Application of chiral quark models to high-energy processes

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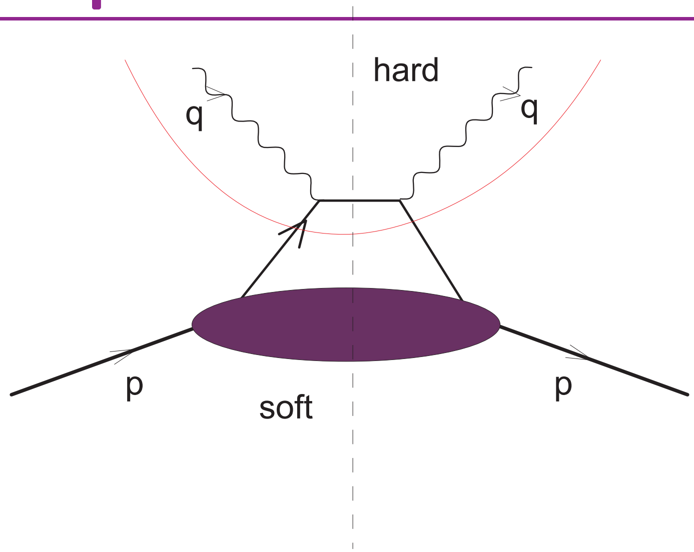
The Alhambra-Wawel collaboration

- Enrique Ruiz-Arriola from Krajnada, talk at Bled'02
- WB+ERA, Impact-parameter dependence of the generalized parton distribution of the pion in chiral quark models, Phys. Lett. **B 574** (2003) 57, hep-ph/0307198
- ERA+WB, Spectral quark model and low-energy hadron phenomenology, Phys. Rev. **D67** (2003) 074021, hep-ph/0301202
- ERA + WB, "Solution of the Kwieciński evolution equations for unintegrated parton distributions using the Mellin transform", hep-ph/0404008, PRD, in press

Intro and outline

- Low-energy quark models can be used to compute low-energy matrix elements of hadronic operators
- DIS
- Generalized and unintegrated parton distributions (GPD and UPD)
- Predictions of chiral quark model for GPD and UPD for the pion
- QCD evolution

Deep Inelastic Scattering



$$Q^2 = -q^2, \quad W^2 = (p+q)^2, \quad x = \frac{Q^2}{2p \cdot q} = \frac{Q^2}{Q^2 + W^2}, \quad Q^2 \to \infty$$

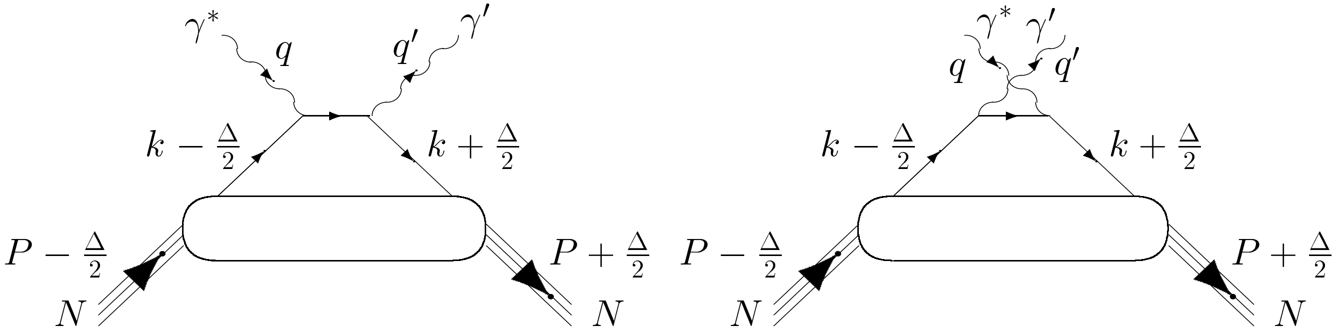
Factorization of soft and hard processess, Wilson expansion (OPE), twist expansion

$$\langle J(q) J(-q) \rangle = \sum_i C_i(Q^2; \mu) \langle \mathcal{O}_i(\mu) \rangle$$

Practical meaning for inclusive processes: $\frac{d\sigma}{dx dQ^2} = \int dx\, f(x) \frac{d\bar{\sigma}(x)}{dx dQ^2}$

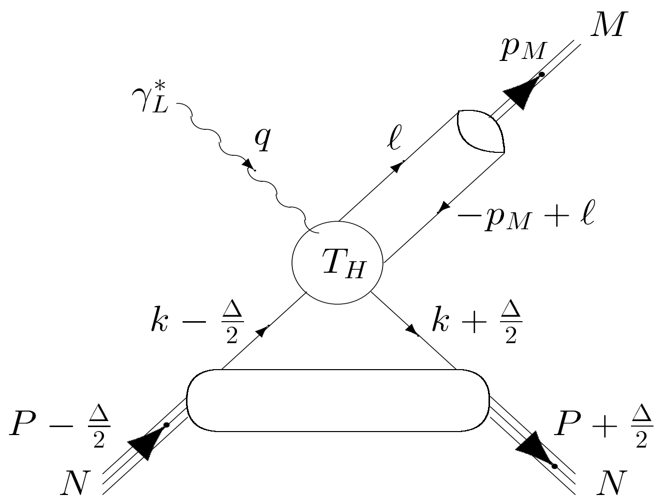
The soft part can be computed in low-energy models

Exclusive processes in QCD



(a) (b)

Deeply Virtual Compton Scattering



Hard Meson Production

non-zero momentum transfer to the target, at least one photon virtual, factorization

Kinematics

Reviews:

K. Goeke, M. V. Polyakov, and M. Vanderhaeghen,
Prog. Part. Nucl. Phys. 47 (2001) 401-515, hep-ph/0106012

M. Diehl, Phys. Rept. 388 (2003) 41-277, hep-ph/0307382

Notation: $P = \frac{p+p'}{2}$, $\Delta = p' - p$, $t = \Delta^2$, $k^+ = xP^+$, $\Delta^+ = -2\xi P^+$

Dictionary:

$t = 0$ & $\xi = 0$	regular PD
$\Delta_\perp = 0$	forward GPD
$\Delta_\perp \neq 0$	off-forward GPD
$\xi = 0$	diagonal GPD (non-skewed GPD)
$\xi \neq 0$	non-diadonal GPD (skewed GPD)

Why interesting?

GPD's provide more detailed information of the structure of hadrons, three-dimensional picture instead of one-dimensional projection of the usual PD, enter sum rules,

Information on GPD may come from such processes as $ep \to ep\gamma$, $\gamma p \to pl^+l^-$, $ep \to epl^+l^-$, or from lattices (hold on!). Small cross sections of exclusive processes require very high accuracy experiments. First results are for the nucleon coming from HERMES and CLAS, also COMPASS, H1, ZEUS

Definition of GPD

The twist-2 GPD of the pion is defined as
(for the case of π^+ $H(x) \equiv H_u(x) = H_{\bar{d}}(1-x)$)

$$H(x, \xi, -\mathbf{\Delta}_\perp^2) = \int \frac{dz^-}{4\pi} e^{ixp^+z^-} \langle \pi^+(p') | \bar{q}(0, -\frac{z^-}{2}, 0) \gamma^+ q(0, \frac{z^-}{2}, 0) | \pi^+(p) \rangle,$$

(Notation: $q(z^+, z^-, z_\perp)$, $z^2 = 0$)

Link operators $P \exp(ig \int_0^z dx^\mu A_\mu$ are implicitly present to ensure gauge invariance

Similar definition for the gluon distribution

Dictionary continued

General structure of the soft matrix element:

$$\langle A \mid \mathcal{O} \mid B \rangle$$

- A = B = one-particle state – PD of A (inclusive DIS)
- A = one-particle state, B = vacuum – distribution amplitude (DA) of A (hadronic form factors, HMP)
- A, B = one-particle state of different momentum – GPD (exclusive DIS, DVCS, HMP)
- A = many-particle state, B = vacuum – GDA (transition form factors)
- ...

