Large-N_c Regge models and the $\langle A^2 \rangle$ condensate*

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Outline

1. Regge models
   - Matching to OPE
   - Strictly linear trajectories
   - Modified model
   - Summary of Regge models
2. The $\langle A^2 \rangle$ condensate
   - Chetyrkin-Narison-Zakharov power corrections
   - Interpretation of $\langle A^2 \rangle$
   - Dimension-2 condensates from the lattice
   - Summary of $\langle A^2 \rangle$

Basic idea

[Golterman 2001, Beane 2001, Simonov 2001, Afonin 2003, ...]

- Use OPE of QCD (here with $m = 0$)
- Incorporate confinement in terms of the radial Regge spectra which satisfy certain constraints at high energies
- Large N_c is assumed, *i.e.* the correlators are saturated with sharp non-interacting meson states • compare correlators

$$\Pi^T_{V+A}(Q^2) = \frac{1}{4\pi^2}\left\{ -\frac{N_c}{3}\log\frac{Q^2}{\mu^2} - \frac{\alpha_S}{\pi}\frac{\lambda^2}{Q^2} + \frac{\pi}{3}\frac{\langle \alpha_S G^2 \rangle}{Q^4} + \ldots \right\}$$

$$\Pi^T_{V-A}(Q^2) = -\frac{32\pi}{9}\frac{\alpha_S \langle \bar{q}q \rangle^2}{Q^6} + \ldots$$

Non-standard $1/Q^2$ term [Chetyrkin, Narison, Zakharov 1999] discussed later

At large-N_c

$$\Pi_V^T(Q^2) = \sum_{n=0}^{\infty} \frac{F_{V,n}^2}{M_{V,n}^2 + Q^2} + c.t., \quad \Pi_A^T(Q^2) = \frac{f^2}{Q^2} + \sum_{n=0}^{\infty} \frac{F_{A,n}^2}{M_{A,n}^2 + Q^2} + c.t.$$

Use radial Regge spectra (fulfilled experimentally [Anisovich 2000])

$$M_{V,n}^2 = M_V^2 + a_V n, \quad M_{A,n}^2 = M_A^2 + a_A n, \quad n = 0, 1, \ldots$$

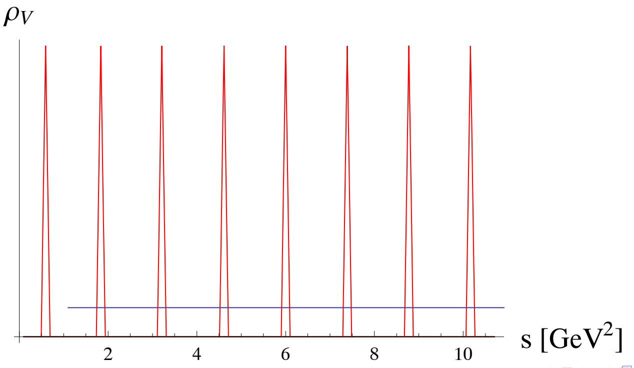
Π_V^T satisfies the once-subtracted dispersion relation $\rightarrow$

$$\Pi_V^T(Q^2) = \sum_{n=0}^{\infty} \left(\frac{F_{V,n}^2}{M_V^2 + a_V n + Q^2} - \frac{F_{V,n}^2}{M_V^2 + a_V n} \right)$$

We need to reproduce the $\log Q^2$ in OPE, for which only the asymptotic part of the spectrum matters $\rightarrow$ at large n the residues are $F_{V,n} \simeq F_V$, similarly $F_{A,n} \simeq F_A$

All highly-excited radial states are coupled to the current with equal strength!

$$\rho_V(s) = F_V^2 \sum_{i=0}^{\infty} \delta(s - M_V^2 + a_V n)$$



Dependence of $F_{V,n}$ or $F_{A,n}$ on n would damage OPE

parton-hadron duality $\to$ linear radial Regge spectra have asymptotically constant residues

Basic identity ($\psi(z) = \Gamma'(z)/\Gamma(z)$):

$$\sum_{n=0}^{\infty} \left(\frac{F^2}{M^2 + an + Q^2} - \frac{F^2}{M^2 + an} \right) = \frac{F^2}{a} \left[\psi\left(\frac{M^2}{a}\right) - \psi\left(\frac{M^2 + Q^2}{a}\right) \right]$$

$$= \frac{F^2}{a} \left[-\log\left(\frac{Q^2}{a}\right) + \psi\left(\frac{M^2}{a}\right) + \frac{a - 2M^2}{2Q^2} + \frac{6M^4 - 6aM^2 + a^2}{12Q^4} + \ldots \right]$$

Π_{V-A} satisfies the unsubtracted dispersion relation (no $\log Q^2$) $\to$

$$F_V^2/a_V = F_A^2/a_A$$

"Chiral symmetry restoration" in the spectra [Glozman 2002]

$$a_V = a_A = a \quad \longrightarrow \quad F_V = F_A = F$$

Exp. [Anisovich 2000]: $\sqrt{\sigma_A} = 464\text{MeV}$, $\sqrt{\sigma_V} = 470\text{MeV}$

$$a_V = a_A = a \quad \longrightarrow \quad F_V = F_A = F$$

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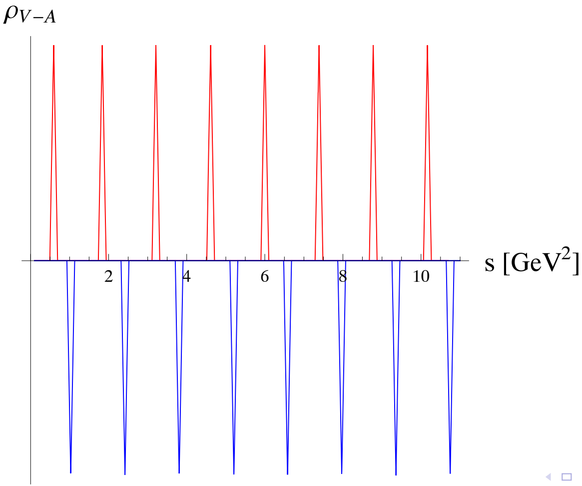


