Baryon inside the pion

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Based on **Pablo Sanchez-Puertas, Enrique Ruiz Arrriola, WB**
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PRD 106 (2022) 036001 [arXiv:2112.11049]

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Some basics of form factors

Concept of the form factor

Elastic scattering cross section on a point-like vs. extended object
e.g., the Ratherford or Mott ($eX \to eX$) scattering

$$\frac{d\sigma}{d\Omega} = \frac{d\sigma}{d\Omega}_{\mathrm{point-like}} |F(\vec{q}^{\,2})|^2$$

momentum transfer $\vec{q} = \vec{p}_f - \vec{p}_i$.

Information on the spatial distribution of scatterers (charge) $\rho(r)$ in the target:

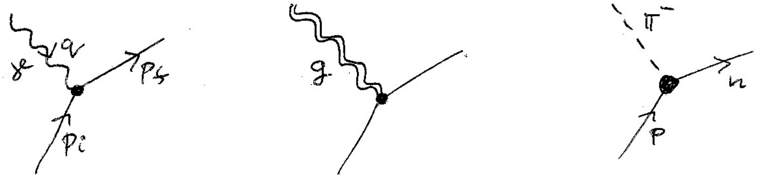
Form factor

$$F(\vec{q}^{\,2}) = \int d^3r\, e^{i\vec{q}\cdot\vec{r}} \rho(r) = \int d^3r\, j_0(|\vec{q}\,|)\rho(r)$$

At low q we have $F(\vec{q}^{\,2}) = \int d^3r\, \rho(r) - \frac{1}{6}\vec{q}^{\,2} \int d^3r\, r^2 \rho(r) + \cdots = \text{"charge"} - \frac{1}{6}\vec{q}^{\,2}\,\mathrm{msr}$

Different probes of the structure

electric magnetic strangeness . . . mass (gravitational) . . . composite operators . . . hadronic



scattering amplitude $= \sum$ tensorial structure $\times$ form factor (scalar function)

Extracted from scattering data and lattice QCD

Relativistic kinematics

$q = p_f - p_i, \quad t = q^2 = q_0^2 - \vec{q}^{\,2} = -Q^2, \quad p_i^2 = p_f^2 = m^2$

$$F = F(t)$$

"Charge"

$$F(0)$$

Mean squared radius

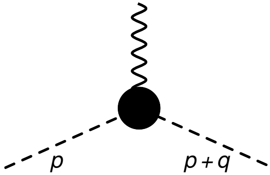
$$\langle r^2 \rangle = 6 \frac{dF(t)}{dt}\Big|_{t=0}$$

Transverse density

$$\rho(b) = \int \frac{d^2 q_\perp}{(2\pi)^2} \, e^{-i\vec{q}_\perp \cdot \vec{b}} F(-\vec{q}_\perp^{\,2})$$

Field theoretic definition

On-shell matrix element of an operator at $x = 0$



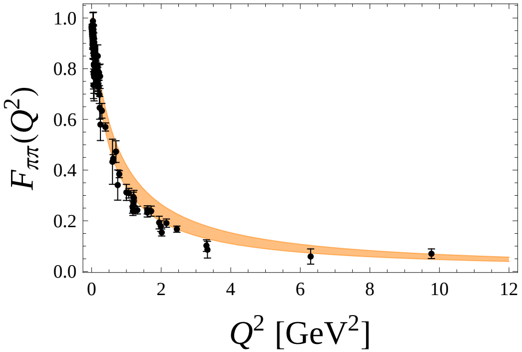
Example: electromagnetic form factor of a (pseudo) scalar particle

$$\langle h(p)|J^\mu(0)|h(p+q)\rangle = (2p^\mu + q^\mu)F(q^2)$$

conserved: $\partial_\mu J^\mu = 0 \to$ Ward-Takahashi identities

$$\to q_\mu(2p^\mu + q^\mu) = (p+q)^2 - p^2 = m^2 - m^2 = 0$$

Pion EM form factor



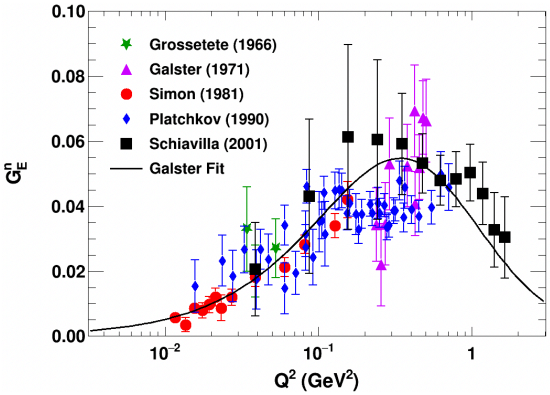
$$F(Q^2) = \frac{m_\rho^2}{1 + Q^2/m_\rho^2}$$

Vector meson dominance model fits the data well

Baryon in the pion?

