Transversity structure of the pion in chiral quark models

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Mini-Workshop Bled 2011: *Understanding Hadronic Spectra*

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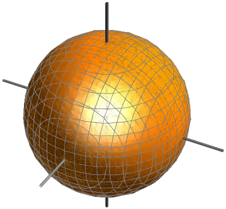
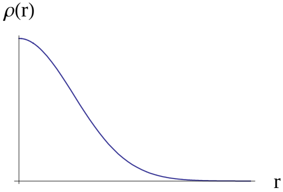
Details in

- *Transversity form factors of the pion in chiral quark models*
  WB, Alexander E. Dorokhov, Enrique Ruiz Arriola,
  PRD 82 (2010) 094001, arXiv:1007.4960 [hep-ph]

Outline

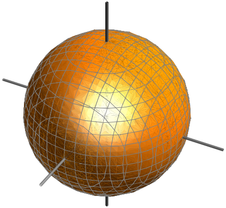
Distribution of charge

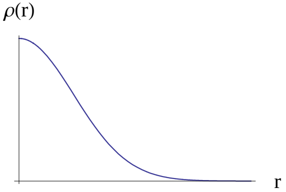




$$Q = \int d^3 r \rho(r), \qquad F(q^2) = \frac{1}{Q} \int d^3 r e^{-i\vec{q}\cdot\vec{r}} \rho(r)$$

Distribution of charge





$$Q = \int d^3 r \rho(r), \qquad F(q^2) = \frac{1}{Q} \int d^3 r e^{-i\vec{q}\cdot\vec{r}} \rho(r)$$

$$= \frac{1}{Q} \int d^3 r \, \rho(r) [1 - i\vec{q}\cdot\vec{r} - \frac{1}{2}(\vec{q}\cdot\vec{r})^2 + \ldots] = 1 - \frac{q^2}{6Q} \int d^3 r \, r^2 \rho(r) + \ldots$$

$$\langle r^2 \rangle = -6 \frac{d}{dq^2} F(q^2)$$

Definition of transversity form factors

FF related to the transversity Generalized Parton Distribution
(maximum-helicity GPD, related to spin distributions)

$$\langle \pi^+(p')|\mathcal{O}_T^{\mu\nu\mu_1\cdots\mu_{n-1}}|\pi^+(p)\rangle = \mathcal{AS}\,\bar{p}^\mu \Delta^\nu \sum_{\substack{i=0\\ \text{even}}}^{n-1} \Delta^{\mu_1}\cdots\Delta^{\mu_i}\,\bar{p}^{\mu_{i+1}}\cdots\bar{p}^{\mu_{n-1}}\,\frac{B_{Tni}^{\pi,u}(t)}{m_\pi}$$

$\bar{p} = \frac{1}{2}(p' + p)$, $\Delta = p' - p$, $t = \Delta^2$, $\mathcal{AS}$ – symmetrization in $\nu, \ldots, \mu_{n-1}$, followed by antisymmetrization in μ, ν, traces in all index pairs are subtracted.

$$\mathcal{O}_T^{\mu\nu\mu_1\cdots\mu_{n-1}} = \mathcal{AS}\,\overline{u}(0)\,i\sigma^{\mu\nu}\,i\overleftrightarrow{D}^{\mu_1}\ldots i\overleftrightarrow{D}^{\mu_{n-1}}u(0)$$

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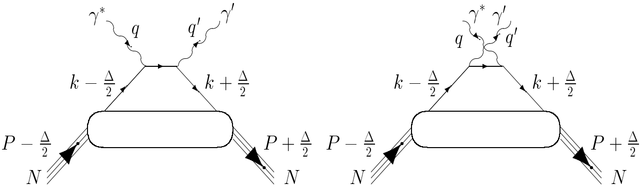
$\bar{p}=\frac{1}{2}(p'+p)$, $\Delta=p'-p$, $t=\Delta^2$, $\mathcal{AS}$ – symmetrization in $\nu,\ldots,\mu_{n-1}$, followed by antisymmetrization in μ,ν, traces in all index pairs are subtracted.