Simplest linear model

Take linear trajectories all the way down:

$$\Pi^T_{V-A}(Q^2) = \frac{F^2}{a}\left[-\psi\left(\frac{M_V^2+Q^2}{a}\right)+\psi\left(\frac{M_A^2+Q^2}{a}\right)\right]-\frac{f^2}{Q^2}$$

$$\simeq \left(\frac{F^2}{a}(M_A^2-M_V^2)-f^2\right)\frac{1}{Q^2}$$

$$+\left(\frac{F^2}{2a}(M_A^2-M_V^2)(a-M_A^2-M_V^2)\right)\frac{1}{Q^4}+\ldots$$

Matching to OPE $\rightarrow$

Weinberg sum rules:

$$f^2 = \frac{F^2}{a}(M_A^2-M_V^2) \qquad \text{(WSR I)}$$

$$0 = (M_A^2-M_V^2)(a-M_A^2-M_V^2) \qquad \text{(WSR II)}$$

$V + A$ needs regularization: carry d/dQ^2, compute the convergent sum, integrate back over Q^2:

$$\Pi^T_{V+A}(Q^2) = \frac{F^2}{a}\left[-\psi\left(\frac{M_V^2+Q^2}{a}\right) - \psi\left(\frac{M_A^2+Q^2}{a}\right)\right] + \frac{f^2}{Q^2} + const$$

$$\simeq -\frac{2F^2}{a}\log\frac{Q^2}{\mu^2} + \left(f^2 + F^2 - \frac{F^2}{a}(M_A^2+M_V^2)\right)\frac{1}{Q^2}$$

$$+\frac{F^2}{6a}\left(a^2 - 3a(M_A^2+M_V^2) + 3(M_A^4+M_V^4)\right)\frac{1}{Q^4} + \ldots$$

Matching of the coefficient of $\log Q^2$ to OPE $\rightarrow$

$$a = 2\pi\sigma = \frac{24\pi^2 F^2}{N_c}$$

σ - (long-distance) string tension

$\rho \rightarrow 2\pi$ [Ecker 1988] $\rightarrow F = 154\text{MeV} \rightarrow \sqrt{\sigma} = 546$ MeV

lattice [Kaczmarek 2005] $\rightarrow \sqrt{\sigma} = 420$ MeV

Dimension-2 and 4 condensates

WSR $\to$

$$M_A^2 = M_V^2 + \frac{24\pi^2}{N_c} f^2, \quad a = M_A^2 + M_V^2 = 2M_V^2 + \frac{24\pi^2}{N_c} f^2$$

Matching higher twists $\to$ dimension-2 and 4 condensates:

$$-\frac{\alpha_S \lambda^2}{4\pi^3} = f^2, \qquad \frac{\alpha_S \langle G^2 \rangle}{12\pi} = \frac{M_A^4 - 4M_V^2 M_A^2 + M_V^4}{48\pi^2}$$

Numerically: $-\frac{\alpha_S \lambda^2}{\pi} = 0.3 \text{ GeV}^2$ as compared to 0.12GeV^2 from [Chetyrkin 1998, Zakharov 2005]. The short-distance string tension $\sigma_0 = -2\alpha_s \lambda^2 / N_c \to \sqrt{\sigma_0} = 782 \text{ MeV}$ - twice as much as σ. The dimension-4 gluon condensate is negative for $M_V \geq 0.46$ GeV, never reaches the QCD sum-rules value

The strictly linear radial Regge model *too restrictive*!

Low-lying states, both their residue and position, depart from the linear trajectories (physical). The OPE condensates are expressed in terms of the parameters of the spectra
(as in [Golterman 2001, Afonin 2006] + dimension-2 condensate)

$$M_{V,0} = m_\rho, \quad M^2_{V,n} = M^2_V + an, \quad n \geq 1$$

$$M^2_{A,n} = M^2_A + an, \quad n \geq 0$$

WSR $\longrightarrow$ ($N_c = 3$ from now on)

$$M^2_A = M^2_V + 8\pi^2 f^2$$

$$a = 8\pi^2 F^2 = \frac{8\pi^2 f^2 \left(4\pi^2 f^2 + {M_V}^2\right)}{4\pi^2 f^2 - {m_\rho}^2 + {M_V}^2}$$