Neutron charge ff

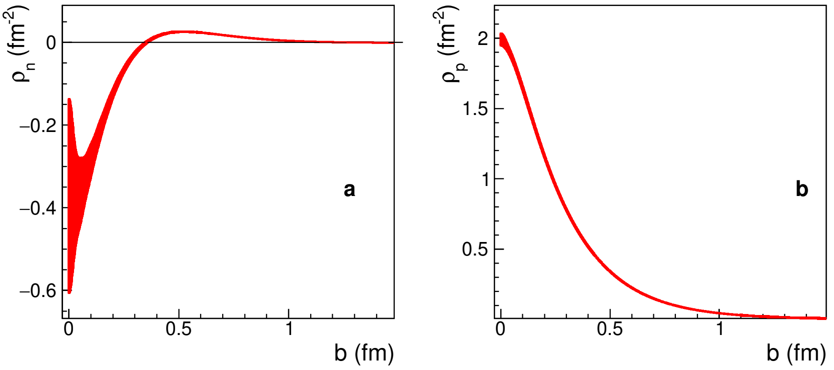
Recall that the neutron, which has no electric charge, has a non zero ff for $q^2 \neq 0$:



unpolarized elastic ed scat.

[Obrecht 2019]

Neutron charge radius

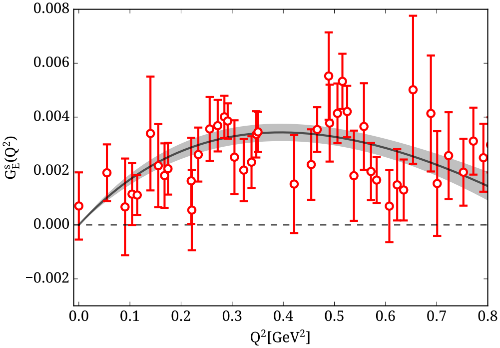


[Atac et al. 2021]

Strangeness in the nucleon

Another example: strange ff's of the nucleon, $G^s_{E,M}$

[Jaffe 1989, Musolf, Burkardt 1993, Forkel, Cohen, Forkel, Nielsen 1993,...]



Alexandrou et. al (ETM Coll.) 2020

Symmetries and the baryon ff of the pion

Symmetries

Divergence of vector currents in QCD

$$\partial_\mu \left[\bar{q}_a(x)\gamma^\mu q_b(x)\right] = i(m_a - m_b)\bar{q}_a(x)q_b(x), \qquad a,b = u,d,s,c,b,t \quad -\text{flavor}$$

$m_a = m_b \to$ conservation of vector currents, quark number of any species conserved

Gell-Mann–Nishijima formula

$$Q = I_3 + \tfrac{1}{2}(B + s + c + b + t)$$

For the pion heavier flavors can be neglected (OZI, large-N_c):

$$J_B^\mu = \frac{1}{N_c}\left(\bar{u}\gamma^\mu u + \bar{d}\gamma^\mu d\right), \quad J_3^\mu = \frac{1}{2}\left(\bar{u}\gamma^\mu u - \bar{d}\gamma^\mu d\right), \quad J_Q^\mu = J_3^\mu + \frac{1}{2}J_B^\mu \quad \text{(all conserved)}$$

Symmetries 2

Baryon, isospin, and charge form factors

$$\langle \pi^a(p) \mid J^\mu_{B,3,Q}(0) \mid \pi^a(p+q)\rangle = (2p^\mu + q^\mu) F^a_{B,3,Q}(q^2), \quad a = 0, +, - \quad \text{(pion isospin)}$$

π^0: $I^G(J^{PC}) = 1^-(0^{-+})$, $\pi^\pm$: $I^G(J^P) = 1^-(0^-)$, $C|\pi^\pm\rangle = |\pi^\mp\rangle$, $G = Ce^{i\pi I_2}$

$J^\mu_{B,3,Q}$ are **odd** under $C \rightarrow$

$F^{\pi^0}_{B,3,Q}(q^2) = 0$ and $F^{\pi^+}_{B,3,Q}(q^2) = -F^{\pi^-}_{B,3,Q}(q^2)$ – always true!

