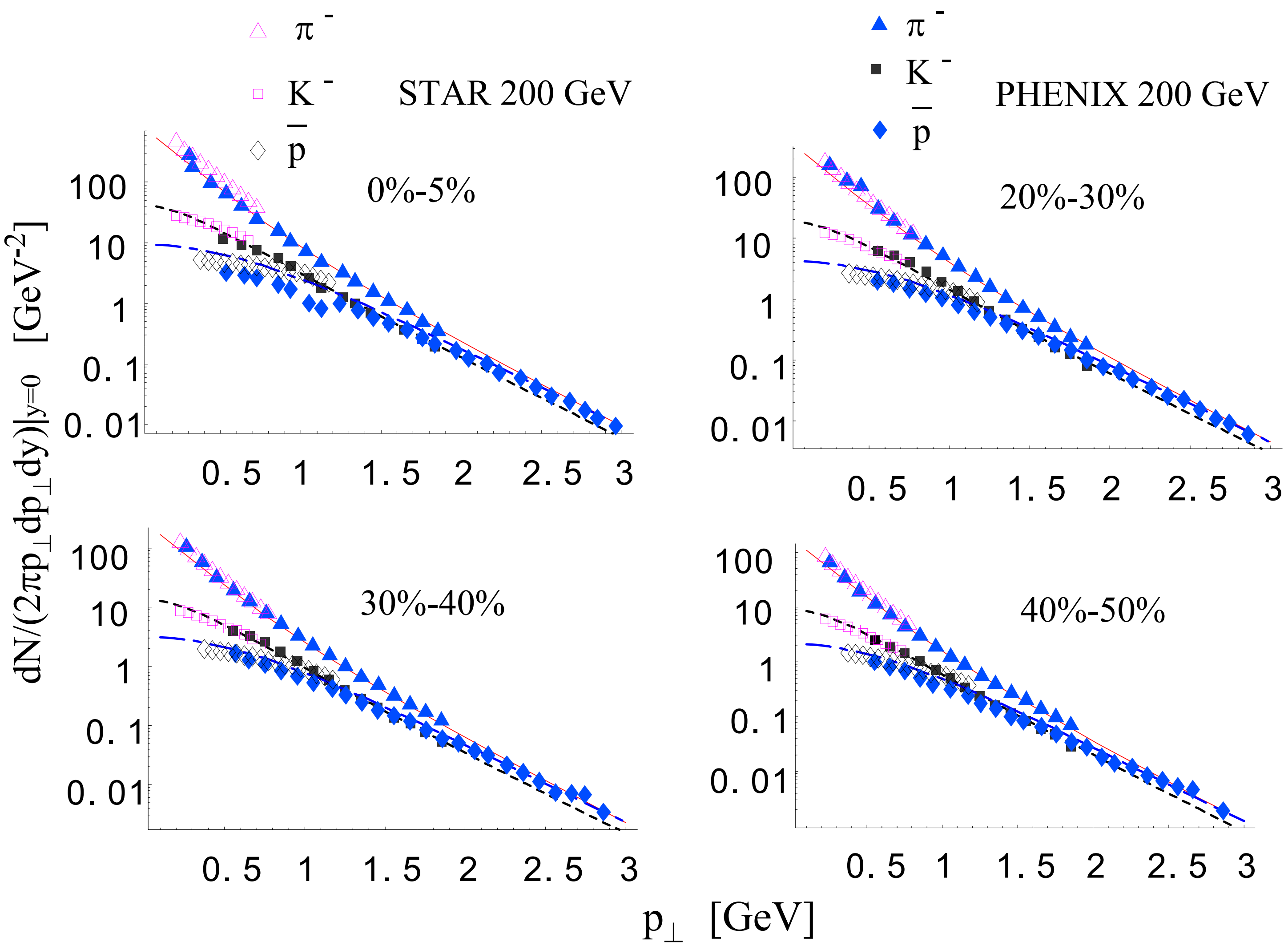


BRAHMS


BRAHMS 10%
PHENIX 5%
PHOBOS 15%
STAR 5%
$(\pi^+ + \pi^-)/2$
π^-
100
10
1
0.1
0.01
0.5
1
1.5
2
2.5
3
$(\bar{p} + p)/2.25$
$\bar{p}$
5
1
0.1
0
$(K^+ + K^-)/2$
K^-
$dN/(2\pi p_\perp dp_\perp dy)|_{y=0}$ [GeV^{-2}]
$p_\perp$ [GeV]



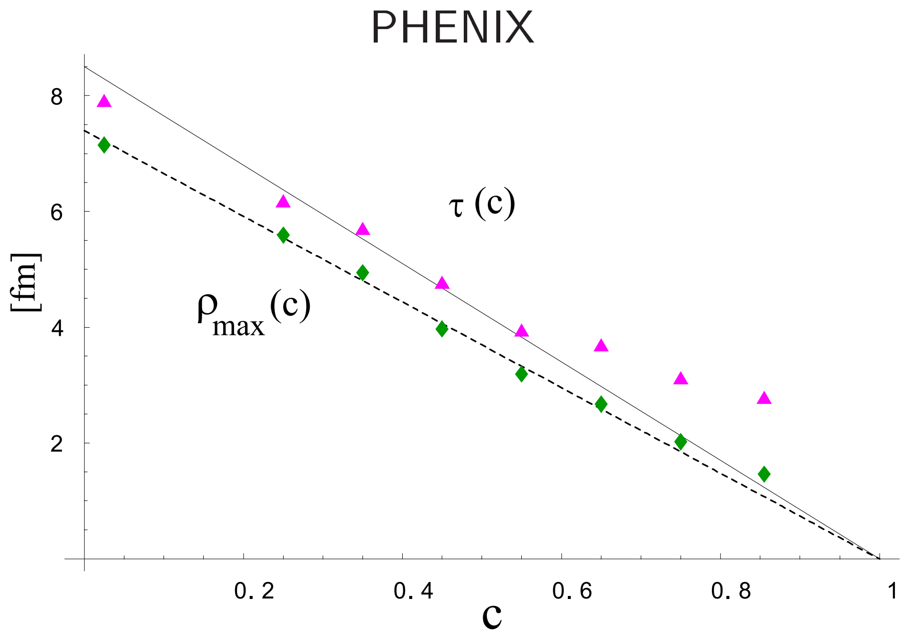
(solid – full feeding, dashed – no feeding from week decays)

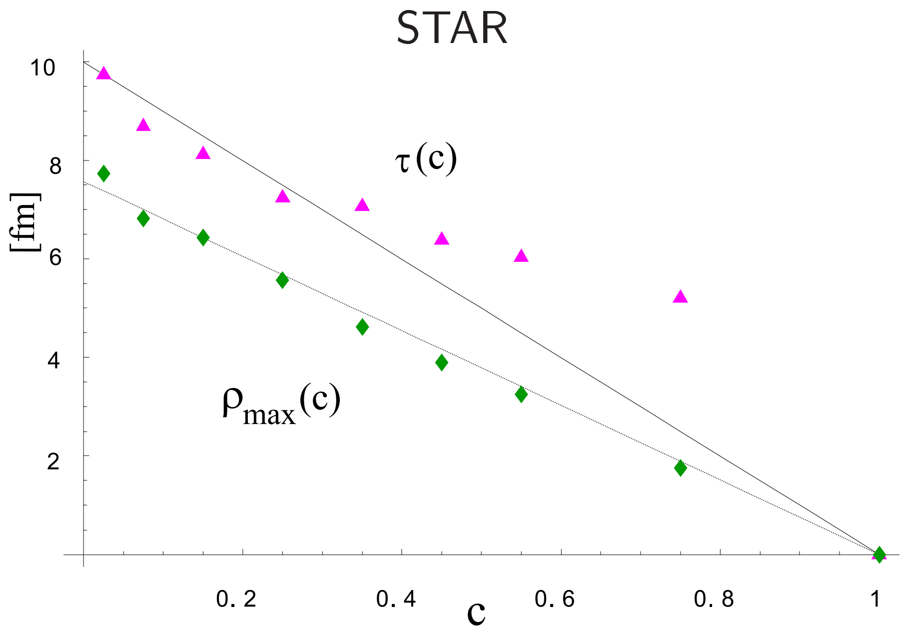


($\bar{p}$ from STAR more flat than from PHENIX)

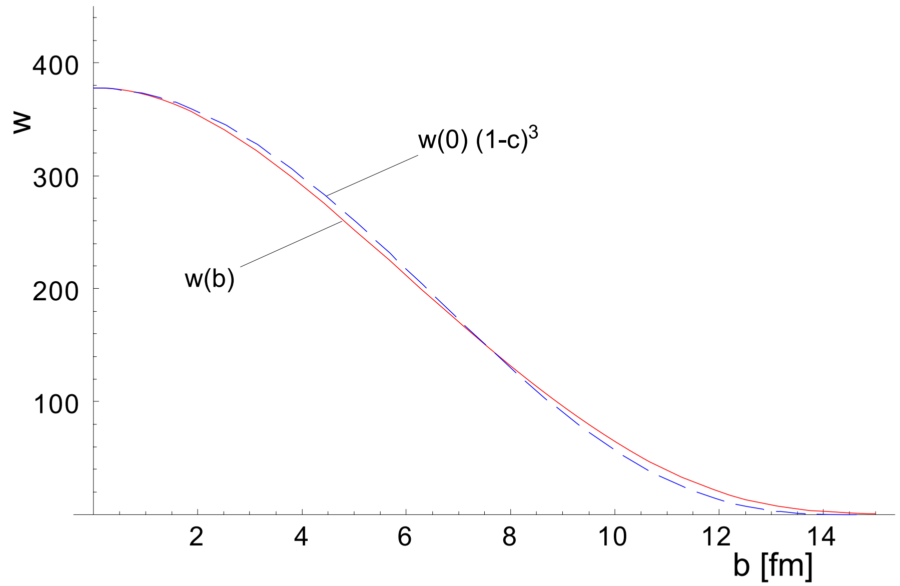
Compilation of geometric parameters (by A. Baran)