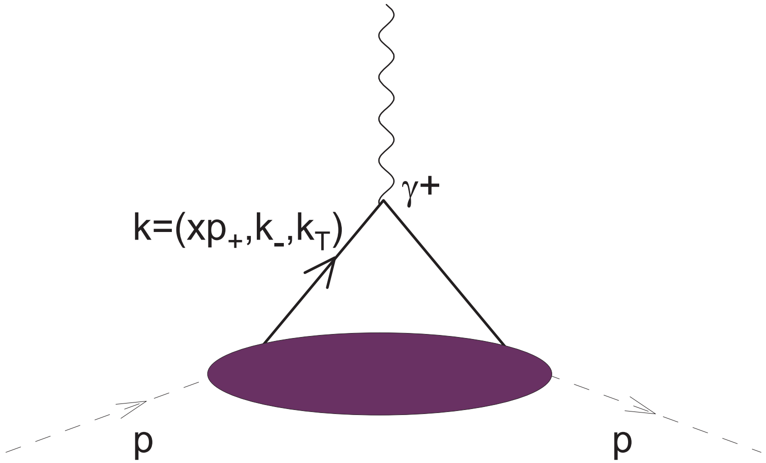
Formal properties of GPD's

$$
\begin{aligned}
H(x,\xi,-\boldsymbol{\Delta}_\perp^2) &= H(x,-\xi,-\boldsymbol{\Delta}_\perp^2) \quad \text{(time reversal)} \\
H(x,\xi,-\boldsymbol{\Delta}_\perp^2)^* &= H(x,-\xi,-\boldsymbol{\Delta}_\perp^2) \quad \text{(reality)} \\
\int_0^1 dx H(x,0,-\boldsymbol{\Delta}_\perp^2) &= F(-\boldsymbol{\Delta}_\perp^2) \quad \text{(form factor)} \\
H(x,0,0) &= q(x) \quad \text{(parton distribution)}
\end{aligned}
$$

GPD "links" the elastic form factor and the parton distribution

Unintegrated Parton Distributions

Leading-twist (=2) UPD



No integration over $k_\perp$! Around since the dawn of QCD (Dokshitzer, Dyakonov, Troyan, 1979), formal definition (Collins, 2003):

$$f(x, k_\perp) = \int \frac{dy^- d^2 y_\perp}{16\pi^3} e^{-ixp^+ y^- + ik_\perp \cdot y_\perp} \langle p \mid \bar{\psi}(0, y^-, y_\perp) W[y, 0] \gamma^+ \psi(0) \mid p \rangle$$

$$\sim \langle p \mid a^\dagger(xp^+, k_\perp) a(xp^+, k_\perp) \rangle$$

Integrated PD:

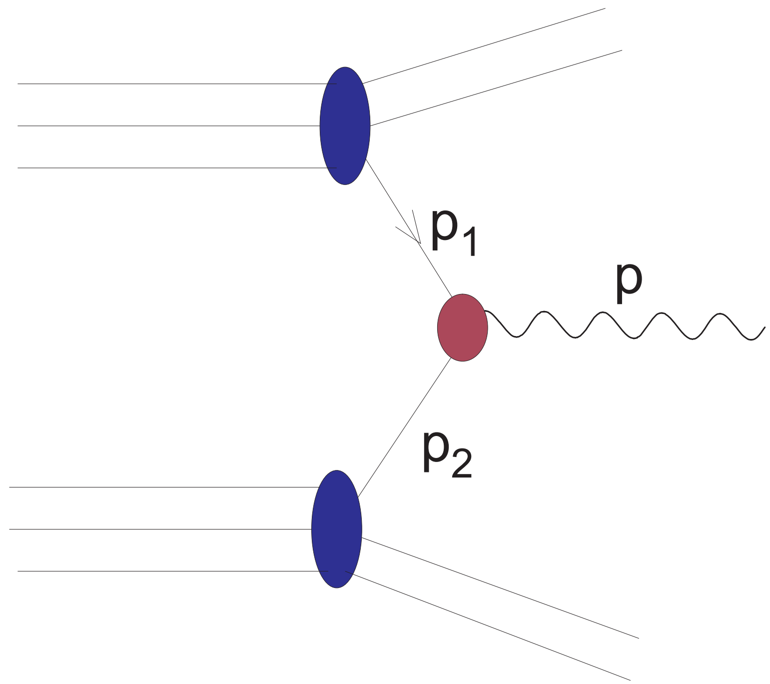
$$f(x) = \int d^2 k_\perp f(x, k_\perp) = \int \frac{dy^-}{4\pi} e^{-ixp^+ y^-} \langle p \mid \bar{\psi}(0, y^-, 0) \gamma^+ \psi(0) \mid p \rangle$$

where $W[y, 0] = P \exp[i \int_0^y ds_\mu A^\mu(s)]$

Significance of UPD's

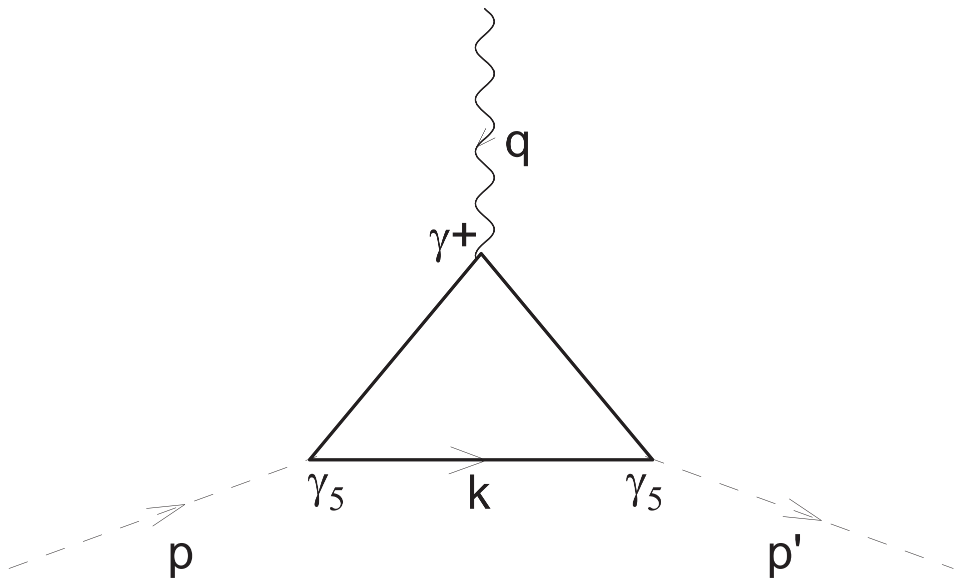
exclusive physical processes: production of W-bosons, Higgs, heavy-flavors, jets

(H. Jung, A. Szczurek, ..., L. Motyka, A. Staśto)



Evaluation of GPD in chiral quark models, $\xi = 0$

In chiral quark models the evaluation of H at the leading-N_c (one-loop) level amounts to the calculation of the diagram



where the solid line denotes the quark of mass ω.

$$
\begin{aligned}
H(x,0,-\mathbf{\Delta}_\perp^2;\omega) &= \frac{iN_c\omega^2}{f_\pi^2}\int\frac{d^4k}{(2\pi)^4}\mathrm{Tr}\left[\gamma^+\frac{1}{\not k-\not p'-\omega}\gamma_5\frac{1}{\not k-\omega}\gamma_5\frac{1}{\not k-\not p-\omega}\right] \\
&\times \quad \delta\left[k^+-(1-x)P^+\right],
\end{aligned}
$$

with $f_\pi = 93$ MeV. The light-cone coordinates are defined as

$$k^+ = k^0 + k^3,\ k^- = k^0 - k^3,\ \vec{k}_\perp = (k^1, k^2)$$

The calculation is done in the Breit frame, and with $\Delta^+ = 0$ and $P = (m_\pi, m_\pi, 0)$. The Cauchy theorem is applied for the k^- integration, yielding in the chiral limit

$$H(x, 0, -\mathbf{\Delta}_\perp^2; \omega) = \frac{N_c \omega^2}{\pi f_\pi^2} \int \frac{d^2 \mathbf{K}_\perp}{(2\pi)^2} \frac{\left[1 + \frac{\mathbf{K}_\perp \cdot \Delta_\perp (1-x)}{\mathbf{K}_\perp^2 + \omega^2}\right]}{(\mathbf{K}_\perp + (1-x)\Delta_\perp)^2 + \omega^2},$$

where $\mathbf{K}_\perp = (1-x)\mathbf{p}_\perp - x\mathbf{k}_\perp$. The integral is $\log$-divergent, and we need regularization

We use two different low-energy quark models which have proven successful in describing soft physics:

1. Nambu–Jona-Lasinio [NJL] model with the Pauli-Villars regulator.
2. Spectral Quark Model [SQM] (ERA + WB). Successful in describing both the low- and high-energy phenomenology of the pion (complies to the chiral symmetry, anomalies, pure twist expansion, quark propagator with no poles!).

Davidson-Arriola '95

For the pion, at the model scale Q_0,

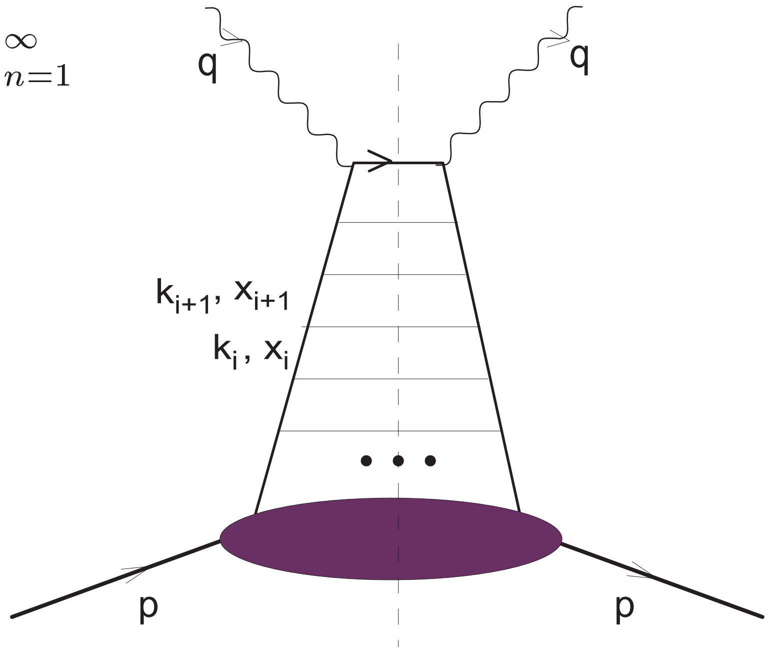
$$q(x) = \theta(x)\theta(1-x)$$

Correct:

- support
- normalization, $2\int_0^1 q(x) = 2$ (two quarks)
- momentum sum-rule, $2\int_0^1 x q(x) = 1$ (no gluons! quarks carry all the momentum)

QCD evolution

ladder, resummation: $\sum_{n=1}^{\infty}$



phase-space restrictions:

$$\mu^2 < k_{\perp,1}^2 < k_{\perp,2}^2 < \ldots < k_{\perp,n}^2 < Q^2$$

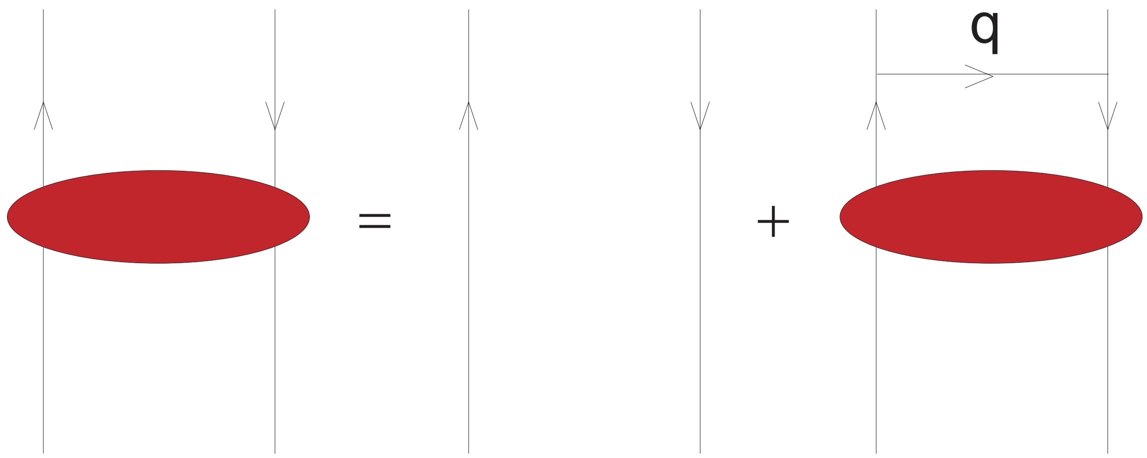
$$1 > x_1 > x_2 > \ldots > x_n = x$$

$$\alpha_s^n \int_{\mu^2}^{Q^2} \frac{dk_{\perp,n}^2}{k_{\perp,n}^2} \int_{\mu^2}^{k_{\perp,n}^2} \frac{dk_{\perp,n-1}^2}{k_{\perp,n-1}^2} \ldots \int_{\mu^2}^{k_{\perp,2}^2} \frac{dk_{\perp,1}^2}{k_{\perp,1}^2} = \frac{\alpha_s^n}{n!} \left(\log \frac{Q^2}{\mu^2} \right)^n$$

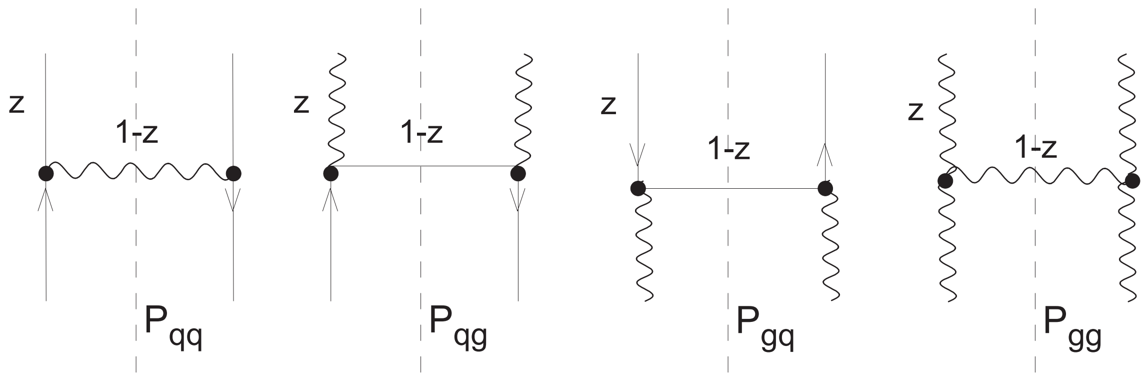
$$\alpha_s(Q^2) = \frac{4\pi}{\beta_0 \log\left(\frac{Q^2}{\Lambda_{QCD}^2}\right)}, \quad \beta_0 = 11/3N_c - 2N_f/3$$

non-planar $\sim 1/Q^2$

Bethe-Salpeter equations



Splitting functions



$$P_{qq} = C_F \frac{1+z^2}{1-z},\ P_{qg} = N_F[z^2 + (1-z)^2],\ P_{gq} = C_F \frac{1+(1-z)^2}{z},$$
$$P_{gg} = 2N_C \left[\frac{z}{1-z} + \frac{1-z}{z} + z(1-z)\right],\quad C_F = \frac{N_C^2-1}{2N_C}$$

DGLAP

Dokshitzer, Gribov, Lipatov, Altarelli, Parisi (70's)

momentum ordering $\equiv$ resummation of $\alpha_s(Q^2)\log Q^2$

$$
\begin{aligned}
f_{NS}(x,Q) &= f_{NS}(x,Q_0) + \int_0^1 dz \int_{Q_0^2}^{Q^2} \frac{dQ'^2}{Q'^2}\frac{\alpha(Q'^2)}{2\pi} P_{qq}(z) \\
&\times \left[\Theta(z-x) f_{NS}\left(\frac{x}{z},Q'\right) - f_{NS}(x,Q')\right]
\end{aligned}
$$

SFSC – "similarly for the singlet channel"

or

$$
Q^2\frac{d}{dQ^2} f_{NS}(x,Q) = \frac{\alpha(Q^2)}{2\pi}\int_0^1 dz P_{qq}(z)\left[\Theta(z-x) f_{NS}\left(\frac{x}{z},Q\right) - f_{NS}(x,Q)\right]
$$

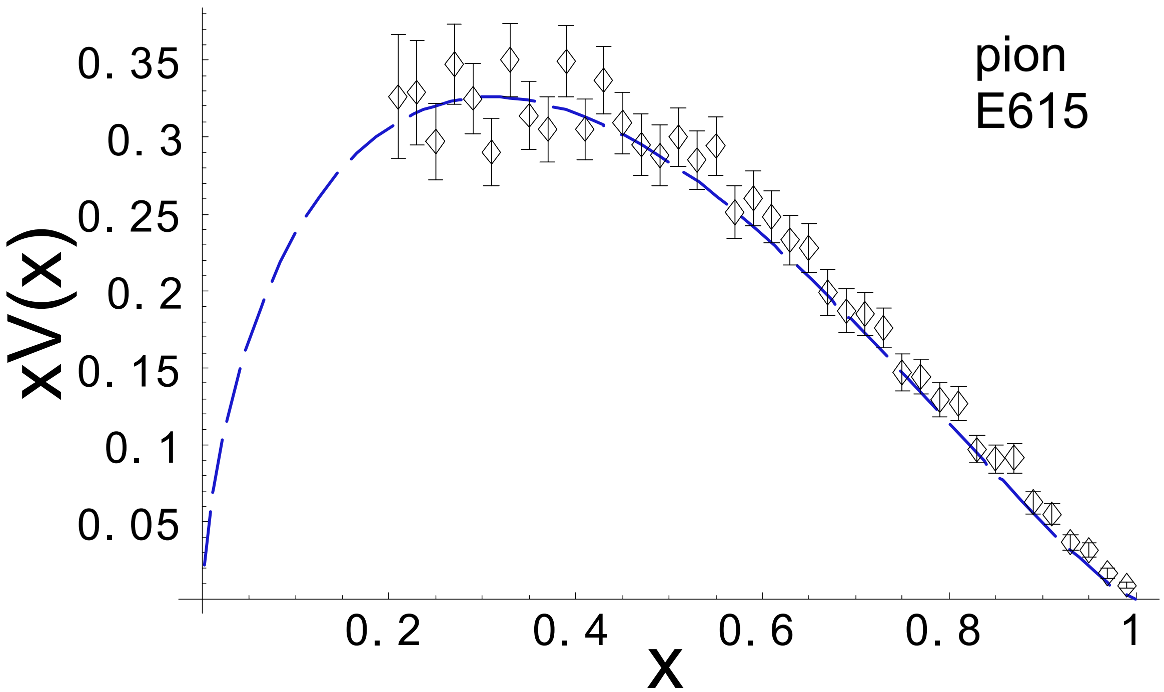
$f_j = \frac{x}{2} p_j$, and *e.g.* for π^+

$$
p_{\rm NS} = \bar{u} - u + d - \bar{d}, \quad p_S = \bar{u} + u + d + \bar{d} + \bar{s} + s + \ldots,
$$

$$
p_{\rm sea} \equiv p_S - p_{\rm NS} = 2\bar{d} + 2u + \bar{s} + s + \ldots, \quad p_G = g
$$

Evolved Davidson-Arriola vs. Fermilab's E615

[J. S. Conway et al., PRD 39 (1989) 92], $\pi^- N \to \mu^+ \mu^- X$
$(V(x, Q_0) = \theta(x)\theta(1-x)$, $Q_0 = 313 MeV$, $Q = 4$ GeV)