e.g., $\langle\pi^0(p)|J^\mu_B(0)|\pi^0(p+q)\rangle = -\langle\pi^0(p)|CJ^\mu_B(0)C|\pi^0(p+q)\rangle = -\langle\pi^0(p)|J^\mu_B(0)|\pi^0(p+q)\rangle = 0$

or $\langle\pi^+(p)|J^\mu_B(0)|\pi^+(p+q)\rangle = -\langle\pi^+(p)|CJ^\mu_B(0)C|\pi^+(p+q)\rangle = -\langle\pi^-(p)|J^\mu_B(0)|\pi^-(p+q)\rangle$

Symmetries 3

Similarly, for **exact** isospin (and G) symmetry (assuming $m_u = m_d$ and neglecting small EM effects)

J_B^μ is **odd** under $G \rightarrow$

$F_B^{\pi^\pm}(q^2) = 0 \qquad (F_3^{\pi^\pm}(q^2) \neq 0$, as J_3^μ is even under G)

However, in the real world the isospin (and G) are broken (a.k.a. charge symmetry breaking) with $m_d > m_u$ and EM

$F_B^{\pi^\pm}(q^2)$ may be (and **is**) nonzero, with $F_B^{\pi^+}(q^2) = -F_B^{\pi^-}(q^2)$

As the baryon charge of the pion is 0, we have

$F_B^{\pi^\pm}(0) = 0 \qquad$ (but not at $q^2 \neq 0$)

On the other hand, $F_3^{\pi^\pm}(0) = \pm 1$ (the 3-component of isospin)

Symmetries 4

- As a rule, if a quantity is not protected by symmetry, hence may be nonzero, it **is nonzero**
- There is the question of magnitude, proportional to the strength of the symmetry breaking
- No probes with baryon number couple directly to the pion (except for lattice QCD) $\to$ we need indirect methods to estimate the effect

Mass splitting

$\Delta m \equiv m_d - m_u = 2.8(2)$MeV ($m_u = 2.01(14)$MeV, $m_d = 4.79(16)$MeV [Davies et al. 2009])

- EM violating effects more tricky to estimate/evaluate, of the order $\alpha_{\mathrm{QED}}/(2\pi) \sim 0.001$

Effective Lagrangian estimate

Order of magnitude from effective Lagrangian

At leading order in the pion momenta and the quark mass splitting

$$J_B^\mu = -2i\frac{c\Delta m}{\Lambda^3}\partial_\nu\left(\partial^\mu\pi^+\partial^\nu\pi^- - \partial^\nu\pi^+\partial^\mu\pi^-\right) + \ldots$$

c – dimensionless number, Λ – typical hadronic scale

J_B^μ is odd under C, trivially conserved, and yields $F_B^{\pi^+}(q^2) = q^2 c\Delta m/\Lambda^3 + \ldots$

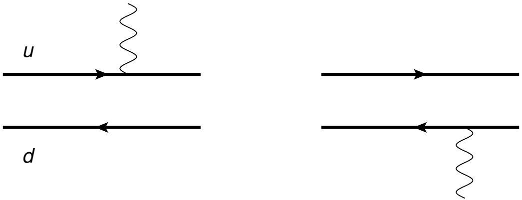
Baryonic ms radius

$\langle r^2\rangle_B^{\pi^+} = 6c\Delta m/m_\rho^3 \simeq c\ 0.002\mathrm{fm}^2 \simeq c(0.04\mathrm{fm})^2$

– small compared to the charge radius $\langle r^2\rangle_Q^{\pi^+} = 0.434(5)\mathrm{fm}^2 = (0.659(4)\mathrm{fm})^2$

Quark-model estimates

Yukawa model (impulse approximation)



$\rho_3(r) = \frac{1}{2}|\Psi_u(\vec{x})|^2 + \frac{1}{2}|\Psi_{\bar{d}}(\vec{x})|^2, \quad \rho_B(r) = \frac{1}{3}|\Psi_u(\vec{x})|^2 - \frac{1}{3}|\Psi_{\bar{d}}(\vec{x})|^2, \quad |\Psi_i(\vec{x})|^2 = \frac{M_i^2}{\pi r} e^{-2M_i r}$

