Baryon inside the pion

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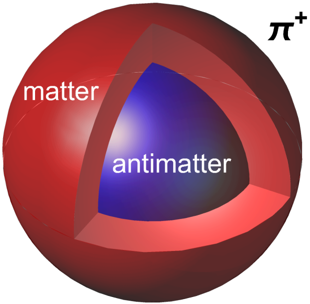
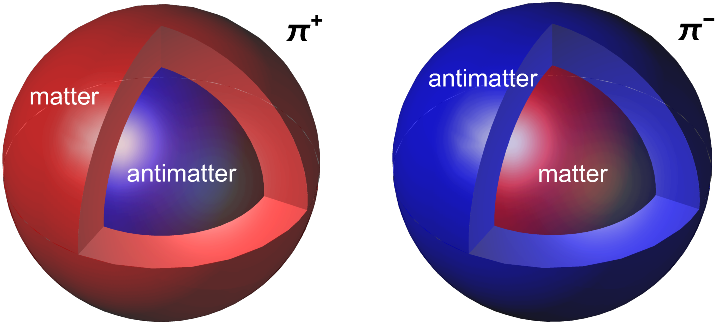
Sneak preview

$\pi^+ = u\bar{d}$, u - baryon charge (matter), $\bar{d}$ - antibaryon charge (antimatter)



Structure of π^+

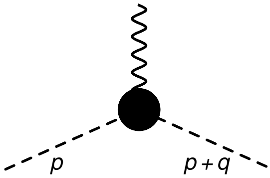
u sticks out more outside, $\bar{d}$ sits more inside


matter
antimatter
π^+
antimatter
matter
π^-

Some basics of form factors

Def. of form factors

On-shell matrix element of an operator at $x = 0$



Example: electromagnetic for a scalar particle

$$\langle h(p)|J^{\mu}(0)|h(p+q)\rangle = (2p^{\mu} + q^{\mu})F_Q^h(q^2)$$

conserved: $\partial_{\mu} J^{\mu} = 0 \to q_{\mu}(2p^{\mu} + q^{\mu}) = (p+q)^2 - p^2 = m_h^2 - m_h^2 = 0$

$F_Q^h(0)$ – charge

$t = q^2 = -Q^2$

Spatial interpretation

Breit frame (no energy transfer): $q^2 = -\vec{q}^{\,2} \equiv -Q^2 \leq 0$

$$\rho(r) = \int \frac{d^3q}{(2\pi)^3} e^{i\vec{q}\cdot\vec{r}} F(-\vec{q}^{\,2}), \quad F(-\vec{q}^{\,2}) = \int d^3r\, j_0(|\vec{q}|r)\rho(r)$$

Expanding in $|\vec{q}|$ near 0 $\rightarrow \int d^3r\,\rho(r) = F(0), \ldots$

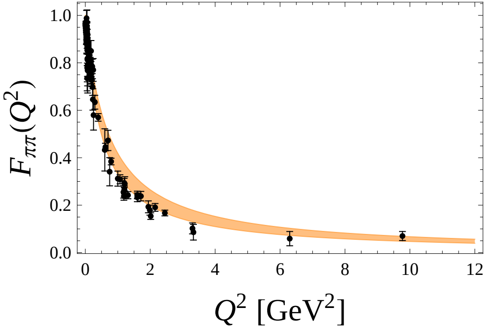
Mean squared radius

$$\langle r^2 \rangle = 6 \frac{dF(t)}{dt}\bigg|_{t=0}$$

Ff's carry information about the size: charge, gravitational, generalized related to GPD's, ...
Extracted from scattering data, lattice QCD

Vector meson dominance (VMD, Sakurai)

$F_3^{\pi^+}(Q^2) = \frac{1}{1+Q^2/m_\rho^2}$ (more generally $\sum_n \frac{c_n}{1+Q^2/m_n^2}$, supported by large N_c)



Works remarkably well!

