Hollowness in pp scattering

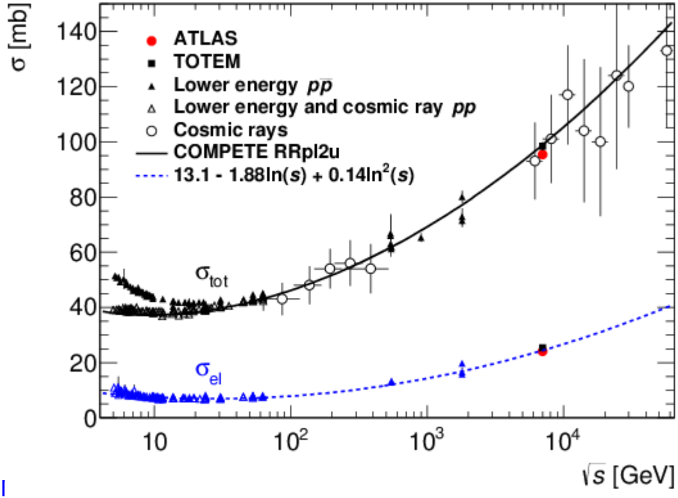
Wojciech Broniowski

Institute of Nuclear Physics PAN
Jan Kochanowski U., Kielce

UW, 27 February 2017

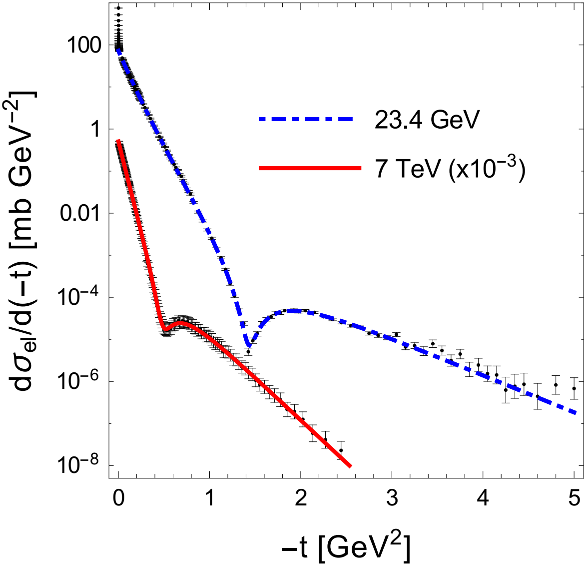
Research with **Enrique Ruiz Arriola** [arXiv:1609.05597]

pp and $p\bar{p}$ cross sections



Surprises!

Elastic scattering from ISR to LHC



Spin-averaged elastic pp scattering amplitude

Parametrization by [Fagundes 2013] based on [Barger-Phillips 1974], motivated by the Regge asymptotics:

$$\frac{|f(s,t)|}{p} = \left| \sum_n c_n(s) F_n(t) s^{\alpha_n(t)} \right| = \left| \frac{i\sqrt{A} e^{\frac{Bt}{2}}}{\left(1 - \frac{t}{t_0}\right)^4} + i\sqrt{C} e^{\frac{Dt}{2} + i\phi} \right|$$

s-dependent (real) parameters are fitted (separately) to all known differential pp cross sections for $\sqrt{s} = 23.4,\ 30.5,\ 44.6,\ 52.8,\ 62.0$, and $7000\ \mathrm{GeV}$ with $\chi^2/\mathrm{d.o.f} \sim 1.2 - 1.7$

