Approaching BaBar with chiral quarks and Regge models

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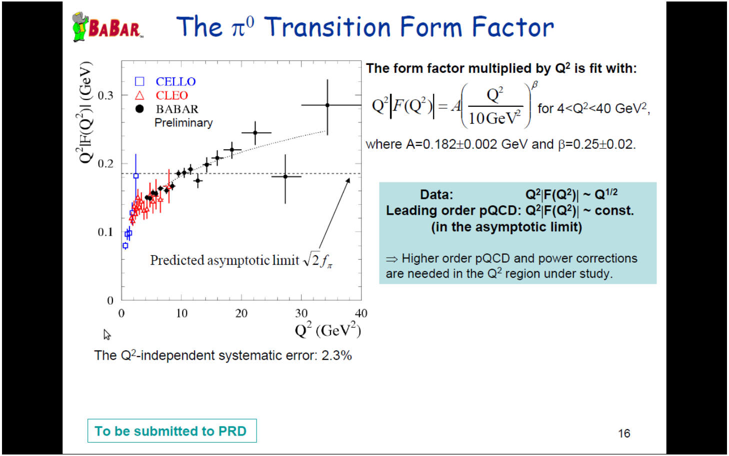
More details in

- *Pion transition form factor in the Regge approach and incomplete vector-meson dominance* E. Ruiz Arriola and WB, Phys. Rev. D81 (2010) 094021
- *Gravitational and higher-order form factors of the pion in chiral quark models*, WB, ERA, Phys. Rev. D78 (2008) 094011
- *Pion electromagnetic form factor, perturbative QCD, and large-N_c Regge models*, ERA, WB, Phys. Rev. D78 (2008) 034031
- *Generalized parton distributions of the pion in chiral quark models and their QCD evolution*, WB , ERA, K. Golec-Biernat, Phys. Rev. D77 (2008) 034023

The BaBar shock



[from Selina Li @ Photon 2009, page 16 (!)]

Why shocking?

Contradicts expectations based on

- factorization
- pQCD evolution (done twist-by-twist)

Brodsky-Lepage:

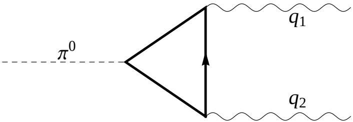
$$Q^2 F_{\pi^0\gamma\gamma^*}(Q^2) \to \frac{2f_\pi}{N_c}\int_0^1 dx \frac{\phi_{\rm as}(x)}{x} = \frac{6f_\pi}{N_c} = 2f_\pi$$

Numerous attempts by [Radyushkin, Polyakov, Dorokhov, Noguera, Vento, Mikhailov, Stefanis, Bakulev, Kotko, Praszalowicz, Kochelev, Diehl, Kroll, Chernyak, Khodjamirian, Li, Nishima, ...]

Outline

1. General constraints: anomaly, Terazawa-West bounds, rare Z decays
2. Can TW bounds be violated? - subtracted dispersion relations
3. Chiral quark models [see Arriola, Polyakov, Dorokhov]
4. Regge models
5. Conclusions

Anomaly



general kinematics:

$$q_1^2 = -\frac{1+A}{2}Q^2,\ q_2^2 = -\frac{1-A}{2}Q^2,\quad -1 \le A \le 1$$

$$F_{\pi^0\gamma\gamma^*}(Q^2 = 0, A) = \frac{1}{4\pi^2 f_\pi}$$

Terazawa-West unitarity bounds

[Terazawa 1972, West 1973, recalled by Dorokhov 2009]

Schwarz inequality involving sums of $\langle 0|J_\mu(0)|n\rangle$ and $\langle \pi^a(q)|J_\mu(0)|n\rangle$
$|\langle\pi|JJ|0\rangle| \leq (|\langle 0|JJ|0\rangle||\langle\pi|JJ|\pi\rangle|)^{1/2}$

$$\mathrm{Im}F_{\pi^0\gamma\gamma^*}(q^2) = \mathcal{O}(1/\sqrt{q^2}) \quad \text{(TW I)}$$

for *time-like* momenta, $q^2 > 4m_\pi^2$
If there are no polynomial terms in the real part of $F_{\pi^0\gamma\gamma^*}$, then

$$\left|F_{\pi^0\gamma\gamma^*}(q^2)\right| = \mathcal{O}(1/\sqrt{q^2})$$

Dispersion relation yields

$$\left|F_{\pi^0\gamma\gamma^*}(Q^2)\right| = \mathcal{O}(1/Q) \quad \text{(TW II)}$$

for all momenta, also large *space-like* momenta Q

The constant in the bound may be given [Terazawa 1973] in terms of the photon spectral density and the pion structure function,

$$\left|F_{\pi^0\gamma\gamma^*}(Q^2)\right| < \frac{2\sqrt{\Pi(\infty)}}{Q}\int_0^1 dx \sqrt{\frac{F_1(x,Q^2)}{x(1-x)}}$$