[Masjuan, ERA, WB, PRD 87 (2013) 014005]

Symmetries and the baryon ff of the pion

Symmetries

Divergence of vector currents in QCD

$$\partial_\mu \left[\bar{q}_a(x)\gamma^\mu q_b(x)\right] = i(m_a - m_b)\bar{q}_a(x) q_b(x), \qquad a,b = u,d,s,c,b,t \quad \text{–flavor}$$

$m_a = m_b \rightarrow$ conservation of vector currents, quark number of any species conserved
For π^+ heavier flavors can be neglected (OZI, large-N_c):

$$J_B^\mu = \frac{1}{N_c}\left(\bar{u}\gamma^\mu u + \bar{d}\gamma^\mu d\right), \quad J_3^\mu = \frac{1}{2}\left(\bar{u}\gamma^\mu u - \bar{d}\gamma^\mu d\right), \quad J_Q^\mu = J_3^\mu + \frac{1}{2}J_B^\mu \quad \text{(all obviously conserved)}$$

Baryon, isospin, and charge form factors

$$\langle \pi^a(p) \mid J^\mu_{B,3,Q}(0) \mid \pi^a(p+q)\rangle = (2p^\mu + q^\mu) F^a_{B,3,Q}(q^2), \quad a = 0,+,- \quad \text{(pion isospin)}$$

Symmetries 2

π^0: $I^G(J^{PC}) = 1^-(0^{-+})$, $\pi^\pm$: $I^G(J^P) = 1^-(0^-)$, $C|\pi^\pm\rangle = |\pi^\mp\rangle$, $G = Ce^{i\pi I_2}$

$J^\mu_{B,3,Q}$ are odd under C $\to$

$F^{\pi^0}_{B,3,Q}(q^2) = 0$ and $F^{\pi^+}_{B,3,Q}(q^2) = -F^{\pi^-}_{B,3,Q}(q^2)$ – always true!

e.g., $\langle\pi^0(p)|J^\mu_B(0)|\pi^0(p+q)\rangle = -\langle\pi^0(p)|CJ^\mu_B(0)C|\pi^0(p+q)\rangle = -\langle\pi^0(p)|J^\mu_B(0)|\pi^0(p+q)\rangle = 0$

Similarly, for **exact** isospin (and G) symmetry ($m_u = m_d$ and neglecting small EM effects)

J^μ_B is odd under G $\to$

$F^{\pi^\pm}_B(q^2) = 0$ ($F^{\pi^\pm}_3(q^2) \neq 0$, as J^μ_3 is even under G)

However, in the real world the isospin (and G) are broken with $m_d > m_u$ and EM

$F^{\pi^\pm}_B(q^2)$ may be (and **is**) nonzero, with $F^{\pi^+}_B(q^2) = -F^{\pi^-}_B(q^2)$

Symmetries 3

The ff at $q = 0$ is the corresponding charge. As the baryon charge of the pion is 0, we have

$$F_B^{\pi^\pm}(0) = 0$$

(but not at $q^2 \neq 0$). On the other hand, $F_3^{\pi^\pm}(0) = \pm 1$ – the 3-component of isospin

- As a rule, if a quantity is not protected by symmetry, hence may be nonzero, it **is nonzero**
- There is the question of magnitude, which is somehow proportional to the strength of the symmetry breaking
- No probes with baryon number couple directly to the pion (except for lattice QCD) $\rightarrow$ we need indirect methods to estimate the effect

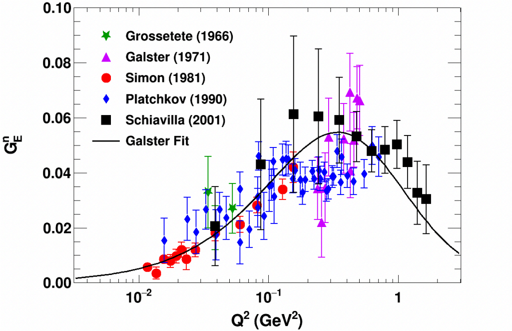
Mass splitting

$\Delta m \equiv m_d - m_u = 2.8(2)$MeV ($m_u = 2.01(14)$MeV, $m_d = 4.79(16)$MeV [Davies et al. 2009])

- EM violating effects more tricky, of the order $\alpha_{\text{QED}}/(2\pi) \sim 0.001$

Neutron

Reminiscent to the neutron, which has no electric charge, but has a non zero (for $q^2 \neq 0$) ff:



unpolarized elastic ed scat.
[Obrecht 2019]

Effective Lagrangian estimate

Order of magnitude from effective Lagrangian

At leading order in the pion momenta and the quark mass splitting

$$J_B^\mu = -2i\frac{c\Delta m}{\Lambda^3}\partial_\nu\left(\partial^\mu\pi^+\partial^\nu\pi^- - \partial^\nu\pi^+\partial^\mu\pi^-\right) + \ldots$$

c – dimensionless number, Λ – typical hadronic scale

J_B^μ is odd under C, trivially conserved, and yields $F_B^{\pi^+}(q^2) = q^2 c\Delta m/\Lambda^3 + \ldots$