	c [%]	τ [fm] (norm)	$\rho_{\max}$ [fm]	$\langle\beta_\perp\rangle$ (slope)
BRAHMS	10	7.68 ± 0.19	7.46 ± 0.05	0.52 ± 0.01
STAR	$0 - 5$	9.74 ± 1.57	7.74 ± 0.68	0.45 ± 0.08
	$5 - 10$	8.69 ± 1.39	7.18 ± 0.64	0.47 ± 0.08
	$10 - 20$	8.12 ± 1.31	6.44 ± 0.57	0.45 ± 0.08
	$20 - 30$	7.24 ± 1.18	5.57 ± 0.50	0.44 ± 0.08
	$30 - 40$	7.07 ± 1.17	4.63 ± 0.39	0.39 ± 0.08
	$40 - 50$	6.38 ± 1.02	3.91 ± 0.33	0.37 ± 0.07
	$50 - 60$	6.19 ± 1.09	3.25 ± 0.28	0.32 ± 0.07
	$70 - 80$	5.48 ± 0.81	4.03 ± 0.10	0.43 ± 0.06
PHENIX	$0 - 5$	7.86 ± 0.38	7.15 ± 0.13	0.50 ± 0.02
	$20 - 30$	6.14 ± 0.32	5.62 ± 0.11	0.50 ± 0.02
	$30 - 40$	5.73 ± 0.16	4.95 ± 0.05	0.48 ± 0.01
	$40 - 50$	4.75 ± 0.28	3.96 ± 0.09	0.47 ± 0.03
	$50 - 60$	3.91 ± 0.23	3.12 ± 0.07	0.45 ± 0.03
	$60 - 70$	3.67 ± 0.12	2.67 ± 0.03	0.42 ± 0.01
	$70 - 80$	3.09 ± 0.11	2.02 ± 0.02	0.39 ± 0.01
	$80 - 91$	2.76 ± 0.20	1.43 ± 0.03	0.32 ± 0.03


PHENIX
8
6
4
2
[fm]
τ (c)
ρ_{max}(c)
0. 2
0. 4
0. 6
0. 8
1
c

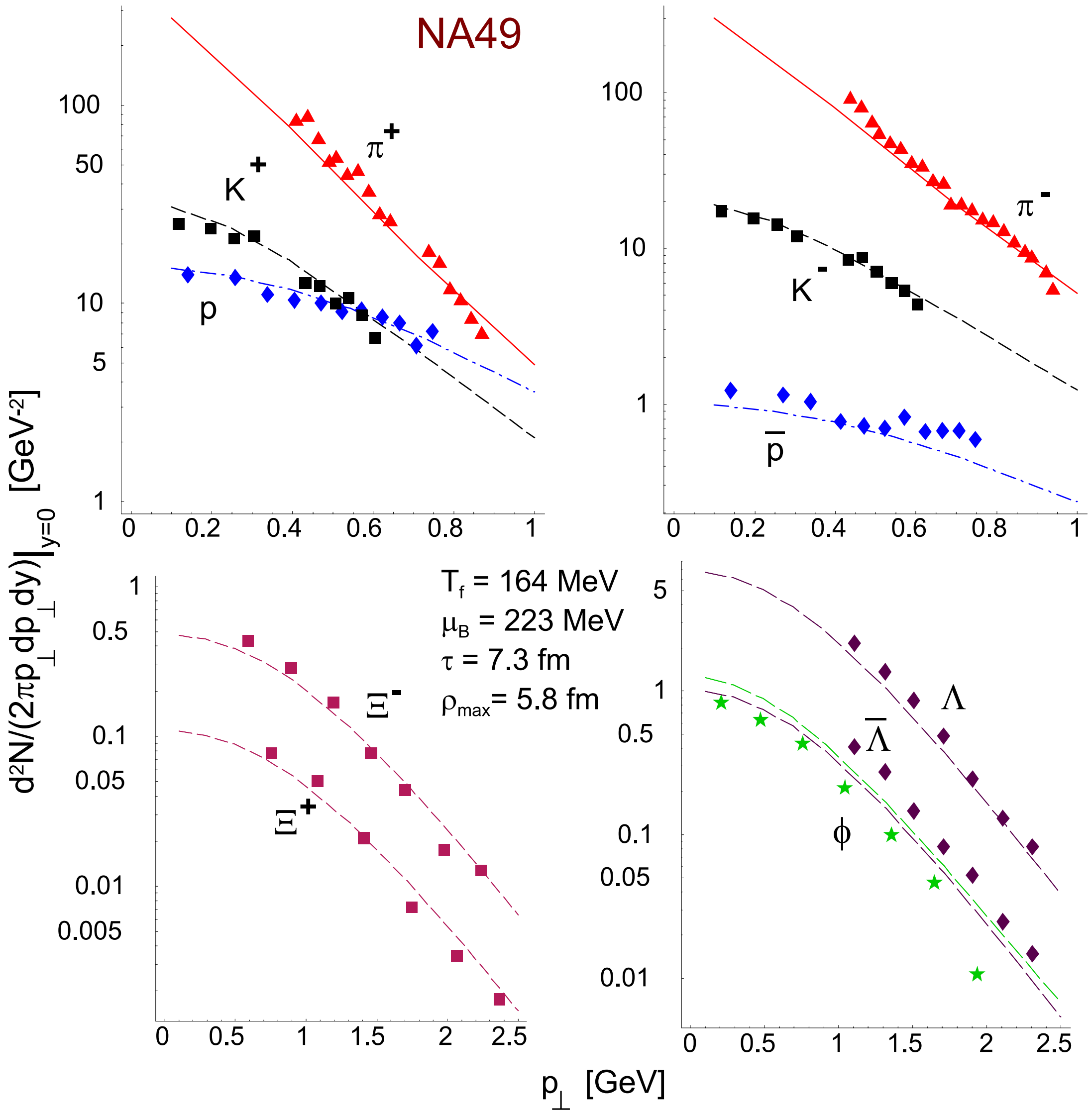

STAR
10
8
6
4
2
[fm]
τ(c)
ρ_{max}(c)
0. 2
0. 4
0. 6
0. 8
1
c

Wounded-nucleon scaling



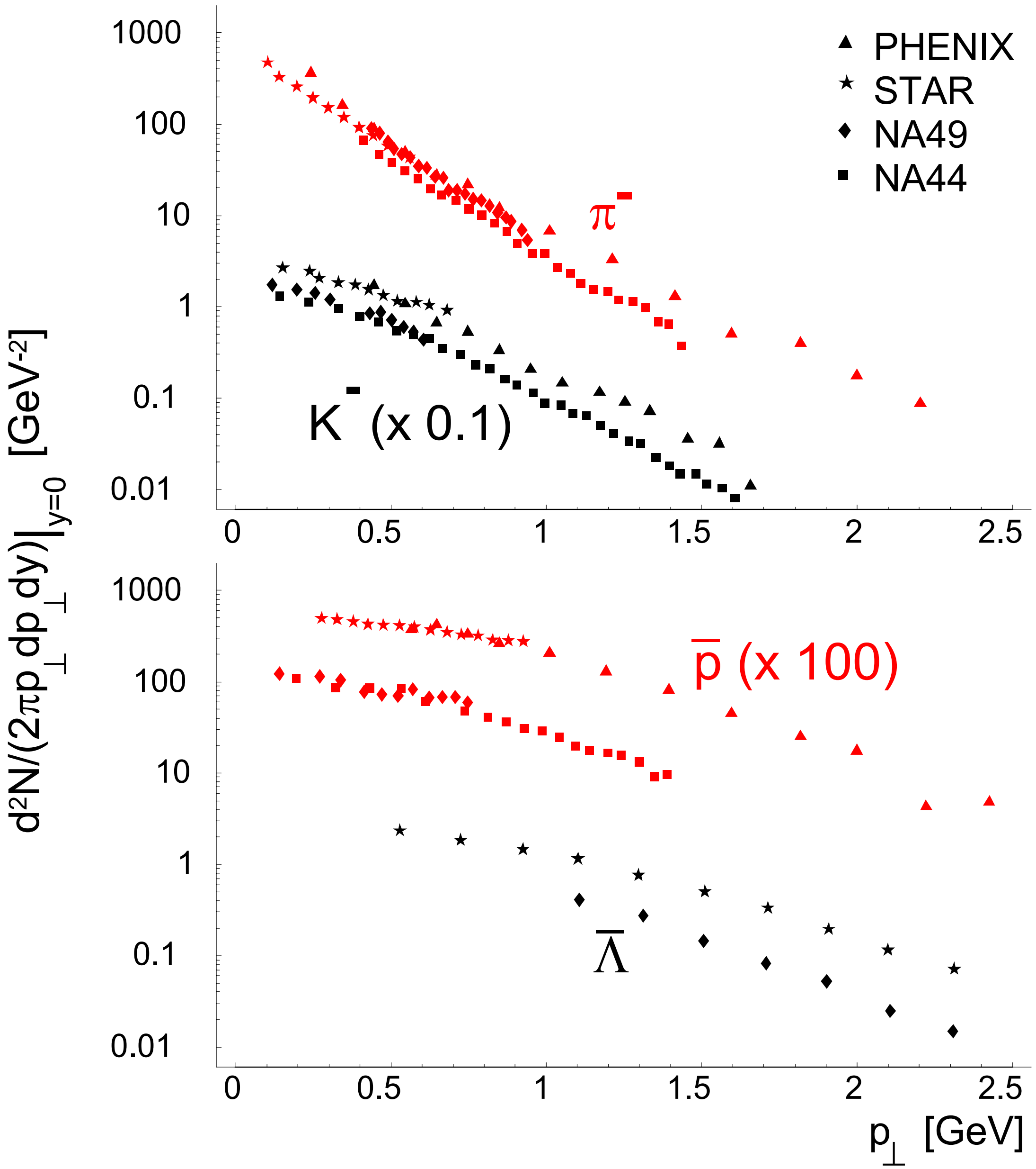
The number of wounded nucleons, $w(b)$ (solid line) and the approximating function $w(0)(1-c(b))^3$ (dashed line), are plotted as functions of the impact parameter b. Since the multiplicity of hadrons produced in our model is proportional to $(1-c)^3$ at low and moderate values of c, the model conforms to the wounded-nucleon scaling

How was it at SPS?