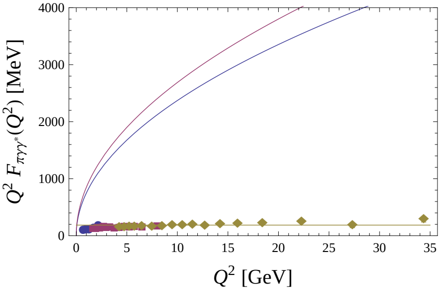
where

$$\Pi(s) = \frac{s}{16\pi^3\alpha_{\rm QED}^2}\sigma_{e^+e^-\to\rm hadrons}(s), \quad \Pi(\infty) = \frac{1}{12\pi^2}\sum_i e_i^2$$

With the SMRS and GRV parameterizations for F_1 we obtain (for Q^2 in the range $10-40\ \mathrm{GeV}^2$)

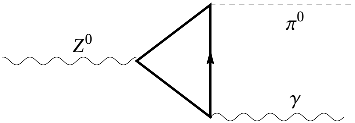
$$\begin{aligned}|F_{\pi^0\gamma\gamma^*}(Q^2)| &< \frac{0.85(1)}{Q} \quad (\mathrm{LO}) \\ &< \frac{0.75(1)}{Q} \quad (\mathrm{NLO})\end{aligned}$$

TW II vs. BaBar



The TW II bound is "inefficient", an order of magnitude above the BaBar data

$Z^0 \to \pi^0\gamma$ decay



Only the vector coupling of the Z^0 boson to the quarks contributes [Jacob, Wu 1989], hence

$$\frac{F_{Z\to\pi^0\gamma}(q^2)}{F_{Z\to\pi^0\gamma}(0)} = \frac{F_{\pi^0\gamma^*\gamma}(q^2)}{F_{\pi_0\gamma^*\gamma}(0)}.$$

The experimental limit
$\Gamma(Z^0 \to \pi^0\gamma) < 5\times 10^{-5}\Gamma_{\rm tot}(Z^0) = 10.25\times 10^{-5}{\rm GeV}$ implies

$$|F_{Z^0\to\pi^0\gamma}(M_Z^2)/F_{Z^0\to\pi^0\gamma}(0)| < 0.17$$

Subtracted dispersion relations

The assumption of the absence of the polynomial terms is equivalent to validity of the unsubtracted dispersion relation
pQCD with factorization $\rightarrow$ $F_{\pi^0\gamma^*\gamma}$ vanishes as $Q \rightarrow \infty$

Below we consider cases where this is not be the case.

Even if the form factor vanishes at infinity, we can write a subtracted relation

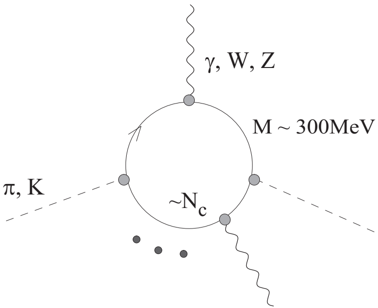
$$F(t) - F(0) = \frac{1}{\pi}\int_{s_0}^{4\Lambda^2} \frac{t}{s}\frac{\mathrm{Im}F(s)}{s-t}ds + \frac{1}{\pi}\int_{4\Lambda^2}^{\infty} \frac{t}{s}\frac{\mathrm{Im}F(s)}{s-t}ds$$

If Λ is large ($\Lambda^2 > Q^2$), the second term is very slowly varying with Q^2 and mimics a constant. In particular, for $\mathrm{Im}F(s) \sim 1/\sqrt{s}$ it behaves as $1/\Lambda + o(1/Q)$

Illustration in chiral quark models

Basic idea of chiral quark models

- illustration how TW II may be violated
- Re and Im parts of the ff can be computed