The quark model scale is low, NLO give similar results

. . . back to GPD

GPD in Nambu–Jona-Lasinio Model

In the NJL model with the Pauli-Villars regularization we get

$$H_{\rm NJL}(x,0,-\mathbf{\Delta}_\perp^2) = 1+\frac{N_c M^2(1-x)|\mathbf{\Delta}_\perp|}{4\pi^2 f_\pi^2 s_i}\sum_i c_i \log\left(\frac{s_i+(1-x)|\mathbf{\Delta}_\perp|}{s_i-(1-x)|\mathbf{\Delta}_\perp|}\right),$$

$$s_i = \sqrt{(1-x)^2\mathbf{\Delta}_\perp^2+4M^2+4\Lambda_i^2},$$

where M is the constituent quark mass, Λ_i are the PV regulators, and c_i are suitable constants. For the twice-subtracted case, explored below, one has, for any regulated function F, the operational definition

$$\sum_i c_i F(\Lambda_i^2) = F(0) - F(\Lambda^2) + \Lambda^2 dF(\Lambda^2)/d\Lambda^2.$$

We use $M = 280$ MeV and $\Lambda = 871$ MeV, which yields the pion decay constant $f_\pi = 93$ MeV

Spectral Quark Model

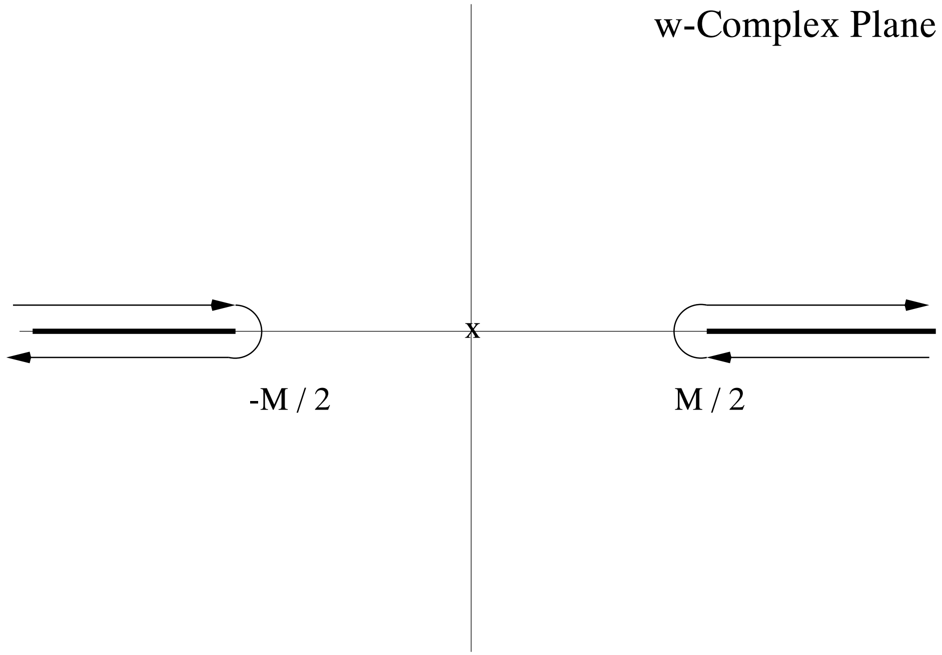
SQM (ERA+WB, 2004) amounts to supplying the quark loop with an integral over the quark mass ω weighted by a quark spectral density $\rho(\omega)$,

$$H_{\rm SQM}(x,0,-\mathbf{\Delta}_\perp^2) = \int_C d\omega\, \rho_V(\omega) H(x,0,-\mathbf{\Delta}_\perp^2;\omega),$$

where

$$\rho_V(\omega) = \frac{1}{2\pi i}\frac{3\pi^2 m_\rho^3 f_\pi^2}{4N_c}\frac{1}{\omega}\frac{1}{(m_\rho^2/4-\omega^2)^{5/2}},$$

and the contour C is given by



with $M = m_\rho$

Then

$$H_{\rm SQM}(x,0,-\boldsymbol{\Delta}_\perp^2) = \frac{m_\rho^2(m_\rho^2 - (1-x)^2\boldsymbol{\Delta}_\perp^2)}{(m_\rho^2 + (1-x)^2\boldsymbol{\Delta}_\perp^2)^2}\theta(x)\theta(1-x).$$

We check that

$$m_\rho^2 = \frac{24\pi^2 f_\pi^2}{N_c}$$

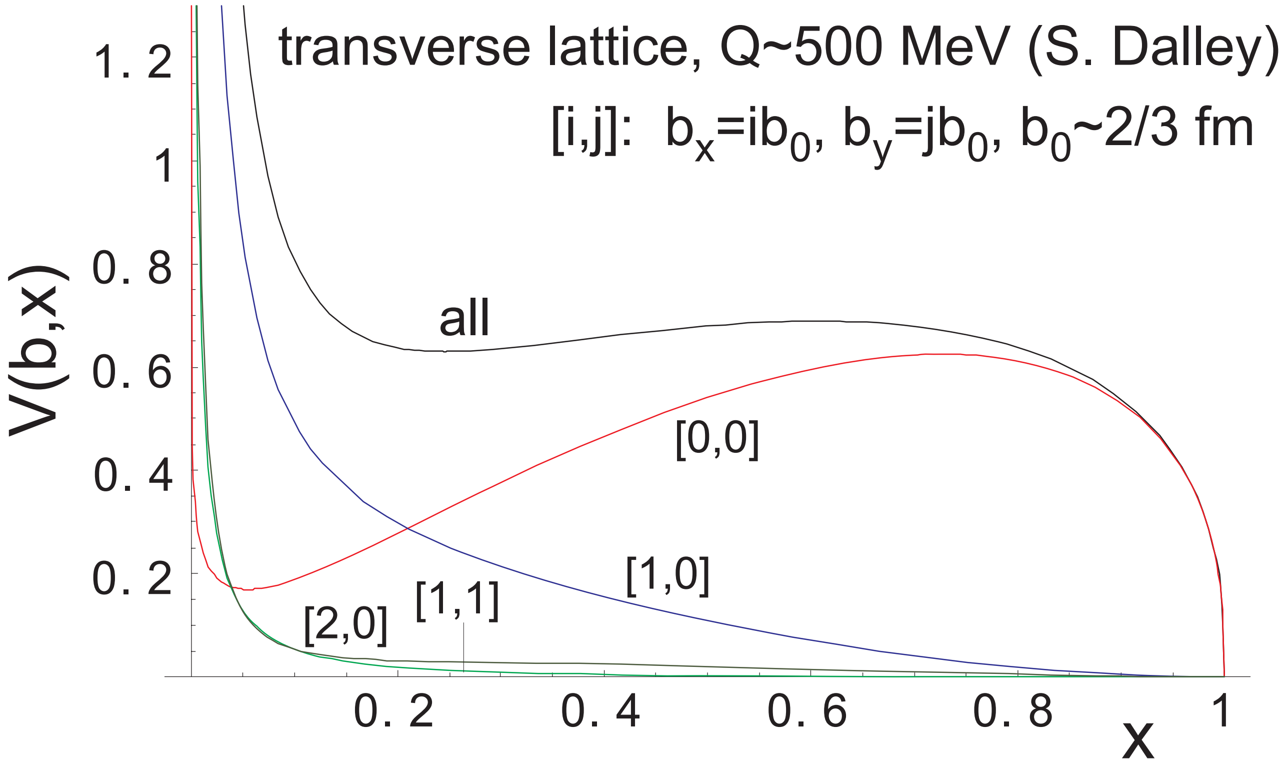
$$F(t) = \int_0^1 dx H_{\rm SQM}(x,0,t) = \frac{m_\rho^2}{m_\rho^2 + t},$$

which is the built-in vector-meson dominance principle. Clearly, $F(0) = 1$, and $H_{\rm SQM}(x,0,0) = \theta(x)\theta(1-x)$ [Davidson-Arriola, 1995]. We pass to the impact-parameter space by the Fourier-Bessel transformation and get

$$q_{\rm SQM}(b,x) = \frac{m_\rho^2}{2\pi(1-x)^2}\left[K_0\left(\frac{bm_\rho}{1-x}\right) - \frac{bm_\rho}{1-x}K_1\left(\frac{bm_\rho}{1-x}\right)\right].$$