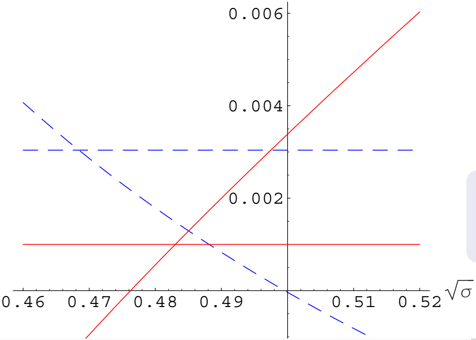
$m_\rho = 0.77\mathrm{GeV}$ - fixed, one free parameter left: M_V or σ

$$M_V^2 = \frac{-16\pi^3 f^4 + 4\pi^2 \sigma f^2 - {m_\rho}^2 \sigma}{4f^2\pi - \sigma}, \quad -\frac{\alpha_S \lambda^2}{4\pi^3} = \frac{16\pi^3 f^4 - \pi\sigma^2 + {m_\rho}^2\sigma}{16 f^2 \pi^3 - 4\pi^2\sigma}$$

$$\frac{\alpha_S \langle G^2 \rangle}{12\pi} = 2\pi^2 f^4 - \pi\sigma f^2 + \frac{3\sigma\left(\frac{{m_\rho}^2\sigma}{(\sigma - 4f^2\pi)^2} - 2\pi\right){m_\rho}^2}{8\pi^2} + \frac{\sigma^2}{12}$$

$$M_V^2 = \frac{-16\pi^3 f^4 + 4\pi^2 \sigma f^2 - {m_\rho}^2 \sigma}{4f^2\pi - \sigma}, \quad -\frac{\alpha_S \lambda^2}{4\pi^3} = \frac{16\pi^3 f^4 - \pi\sigma^2 + {m_\rho}^2\sigma}{16 f^2 \pi^3 - 4\pi^2\sigma}$$

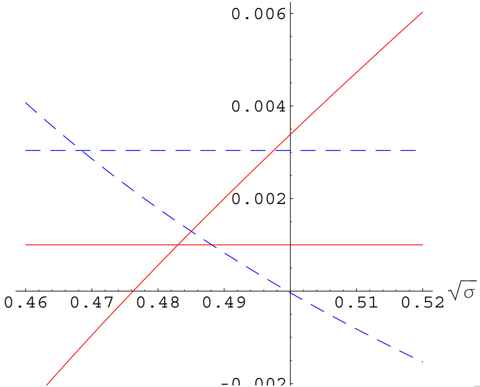
$$\frac{\alpha_S \langle G^2 \rangle}{12\pi} = 2\pi^2 f^4 - \pi\sigma f^2 + \frac{3\sigma \left(\frac{{m_\rho}^2 \sigma}{(\sigma - 4f^2\pi)^2} - 2\pi \right) {m_\rho}^2}{8\pi^2} + \frac{\sigma^2}{12}$$



dim.-2 [GeV^2] and -4 [GeV^4] condensate
horizontal lines - "physical" values:

$$-\alpha_s \lambda^2/(4\pi^3) = 0.003 \text{ GeV}^2$$
$$\alpha_S \langle G^2 \rangle/(12\pi) = 0.001 \text{ GeV}^4$$

The window for which both condensates are positive yields very acceptable values of σ [Anisovich 2000]. The consistency check $\sigma = \sigma_0$ holds for $\sqrt{\sigma} = 497\mathrm{MeV}$ The magnitude of the condensates is in the ball park of the "physical" values. The value of M_V in the "fiducial" range is around 0.82 GeV – a very reasonable value



dim. 2 [GeV^2] and 4 [GeV^4] condensates
horizontal lines - "physical" values:
$-\alpha_s \lambda^2/(4\pi^3) = 0.003 \text{ GeV}^2$,
$\alpha_S \langle G^2 \rangle/(12\pi) = 0.001 \text{ GeV}^4$

ρ and a_1 spectra

ρ states: Exp.: 770, 1450, 1700, 1900*, 2150* MeV
model: 770, 1355, 1795, 2147 MeV ($\sigma = (0.47\ \mathrm{GeV}^2)$
a_1 states: Exp.: 1260, 1640 MeV, model: 1015, 1555 MeV
$V - A$ channel well reproduced with radial Regge models

Das-Mathur-Okubo sum rule $\rightarrow L_{10}$
Das-Guralnik-Mathur-Low-Yuong s.r. $\rightarrow$ e.m. pion mass splitting

Linear model with $M_A^2 = 2M_V^2$ and $M_V = \sqrt{24\pi^2/N_c} f = 764\mathrm{MeV}$:

$$\sqrt{\sigma} = \sqrt{3/2\pi} M_V = 532\mathrm{MeV},\ F = \sqrt{3} f = 150\mathrm{MeV}$$
$$L_{10} = -N_c/(96\sqrt{3}\pi) = -5.74 \times 10^{-3} (-5.5 \pm 0.7 \times 10^{-3})_{\mathrm{exp}}$$
$$m_{\pi^\pm}^2 - m_{\pi_0}^2 = (31.4\mathrm{MeV})^2\ (35.5\mathrm{MeV})^2_{\mathrm{exp}}$$

Our second model with $\sigma = (0.48\mathrm{GeV})^2$:

$$L_{10} = -5.2 \times 10^{-3} \text{ and } m_{\pi^\pm}^2 - m_{\pi_0}^2 = (34.4\mathrm{MeV})^2$$

- significance of confinement also for the short-distance expansion [Chetyrkin 1998, Zakharov 2005]
- Effective low-energy models produce $1/Q^2$ corrections (provide dim. 2 scale), *e.g.* the instanton-based chiral quark model [Dorokhov+WB 2003]

$$-\frac{\alpha_S}{\pi}\frac{\lambda^2}{Q^2} = -2N_c \int du \frac{u}{u + M(u)^2} M(u) M'(u) \simeq 0.2 \text{ GeV}^2$$