$$\frac{d\sigma_{\mathrm{el}}(s,t)}{dt} = \frac{\pi}{p^2} |f(s,t)|^2$$

ρ parameter

$$\rho(s,t) = \frac{\mathrm{Re} f(s,t)}{\mathrm{Im} f(s,t)}$$

$$f(s,t) = \frac{i + \rho(s,t)}{\sqrt{1+\rho(s,t)^2}} |f(s,t)|$$

From the optical theorem

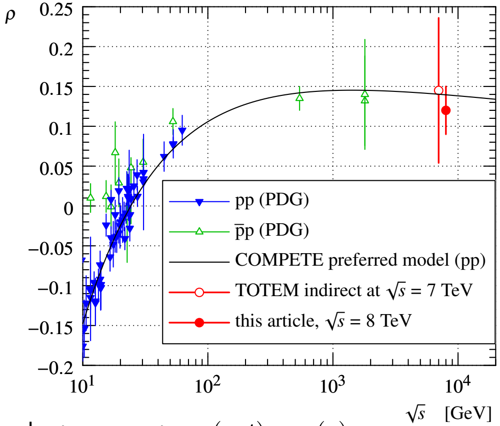
$$\sigma_{\mathrm{tot}}(s) = \frac{4\pi}{p} \mathrm{Im} f(s,0) = \frac{4\sqrt{\pi d\sigma_{\mathrm{el}}/dt|_{t=0}}}{\sqrt{1+\rho(s,0)^2}}$$

or

$$\rho(s)^2 \equiv \rho(s,0)^2 = \frac{16\pi \frac{d\sigma_{\mathrm{el}}(s,t)}{dt}|_{t=0}}{\sigma_{\mathrm{tot}}(s)^2} - 1$$

Up to a sign $\rho(s,0)$ determined from the measured cross sections (sign may be determined from the interference with the Coulomb amplitude)

ρ parameter from the experiment



We take a t-independent parameter $\rho(s,t) = \rho(s)$
Results similar for the Bailly et al. parametrization

$$\rho(s,t) = \frac{\rho(s)}{1 - t/t_0(s)}$$

$t_0(s)$ – position of the diffractive minimum

How well it works?

$\sqrt{s}$ [GeV]	$\sigma_{\rm el}$ [mb]	$\sigma_{\rm in}$ [mb]	$\sigma_{\rm T}$ [mb]	B [GeV^{-2}]	ρ
23.4	6.6	31.2	37.7	11.6	0.00
exp.	6.7(1)	32.2(1)	38.9(2)	11.8(3)	0.02(5)
200	10.0	40.9	50.9	14.4	0.13
exp.			54(4)	16.3(25)	
7000	25.3	73.5	98.8	20.5	0.140
exp.	25.4(11)	73.2(13)	98.6(22)	19.9(3)	0.145(100)

(B is the slope parameter)

Eikonal approximation

$$f(s,t) = \sum_{l=0}^{\infty}(2l+1)f_l(p)P_l(\cos\theta)$$
$$= \frac{p^2}{\pi}\int d^2b\, h(\vec{b},s)\, e^{i\vec{q}\cdot\vec{b}} = 2p^2\int_0^{\infty} bdbJ_0(bq)h(b,s)$$

$t = -\vec{q}^2$, $q = 2p\sin(\theta/2)$, $bp = l + 1/2 + \mathcal{O}(s^{-1})$, $P_l(\cos\theta) \to J_0(qb)$

(would need 40000 partial waves at the LHC!)

In the impact-parameter representation

$$h(b,s) = \frac{i}{2p}\left[1 - e^{i\chi(b)}\right] = f_l(p) + \mathcal{O}(s^{-1})$$

The eikonal approximation works well for $b < 2$ fm and $\sqrt{s} > 20$ GeV

Procedure: $f(s,t) \to h(b,s) \to \chi(b)$...

Eikonal approximation 2

The standard formulas for the total, elastic, and total inelastic cross sections read

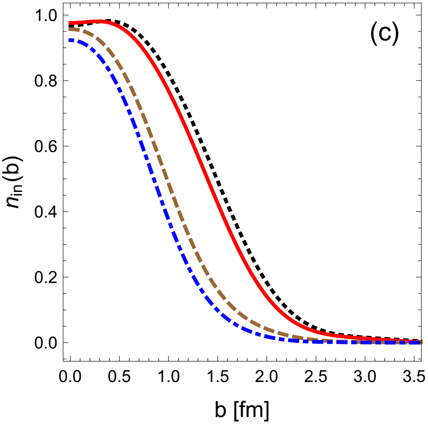
$$\begin{aligned}
\sigma_{\rm tot} &= \frac{4\pi}{p}{\rm Im} f(s,0) = 4p\int d^2b{\rm Im}h(\vec{b},s) = 2\int d^2b\left[1-{\rm Re}\,e^{i\chi(b)}\right] \\
\sigma_{\rm el} &= \int d\Omega|f(s,t)|^2 = 4p^2\int d^2b|h(\vec{b},s)|^2 = \int d^2b|1-e^{i\chi(b)}|^2 \\
\sigma_{\rm in} &\equiv \sigma_{\rm tot}-\sigma_{\rm el} = \int d^2b n_{\rm in}(b) = \int d^2b\left[1-e^{-2{\rm Im}\chi(b)}\right]
\end{aligned}$$