Lattice results

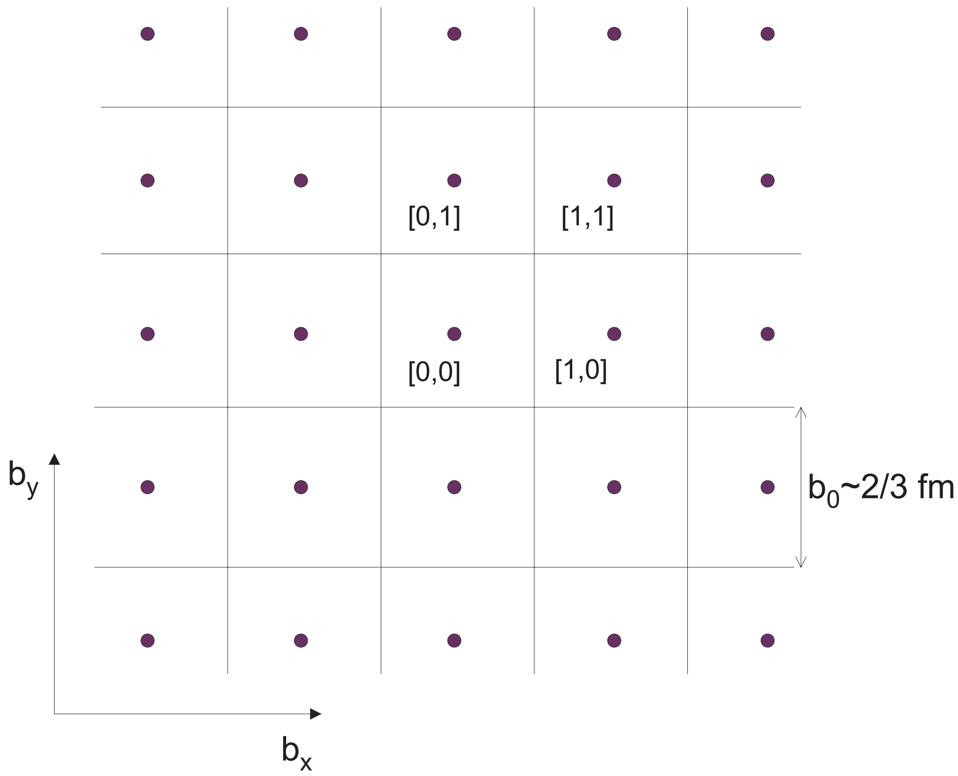
Simon Dalley



$(V(b,x)$ – nonsinglet (valence) quark distribution)

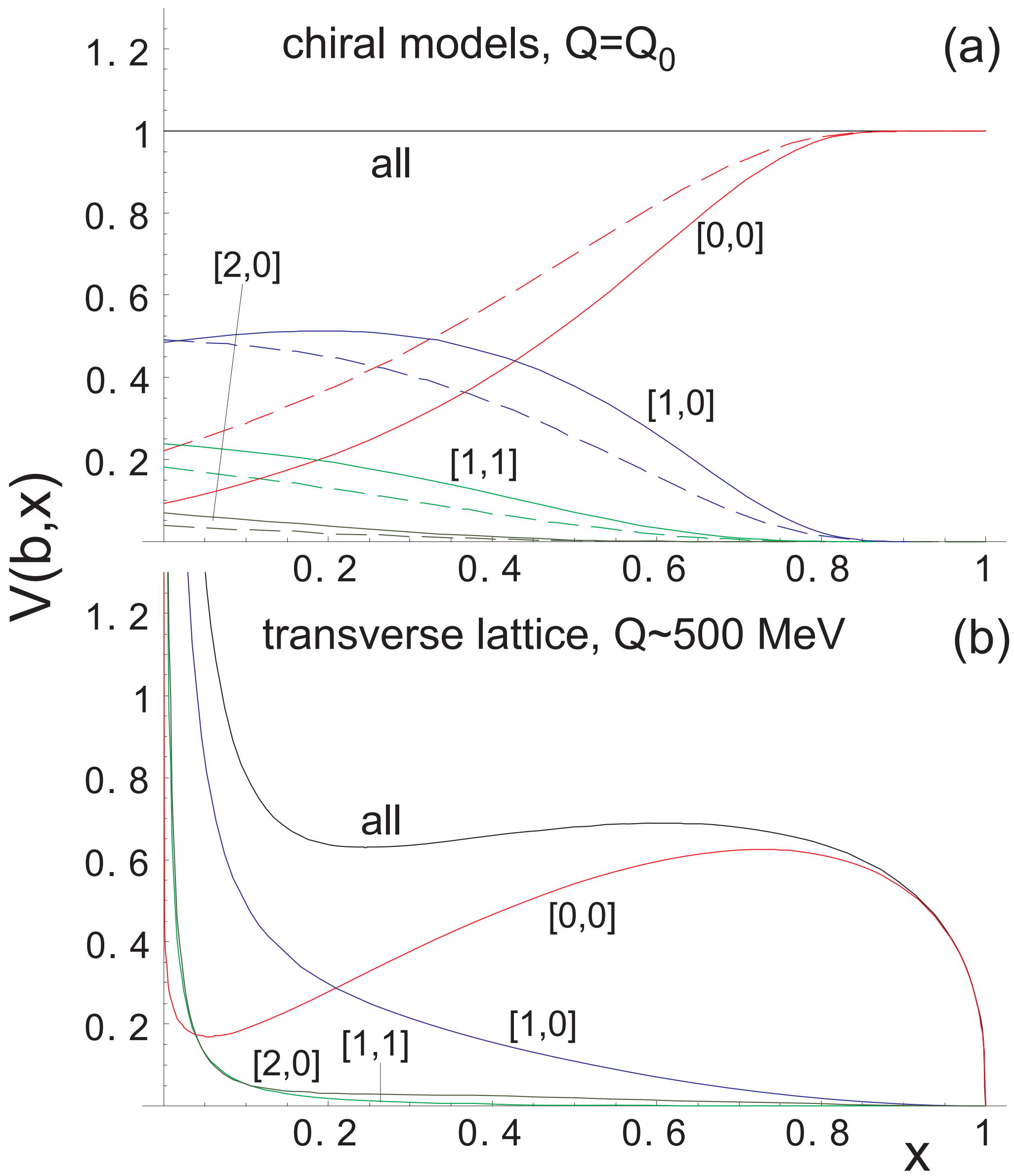
For π^+ we have $V = u - \bar{u} - d + \bar{d}$

Smearing over b



$$V(x,[i,j]) \equiv \int_{(i-1/2)b_0}^{(i+1/2)b_0} db_x \int_{(j-1/2)b_0}^{(j+1/2)b_0} db_y V(x, \sqrt{b_x^2 + b_y^2}).$$

The degeneracy factor for plaquettes equidistant from the origin is included, *i.e.* the $[1,0]$, $[1,1]$, and $[2,0]$ plaquettes are multiplied by 4, $[2,1]$ by 8, *etc.*



(a) SQM (solid) and NJL (dashed) at $Q = Q_0 = 313$ MeV. (b) Transverse lattice [Dalley 2003]. The initial condition of (a) needs to be evolved to a higher scale!

QCD evolution and the quark-model scale, Q_0

The models have produced GPD coresponding to a low, a priori unknown quark model scale, Q_0. A way to estimate it is to run the QCD evolution starting from various Q_0's up to a scale Q where data can be used.

QCD EVOLUTION IS OBVIOUSLY A NECESSARY STEP!

Determination of Q_0

The scale Q_0 (the quark-model scale) is defined as the scale where all momentum of the hadron is carried by the valence quarks. The valence contribution to the energy momentum tensor evolves as

$$\frac{V_1(Q)}{V_1(Q_0)} = \left(\frac{\alpha(Q)}{\alpha(Q_0)}\right)^{32/81} .$$

In [SMRS, 1992] at $Q = 2$ GeV the valence quarks carry 47% of the total momentum of the pion. Downward LO evolution requires

$$V_1(Q_0) = 1, \quad G_1(Q_0) + S_1(Q_0) = 0,$$

which gives

$$Q_0 = 313^{+20}_{-10} \text{ MeV}.$$

Rather low! One can hope that the typical expansion parameter $\alpha(Q_0)/(2\pi) \sim 0.34 \pm 0.04$ makes the perturbation theory still meaningful. NLO supports this assumption [Davidson + ERA, 2002]. Similar estimate for Q_0 has been obtained from an analysis of pion DA [ERA + WB,2002].

QCD evolution of the diagonal non-forward $V(x, Q, b)$

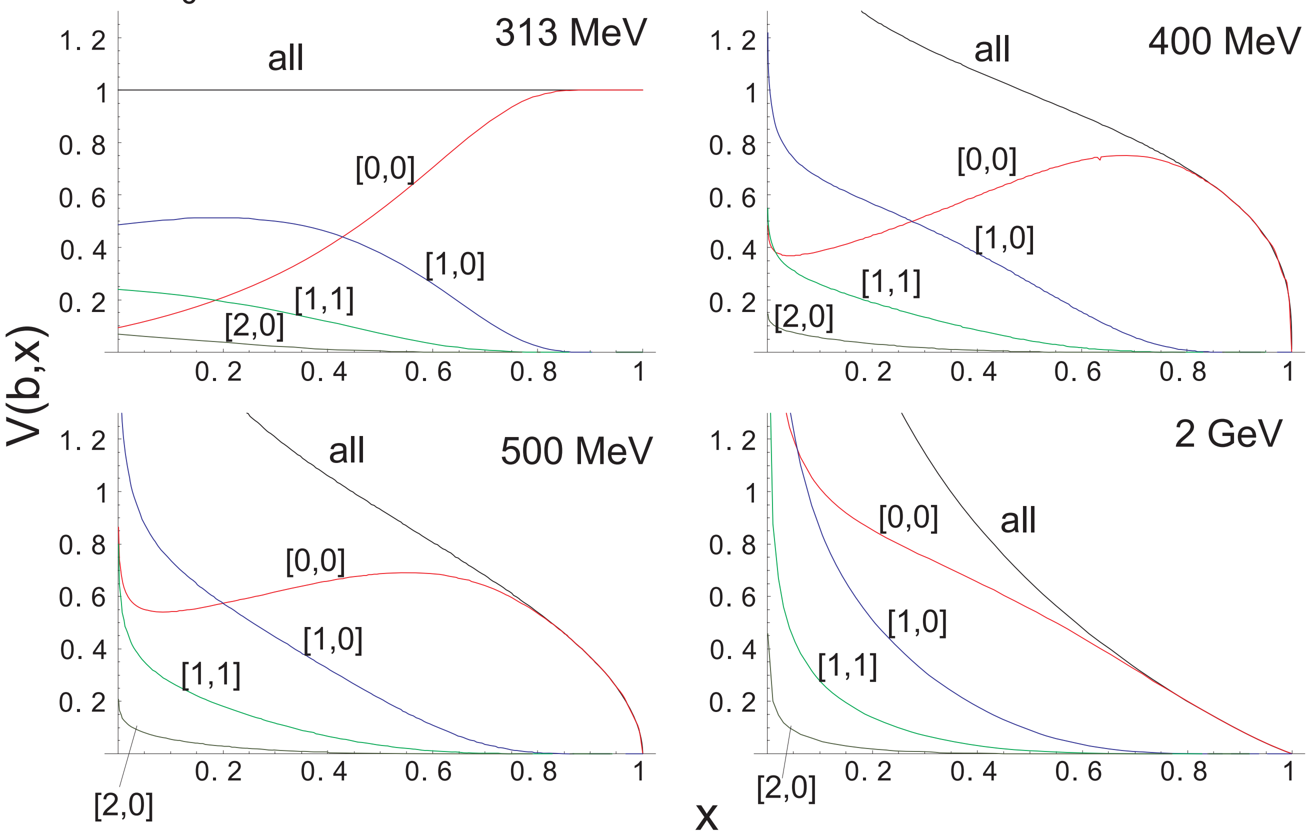
We apply the evolution to the smeared functions,