- covariant Lagrangian-form calculation, no factorization
- soft regime → chiral symmetry breaking
- NJL, instanton-motivated [Dorokhov]
- relatively few parameters (traded for f_π, m_π, ...)
- numerous processes with pions, γ, ...
- no confinement - careful not to open the $q\bar{q}$ threshold
- quark model scale low - need for QCD evolution if higher scales are involved

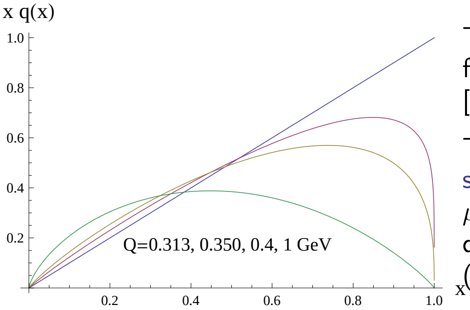
Glossary of results showing that the approach is reasonable for computing soft matrix elements

Parton Distribution Function of the pion

NJL gives [Davidson, Arriola, 1995]

$$q(x) = 1$$

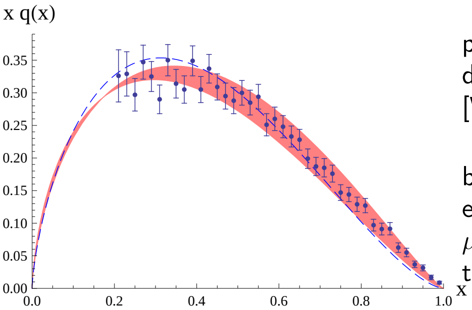
LO DGLAP QCD evolution (good at intermediate x) of the non-singlet part to growing scales



The same constant PDF of the pion follows from AdS/CFT [Brodsky, Teramond 2008]

The question of renormalization scale: momentum sum-rule → $\mu_0 \sim 320$ MeV → at 2 GeV valence quarks carry 47% of the momentum (Durham), $\alpha(\mu_0)/\pi = 0.68$

Valence PDF from NJL vs. E615

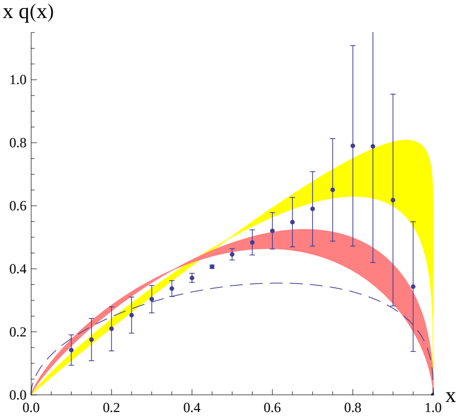


points: Drell-Yan from E615
dashed: reanalysis of the data [Wijesooriya et al., 2005]

band: valence PDF from NJL evolved from the QM scale $\mu_0 = 313^{+20}_{-10}$ MeV to $\mu = 2$ GeV of the experiment

Valence PDF from NJL vs. transverse lattice

transverse lattices: [Burkardt, Dalley, Van de Sande]



points: transverse lattice [Dalley, Van de Sande, 2003]
yellow: NJL evolved to $\mu = 0.35$ GeV
pink: NJL evolved to $\mu = 0.5$ GeV
dashed: GRS parametrization at $\mu = 0.5$ GeV

Pion Distribution Amplitude

$$\langle 0|\overline{q}(z)\gamma_\mu\gamma_5[z,-z]q(-z)|\pi^0(q)\rangle = i\sqrt{2}f_\pi(q^2)q_\mu\int_0^1 dx e^{i(2x-1)q\cdot z}\phi(x)$$

($z^2 = 0$, in the light-cone gauge $[z,-z] = 1$)

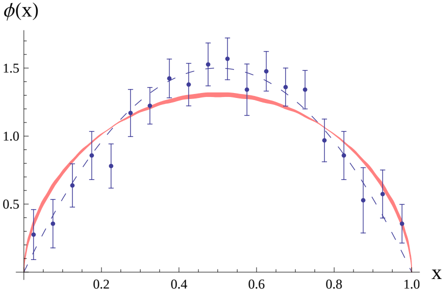
NJL [ERA, WB 2003)]: $\phi(x) = 1 \quad \text{(at QM scale)}$

(different from AdS/CFT: $\sim \frac{8}{\pi}\sqrt{x(1-x)}$ or $6x(1-x)$)