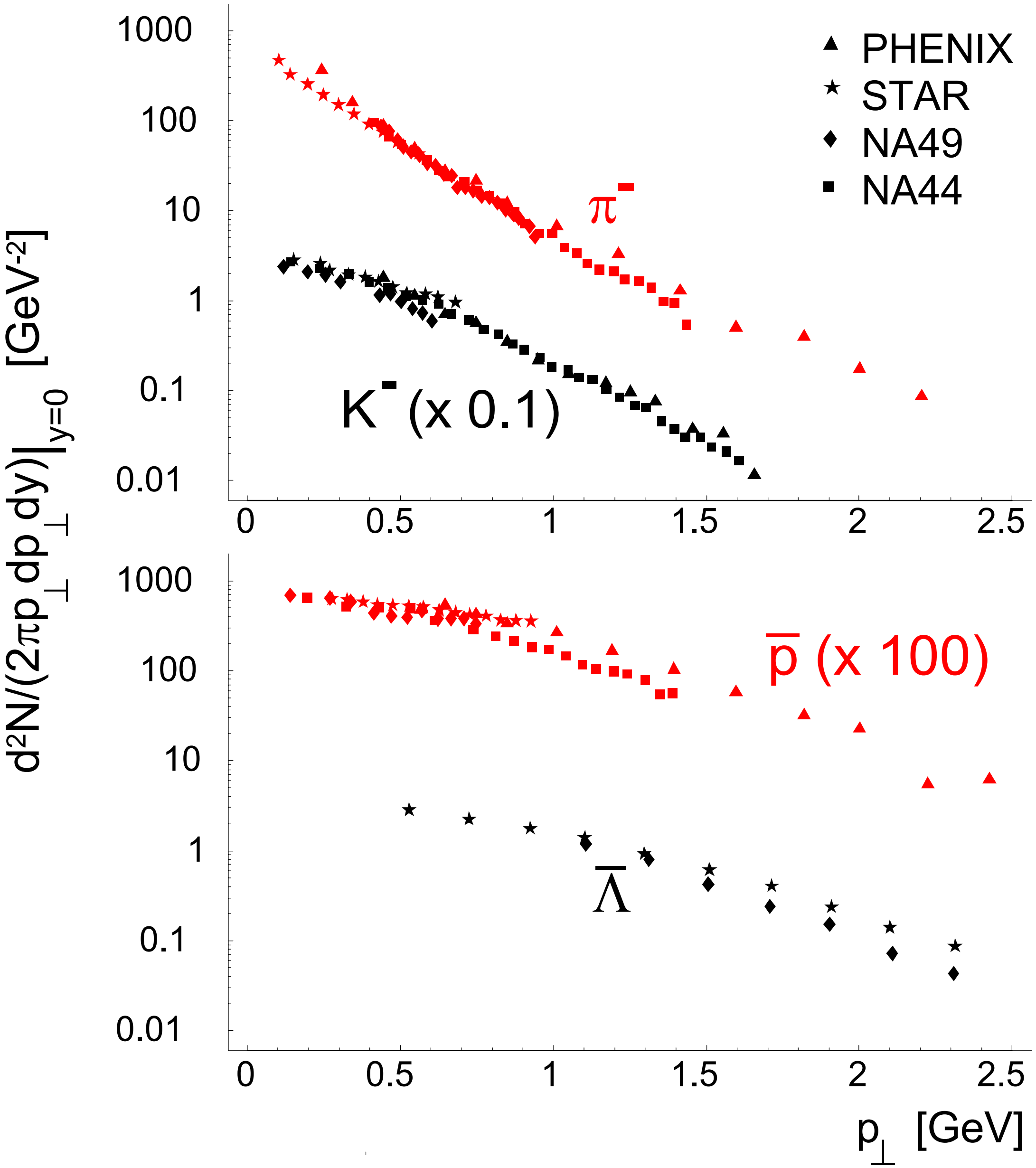


(Ω^- did not work, exp. much steeper)

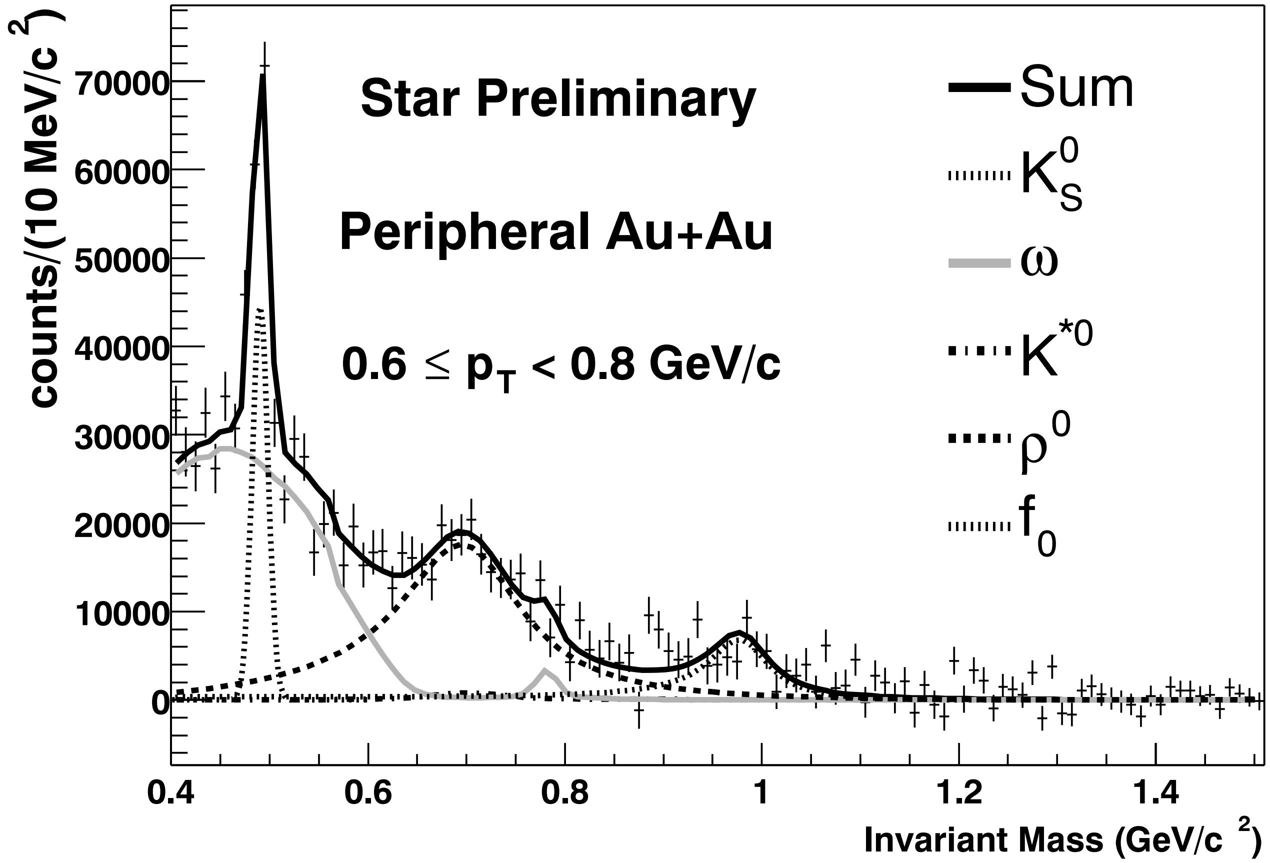
SPS vs. RHIC @ 130 on one plot



The same with spectra rescaled with the factors $e^{-\mu/T}$ + for NA44 centrality correction



$\pi^+\pi^-$ pairs from STAR



(from J. Adams et al., nucl-ex/0307023; P. Fachini, nucl-ex/0305034)

(Brown+Shuryak, Kolb-Prakash, Rapp, Pratt+Bauer)

The phase-shift formula for the density of resonances

Resonances provide kinematic correlations

Beth,Uhlenbeck (1937); Dashen, Ma, Bernstein, Rajaraman (1974); **Weinhold (1998)**, Friman, Nörenberg; **WB, WF, B. Hiller**, PRC **68** (2003) 034911; Pratt, Bauer, nucl-th/0308087