$$\mathcal{O}_T^{\mu\nu\mu_1\cdots\mu_{n-1}}=\mathcal{AS}\,\overline{u}(0)\,i\sigma^{\mu\nu}i\overleftrightarrow{D}^{\mu_1}\ldots i\overleftrightarrow{D}^{\mu_{n-1}}u(0)$$

$$\langle\pi^+(p')|\overline{u}(0)\sigma^{\mu\nu}u(0)|\pi^+(p)\rangle=\mathcal{AS}\,\bar{p}^\mu\Delta^\nu\frac{B_{T10}^{\pi,u}(t)}{m_\pi}$$

$$\mathcal{AS}\,\langle\pi^+(p')|\overline{u}(0)\sigma^{\mu\nu}\overleftrightarrow{D}^{\mu_1}u(0)|\pi^+(p)\rangle=\mathcal{AS}\,\bar{p}^\mu\Delta^\nu\bar{p}^{\mu_1}\frac{B_{T20}^{\pi,u}(t)}{m_\pi}$$

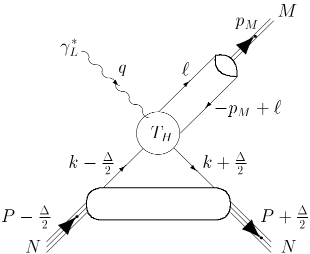
Exclusive processes in QCD



(a) (b)

Deeply Virtual Compton Scattering

non-zero momentum transfer to the target, at least one photon virtual



Hard Meson Production

Dictionary of matrix elements

General structure of the soft matrix elements: $\langle A \mid \mathcal{O} \mid B \rangle$

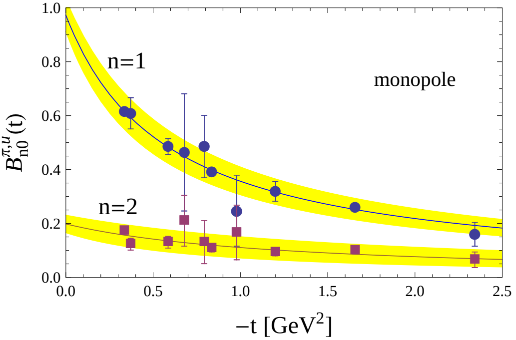
- A = B = one-particle state – Parton Distribution of A (inclusive DIS)
- A = one-particle state, B = vacuum – distribution amplitude (DA) of A (hadronic form factors, HMP)
- A, B = one-particle state of different momentum – GPD (exclusive DIS, DVCS, HMP)
- A = many-particle state, B = vacuum – GDA (transition form factors)
- A $\neq$ B (A, B – different hadronic states) – Transition Distribution Amplitude ($h\bar{h} \to \gamma\gamma^*$, Pire & Szymanowski 2004)
- ...

Lattice results

Many results which come from lattices, especially for the **pion**, will not be accessible in physical experiments. We want to compute them!

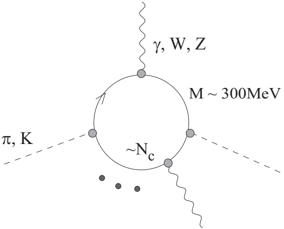
Motivation: QCDSF data

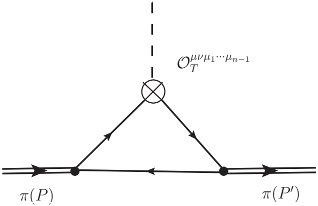
[data from D. Brommel et al. (QCDSF), PRL 101 (2008) 122001]



Monopole fit: $m_1 = 760 \pm 50$ MeV (ρ), $m_2 = 1120 \pm 250$ MeV (f_2) (meson dominance)

Basic idea of chiral quark models





+ crossed

- one-quark-loop, large N_c
- covariant Lagrangian calculation
- soft regime → chiral symmetry breaking
- NJL, local and nonlocal, instanton-motivated
- few parameters (traded for f_π, m_π, . . .)
- numerous processes with pions, γ, . . .
- no confinement - careful not to open the $q\overline{q}$ threshold
- quark model scale low - need for QCD evolution to higher scales

Pion develops structure via the quark loop

Pion develops structure via the quark loop

approach = model + QCD evolution

Evolution generates gluons and the sea quarks as the scale is increased

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All features satisfied: support, polynomiality, positivity, charge and momentum sum rules, Callan-Gross ($F_2 = 2xF_1$), . . .)