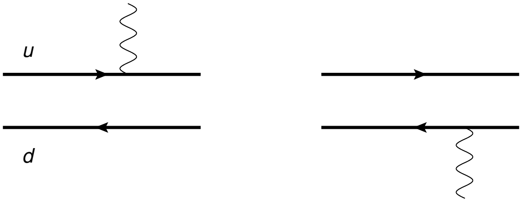
Baryonic ms radius

$\langle r^2\rangle_B^{\pi^+} = 6c\Delta m/m_\rho^3 \simeq c\ 0.002\mathrm{fm}^2 \simeq c(0.04\mathrm{fm})^2$

– small compared to the charge radius $\langle r^2\rangle_Q^{\pi^+} = 0.434(5)\mathrm{fm}^2 = (0.659(4)\mathrm{fm})^2$

Quark-model estimates

Yukawa model (impulse approximation)



$\rho_3(r) = \frac{1}{2}|\Psi_u(\vec{x})|^2 + \frac{1}{2}|\Psi_{\bar{d}}(\vec{x})|^2, \quad \rho_B(r) = \frac{1}{3}|\Psi_u(\vec{x})|^2 - \frac{1}{3}|\Psi_{\bar{d}}(\vec{x})|^2, \quad |\Psi_i(\vec{x})|^2 = \frac{M_i^2}{\pi r} e^{-2M_i r}$

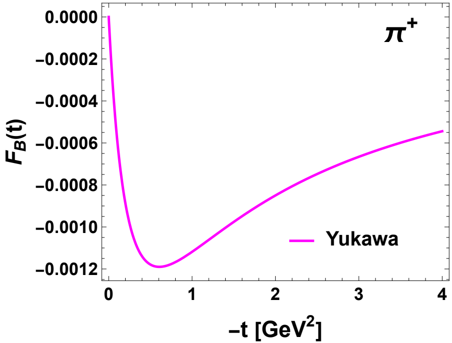
$M_{u,d} = M \mp \frac{1}{2}\Delta m$, $M \simeq \frac{1}{2} m_\rho$ – constituent quark masses

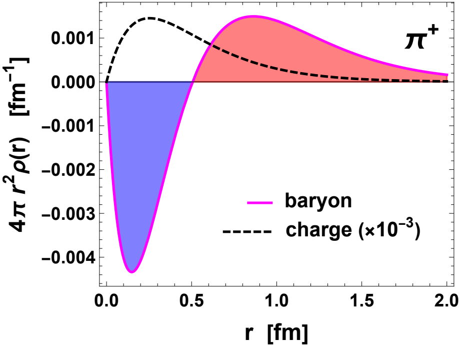
$\Delta m = 0 \to F_3^{\pi^+} = 1/(1 + Q^2/m_\rho^2)$ (VMD)

Baryon ff

$$F_B^{\pi^+}(-Q^2) = \frac{1}{N_c}\left[\frac{4M_u^2}{4M_u^2 + Q^2} - \frac{4M_d^2}{4M_d^2 + Q^2}\right] \simeq -\frac{4\Delta m\, m_\rho Q^2}{3(m_\rho^2 + Q^2)^2}$$

Yukawa model 2





$$\langle r^2 \rangle_B^{\pi^+} \simeq \frac{8\Delta m}{m_\rho^3} \simeq (0.04\ \text{fm})^2$$

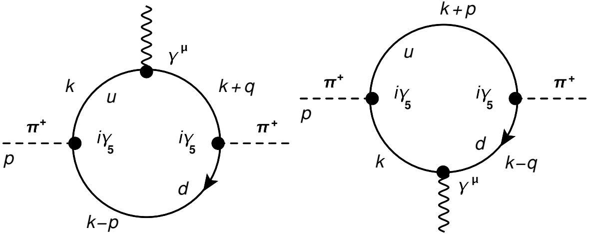
Mechanistic explanation

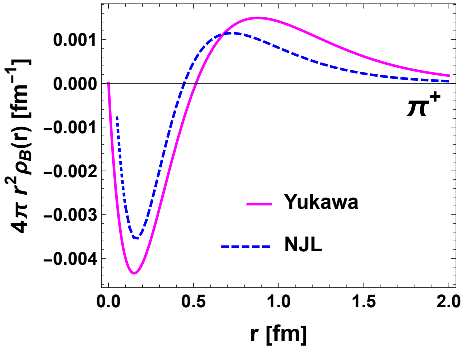
Trivializing the story...

$\bar{d}$ is a bit heavier than u, hence its distribution is somewhat more compact.
Center-of-mass argument, more and more apparent for larger mass asymmetry.