- Matching OPE to the radial Regge models produces in a natural way the $1/Q^2$ correction to the V and A correlators. Appropriate conditions are satisfied by the asymptotic spectra, while the parameters of the low-lying states are tuned to reproduce the values of the condensates

- In principle, these parameters of spectra are measurable, hence the information encoded in the low-lying states is the same as the information in the condensates
- Yet, sensitivity of the values of the condensates to the parameters of the spectra, as seen by comparing the two explicit models considered in this paper, makes such a study difficult or impossible at a more precise level
- Regge models work very well in the $V - A$ channel
- Other applications: pion distribution amplitude $\phi(x) = 1$ at the low-energy model scale (as in chiral quark models) [Arriola+WB, 2006]

Condensates in QCD

A correlator of two currents,

$$\Pi_{AB}(q) \equiv i \int d^4 x e^{iq\cdot x} \langle 0 \mid T\{J_A(x), J_B^\dagger(0)\} \mid 0 \rangle$$

can be expanded at large Euclidean momenta with the help of the Wilson expansion. For vector currents, $J_{\mu,a}^{V\pm A} = \bar{q}(1 \pm \gamma_5)\frac{\tau_a}{2}q$, one gets explicitly $\Pi_{\mu\nu,ab}^{V\pm A} = (q_\mu q_\nu - g_{\mu\nu}q^2)\delta_{ab}\Pi^{V\pm A}$ with

$$\Pi^{V+A} = -\frac{1}{4\pi^2}\left(1+\frac{\alpha_s}{\pi}\right)\log(Q^2/\mu^2) + \frac{1}{12}\frac{\langle \frac{\alpha_S}{\pi}(G_{\mu\nu}^a)^2\rangle}{Q^4} + \frac{64\pi}{81}\frac{\alpha_S\langle\bar{q}q\rangle^2}{Q^6} + \ldots$$

$$\Pi^{V-A} = \frac{2m_c\langle\bar{q}q\rangle}{Q^4} - \frac{32\pi}{9}\frac{\alpha_S\langle\bar{q}q\rangle^2}{Q^6} + \ldots$$

(for other channels similar expressions)

Parameterization of non-perturbative physics in terms of:

$$m_c \langle \bar{q} q \rangle = -\frac{1}{2} m_\pi^2 f_\pi^2 = -0.8 \times 10^{-4} \mathrm{GeV}^4, \ \frac{\alpha_S}{\pi} \langle G^2 \rangle = (0.31^{+0.05}_{-0.10} \mathrm{GeV})^4, \ \ldots$$

Vast applications in hadronic physics: QCD sum rules, lattice, ...

Structures selected for condensates were dictated by gauge invariance of QCD

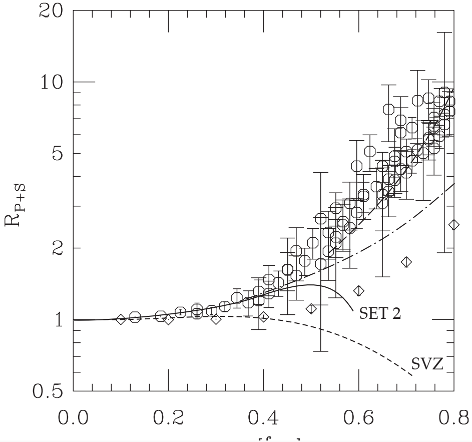
Chetyrkin, Narison, and Zakharov (1999) append the standard OPE with the $1/Q^2$ power correction, introducing a "gluon-mass" term (3 slides down). Subsequently, Narison and Zakharov (2001) demonstrated that some lattice data are much better reproduced with this term

$$\Pi^{V+A} = -\frac{1}{4\pi^2}\left(1 + \frac{\alpha_s}{\pi}\right) \log(Q^2/\mu^2) - \frac{\alpha_S}{4\pi^3} \frac{\lambda^2}{Q^2} + \frac{1}{12} \frac{\langle \frac{\alpha_S}{\pi} G^2 \rangle}{Q^4} + \ldots$$

$$\Pi^{V-A} - \quad \text{unchanged}$$

In coordinate space this leads to quadratic terms

$$R_{P+S} \equiv \frac{1}{2}\left(\frac{\Pi^P}{\Pi^P_{\rm pert}} + \frac{\Pi^S}{\Pi^S_{\rm pert}}\right) \to 1 - \frac{\alpha_S}{2\pi}\lambda^2 x^2 - \frac{\pi^2}{96}\langle \frac{\alpha_S}{\pi} G^2 \rangle x^4 + \ldots$$



Narison+Zakharov, PLB 522 (2001)266

Fit: $-\frac{\alpha_S}{\pi}\lambda^2 = 0.12 \mathrm{GeV}^2$

Lattice data vs. OPE predictions for two sets of QCD condensate values. The dot-dashed curve – SET 3 where the contribution of the x^2-term has been added to SET 2. The bold dashed curve is SET 3 + a fitted value of the $D = 8$ condensate contributions. The diamond curve – the instanton liquid model

Interpretation via the gluon mass

Attempt to put confinement in OPE: the potential

$$V(r) = -\frac{4}{3}\frac{\alpha_S}{r} + \sigma_0 r$$

is obtained as the Fourier transform of the Coulomb gluon propagator

$$G(Q^2) = \frac{1}{Q^2 - \lambda^2} = \frac{1}{Q^2} + \frac{\lambda^2}{Q^4} + \ldots, \quad (Q^2 \gg \lambda^2 \sim \Lambda^2_{\rm QCD})$$