Pion develops structure via the quark loop

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Next: glossary of our old results showing that the approach is reasonable for computing soft matrix elements appearing in high-energy experiments and lattices

Parton Distribution Function of the pion

NJL gives the constant valence PDF [Davidson, Arriola, 1995]:

$$q(x) = 1$$

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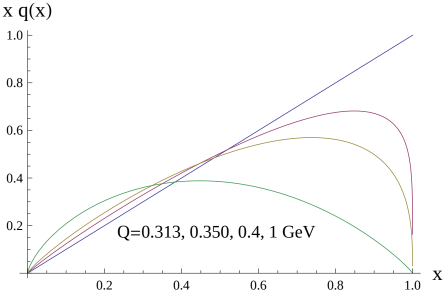
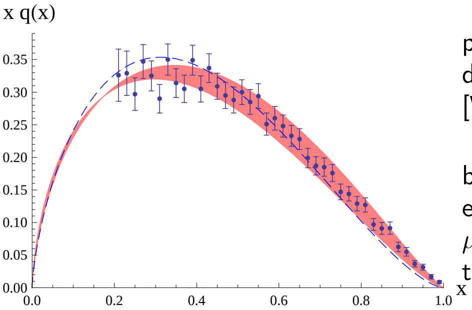
LO DGLAP QCD evolution (good at intermediate x) to higher scales



constant PDF of the also pion follows from AdS/CFT models [Brodsky, Teramond 2008] $\rightarrow$ BaBar pion-photon transition form factor

The question of renormalization scale: momentum sum-rule $\rightarrow$ $\mu_0 \sim 320$ MeV $\rightarrow$ at 2 GeV valence quarks carry 47% of the momentum (Durham), $\alpha(\mu_0)/\pi = 0.68$

Valence PDF from NJL vs E615



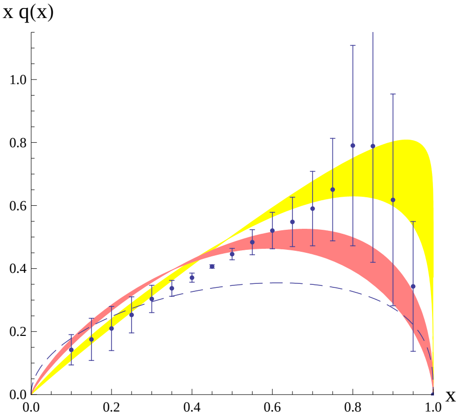
points: Drell-Yan from E615
dashed: reanalysis of the data [Wijesooriya et al., 2005]

band: valence PDF from NJL evolved from the QM scale $\mu_0 = 313^{+20}_{-10}$ MeV to $\mu = 2$ GeV of the experiment

(last year's analysis of Aicher, Schafer, and Vogelsang, including the soft gluon resummation, moves the strength to lower x)

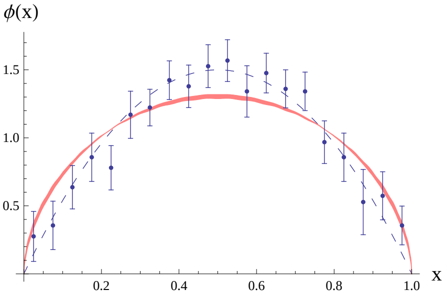
Valence PDF from NJL vs. transverse lattice

transverse lattices: [Burkardt, Dalley, Van de Sande]

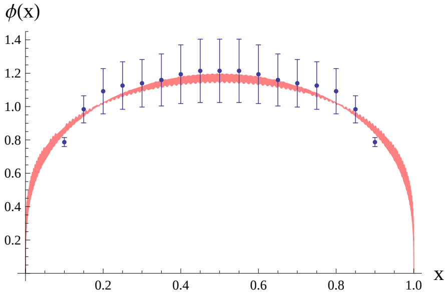


points: transverse lattice [Dalley, Van de Sande, 2003]
yellow: NJL evolved to $\mu = 0.35$ GeV
pink: NJL evolved to $\mu = 0.5$ GeV
dashed: GRS parametrization at $\mu = 0.5$ GeV