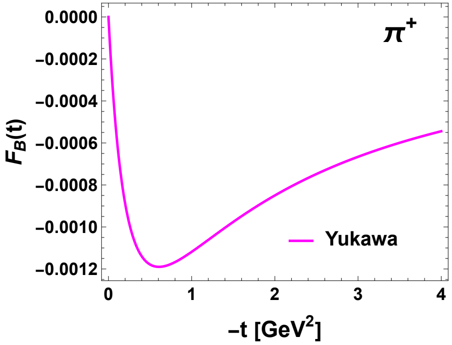
$M_{u,d} = M \mp \frac{1}{2}\Delta m$, $M \simeq \frac{1}{2} m_\rho$ – constituent quark masses

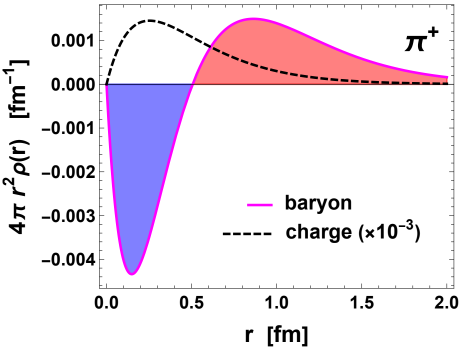
$\Delta m = 0 \to F_3^{\pi^+} = \frac{1}{1+Q^2/m_\rho^2}$ (vector meson dominance)

Baryon ff

$$F_B^{\pi^+}(-Q^2) = \frac{1}{N_c}\left[\frac{4M_u^2}{4M_u^2+Q^2} - \frac{4M_d^2}{4M_d^2+Q^2}\right] \simeq -\frac{4\Delta m\, m_\rho Q^2}{3(m_\rho^2+Q^2)^2}$$

Yukawa model 2



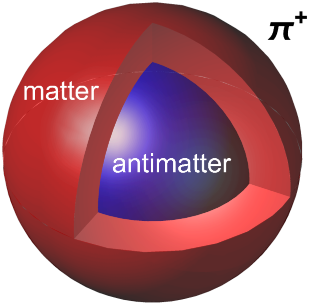


$$\langle r^2 \rangle_B^{\pi^+} \simeq \frac{8\Delta m}{m_\rho^3} \simeq (0.04\ \mathrm{fm})^2$$

Mechanistic explanation

$\bar{d}$ is a bit heavier than u, hence its distribution is somewhat more compact.

$\pi^+ = u\bar{d}$, u - baryon charge (matter), $\bar{d}$ - antibaryon charge (antimatter)



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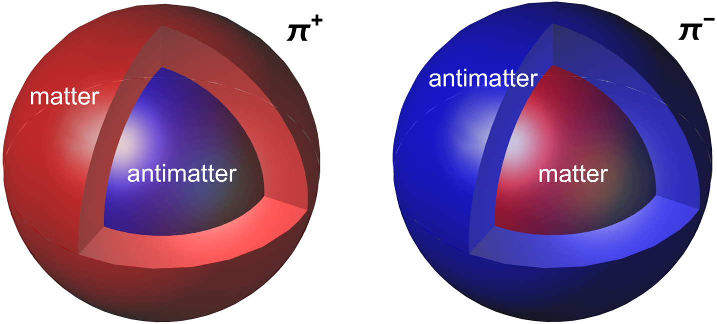
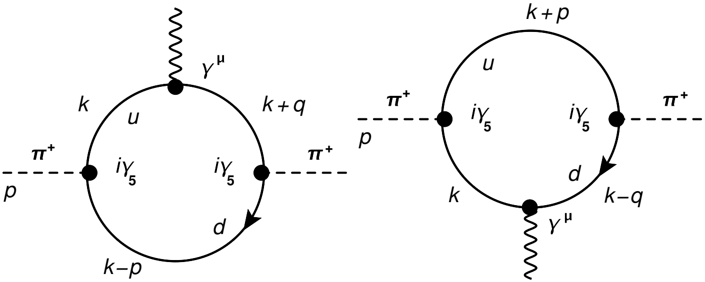


Nambu–Jona-Lasinio (NJL) model

Covariant field-theoretic model. Dynamical chiral symmetry breaking, point-like interaction, large-N_c (one-loop), regularization. Generally very successful in pion low-energy phenomenology

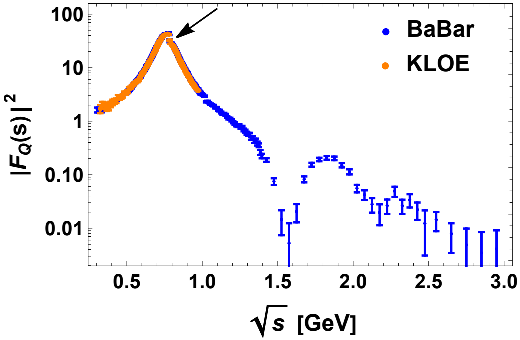


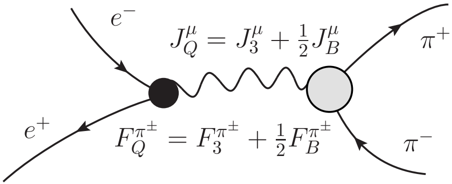
NJL: $\langle r^2 \rangle_B^{\pi^+} \simeq (0.06 \text{ fm})^2$

Determination from exp. data (!)