There remains the question of the size of the effect

Covariant field-theoretic model. Dynamical chiral symmetry breaking, point-like interaction, large-N_c (one-loop), regularization. Generally very successful in pion low-energy phenomenology



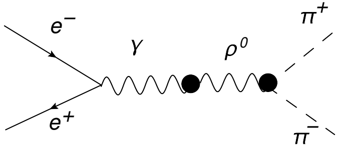


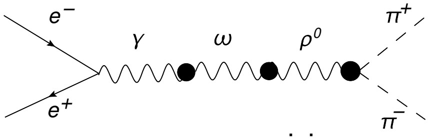
NJL: $\langle r^2 \rangle_B^{\pi^+} \simeq (0.03 \text{ fm})^2$

Determination from exp. data (!)

$e^+e^- \to \pi^+\pi^-$

(relevance for HVP in $g-2$)





$\rho_0 - \omega$ mixing

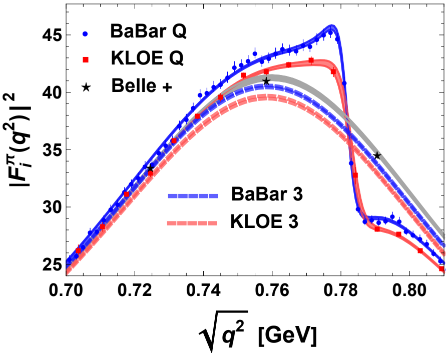
$$F_3^{\pi^+}(s) = \frac{1}{1+c'+c''+c'''}[D_{\rho^0}(s) + c' D_{\rho'^0}(s) + c'' D_{\rho''^0}(s) + c''' D_{\rho'''^0}(s)]$$

$$\frac{1}{2} F_B^{\pi^+}(s) = c_{\rho^0\omega}\, s\, D_{\rho^0}(s) D_\omega(s), \qquad D_V(s) = \frac{1}{m_V^2 - s - i\, m_V \Gamma_V(s)}$$

[Gounaris-Sakurai 1968]

KLOE and BaBar

Result of the fit in the relevant range of s (here $q^2 = s$)



solid bands along the data – $F_Q^{\pi^+}(s) = F_3^{\pi^+}(s) + \frac{1}{2}F_B^{\pi^+}(s)$

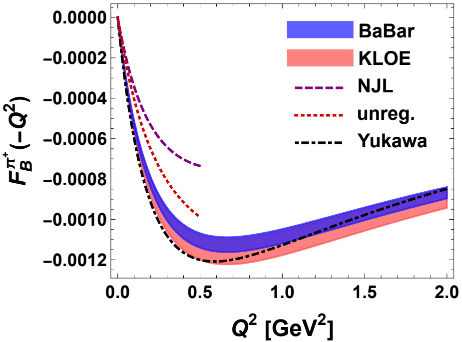
dashed bands – $F_3^{\pi^+}(s)$

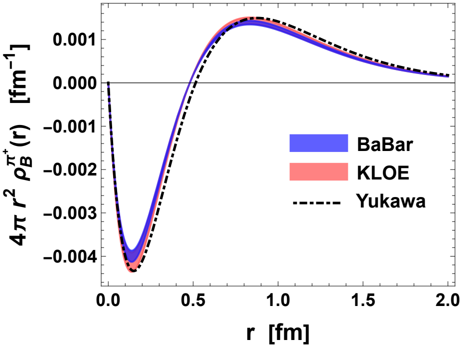
gray band - see the following

Continuation with the dispersion relation

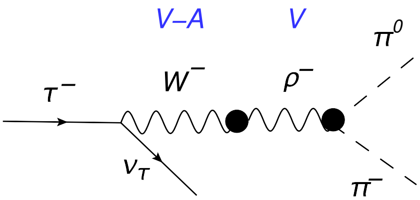
$Q^2 = -q^2 = s$

$$F_B^{\pi^\pm}(-Q^2) = \frac{1}{\pi}\int_{4m_{\pi^+}^2}^{\infty} ds \frac{\mathrm{Im} F_B^{\pi^\pm}(s)}{s+Q^2} = \frac{q^2}{\pi}\int_{4m_{\pi^+}^2}^{\infty} ds \frac{\mathrm{Im} F_B^{\pi^\pm}(s)}{s(s+Q^2)}$$





BaBar: $\langle r^2\rangle_B^{\pi^+} \simeq (0.0411(7)\ \mathrm{fm})^2$, KLOE: $\langle r^2\rangle_B^{\pi^+} \simeq (0.0412(12)\ \mathrm{fm})^2$ (stat. only)