λ - (tachyonic) gluon mass: $-(3/N_c)^2\lambda^2 = m_A^2 = \frac{N_c}{4(N_c^2-1)} g^2 \langle A^2 \rangle_{\rm Eucl.}$
short-distance string tension: $\sigma_0 = -2\alpha_s \lambda^2 / N_c$

Far reaching consequences:

- Standard OPE may be improved
- Interpretation of the gluon mass via $\sim \langle A^a_\mu A^\mu_a \rangle$ (comes next)
- This apparently gauge-dependent condensate may be written in terms of a gauge-independent expression! (but non-local)

Gauge-independent meaning of gauge-dependent operators

[Gubarev, Stodolsky, Zakharov, PRL **86** (2001) 2220]
Example from magnetostatics:
... since ... nonzero magnetic field $\mathbf{B} = \nabla \times \mathbf{A}$*, we know some nonzero* $\mathbf{A}$ *must be present;* $\mathbf{A}$ *cannot be zero everywhere. Now consider* $\int \mathbf{A}^2 d^3x$*. It is a positive quantity and cannot be zero. It must then have some minimum value. Therefore of all the possible* $\mathbf{A}$ *configurations which yield the given* $\mathbf{B}$ *the one (or the ones) with the smallest integral of* $\mathbf{A}^2$ *has in a sense an invariant significance (*$\langle A^2_{\rm min} \rangle$*). Suppose ... that* $\int \mathbf{A}^2 d^3x$ *is at its minimum value; then under a gauge transformation it is stationary. Considering* $\mathbf{A} \to \mathbf{A} + \nabla\phi$ *for infinitesimal* ϕ *we have* $\int \mathbf{A} \cdot \nabla\phi \, d^3x = 0$ *and integrating by parts*

$$\int \phi \nabla \cdot \mathbf{A} \; d^3x + \text{surface terms} = 0.$$

Since ϕ *is arbitrary ... the "minimum* A^2*" condition is equivalent to the familiar gauge condition* $\nabla \cdot \mathbf{A} \; = \; 0$

There is the vector relation (from $(\mathbf{k} \times \mathbf{A})^2 = k^2 A^2 - (\mathbf{k} \cdot \mathbf{A})^2$)

$$\begin{aligned} \int \mathbf{A}^2(x) d^3x &= \frac{1}{4\pi} \int d^3x d^3x' \left(\frac{[\nabla \times \mathbf{A}(x)] \cdot [\nabla \times \mathbf{A}(x')]}{|\mathbf{x} - \mathbf{x}'|} \right. \\ &+ \left. \frac{[\nabla \cdot \mathbf{A}(x)][\nabla \cdot \mathbf{A}(x')]}{|\mathbf{x} - \mathbf{x}'|} \right) + \text{surface terms} \end{aligned}$$

Hence

$$A^2_{\min} = \frac{1}{4\pi V} \int d^3x d^3x' \frac{\mathbf{B}(\mathbf{x}) \cdot \mathbf{B}(\mathbf{x}')}{|\mathbf{x} - \mathbf{x}'|} + \text{surface terms}$$

Locality traded for manifest gauge invariance!

4 dimensions: $\frac{1}{2}(\epsilon_{\mu\nu\lambda\rho}k_\lambda A_\rho)^2 = k^2A^2 - (k\cdot A)^2$, minimization equivalent to the Landau (*aka* the Lorentz) gauge condition

$$\begin{aligned}\int A^2(x)d^4x &= \frac{1}{2\pi^2}\int d^4x d^4x' \frac{[F_{\mu\nu}(x)][F^{\mu\nu}(x')]}{(x-x')^2} \\ &+ \frac{1}{2\pi^2}\int d^4x d^4x' \frac{[\partial_\mu A_\mu(x)][\partial_\nu A_\nu(x')]}{(x-x')^2} + \text{surface terms}\end{aligned}$$

Non-abelian case:

$$A^2_{\min} = \frac{1}{VT}\min_g \int d^4x \left(gA_\mu g^\dagger - \frac{i}{g_s} g\partial_\mu g^\dagger\right)^2$$

g – the group element, g_s – coupling constant. Minimization gives the function $g_{\min}(x;A)$. We can compensate the gauge transformation $A_\mu \to g' A_\mu g'^\dagger + g'\partial_\mu g'^\dagger$ with $g_{\min} \to g_{\min} g'^{-1}$, then $A^2_{\min}$ remains invariant

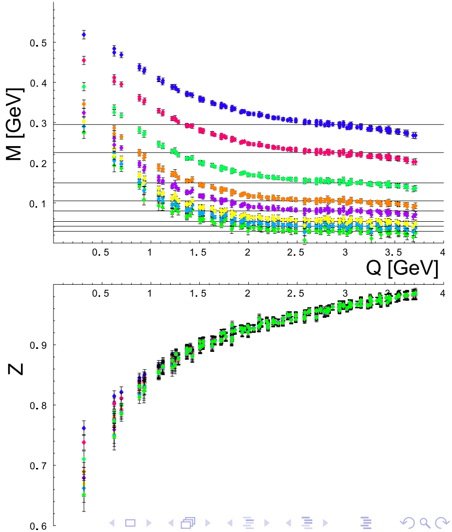
Quark propagator in the Landau gauge from the lattice