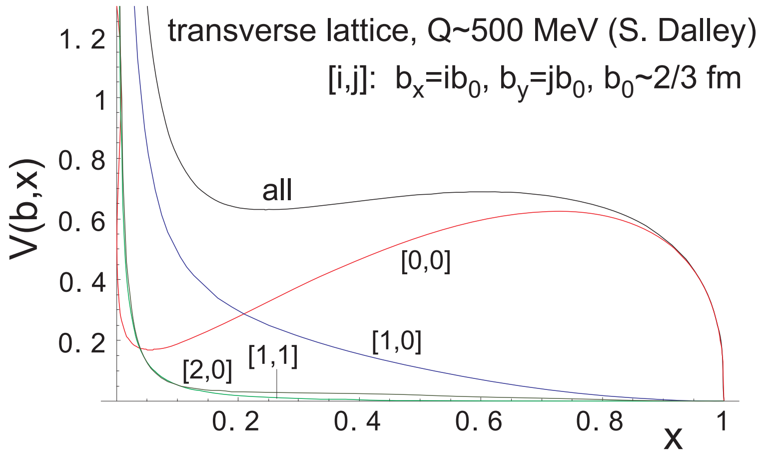
$$V(x, Q, [i, j]) = \int_{-i\infty}^{+i\infty} \frac{\mathrm{d}n}{2\pi i} x^{-n} \left(\frac{\alpha(Q)}{\alpha(Q_0)} \right)^{\gamma n/(2\beta_0)} \int_0^1 dx' \, x'^{n-1} V(x', Q_0, [i, j])$$

where the distribution at the scale Q_0 is the prediction of either of the two considered chiral quark models

... and the results are ...

SQM, b_0=2/3 fm





qualitative agreement

UPD's of the pion

NJL with PV regularization:

$$q(x, k_\perp, Q_0) = \frac{\Lambda^4 M^2 N_c}{4 f_\pi^2 \pi^3 \left(k_\perp^2 + M^2\right) \left(k_\perp^2 + \Lambda^2 + M^2\right)^2} \theta(x)\theta(1-x)$$

$$F_{\rm NJL}^{\rm NP}(b) = \frac{M^2 N_c}{4 f_\pi^2 \pi^2} \left(2K_0(bM) - 2K_0(b\sqrt{\Lambda^2 + M^2}) - \frac{b\Lambda^2 K_1(b\sqrt{\Lambda^2 + M^2})}{\sqrt{\Lambda^2 + M^2}} \right)$$

$$\langle k_\perp^2 \rangle_{\rm NP}^{\rm NJL} = (626 \text{ MeV})^2 \qquad (M = 280 \text{ MeV}, \Lambda = 871 \text{ MeV})$$

SQM:

$$q(x, k_\perp, Q_0) = \frac{6 m_\rho^3}{\pi (k_\perp^2 + m_\rho^2/4)^{5/2}} \theta(x)\theta(1-x),$$

$$F_{\rm SQM}^{\rm NP}(b) = \left(1 + \frac{b m_\rho}{2}\right) \exp\left(-\frac{m_\rho b}{2}\right)$$

$$\langle k_\perp^2 \rangle_{\rm NP}^{\rm SQM} = \frac{m_\rho^2}{2} = (544 \text{ MeV})^2$$

(the meaning of b different, conjugated to $k_\perp$)

The Kwieciński equations

(CCFM framework, generalization of DGLAP)

$$f_{NS}(x, k_\perp, Q) = f_{NS}(x, k_\perp, Q_0) + \int_0^1 dz \int_{Q_0^2}^{Q^2} \frac{d^2Q'}{\pi Q'^2} \frac{\alpha(Q'^2)}{2\pi} P_{qq}(z) \times \left[\Theta(z - x) f_{NS}\left(\frac{x}{z}, k_\perp + (1 - z)Q', Q\right) - f_{NS}(x, k_\perp, Q)\right]$$

SFSC

The Fourier-Bessel transformation

$$f_j(x, b, Q) \equiv \int d^2k_\perp \, e^{-ik_\perp \cdot b} f_j(x, k_\perp, Q) = \int_0^\infty 2\pi dk_\perp \, k_\perp J_0(bk_\perp) f_j(x, k_\perp, Q)$$

diagonalizes the equations in the transverse coordinate b:

$$Q^2 \frac{\partial f_{\rm NS}(x, b, Q)}{\partial Q^2} = \frac{\alpha(Q^2)}{2\pi} \int_0^1 dz \, P_{qq}(z) \left[\Theta(z - x) \, J_0((1 - z)Qb) \, f_{\rm NS}\left(\frac{x}{z}, b, Q\right) - f_{\rm NS}(x, b, Q)\right]$$

SFSC

Remarks:

$b = 0 \to J_0 = 1 \longrightarrow$ equations identical to DGLAP, with the distributions f_j at $b = 0$ becoming the integrated PD's:

$$f_j(x, b = 0, Q) = \frac{x}{2} p_j(x, Q)$$

"b-factorization": $f(x, b, Q)$-solution $\longrightarrow F(b) f(x, b, Q)$-solution

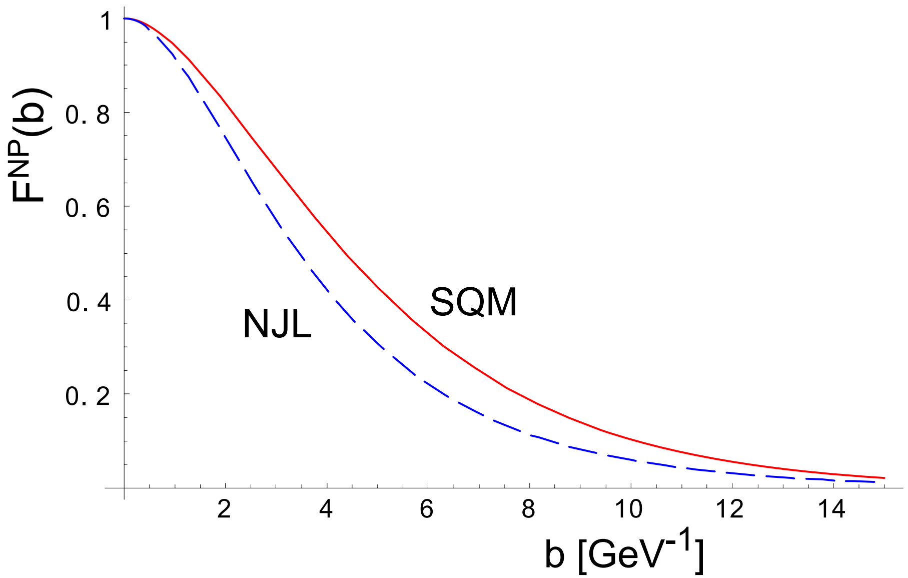
For each b at an initial scale Q_0 the non-perturbative UPD's depending on x and b ($k_\perp$) are perturbatively evolved to a higher scale Q