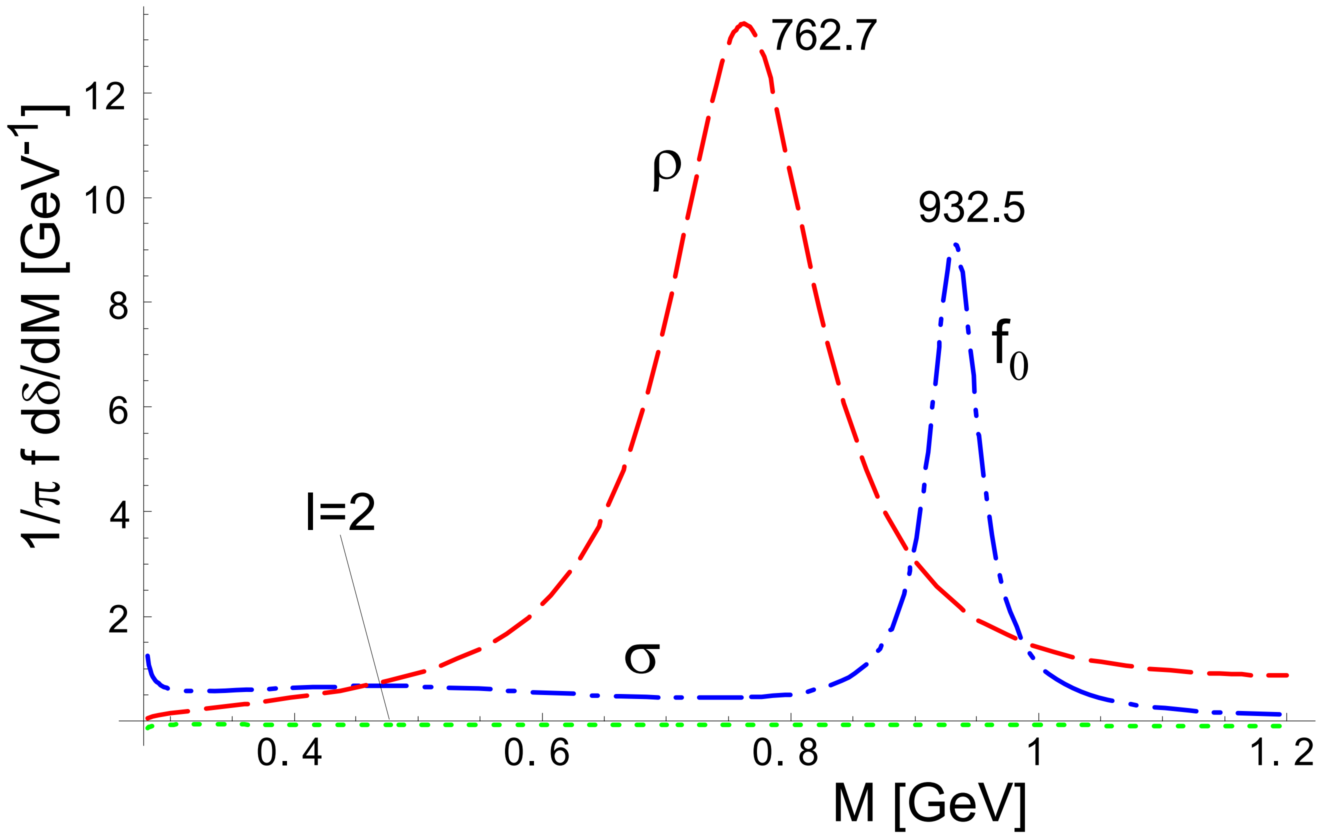
$$\frac{dn}{dM} = f \int \frac{d^3p}{(2\pi)^3} \frac{d\delta_{\pi\pi}(M)}{\pi dM} \frac{1}{\exp\left(\frac{\sqrt{M^2+\boldsymbol{p}^2}}{T}\right) \pm 1}$$

For narrow resonances $d\delta(M)/dM \simeq \pi\delta(M - m_R)$, and

$$n^{\mathrm{narrow}} = f \int \frac{d^3p}{(2\pi)^3} \frac{1}{\exp\left(\frac{\sqrt{m_R^2+\boldsymbol{p}^2}}{T}\right) \pm 1}$$

In a thermal system the density of states changes $\to$ phase shifts appear (not the spectral function) [S. Pratt, Warsaw Meeting on Particle Correlations, 2003]

$d\delta_{\pi\pi}(M)/dM$ **from experiment**

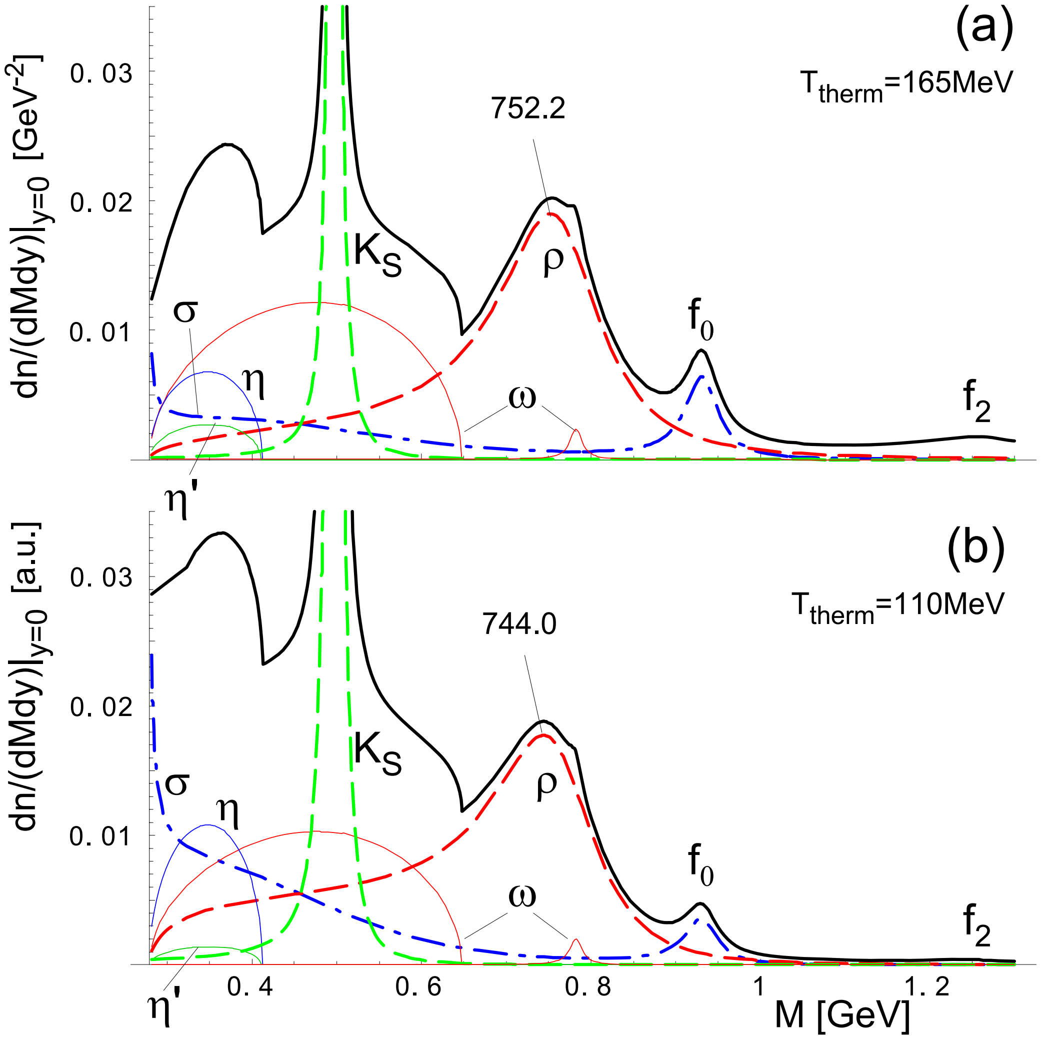


Small contribution from σ, negative and tiny contribution from $I = 2$, ρ-peak slightly shifted to lower M, $1/\sqrt{M - 4m_\pi^2}$ behavior for the σ

Warm-up calculation - static source

We compute the spectra at mid-rapidity, hence

$$\left.\frac{dn}{dM\,dy}\right|_{y=0} = \sum_i f_i \int_{0.2\mathrm{GeV}}^{2.2\mathrm{GeV}} \frac{p_\perp dp_\perp}{(2\pi)^2} \frac{d\delta_i(M)}{\pi dM} \frac{\sqrt{M^2+p_\perp^2}}{\exp\left(\frac{\sqrt{M^2+\boldsymbol{p}_\perp{}^2}}{T}\right)-1}$$