PDA from NJL vs. E791 and lattice data

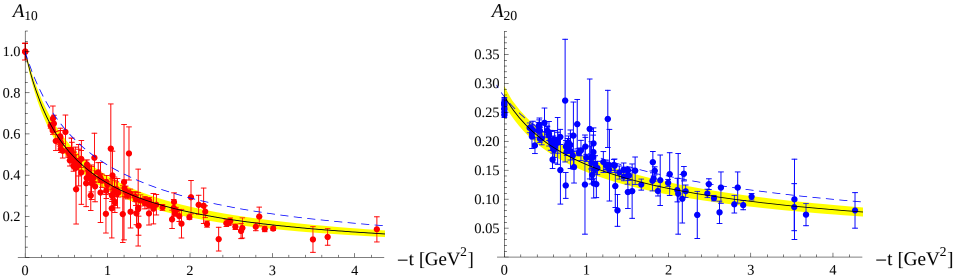


points: E791 data from di-jet production in $\pi + A$
band: NJL evolved to $\mu = 2$ GeV
dashed line: asymptotic form ($\mu \to \infty$)



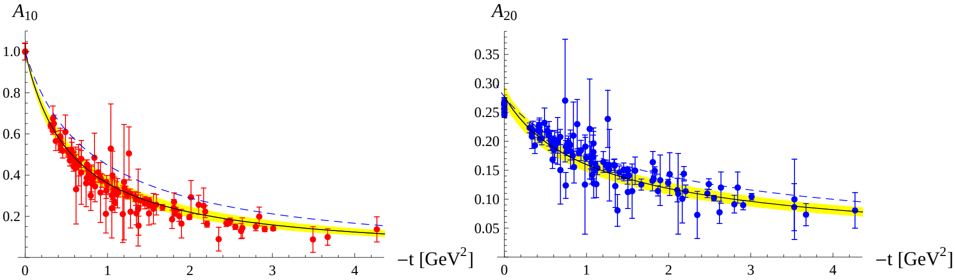
points: transverse lattice data [Dalley, Van de Sande, 2003]
band: NJL evolved to $\mu = 0.5$ GeV

Gravitational form factors



Pion charge ff (left) and the quark part of the spin-2 gravitational ff (right) in SQM (solid line) and NJL (dashed line) [WB, ERA 2008], compared to the data [Brömmel et al., 2005-7]

Gravitational form factors



Pion charge ff (left) and the quark part of the spin-2 gravitational ff (right) in SQM (solid line) and NJL (dashed line) [WB, ERA 2008], compared to the data [Brömmel et al., 2005-7]

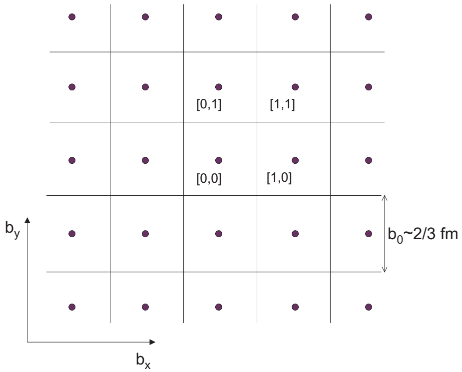
Quark-model relation: $\langle r^2 \rangle_\Theta = \frac{1}{2} \langle r^2 \rangle_V$

Matter more concentrated than charge!

(later also found in soft-wall AdS/CFT by Brodsky and Teramond)

Forward ($\xi = 0$) GPDs and transverse lattices

WB and ERA, *Impact parameter dependence of the generalized parton distribution of the pion in chiral quark models*, PLB 574 (2003) 57]
data from S. Dalley, PLB570 (2003) 191

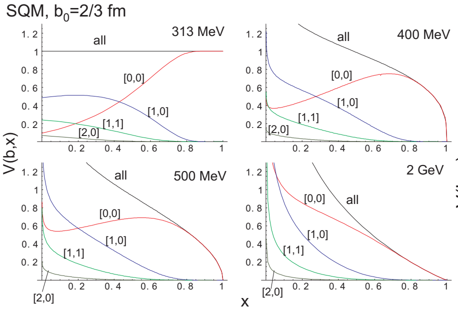


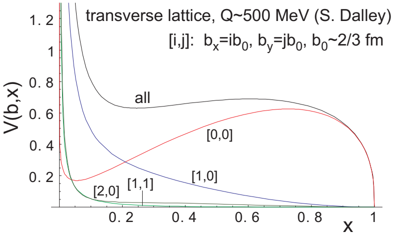
b conjugate to momentum Δ, labeling of lattice plaquettes