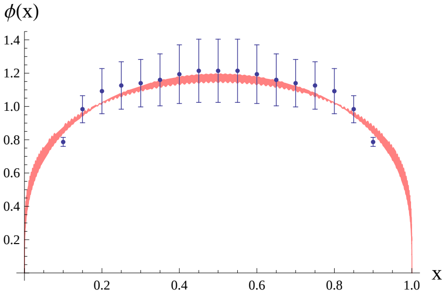
Nonvanishing at the end points [see Polyakov's talk for implications]
LO ERBL evolution makes $\phi(x)$ vanish at the end-points [WB, ERA, Golec-Biernat, 2008]

$$\phi(x) \sim x^{2C_F/\beta_0 \, \log[\alpha(\mu_0)/\alpha(\mu)]}$$

PDA from NJL vs. E791 and lattice data

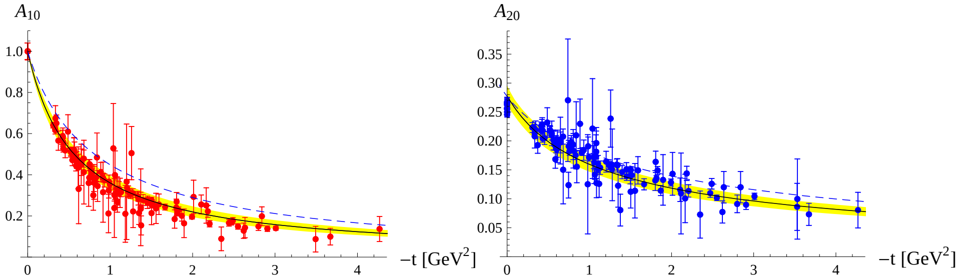


points: E791 data from di-jet production in $\pi + A$
band: NJL evolved to $\mu = 2$ GeV
dashed line: asymptotic form ($\mu \to \infty$)



points: transverse lattice data [Dalley, Van de Sande, 2003]
band: NJL evolved to $\mu = 0.5$ GeV

NJL vs. full-QCD Euclidean lattice



Pion charge ff (left) and the quark part of the spin-2 gravitational ff (right) in SQM (solid line) and NJL (dashed line) [WB, ERA 2008], compared to the data [Brömmel et al., 2005-7]

Quark-model relation: $\langle r^2 \rangle_\Theta = \frac{1}{2} \langle r^2 \rangle_V$
Matter more concentrated than charge!
(also found in soft-wall AdS/CFT [talks by Brodsky and Teramond])

Violating the TW II bound

Georgi-Manohar model as a TW II-violating model

$$L = \bar{q}\left(i\partial\!\!\!/ + g_A^Q A\!\!\!/\gamma_5 - M\right)q + \frac{f^2}{4}\mathrm{Tr}\left(\partial_\mu U^\dagger \partial^\mu U\right) + \mathrm{WZW}$$

$$A_\mu = \frac{i}{2}(u^\dagger \partial_\mu u - u\partial_\mu u^\dagger), \quad u = e^{i\vec{\pi}\cdot\vec{\tau}/(2f)}, \quad U = u^2$$

$$F_{\pi^0\gamma\gamma^*}(Q^2) = \frac{1}{4\pi^2 f_\pi} + \frac{g_A^Q}{4\pi^2 f_\pi}\left[G(Q^2) - 1\right]$$

$$G(Q^2) = \frac{2M^2}{Q^2}\int_0^1 \frac{dx}{x}\log\left[1 + x(1-x)\frac{Q^2}{M^2}\right]$$

Anomaly satisfied, but for $g_A^Q \neq 1$ no vanishing at $Q^2 \to \infty$:

$$F_{\pi^0\gamma\gamma^*}(Q^2) = \frac{1 - g_A^Q}{4\pi^2 f_\pi} + \frac{g_A^Q M^2}{4\pi^2 f_\pi}\frac{\left[\log(Q^2/M^2)\right]^2}{Q^2} + \dots$$

Fulfills dispersion relation and TW I but not TW II.