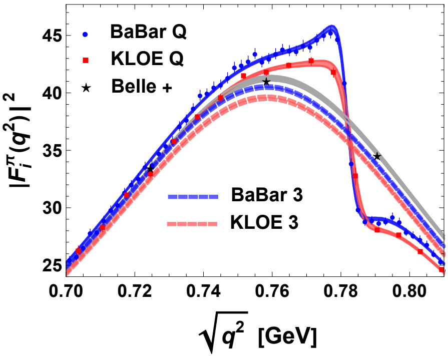


$$J^{\mu}_{\pm} = \frac{1}{2}\bar{q}\gamma^{\mu}\tau^{\pm}q \qquad \tau - \text{ (flavor Pauli matrix)}$$

$$\langle \pi^0(p) \mid J^{\mu}_{\pm}(0) \mid \pi^{\mp}(p+q)\rangle \rightarrow \text{ form factor}$$

$$F^{\pi}_{\pm}(s) = \frac{1}{1+c'+c''}[D_{\rho^{\pm}}(s) + c' D_{\rho'^{\pm}}(s) + c'' D_{\rho''^{\pm}}(s)]$$

Consistency check



- The solid and dashed BaBar and KLOE bands should overlap – some systematic inconsistency between the experiments is apparent
- The gray band should overlap with the dashed bands only in the strict isospin limit – departure measures the isospin breaking

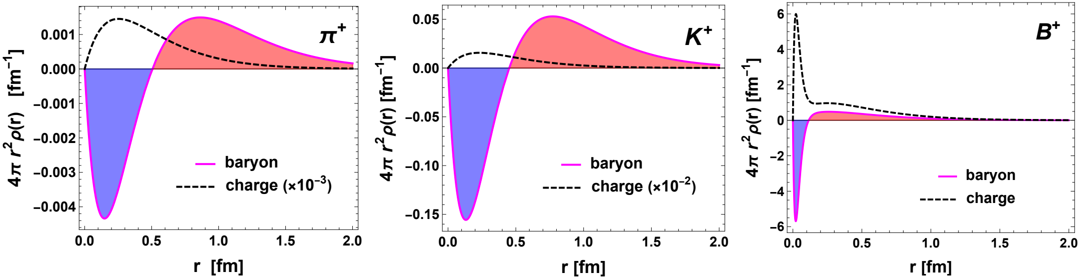
Comparison of our various estimates

approach	$\langle r^2 \rangle_B^{\pi^+}$	comment
effective Lagrangian	$c(0.04\ \mathrm{fm})^2$	c - number of order 1
toy Yukawa model	$(0.04\ \mathrm{fm})^2$	
NJL with PV reg.	$(0.03\ \mathrm{fm})^2$	
NJL without reg.	$(0.03\ \mathrm{fm})^2$	
BaBar	$(0.041(1)\ \mathrm{fm})^2$	exp. statistical error only
KLOE	$(0.041(1)\ \mathrm{fm})^2$	

- Remarkable agreement between very different methods
- BaBar and KLOE extractions incorporate both Δm and EM breaking

Heavy-light mesons

Heavy-light mesons – much stronger effect



$K^0 = d\bar{s}$ – charge of each quark = minus baryon number! $\rightarrow \langle r^2 \rangle_B^{K^0} = -\langle r^2 \rangle_Q^{K^0}$

PDG: $\langle r^2 \rangle_Q^{K^0} = -(0.277(2)\ \mathrm{fm})^2 = -0.077(10)\ \mathrm{fm}^2$

Yukawa model: $-\langle r^2 \rangle_B^{K^0} = \langle r^2 \rangle_Q^{K^0} \simeq -(0.22\ \mathrm{fm})^2 \simeq -0.05\ \mathrm{fm}^2$

Conclusions

Outlook

1. Fundamental feature of the pion, eventually should end up in PDG Tables
2. Small, but possible to extract from the present experimental data – could be elevated to strict determination after some experimental and theoretical systematic issues are resolved
3. Estimates from very different approaches yield
   $\langle r^2 \rangle_B^{\pi^+} = (0.03 - 0.04 \text{ fm})^2 = 0.001 - 0.002 \text{ fm}^2$,
   sign follows from mechanistic interpretation
4. Lattice QCD: $\langle r^2 \rangle_Q^{\pi} = (0.648(15) \text{ fm})^2 = 0.42(2) \text{ fm}^2$ – our signal for the baryon ff is a factor of ~ 10 too small (0.002 vs 0.02) to be currently detected (but still could be tried)
5. Good lattice prospects for the kaon or heavy-light mesons

THANKS FOR YOUR ATTENTION!