$e^+e^- \to \pi^+\pi^-$

Long tradition of $e^+e^- \to \pi^+\pi^-$ measurements





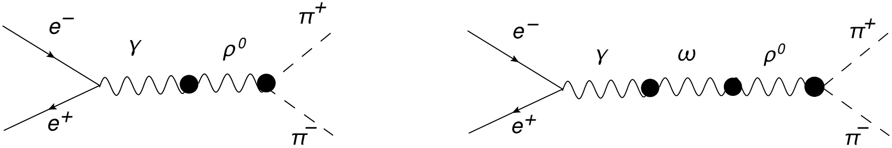
Arrow indicates a wiggle due to $F_B^{\pi^\pm} \neq 0$!

(relevant for hadronic vacuum polarization in $g-2$)

Vector meson dominance

isospin part

baryonic part



$\rho_0 - \omega$ mixing

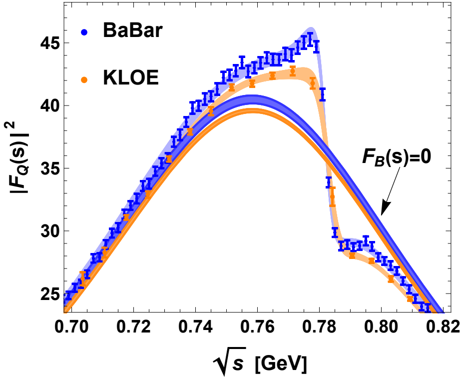
$$F_3^{\pi^+}(s) = \frac{1}{1+c'+c''+c'''}[D_{\rho^0}(s) + c' D_{\rho'^0}(s) + c'' D_{\rho''^0}(s) + c''' D_{\rho'''^0}(s)]$$

$$\frac{1}{2}F_B^{\pi^+}(s) = c_{\rho^0\omega}\, s\, D_{\rho^0}(s) D_\omega(s), \qquad D_V(s) = \frac{m_V^2}{m_V^2 - s - i\, m_V \Gamma_V(s)}$$

[Gounaris-Sakurai 1968, largely used by exp. groups]

Our fit to KLOE and BaBar

... shown in the relevant range of s

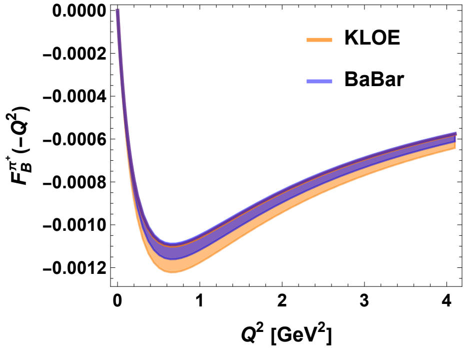


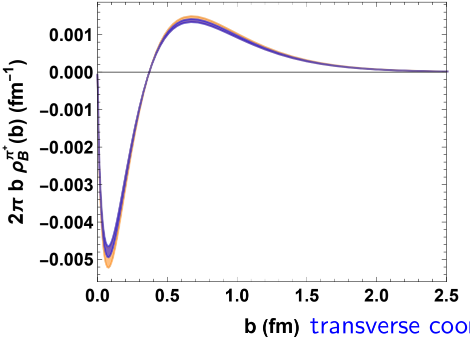
Necessity of $F_B^{\pi^\pm} \neq 0$ (or $\rho - \omega$ mixing)

Unresolved disrepancy between KLOE and BaBar!