Spectral Quark Model:

$$G(Q^2) = \frac{1}{3}\left[\frac{2m_\rho^2}{m_\rho^2 + Q^2} + \frac{m_\rho^2}{Q^2}\log\left(\frac{m_\rho^2 + Q^2}{m_\rho}\right)\right]$$

With $g_A^Q = 1$ this model fulfills the result of [Radyushkin 2009] with a similar mass scale, m_ρ:

$$F_{\pi^0\gamma\gamma^*}(Q^2) = \frac{1 - g_A^Q}{4\pi^2 f_\pi} + \frac{g_A^Q m_\rho^2}{12\pi^2 f_\pi}\frac{[\log(Q^2/m_\rho^2)]}{Q^2} + \dots$$

No factorization within chiral quark models →

NJL: $Q^2 F_{\pi^0\gamma\gamma^*}(Q) \sim (\log(Q^2/\mu^2))^2$,
Spectral Quark Model: $Q^2 F_{\pi^0\gamma\gamma^*}(Q) \sim \log(Q^2/\mu^2)$

Precise fits in the whole Q^2 range tricky
[see talks by Polyakov and Dorokhov]

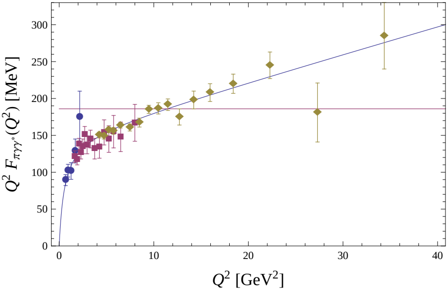
Regge models incorporate large N_c and confinement

Incomplete VMD in a one-state model

Subtracted dispersion relation for $F_{\pi^0\gamma\gamma^*}$ saturated with one resonance:

$$F_{\pi^0\gamma\gamma^*}(-Q^2) = \frac{1}{4\pi^2 f_\pi}\left[1 - c\frac{Q^2}{M_V^2 + Q^2}\right]$$

CLEO only: $c = 0.998(18)$, $M_V = 777(44)$ MeV, $\chi^2/\mathrm{DOF} = 0.54$
all: $c = 0.986(2)$, $M_V = 748(14)$ MeV, $\chi^2/\mathrm{DOF} = 0.7.$



(CLEO and BaBar data statistically compatible)

TW II violated

b_π

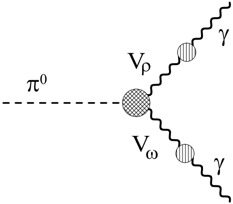
radius squared:

$$b_\pi = -\left[\frac{1}{F_{\pi^0\gamma\gamma^*}(Q)}\frac{d}{dQ^2}F_{\pi^0\gamma\gamma^*}(Q)\right]\Big|_{Q^2=0}$$

Our fit: $b_\pi = \frac{c}{M_V^2} = 1.76(7)\ \mathrm{GeV}^{-2}$
PDG: $b_\pi = (1.76 \pm 0.22)\mathrm{GeV}^{-2}$
CELLO: $b_\pi = (1.4 \pm 0.8 \pm 1.4)\mathrm{GeV}^{-2}$ [Meijer et al. 1992].

$$|F_{Z\to\pi^0\gamma}(M_Z^2)/F_{Z\to\pi^0\gamma}(0)| = 0.014(2) \ll 0.17$$

Radial Regge models



Large-N_c QCD involves tree-level diagrams with infinitely many states, including the radial excitations

(soft-wall AdS/CFT $\rightarrow$ Regge phenomenology)

Radial trajectory: $M_n^2 = M_V^2 + an$ - need to model residues/coupling constants

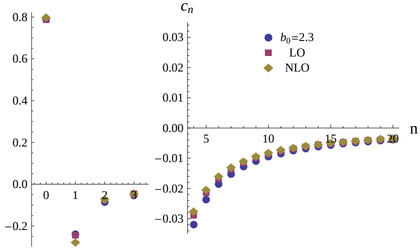
First the charge form factor

Veneziano-Dominguez models

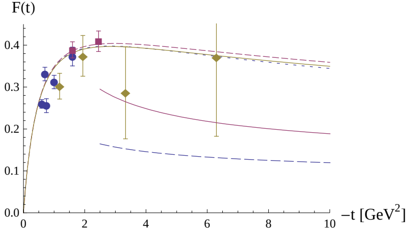
$$f_b(t) = \frac{1}{B(b-1, \frac{M_V^2}{a})} \sum_{n=0}^{\infty} \frac{\Gamma(2-b+n)}{\Gamma(n+1)\Gamma(2-b)} \frac{1}{M_n^2 - t}$$

$$f_b(0) = 1, \quad f_b(t=-Q^2) \sim (Q^2)^{1-b}$$

(approach used previously to describe the pion charge ff [ERA, WB 2008]