Forward GPD of the pion in NJL vs lattice

model

lattice data



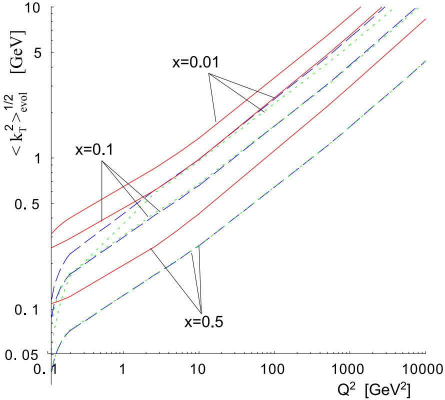


fair agreement for scales $\mu \sim 400$ MeV

TMD

TMD = k_T-unintegrated PDF

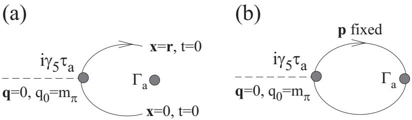
[Kwiecinski 2002; Gawron, Kwiecinski, WB, 2003; ERA, WB, 2004]

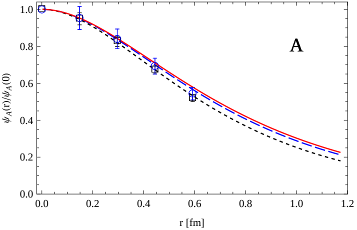


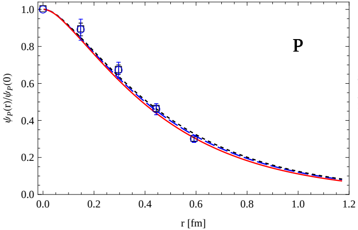
solid – valence
dotted – sea
dashed – gluons

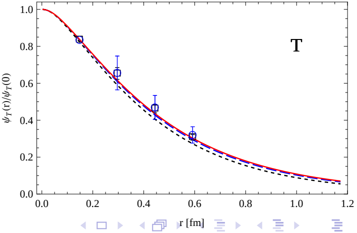
Pion wave function

[Pion wave function from lattice QCD vs. chiral quark models, WB, S. Prelovsek, L. Santelj, E. Ruiz Arriola, PLB 686 (2010) 313, arXiv:0911.4705]









. . . back to transversity form factors

Evolution of transversity GFFs

[WB, ERA, PRD 79 (2009) 057501] → LO DGLAP-ERBL evolution of GFFs

$$\gamma_n^T = -2C_F\left(3 - 4\sum_1^n \frac{1}{k}\right), \quad L_n = \left(\frac{\alpha(\mu)}{\alpha(\mu_0)}\right)^{\gamma_n^T/(2\beta_0)}$$

$$\begin{aligned}
B_{T10}(t;\mu) &= L_1 B_{T10}(t;\mu_0) \\
B_{T20}(t;\mu) &= L_2 B_{T20}(t;\mu_0) \\
B_{T30}(t;\mu) &= L_3 B_{T30}(t;\mu_0) \\
B_{T32}(t;\mu) &= \frac{1}{5}\left(L_1 - L_3\right) B_{T30}(t;\mu_0) + L_3 B_{T32}(t;\mu_0) \\
&\cdots
\end{aligned}$$

Multiplicative evolution for B_{Tn0}, absolute predictions for FF at the origin

Evolution of transversity GFFs

[WB, ERA, PRD 79 (2009) 057501] $\to$ LO DGLAP-ERBL evolution of GFFs

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B_{T32}(t;\mu) &= \frac{1}{5}\left(L_1 - L_3\right) B_{T30}(t;\mu_0) + L_3 B_{T32}(t;\mu_0) \\
&\dots
\end{aligned}$$