Continuation space-like Q^2 with the dispersion relation

$$F_B^{\pi^\pm}(-Q^2) = \frac{1}{\pi}\int_{4m_{\pi^+}^2}^{\infty} ds \frac{\mathrm{Im} F_B^{\pi^\pm}(s)}{s+Q^2} = -\frac{Q^2}{\pi}\int_{4m_{\pi^+}^2}^{\infty} ds \frac{\mathrm{Im} F_B^{\pi^\pm}(s)}{s(s+Q^2)}$$





transverse coordinate

$e^+e^- \to \pi^+\pi^-$:

BaBar: $\langle r^2 \rangle_B^{\pi^+} \simeq (0.0411(7)\ \mathrm{fm})^2$, KLOE: $\langle r^2 \rangle_B^{\pi^+} \simeq (0.0412(12)\ \mathrm{fm})^2$ (stat. errors only)

Comparison of our various estimates

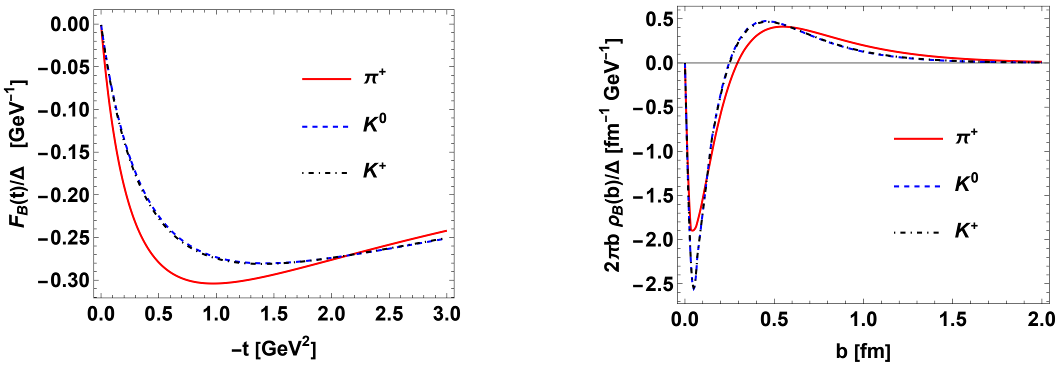
approach	$\langle r^2 \rangle_B^{\pi^+}$	comment
effective Lagrangian	$c(0.04\ \mathrm{fm})^2$	c - number of order 1
toy Yukawa model	$(0.04\ \mathrm{fm})^2$	
NJL	$(0.06\ \mathrm{fm})^2$	
BaBar	$(0.041(1)\ \mathrm{fm})^2$	exp. statistical error only
KLOE	$(0.041(1)\ \mathrm{fm})^2$	

- Agreement between very different methods
- BaBar and KLOE extractions incorporate both Δm and EM breaking (but EM canceled from the initial and final state interactions)

Baryon in the kaon

Kaon in NJL

Full analogy to π^+: for $K^+ = u\bar{s}$ replace $d \to s$, for $K^0 = d\bar{s}$ replace $u \to d$ and $d \to s$
NJL: $m_s/m = 26$ (fits m_K), PDG: $m_s/m = 27.3^{+0.7}_{-1.3}$



(for π^+, K^0, $K+$, correspondingly, $\Delta = M_d - M_u$, $\Delta = M_s - M_d$, $\Delta = M_s - M_u$)

Kaon baryonic radius

NJL:

$$\langle r^2 \rangle_B^{K^+} = (0.24(1)\ \mathrm{fm})^2, \quad \langle r^2 \rangle_B^{K^0} = (0.23(1)\ \mathrm{fm})^2$$

In NJL, $\langle r^2 \rangle_B^{K^0} = -\langle r^2 \rangle_Q^{K^0}$, since the baryon number and electric charge of d and $\bar{s}$ quarks are equal and opposite

PDG:

$\langle r^2 \rangle_Q^{K^0} = -(0.28(2)\ \mathrm{fm})^2$, of the same sign and close in magnitude to NJL

Within the reach of the lattice

Conclusions

Outlook

1. Fundamental feature of the pion, eventually should end up in the PDG Tables
2. Small, but as shown, possible to extract from the present experimental data – could be elevated to strict determination after some experimental and theoretical systematic issues are resolved
3. Estimates from very different approaches yield $\langle r^2 \rangle_B^{\pi^+} = (0.03 - 0.06 \text{ fm})^2$, the sign agrees with the mechanistic interpretation
4. Lattice QCD: $\langle r^2 \rangle_Q^{\pi} = (0.648(15) \text{ fm})^2 = 0.42(2) \text{ fm}^2$ – our signal for the baryon ff is a factor of ~ 10 too small (0.002 vs the accuracy of 0.02) to be currently detected (but still could be tried)
5. Good lattice prospects for the kaon or heavy-light mesons

THANKS FOR YOUR ATTENTION!