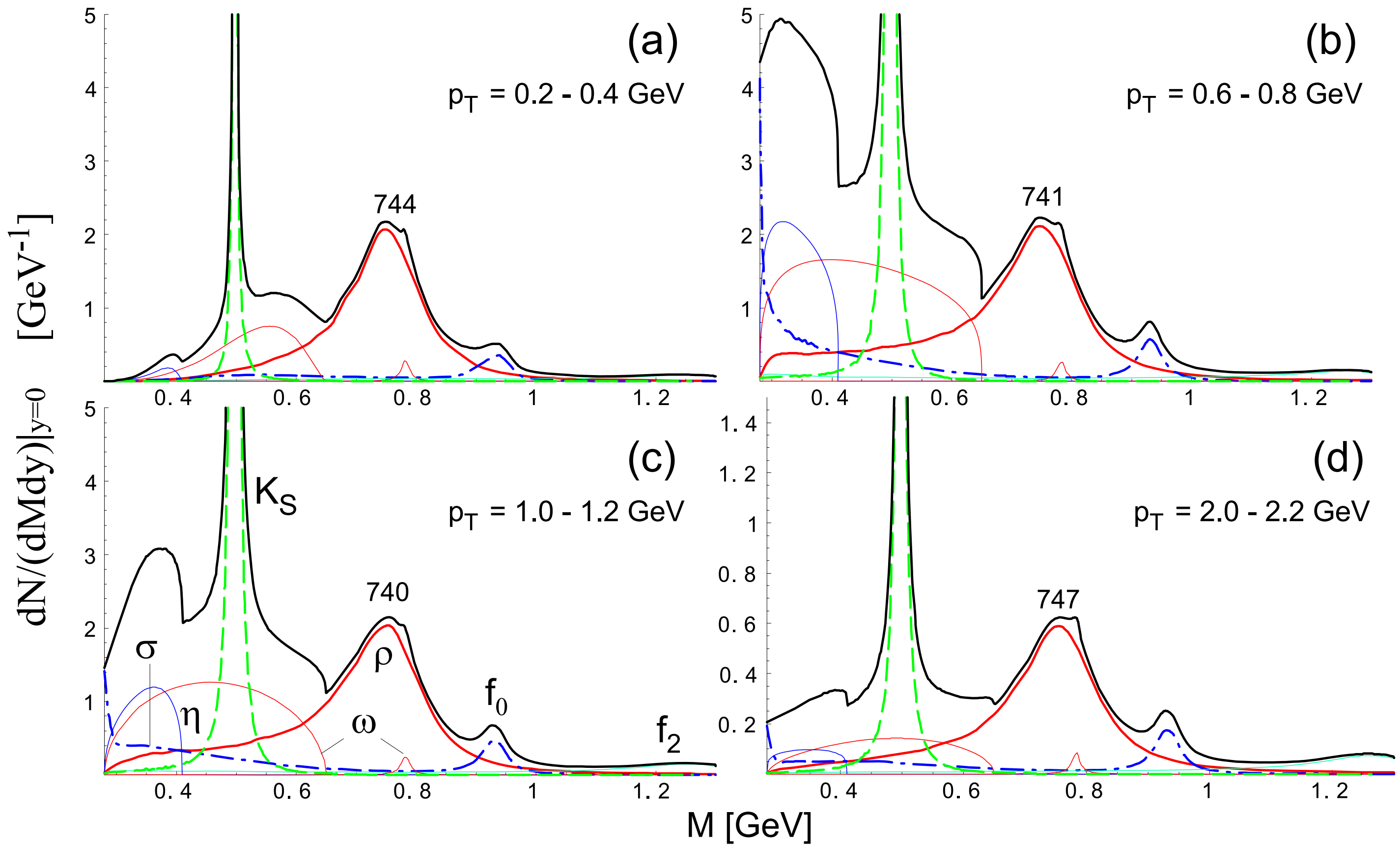
flat

steep

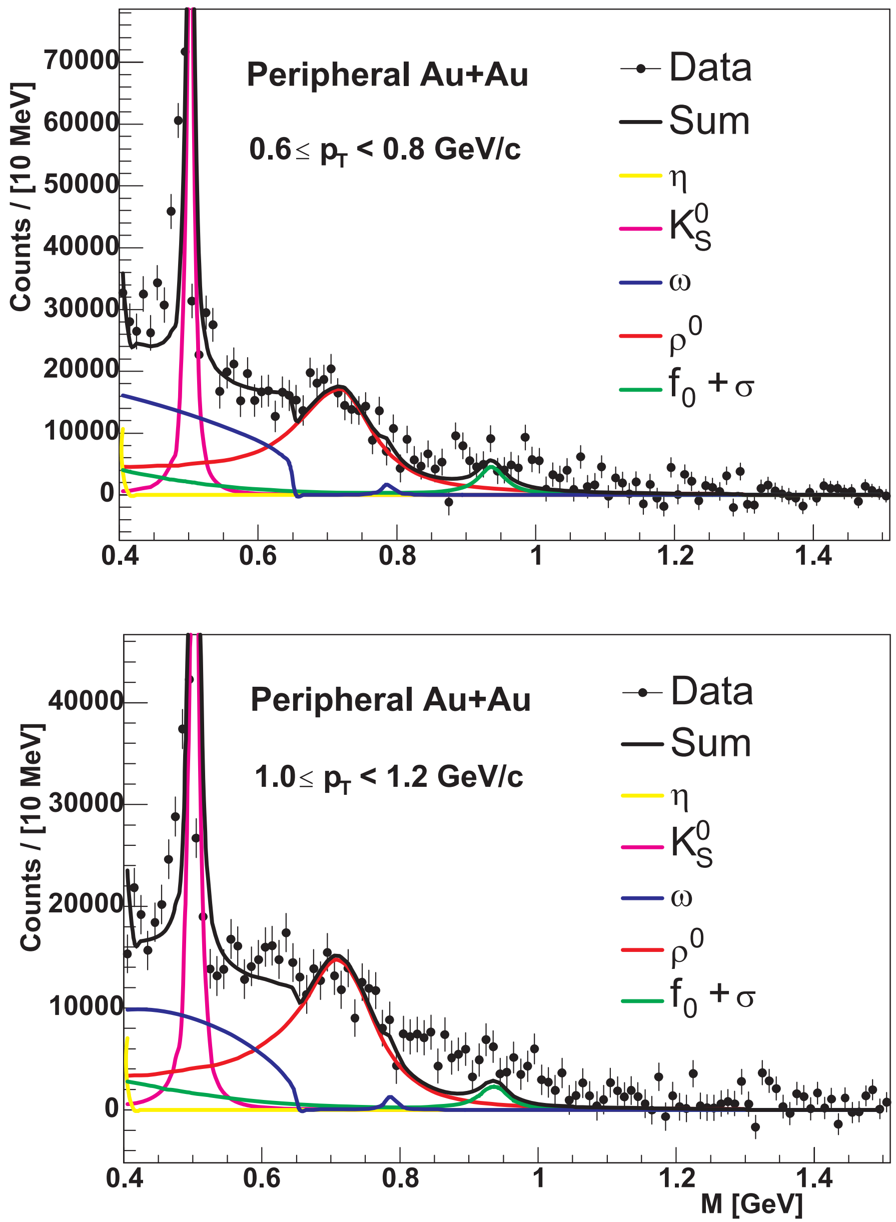
Cuts/flow + feeding from resonances

Flow has no effect on the invariant mass of a pair of particles produced in a resonance decay, since the quantity is Lorentz-invariant. Neverthelss, it affects the results since the kinematic cuts in an obvious manner break this invariance

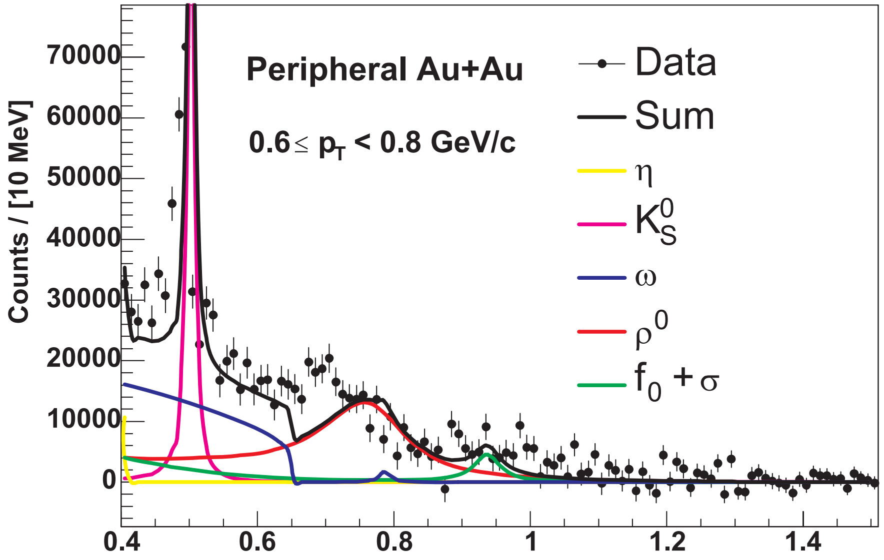
$\sim$ 30 MeV shift of the ρ peak



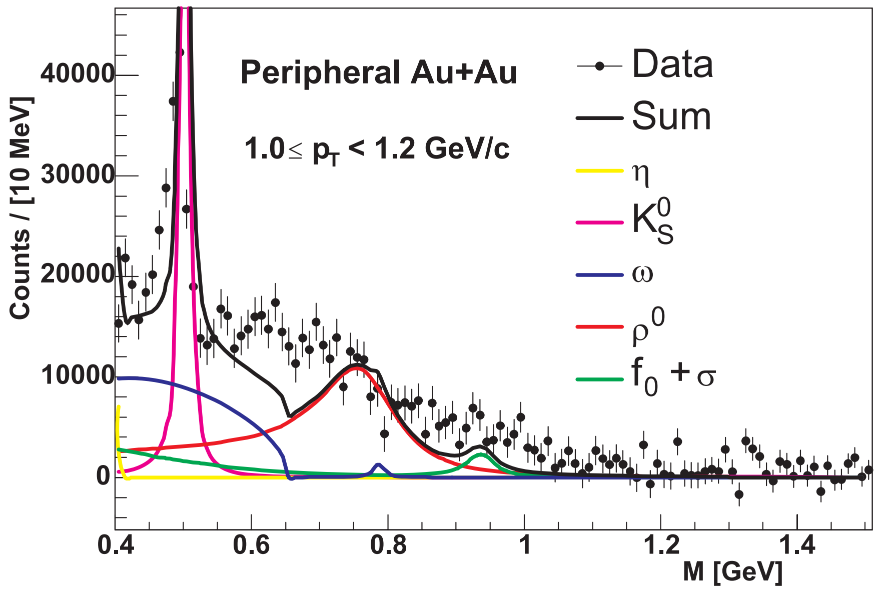
STAR vs. thermal model, lowered ρ



(prepared by P. Fachini)