The inelasticity profile

$$n_{\rm in}(b) = 4p{\rm Im}h(b,s) - 4p^2|h(b,s)|^2$$

satisfies $0 \geq n_{\rm in}(b) \leq 1$ (unitarity)

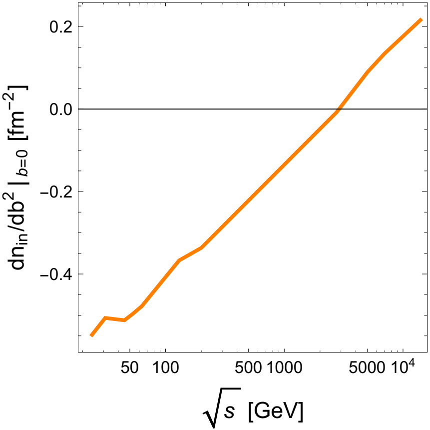
Dip (or flattening) in the inelasticity profile at $b = 0$



From top to bottom: $\sqrt{s} = 14000, 7000, 200, 23.4$ GeV

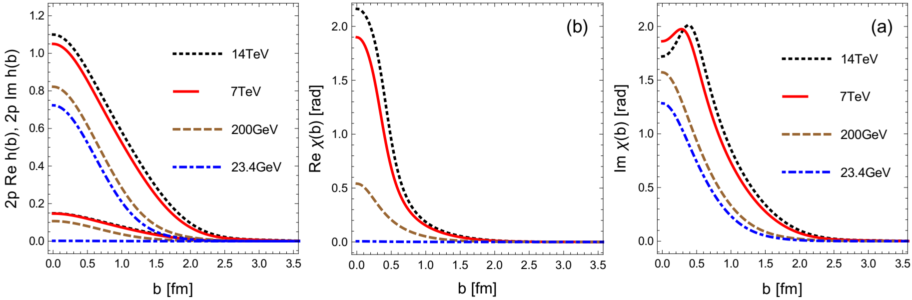
Dip: collisions more destructive at $b > 0$ than for head-on!

Curvature at the origin of the inelasticity profile



Transition around $\sqrt{s} = 3$ TeV

Amplitude and eikonal phase



$$2p\,h(b) = i\left[1 - e^{i\chi(b)}\right]$$

$$k(b) \equiv \mathrm{Im}[2p\,h(b)] = 1 - \cos\left(\mathrm{Re}[\chi(b)]\right) e^{-\mathrm{Im}[\chi(b)]}$$
$$\mathrm{Re}[2p\,h(b)] = \sin\left(\mathrm{Re}[\chi(b)]\right) e^{-\mathrm{Im}[\chi(b)]}$$

At the LHC $\mathrm{Re}[\chi(b)] > \pi/2 \to \cos\left(\mathrm{Re}[\chi(b)]\right) < 0 \to k(b) > 1$

In our model $n_{in}(b) = 2k(b) - (1+\rho^2)k(b)^2$

$$\frac{dn_{in}(b)}{db^2} = 2\frac{dk(b)}{db^2}\left[1 - (1+\rho^2)k(b)\right] < 0 \text{ if } k(b) < \frac{1}{1+\rho^2}$$

Quantum nature

Large real part of the eikonal phase $\rightarrow$ minimum develops at $b = 0$

Glauber (1959): *The eikonal phase is additive in scattering of composite objects. The (potentially small) eikonal phases of the constituents may add up to a large eikonal phase of the whole composite object*

Quantum interference is essential

Gaussian model

[adaptation of the model by Dremin, 2014]

$$\mathrm{Im}(2p\,h(p)) = A e^{-\frac{2b^2}{2B}}$$

$$A = \frac{4\sigma_{\rm el}}{\left(1+\rho^2\right)\sigma_{\rm tot}}, \quad B = \frac{\left(1+\rho^2\right)\sigma_{\rm tot}^2}{16\pi\sigma_{\rm el}}$$