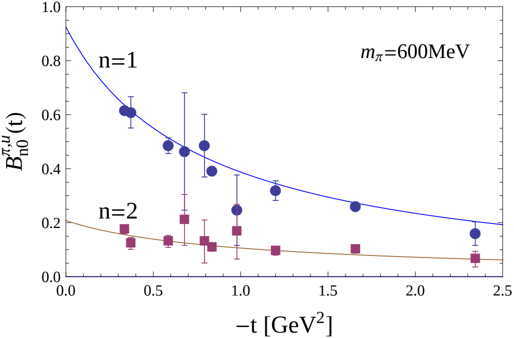
Multiplicative evolution for B_{Tn0}, absolute predictions for FF at the origin

$$\gamma_1^T = \frac{8}{3}, \quad \gamma_2^T = 8$$

$$B_{T10}(t;2\ \text{GeV}) = 0.75 B_{T10}(t;313\ \text{MeV})$$

$$B_{T20}(t;2\ \text{GeV}) = 0.43 B_{T20}(t;313\ \text{MeV})$$

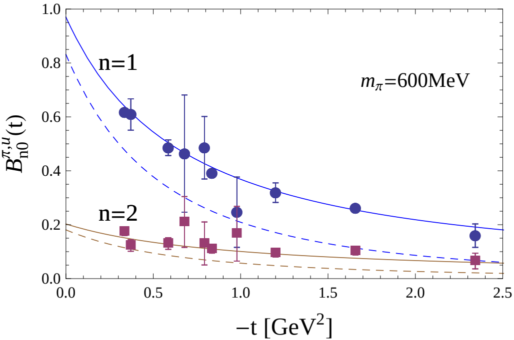
B_{T10} and B_{T20} in NJL



NJL, $M = 250$ MeV $m_\pi = 600$ MeV, evolved to the lattice scale of 2 GeV [data from Brommel et al.]

- correct fall-off and central values

Transversity GFFs in nonlocal models

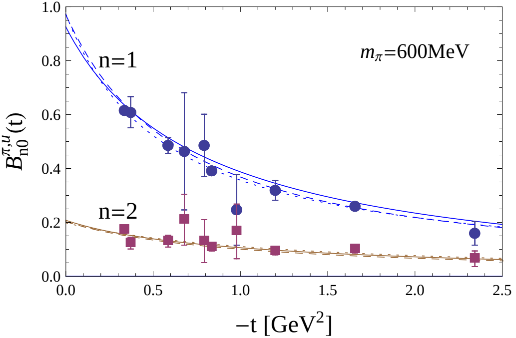


dashed – instanton-based model, $F = \sqrt{M(k_+^2)M(k_-^2)}$

solid – Holdom-Terning-Verbeek (HTV), $F = \frac{1}{2}\left(M(k_+^2) + M(k_-^2)\right)$
[PLB 245 (1990) 612]

- HTV in better agreement

HTV vs NJL

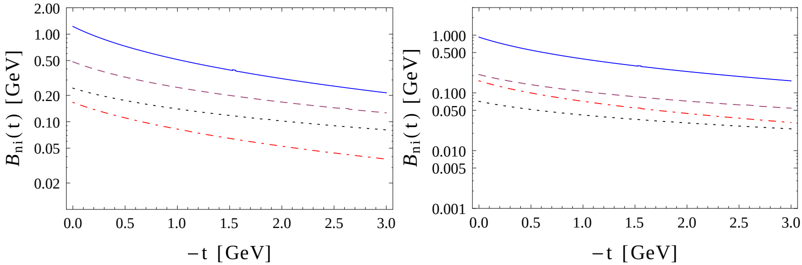


solid – local NJL dashed – HTV

- HTV very close to local NJL

Higher form factors

$\mu = 313$ MeV $\qquad\qquad \mu = 2$ GeV



solid: $B^{\pi,u}_{T10}$, dashed: $B^{\pi,u}_{T20}$, dotted: $B^{\pi,u}_{T30}$, dot-dash: $B^{\pi,u}_{T32}$