... now we go on the lattice!
[Arriola+Bowman+WB 2004]

$$S(p) = \frac{Z(p)}{\not{p} - M(p)}$$

Landau gauge $\partial \cdot A = 0$
[Bowman *et al.* 2002]

Sets of points for $m_c = 29$, 42, 54, 80, 105, 150, 225, and 295 MeV (horizontal lines). Asqtad improved staggered action, gauge ensemble made of 100 quenched $16^3 \times 32$ lattices, $a = 0.124$ fm

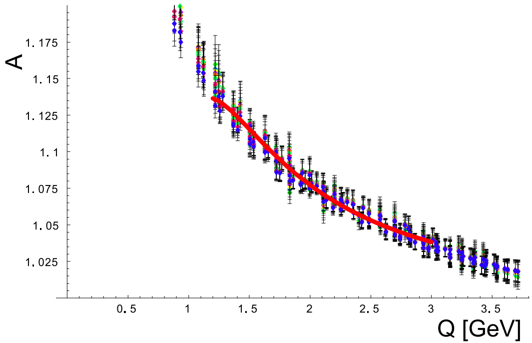


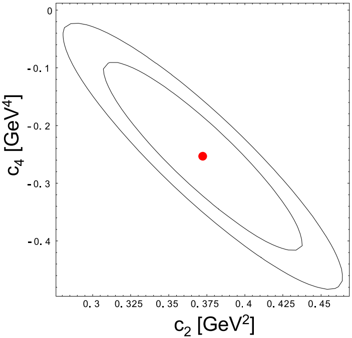
Lavelle propagators

Lavelle and Schaden (1988) and Lavelle and Oleszczuk (1992) worked out the quark propagator in presence of condensates in the general covariant gauge. In the Landau gauge $\partial \cdot A^a = 0$ this propagator yields

$$\begin{aligned} A(Q) \equiv Z^{-1}(Q) &\to 1 + \frac{\pi\alpha_s \langle A^2 \rangle}{N_c Q^2} - \frac{\pi\alpha_s \langle G^2 \rangle}{3 N_c Q^4} + \frac{3\pi\alpha_s \langle \bar{q} g_s \not{A} q \rangle}{4Q^4} + \dots \\ &= 1 + \frac{c_2}{Q^2} + \frac{c_4}{Q^4} + \dots \end{aligned}$$

... now we have the lattice data to compare to!
[Ruiz Arriola, Bowman, WB, PRD **70** (2004) 097505]
[Furui, Nakajima (2006) - full QCD]





The optimum values for c_2 and c_4 yield

$$\alpha_s \langle A^2 \rangle = (0.36 \pm 0.04)\ \mathrm{GeV}^2 \quad \text{or} \quad g_s^2 \langle A^2 \rangle = (2.1 \pm 0.1\ \mathrm{GeV})^2$$

$$\alpha_s \langle \bar{q} g_s \not{A} q \rangle - \frac{4\pi}{27} \langle \frac{\alpha_s}{\pi} G^2 \rangle = (-0.11 \pm 0.03)\ \mathrm{GeV}^4$$

Since $\langle \frac{\alpha_s}{\pi} G^2 \rangle \simeq 0.01 \text{ GeV}^4$, its contribution is negligible compared to the mixed condensate term. Thus

$$\alpha_s \langle \bar{q} g_s \not{A} q \rangle = (-0.11 \pm 0.03) \text{ GeV}^4$$

(first estimate of this quantity).
The errors are statistical. In addition, there are systematic errors originating from the choice of the fitted function $A(Q)$ and the "fiducial" region in Q. Quantities quoted in physical units are also subject to the uncertainty in the scale of the lattice simulations

Running of $\langle A^2 \rangle$

Operators in QCD run with appropriate anomalous dimesions. The anomalous dimension for the A^2 condensate has been worked out by Gracey and Boucaud *et al.* with the result

$$\alpha_s(\mu^2)\langle A^2 \rangle_\mu \sim \alpha_s(\mu^2)^{1-\gamma_{A^2}/\beta_0},$$

where $\gamma_{A^2} = 35/4$ and $\beta_0 = 11$ for $N_f = 0$, hence $1 - \gamma_{A^2}/\beta_0 = 9/44$ and the evolution is very slow. For instance, the change of μ^2 from 1 GeV^2 up to 10 GeV^2 results in a reduction of $\alpha_s \langle A^2 \rangle$ by 10% only

(we use $\alpha_s(\mu^2) = 4\pi/(9 \log[\mu^2/\Lambda^2])$, with $\Lambda = 226 \text{ MeV}$ for the LO evolution.)

The gluon mass

[Celenza+Shakin 1986]
Many estimates in the literature refer the gluon mass ($\rightarrow$ the Lavelle propagator for the gluon)

$$m_A^2 = \frac{3}{32} g_s^2 \langle A^2 \rangle$$

Our estimate for $\langle A^2 \rangle$, when evolved from 2 GeV^2 (assumed lattice scale) to 10 GeV^2 (physical scale), yields

$$m_A^2 = (625 \pm 33)^2 \text{ MeV}^2 \quad (\text{at } 10 \text{ GeV}^2)$$

Evolution from 1 to 10 GeV^2 gives $m_A^2 = (611 \pm 32)^2$ MeV^2, while evolution from 4 to 10 GeV^2 $m_A^2 = (635 \pm 34)^2$ MeV^2
(consistent with numerous estimates from other methods)

Summary of $\langle A^2 \rangle$

- One can obtain the Landau-gauge condensates from the lattice propagators
- Our value of $\langle A^2 \rangle$ compatible with other estimates (lattice gluon propagator, gluon mass). We find from the quark propagator the gluon mass of $600 - 650$ MeV
- First estimate of the mixed condensate $\langle \bar{q} \not{A} q \rangle$
- Saving the gauge invariance, or interpreting the apparently gauge-variant quantity in a gauge-independent way: minimum over g, Kondo, Slavnov
- Deeper meaning of the A^2 condensate: possible modification of OPE, Gribov copies, topological structure of the vacuum and confinement, lattice calculations, ...