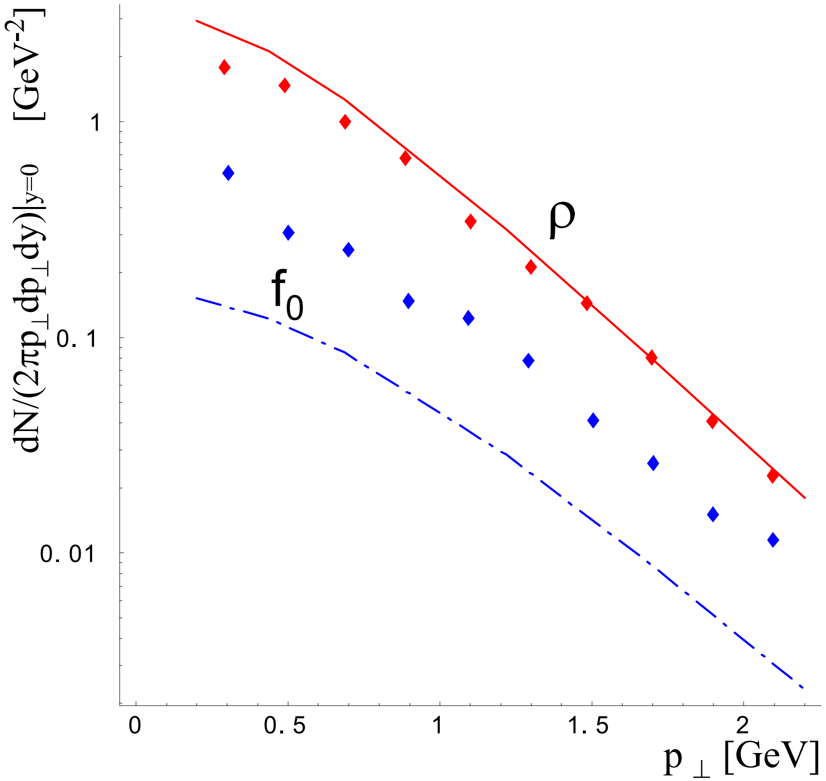


vacuum ρ



(worse agreement)

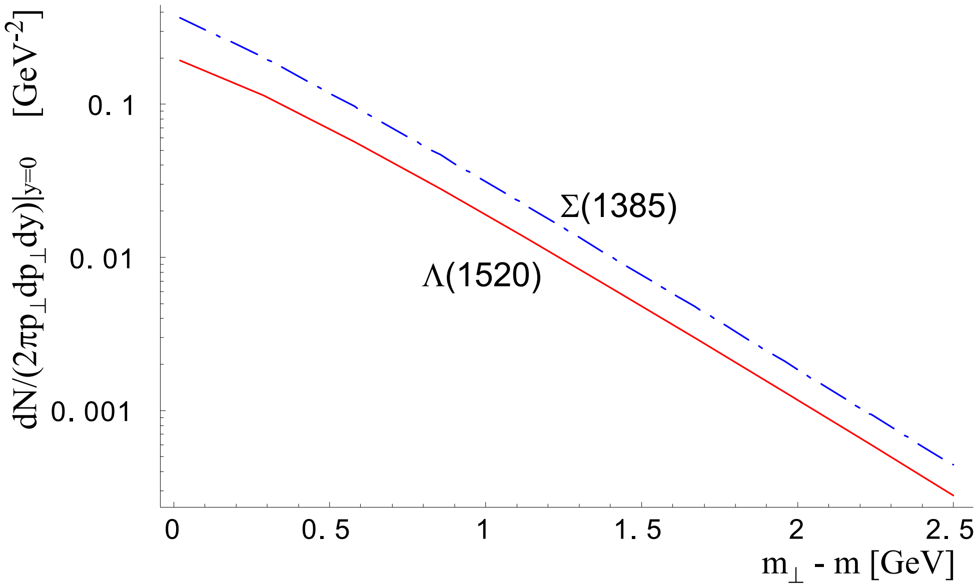
$p_\perp$ spectra of resonances



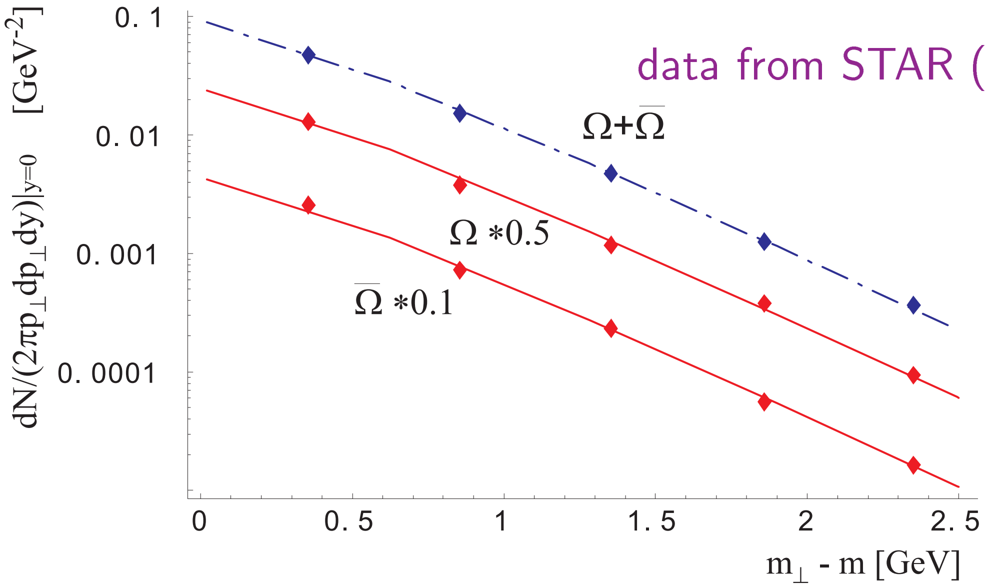
(model parameters: $\tau = 5$ fm and $\rho_{\max} = 4.2$ fm)

For f_0 experiment > thermal model!

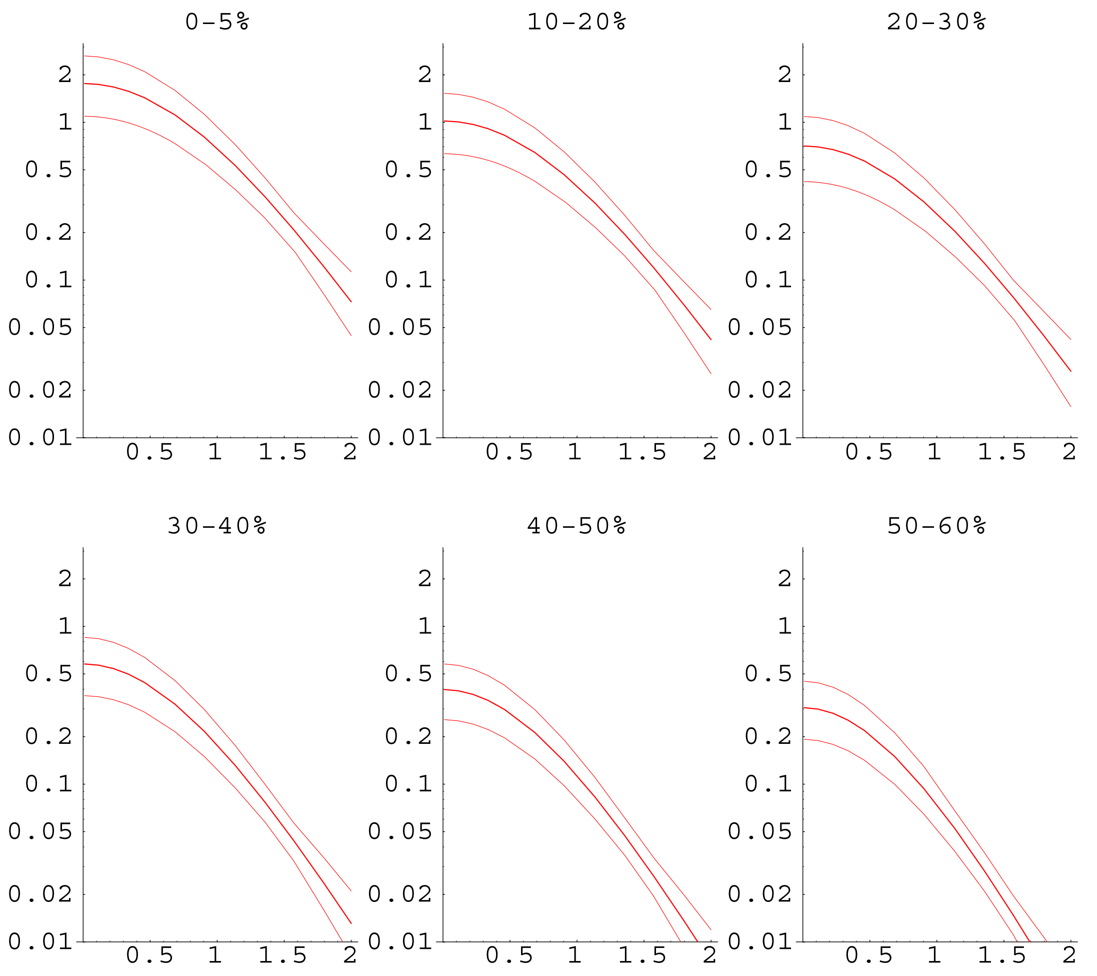
Predictions



data from STAR (C. Suire, QM2002)



Prediction for Δ^{++}



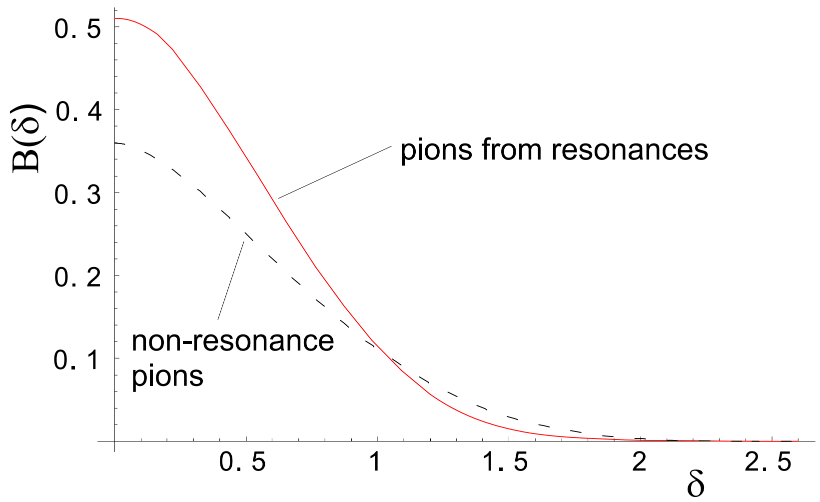
$p_\perp$ spectra for Δ^{++}. The bands indicate the uncertainty of τ and $\rho_{\max}$ from the Table given above.