Initial profile

1. (Kwieciński + Gawron + WB, '03):

$$p_j(x, Q_0) = \text{GRV/GRS}, \; F(b) = e^{-\frac{b^2}{b_0^2}}$$

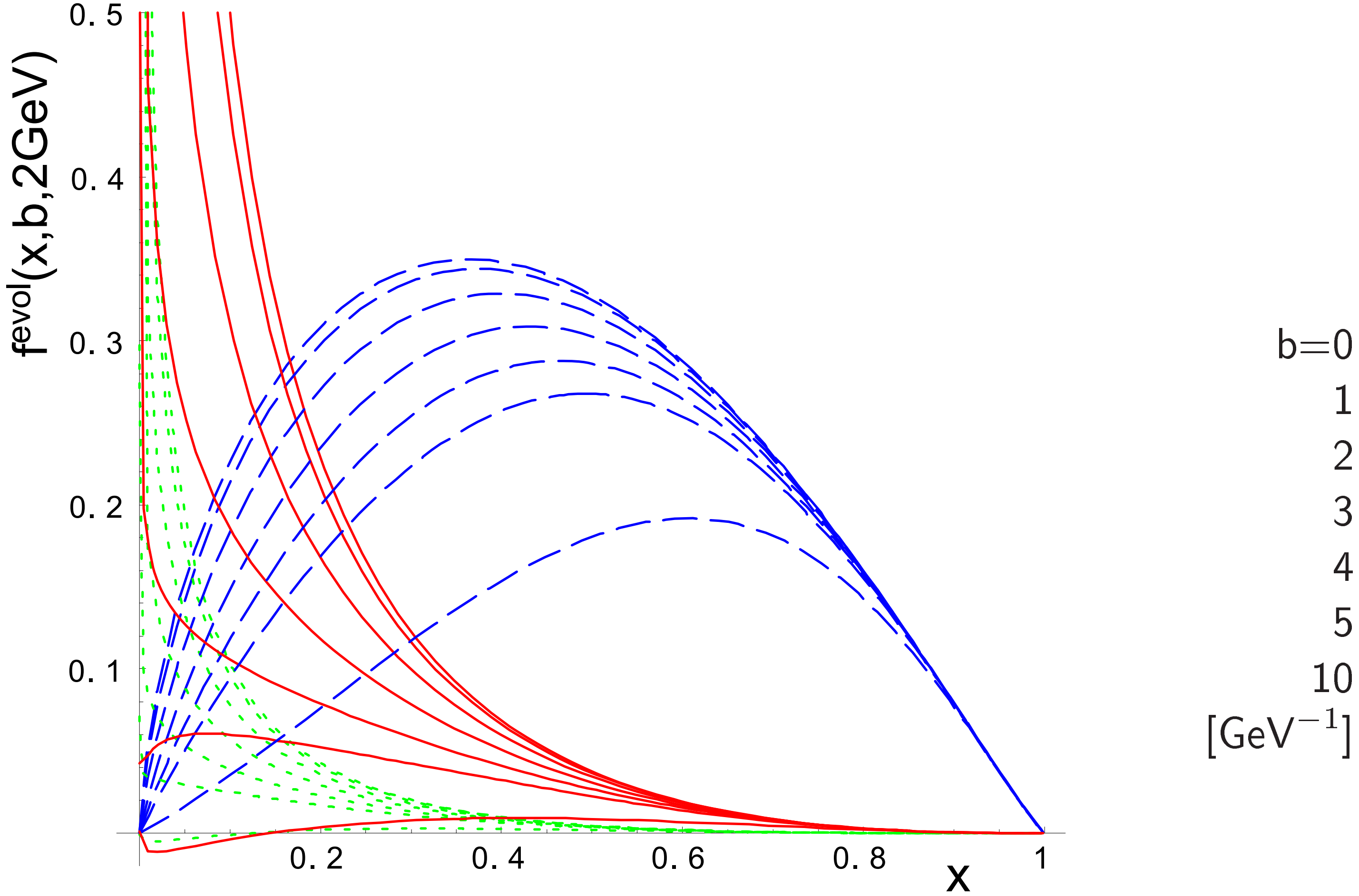
2. (ERA+WB, '04): Chiral quark models give predictions for the pion



At large b fall off exponentially, at large $k_\perp$ fall off as a power law

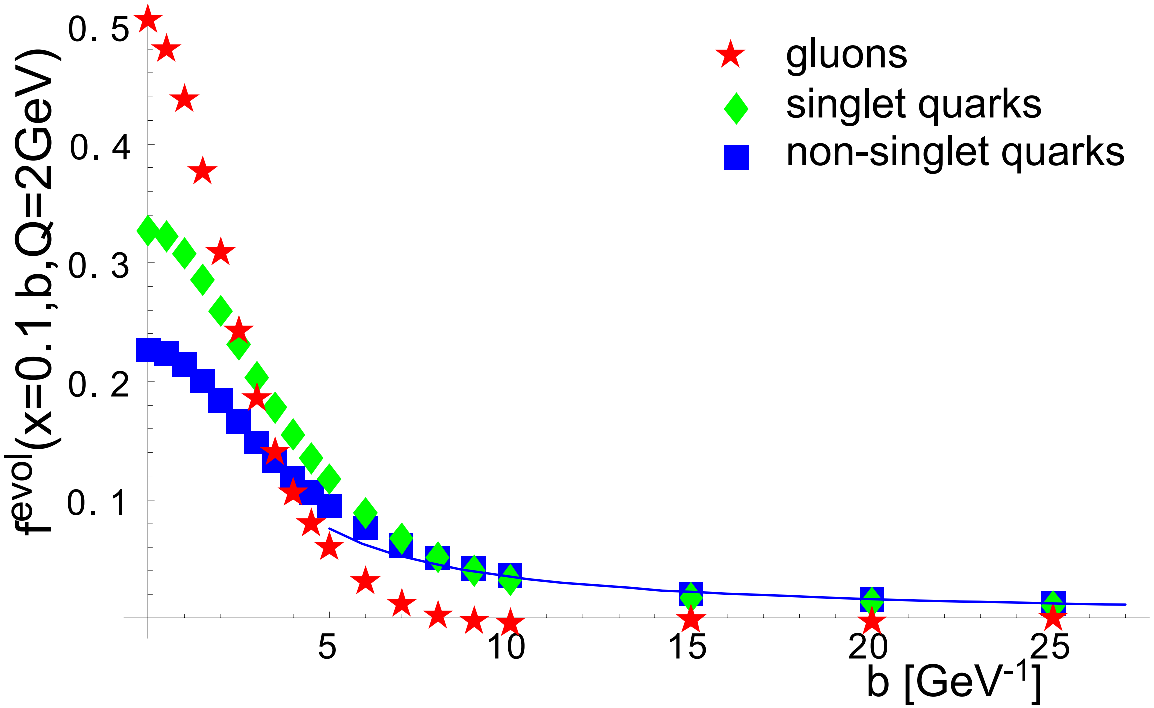
. . . now we run the evolution

Numerical solution, $Q^2 = 4\ \mathrm{GeV}^2$



(non-singlet (valence) quarks, sea quarks $(S - NS)$, and gluons)

Numerical solution, $Q^2 = 4\ \mathrm{GeV}^2$, $x = 0.1$

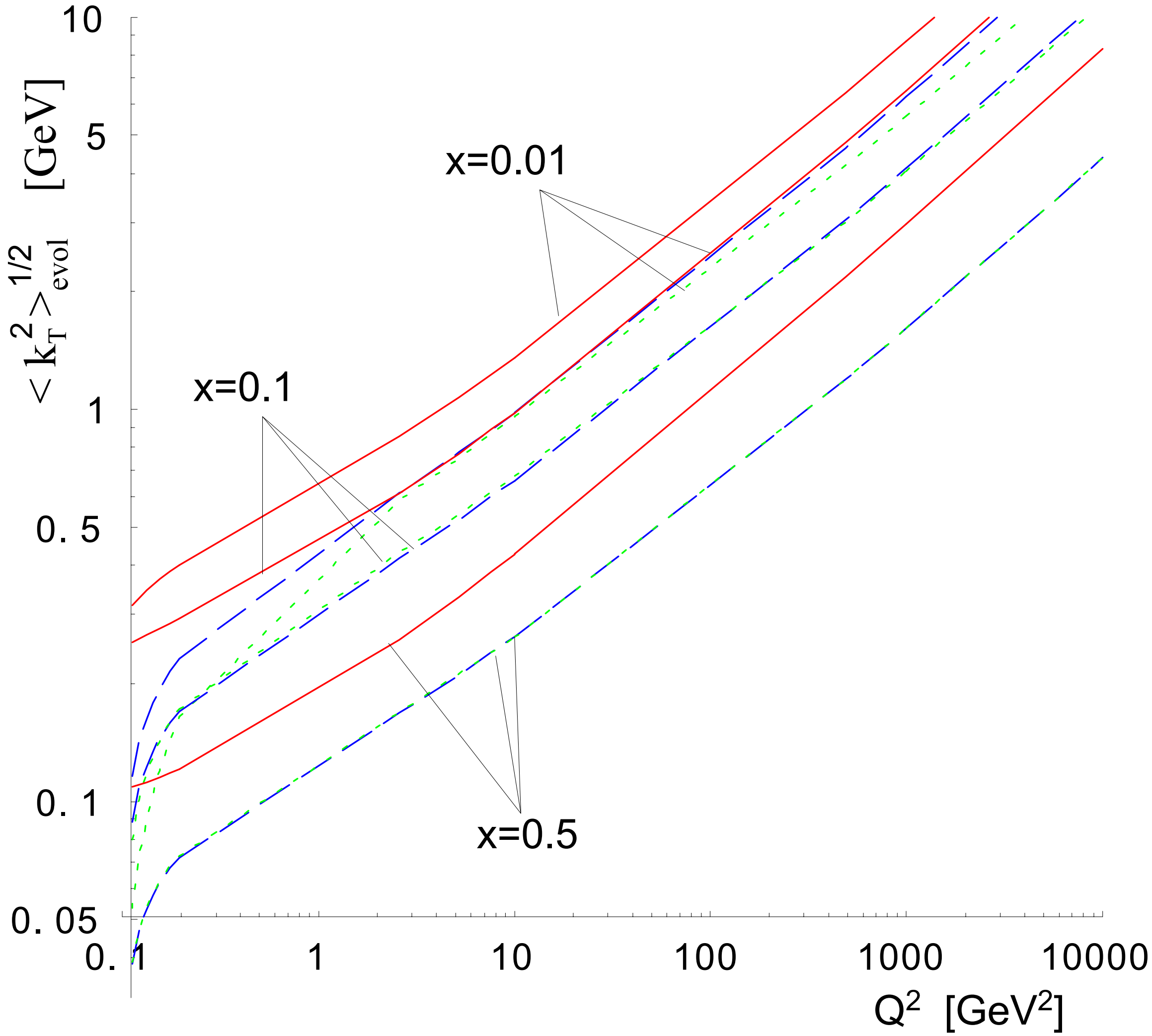


Shrinking in b (spreading in $k_\perp$) as Q grows!