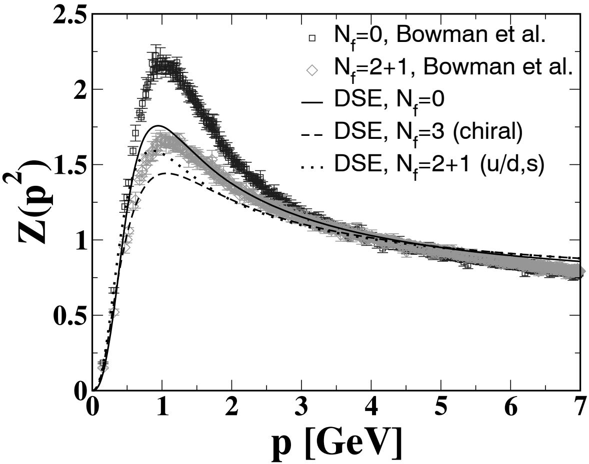
Status 2007

[Kharzeev 2007]:
Since the linear behavior of confining potential has been firmly established by now in lattice calculations, we have to understand the origin of the linear confinement from the point of view of OPE ... Several interesting scenarios have been proposed to achieve this goal ... all of them involve a modification of the gluon propagator, or the QCD coupling, in the infrared region ... In terms of the OPE these modifications amount to the presence of dimension two operator; this means that confinement effects manifest themselves already at relatively short distances

Backup slides / comments

Lattice gluon propagator in the Landau gauge

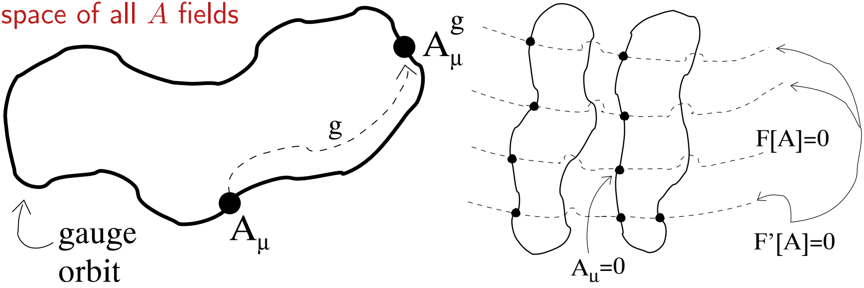
$$D_{\mu\nu}(p) = \left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right) \frac{Z(p^2)}{p^2}$$



Gribov copies

Non-abelian and non-perturbative: no local gauge fixing is free of Gribov copies, *i.e.* configuration of fields from the same gauge orbit entering the path integration [from Williams 2003]

space of all A fields



F - complete gauge fixing

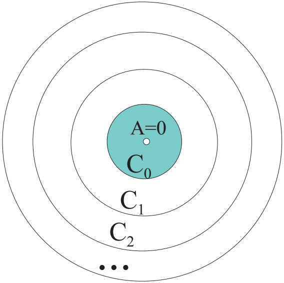
F' - incomplete gauge fixing, *e.g.* $F'(A(x)) = \partial \cdot A(x)$

Gribov regions and horizons

Divide the gauge-fixed space of A into regions containing n negative values of the Fadeev-Popov operator for the Landau gauge, $-\partial^\mu(\partial_\mu \cdot + [A_\mu, \cdot])$

space of

$A : F'(A) = 0$

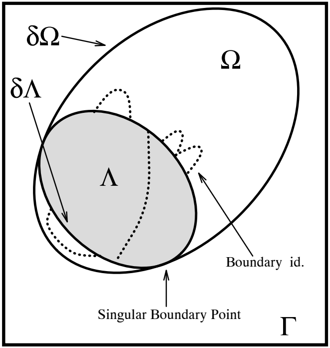


Boundaries (where det of the FP operator vanishes) are called the Gribov horizons. Gribov showed that neighboring regions $C_n - C_{n-1}, \ldots, C_2 - C_1$ contain copies. He suspected that C_0 is free of copies. However, Zwanziger (1989) showed that this conjecture is not true

- $A = 0$ belongs to C_0
- C_0 is convex
- Every gauge orbit passes throught C_0

Fundamental modular region

Γ - space of all A subject to the gauge-fixing constraint
$\Omega = C_0$ - first Gribov region
$\delta\Omega$ - first Gribov horizon
Λ - fundamental modular region (FMR), dashed lines - group orbits connecting Gribov copies
[from Stodolsky, van Baal, Zakharov 2002]



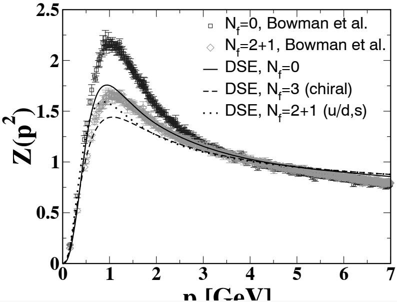
FMR is free of Gribov copies! Furthermore, Zwanziger showed that the first Gribov region (Ω) corresponds to all mimima of A^2 with respect to g (local and absolute), while FMR (Λ) corresponds to the absolute minima of A^2! Further Gribov regions correspond to critical points with 1, 2, . . . negative eigenvalues.