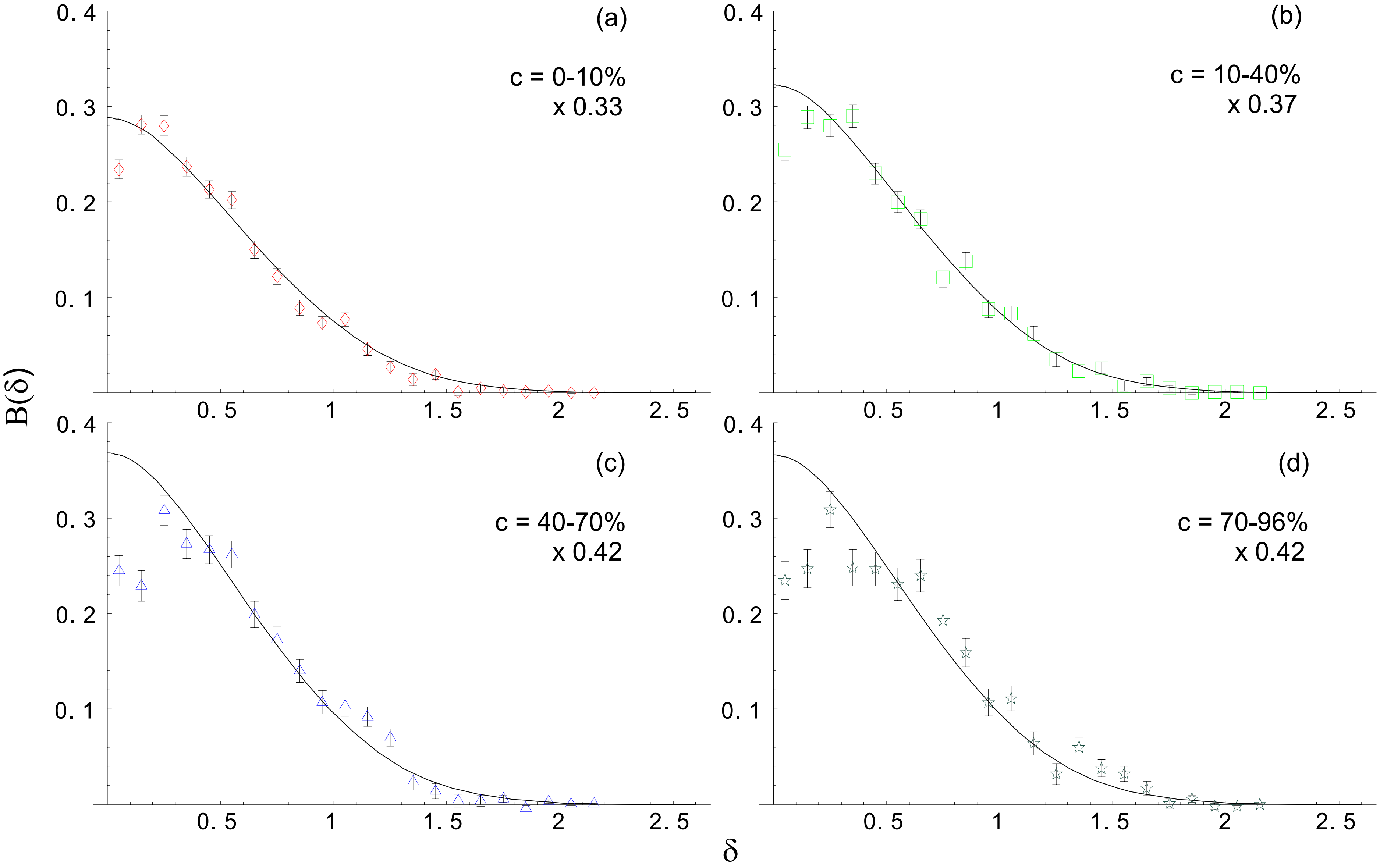
Balance functions in the thermal model

$$B(\delta, Y) = \frac{1}{2}\left\{\frac{\langle N_{+-}(\delta)\rangle - \langle N_{++}(\delta)\rangle}{\langle N_+\rangle} + \frac{\langle N_{-+}(\delta)\rangle - \langle N_{--}(\delta)\rangle}{\langle N_-\rangle}\right\},$$

where $N_{+-}(\delta)$ counts the opposite-charge pairs when both members of the pair fall into the rapidity window Y, $|y_2 - y_1| \equiv \delta$, and N_+ is the number of positive particles in Y.

$$B(\delta, Y) = B_{\mathrm{R}}(\delta, Y) + B_{\mathrm{NR}}(\delta, Y)$$





(data from STAR, PRL 90 (2003) 172301)

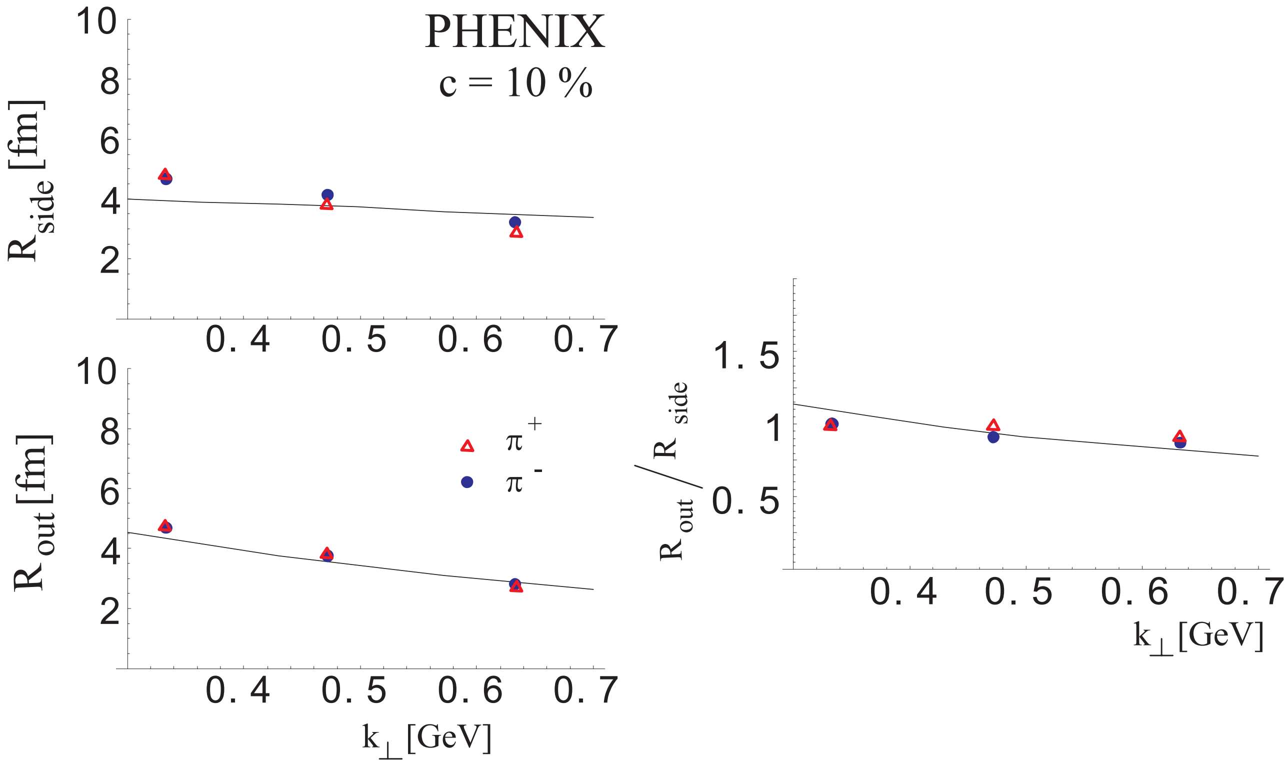
The widths of the balance functions, $\langle\delta\rangle$, are obtained (as in experiment) for the range $0.2 < \delta < 2.6$

Model				
$\rho_{\rm max}/\tau$	$\langle\beta_\perp\rangle$	$\langle\delta\rangle_{\rm res}$	$\langle\delta\rangle_{\rm therm}$	$\langle\delta\rangle_{\rm tot}$
0.9	0.50	0.59	0.67	0.63
Experiment				
$c = 0 - 10\%$				0.594 ± 0.019
$c = 10 - 40\%$				0.622 ± 0.020
$c = 40 - 70\%$				0.633 ± 0.024
$c = 70 - 96\%$				0.664 ± 0.029

HBT radii

$$S(x,p) = \int d\Sigma_\mu p^\mu \delta(x'-x) f(x',p)$$

$$C(\vec{q},\vec{P}) = 1 + \frac{\left|\int d\Sigma(x)\cdot u(x) e^{iq\cdot x} S(P\cdot u(x))\right|^2}{\int d\Sigma \cdot u S((P+\frac{q}{2})\cdot u(x)) \int d\Sigma \cdot u S((P-\frac{q}{2})\cdot u(x))}$$



The pionic HBT radii for most-central collisions @130 GeV, and their ratio, as predicted by the model + excluded volume corrections (∼30% enhancement of model radii) and measured by PHENIX