- Can be measured on the lattice

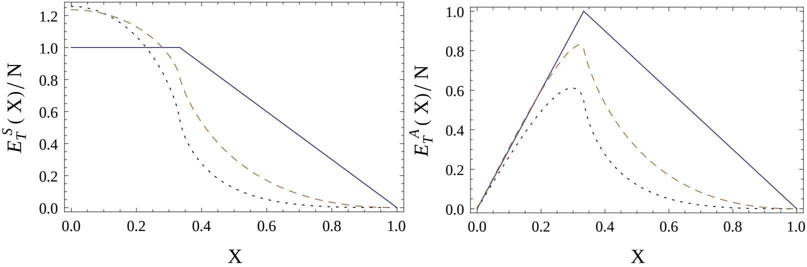
Transversity GPD

Full GPDs

GPD = infinite collections of generalized form factors

$$\int_{-1}^{1} dX\, X^{n-1} E_T^{\pi}(x,\xi,t) = \sum_{\substack{i=0,\\ \text{even}}}^{n-1} (2\xi)^i\, B_{Tni}^{\pi}(t)$$

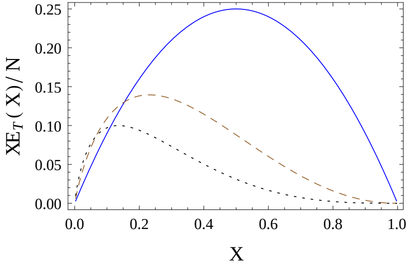
Evolution of transversity GPDs, $\xi = 1/3$, $t = 0$



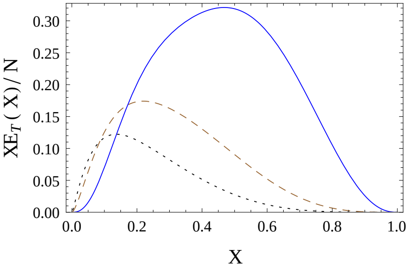
local NJL, $t = 0$, $\xi = 1/3$, $\mu = 313$ MeV, 2 GeV, 1 TeV

Same for $\xi = 0$

local NJL



nonlocal, instanton vertex



local NJL vs nonlocal instanton, $t = 0$, $\xi = 0$, $\mu = 313$ MeV, 2 GeV, 1 TeV

- different end-point behavior, related to the non-locality

Meson dominance of form factors

charge pion ff – m_ρ
gravitational pion ff – $m_{f_2(1270)}$ (spin 2), m_σ (spin 0)
B^π_{T10} – m_ρ
B^π_{T20} – $m_{f_2(1270)}$

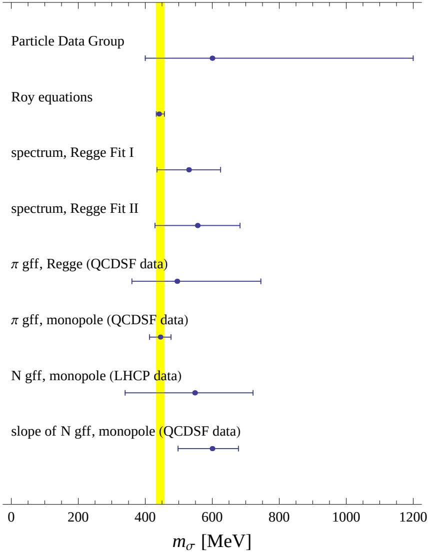
Monopole fit to the lattice data:

$$M_1 = 760(50)\ \text{MeV}, \quad B^\pi_{T10}(t=0) = 0.97(6)$$
$$M_2 = 1120(250)\ \text{MeV}, \quad B^\pi_{T20}(t=0) = 0.20(3)$$

• Values at the origin are model predictions, QCD evolution necessary

Meson dominance seems to be working very well in all channels!

$\sigma(600)$ compilation



[ERA, WB, PRD 81 (2010) 054009]

results consistent with the Bern and Madrid analyses (Roy equations)

Summary

- "Model + evolution"
- GPDs in various channels are collections of generalized form factors
- Computed on the lattice
- Chiral quark models compare nicely to lattice results, link between the high- and low-energy analyses
- The QCD evolution is necessary, the quark-model scale μ_0 is low, ~ 320 MeV (no gluons)
- B_{Tn0} evolve multiplicatively, absolute predictions
- NJL works $\to$ chiral symmetry breaking is the key dynamical factor for soft physics
- Lattice form factors give the meson masses pretty accurately! Meson dominance works remarkably well for the vector, 2 gravitational, and 2 transversity ff (it also works in the nucleon channel)
- $\to$ constituent quark – hadron duality
- "All" works!