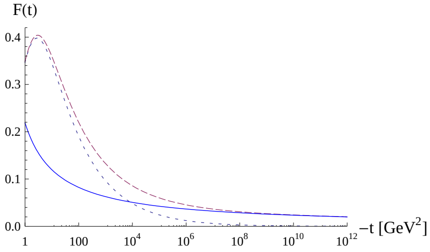
Residues in the pion charge ff



$F(t) \equiv -tF_V(t)$, NLO (solid), LO (dashed), TJLAB (circles and squares), Cornell (diamonds), lower curves: NLO (solid) and (LO) pQCD

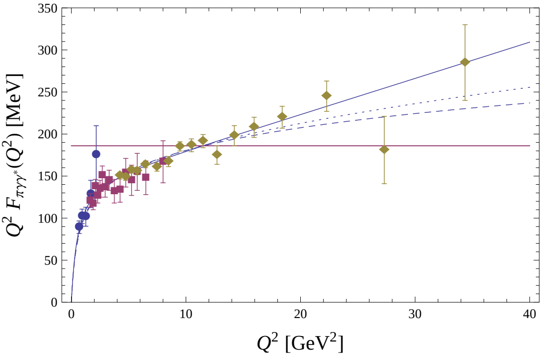
Reaching pQCD in the charge ff

The pQCD result is reached at very high scales



$F(t) \equiv -tF_V(t)$, solid - asymptotic LO pQCD, dashed - Regge model.

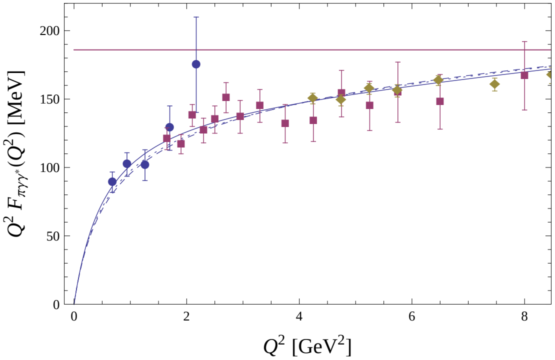
Regge fits to the transition ff



dashed line: Veneziano-Dominguez model $b = 1.81$ (TW II satisfied)
dotted line: first pole separated and fixed $b = 1.5$ (TW II satisfied)

solid line: single-state subtracted model (TW II violated, works best!)

Regge fits to the transition ff - low Q^2



at low Q^2 all models compatible

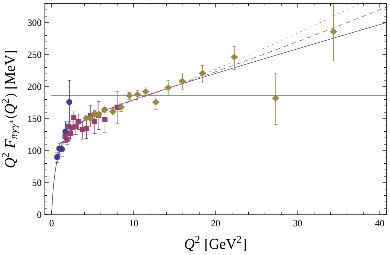
Dependence on momentum asymmetry

BaBar kinematic setup $-q_1^2 < 0.6\ \mathrm{GeV}^2$ and $-q_2^2 > 3\ \mathrm{GeV}^2$ $\rightarrow$

$$|A| = \left| \frac{q_1^2 - q_2^2}{q_1^2 + q_2^2} \right| \sim 0.9 - 0.97 \neq 1$$

Significant for precision fits
IMVD Regge model becomes

$$F_{\pi^0\gamma\gamma^*}(-Q^2) = \frac{1}{4\pi^2 f_\pi}\left[1 - c\left(1 - \frac{4M_V^4}{4M_V^4 + 4M_V^2 Q^2 + (1-A^2)Q^4}\right)\right]$$



$A = 1$ (solid), 0.975 (dashed), and 0.95 (dotted) yield, respectively,
$c = 0.986, 0.978, 0.974$,
$M_V = 748, 754, 768$ MeV.

Summary

- TW I always satisfied, TW II not necessarily (e.g. GM model) - subtractions possible
- TW bounds, estimated with phenomenological parametrization of the pion PDF, extend an order of magnitude above the BaBar data (ineffective)
- Constraint in the time-like region from the experimental bounds on the rare $Z \to \pi^0\gamma$ decay is comfortably satisfied in our models
- Incomplete Vector-Meson Dominance with a single state reproduces the data in the whole experimental range, $0 < Q^2 < 35\ \mathrm{GeV}^2$.
- Within the Regge approach with infinitely many states (cf. soft-wall AdS/CFT) the data can be fitted with or without a subtraction constant
- The model fits are sensitive to the photon momentum asymmetry parameter A, affecting, e.g., the fitted vector meson mass or other parameters