Excluded-volume (Van der Waals) corrections

Such effects were found important in previous studies of the particle multiplicities in ultra-relativistic heavy-ion collisions, leading to a significant dilution of system. They bring in a factor (Gorenstein)

$$\frac{e^{-Pv_i/T}}{1+\sum_j v_j e^{-Pv_j/T} n_j},$$

into phase-space integrals, where P denotes the pressure, $v_i = 4\frac{4}{3}\pi r_i^3$ is the excluded volume, and n_i is the density of particles of species i. The pressure is calculated self-consistently from the equation

$$P = \sum_i P_i^0(T, \mu_i - Pv_i/T) = \sum_i P_i^0(T, \mu_i) e^{-Pv_i/T}$$

where P_i^0 is the partial pressure of the ideal gas of hadrons of species i. If $r_i = r$, $v_i = v$, the excluded-volume correction produces a common scale factor, S^{-3}. Then

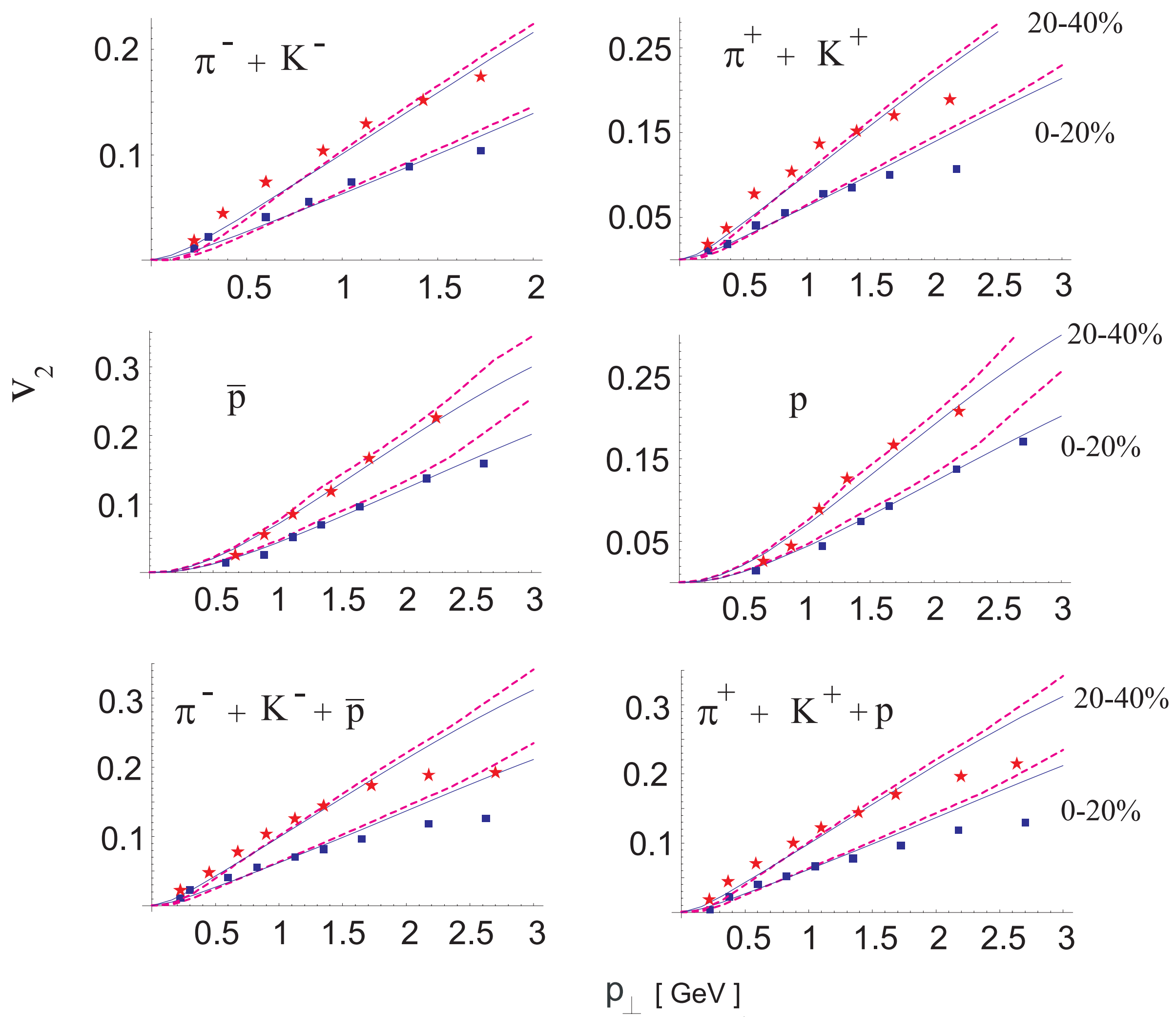
$$\frac{dN_i}{d^2p_\perp dy} = \tau^3 \int_{-\infty}^{+\infty} d\alpha_\parallel \int_0^{\rho_{\max}/\tau} \sinh\alpha_\perp d\left(\sinh\alpha_\perp\right) \int_0^{2\pi} d\xi \, p\cdot u \, S^{-3} f_i\left(p\cdot u\right)$$

The presence of the factor S^{-3} is compensated by rescaling ρ and τ by the factor S. That way, we retain all the previously obtained results for the particle abundances and the momentum spectra. However, now the system is more dilute and larger in size.

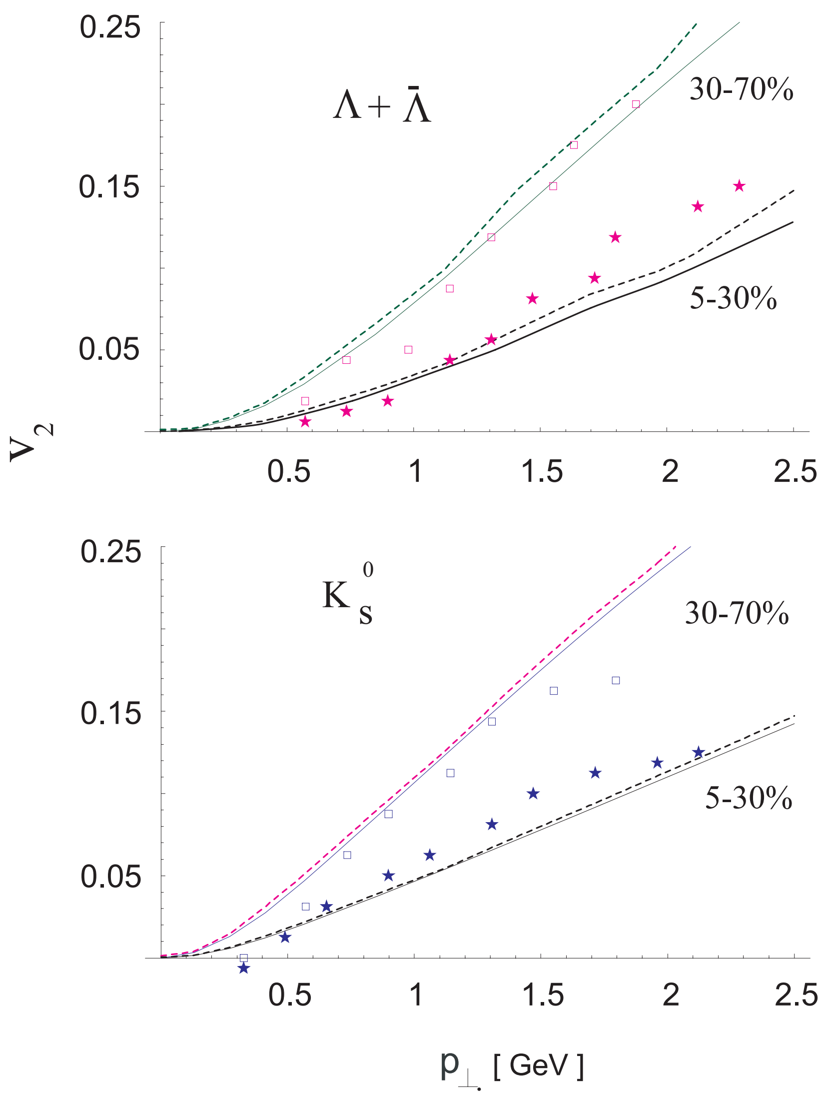
With our values of the thermodynamic parameters we have $\sum_i P_i^0(T,\mu_i) = 80\text{MeV/fm}^3$, which leads to $S = 1.3$ with $r = 0.6$fm. Values of this order have been typically obtained in other works. Thus, the excluded-volume corrections can increase the size parameters at freeze-out by about 30% and help to alleviate the problem with the HBT radii. Hadrons have sizes!

Elliptic flow

(Anna Baran, to be published) Idea: fix azimuthal asymmetry of shape/flow with the data on pions, kaons, ... and then make predictions for other particles. solid (dashed): with (without) resonance decays. (data from PHENIX @ 200 GeV)



v_2 for strange particles



(data from STAR)

Summary

1. Works for abundances, $p_\perp$-spectra, including strange particles and resonances
2. Lower $T_{\rm kin}$ would lead to much less resonances!
3. Resonances are an important source of correlations
4. Shape of the $\pi\pi$ "spectral line" - new thermometer, derivative of phase shifts must be used, full model gives similar results at 165 MeV to the naive calculation at 110 MeV (cooling via decays)
5. Not possible to place the ρ peak at the experimental value. Medium effects? (Brown-Rho-Shuryak)
6. By summing up the resonance and non-resonance contributions we obtain the pion balance function with the shape similar to the data
7. $R_{\rm out}/R_{\rm side} \sim 1$
8. v_2 similar to hydro

Soft physics ($p_\perp < 1.5 - 2$) GeV is well desribed by the thermal approach with the single-freezeout approximation and resonance decays