Significance for calculations of propagators on the lattice

Modifications in IR occur when restrictions to Ω or Λ regions are made:

$$D_{\mu\nu}(p) = \left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right) \frac{Z(p^2)}{p^2}, \quad D_G(p) = -\frac{G(p^2)}{p^2}$$

Dyson-Schwinger equations in the Landau gauge for the ghost and gluon dressing functions, $G(p^2)$ and $Z(p^2)$ yield simple power laws $Z(p^2) \sim (p^2)^{2\kappa}, \;\; G(p^2) \sim (p^2)^{-\kappa}$ (with $\kappa \simeq 0.6$)



Ghost IR enhanced, gluon IR vanishing!
Kugo-Ojima (1979) confinement criterion (in Landau gauge): Ghost propagator more singular than the pole $\equiv$ confinement. Seen on the lattice!

Other recent developments

- Restriction to FMR is not trivial to accomplish. Silva and Oliveira (2004) showed that the difference for the gluon propagator when restricting from Ω further to Λ is at the level of a few percent at soft momenta
- Boucaud *et al.* (2000), Gubarev et al. (2001) - topological structure of the vacuum, relevance for confinement
- "The minimal value of the potential squared, $\langle A^2_{\rm min} \rangle$, encodes information on the topological defects in gauge theories"
- Slavnov (2004) - proof of gauge invariance of the *expectation value* of A^2 (uses non-commutative geometry
- *"The gauge invariance of the condensate follows from the hidden symmetry of Yang-Mills theory, which becomes explicit if one considers it as a limit of the noncommutative gauge model"*

Lattice details

Asqtad action - improved Kogut-Susskind action, with errors of the order $\mathcal{O}(a^4, a^2 g^2)$, with tadpoles summed up

Enforcing Landau gauge - minimizing $\int d^4x \mathrm{Tr}[A^g_\mu(x) A^g_\mu(x)]$ over the group g is equivalent to maximizing

$$\sum_{x,\mu} \mathrm{Re}\left(\mathrm{Tr}[g(x) U_\mu(x) g^\dagger(x+\hat{\mu})] \right) .$$

The algorithm may produce global as well as many local minima. That way one may test the numerical significance of restricting to FMR. Typically, it is a few percent effect for the gluon propagator in the soft region [Silva and Oliveira, 2004].

BRST

Kondo (2001) - gauge-covariant redefinition of the gluon field and the discovery of a BRST invariant realization of the A^2 condensate:

$$\frac{1}{VT}\langle \int d^4x \left(\frac{1}{2} A_\mu(x) A^\mu(x) - i\alpha c(x)\bar{c}(x) \right) \rangle$$

α - gauge-fixing parameter, c, $\bar{c}$- ghosts, in the Landau gauge $\alpha = 0$

Sum rules

The first Weinberg sum rule:

$$\frac{1}{4\pi^2}\int_0^{s_0\to\infty} ds\left[v_1(s)-a_1(s)\right]=\left[-Q^2\Pi^{V-A}\left(Q^2\right)\right]_{Q^2\to 0}=f_\pi^2$$

The second Weinberg sum rule:

$$\begin{aligned}\frac{1}{4\pi^2}\int_0^{s_0\to\infty} ds\, s\left[v_1(s)-a_1(s)\right] &= Q^2\left[-Q^2\Pi^{V-A}\left(Q^2\right)\right]_{Q^2\to\infty}\\ &= -2m_c\langle\overline{q}q\rangle\end{aligned}$$

The Das-Mathur-Okubo (DMO) sum rule:

$$\begin{aligned}\frac{1}{4\pi^2}\int_0^{s_0\to\infty} ds\frac{1}{s}\left[v_1(s)-a_1(s)\right] &= \frac{\partial}{\partial Q^2}\left[Q^2\Pi^{V-A}\left(Q^2\right)\right]\Big|_{Q^2\to 0}\\ &= f_\pi^2\frac{\left\langle r_\pi^2\right\rangle}{3}-F_A=-4L_{10}\end{aligned}$$

DGLMY sum rule

The Das-Guralnik-Mathur-Low-Yuong (DGMLY) sum rule:

$$-\frac{1}{4\pi^2}\int_0^{s_0\to\infty} ds\, s \ln\frac{s}{\mu^2}\left[v_1(s) - a_1(s)\right] = \int_0^{\infty} dQ^2\left[-Q^2\Pi^{V-A}(Q^2)\right] = \frac{4\pi f_\pi^2}{3\alpha_{\rm QED}}\left[m_{\pi^\pm}^2 - m_{\pi^0}^2\right]$$

No dependence on μ^2 thanks to WSR II. Witten: positive electromagnetic mass shift of the charged pions follows from DGMLY and positivity of the $V-A$ correlator,

$$-Q^2\Pi^{V-A}(Q^2) \geqslant 0, \qquad \text{for} \qquad 0 \leqslant Q^2 \leqslant \infty.$$

If $m_u = m_d = 0$ quarks were massless ($m_{\pi^0} = 0$) and the mass shift were negative, $\pi^\pm$ would become tachyons