effect increases with increasing Q and dropping x, largest for gluons

Long, power-law tail in b

Spreading in $k_\perp$



(non-singlet (valence) quarks, singlet quarks, and gluons)

Asymptotically $\langle k_\perp^2 \rangle_{\rm evol} \sim Q^2 \alpha(Q^2)$

Full width: $\langle k_\perp^2 \rangle = \langle k_\perp^2 \rangle_{\rm NP} + \langle k_\perp^2 \rangle_{\rm evol}$

Pion distribution amplitude (PDA)

ERA+WB, PRD 66 (2002) 094016

The pion light-cone wave function (the axial-vector component) is

$$\Psi_\pi(x, \vec{k}_\perp) = -\frac{i\sqrt{2}}{4\pi f_\pi} \int d\xi^- d^2\xi_\perp e^{i(2x-1)\xi^- p^+ - \xi_\perp \cdot k_\perp} \langle \pi^+(p) | \bar{u}(\xi^-, \xi_\perp) \gamma^+ \gamma_5 d(0) | 0 \rangle .$$

where $p^\pm = m_\pi$. The pion distribution amplitude is defined as

$$\varphi_\pi(x) = \int d^2k_\perp \Psi_\pi(x, \vec{k}_\perp)$$

Physical importance:

$$\frac{Q^2 F_{\gamma^* \to \pi\gamma}(Q)}{2f_\pi}\Big|_{\mathrm{twist}-2} = \int_0^1 dx \frac{\varphi_\pi(x, Q)}{6x(1-x)} \qquad (1)$$

Quark-model predictions

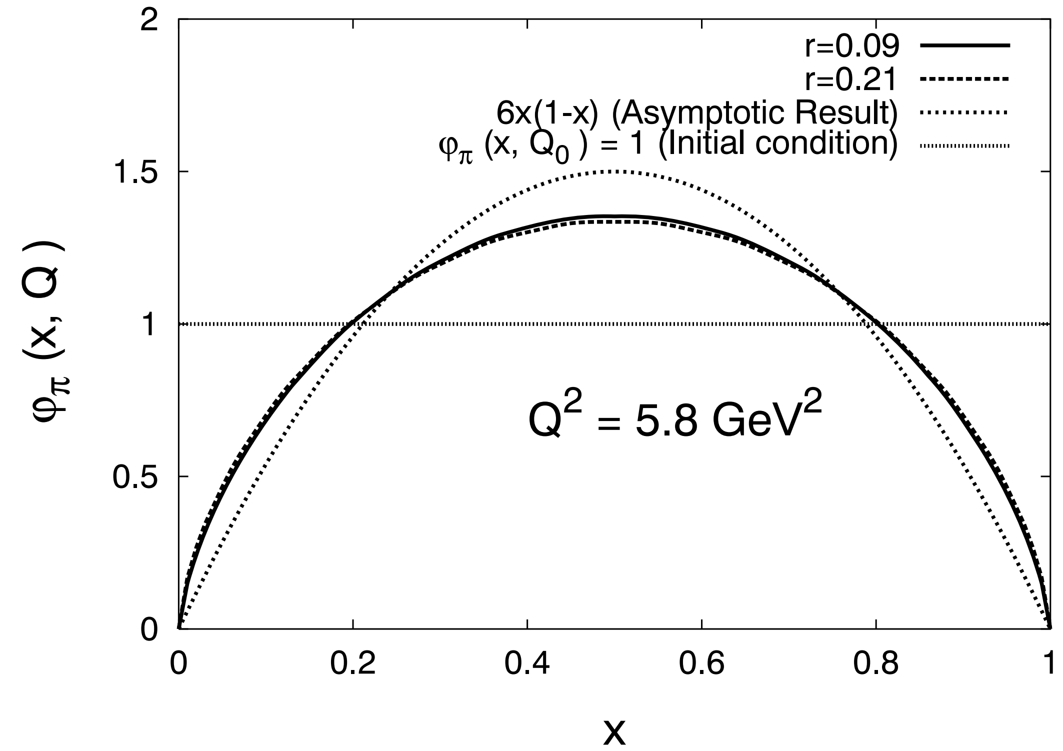
NJL (in the chiral limit):

$$
\begin{aligned}
\Psi_\pi(x,k_\perp) &= \frac{4N_c M^2}{16\pi^3 f_\pi^2}\sum_i c_i \frac{1}{k_\perp^2+\Lambda_i^2+M^2}\theta(x)\theta(1-x)\\
\varphi_\pi(x) &= \theta(x)\theta(1-x)\\
\langle \vec{k}_\perp^2\rangle &= -\frac{M\langle \bar{u}u\rangle}{f_\pi^2}.
\end{aligned}
$$

SQM:

$$
\begin{aligned}
\Psi(x,k_\perp) &= \frac{3M_V^3}{16\pi(k_\perp^2+M_V^2/4)^{5/2}}\theta(x)\theta(1-x)\\
\varphi_\pi(x) &= \theta(x)\theta(1-x)\\
\langle \vec{k}_\perp^2\rangle &= m_\rho^2/2
\end{aligned}
$$

QCD evolution of PDA



The pion distribution amplitude in the chiral limit evolved to the scale $Q^2 = (2.4\mathrm{GeV})^2$. The two values for the evolution ratio $r = \alpha(Q)/\alpha(Q_0)$ reflect the uncertainties in the values of based on an analysis of the CLEO data. We also show the unvolved PDA, $\varphi_\pi(x, Q_0) = 1$, and the asymptotic PDA, $\varphi_\pi(x, \infty) = 6x(1-x)$.

$Q_0 = 322 \pm 45$ MeV (PD $\rightarrow 313^{+20}_{-10}$ MeV)

Results and conclusions

- Chiral quark models may be used to provide GPD, UPD, PDA, ... at a low, a priori unknown scale, Q_0
- In "good" models the results have correct formal features and are simple
- The quark model scale Q_0 can be estimated with the help of the momentum fraction carried by the valence quarks
- Appropriate QCD evolution must be carried over
- At $Q = 2$ GeV the result for the forward distribution agrees very well with the SMRS parameterization of the pion structure function [Davidson-Arriola, 1995]. At $Q = 4$ GeV it agrees with the Fermilab E615 data
- The results for GPD agree qualitatively with the data from transverse lattices
- For PDA reasonable agreement with the data from CLEO. Extraction of Q_0 consistent with the earlier value from PD
- Obviously, most data are for the nucleon
- In summary, very reasonable results for PD, GPD, UPD, PDA. QCD evolution is a necessary step – challange to any model of the hadron structure!

BACKUP slides

Behavior at $x \to 1$

A function that initially behaves as $V(x, Q_0, b) \to C(b)(1-x)^p$ evolves into

$$V(x, Q, b) \to C(b)(1-x)^{p - \frac{4C_F}{\beta_0} \log \frac{\alpha(Q)}{\alpha(Q_0)}}, \qquad x \to 1.$$

Moments in SQM

In the Spectral Quark Model

$$V_n(Q_0, b) =$$
$$\frac{m_\rho^2 \Gamma(n+1)}{\pi 2^{n+3}} \left[b m_\rho G^{4,0}_{2,4}\left(\frac{b^2 m_\rho^2}{4} \middle| \begin{array}{c} \frac{n-1}{2}, \frac{n}{2} \\ -1, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2} \end{array} \right) - G^{4,0}_{2,4}\left(\frac{b^2 m_\rho^2}{4} \middle| \begin{array}{c} \frac{n}{2}, \frac{n+1}{2} \\ -\frac{1}{2}, 0, 0, 0 \end{array} \right) \right],$$

where G denotes the Meijer G function. This form can be useful for further analytic considerations.

Scaling in $b/(1-x)$

Generally, the chiral quark model (one-loop) results depend on $\mathbf{\Delta}_\perp$ and x only through the combination $(1-x)^2\mathbf{\Delta}_\perp^2$. Consequently, in the b space they depend on the combination $b^2/(1-x)^2$. Due to this property

$$\frac{\int d^2b\, b^{2n} q(b,x)}{\int d^2b\, q(b,x)} \equiv \langle b^{2n}\rangle(x) = (1-x)^{2n}\langle b^{2n}\rangle(0).$$

This means, that all the moments except for $n=0$ vanish as $x \to 1$, or $q(b,x)$ becomes a $\delta(b)$ function in this limit. This behavior is seen in the transverse lattice data.