Curvature of inelasticity profile at the origin

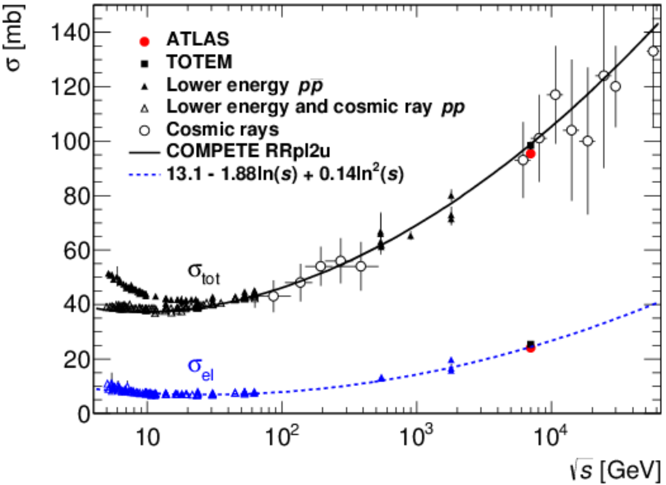
$$\frac{1}{2}\left.\frac{d^2 n_{\rm in}(b)}{db^2}\right|_{b=0} = \frac{64\pi\sigma_{\rm el}^2(4\sigma_{\rm el}-\sigma_{\rm tot})}{\left(r^2+1\right)^2\sigma_{\rm tot}^4}$$

– curvature changes sign when $\sigma_{\rm el} = \frac{1}{4}\sigma_{\rm tot}$!

Value at the origin:

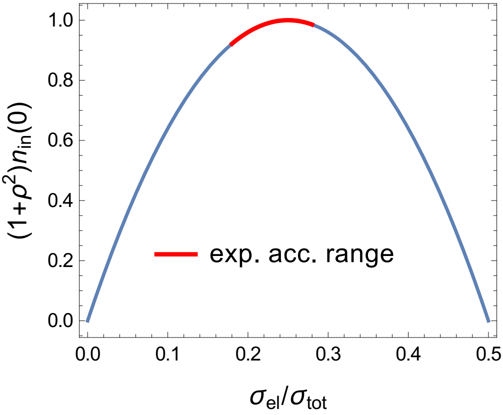
$$n_{\rm in}(0) = \frac{8\sigma_{\rm el}}{(1+\rho^2)\sigma_{\rm tot}}\left(1 - 2\frac{\sigma_{\rm el}}{\sigma_{\rm tot}}\right)$$

Cross sections



$\sigma_{\rm el}$ grows relatively faster than $\sigma_{\rm tot}$ $\rightarrow$ ratio goes above 1/4 as s increases!

Gaussian model 2



Data are in the central region of the abscissa

Hollowness effect expected to increase with the collision energy

Inadequacy of folding models

In many models incoherent superposition is assumed, i.e., inelasticity of a composite object is obtained from inelasticities of components:

$$
\begin{aligned}
n_{\rm in}(b) &\propto \int d^2b_1 d^2b_2 \rho(\vec{b}_1+\vec{b}/2) w(\vec{b}_1-\vec{b}_2)\rho(\vec{b}_2-\vec{b}/2) \\
&= \int d^3b_1 d^3b_2 \rho(\vec{b}_1) w(\vec{b}_1-\vec{b}_2)\rho(\vec{b}_2) \\
&- \frac{1}{2}\int d^3b_1 d^3b_2 [\vec{b}\cdot\nabla\rho(\vec{b}_1)] w(\vec{b}_1-\vec{b}_2)[\vec{b}\cdot\nabla\rho(\vec{b}_2)] + \ldots
\end{aligned}
$$

For a positive kernel $w(\vec{b}_1-\vec{b}_2)$ both integrals are necessarily positive $\rightarrow$ $n_{\rm in}(b)=\alpha^2-\beta^2 b^2$ has a local maximum at $b=0$, in contrast to the phenomenological hollowness result

(folding models usually take $w(\vec{b}_1-\vec{b}_2) \propto \delta(\vec{b}_1-\vec{b}_2)$)

Remarks

Note that the "no hollowness from folding" result holds for any shape of the colliding proton, i.e., the distribution of elementary scatterers $\rho(\vec{b}_{1,2})$ can have any form. Even collision of two donuts resets in most damage when their CM's are aligned

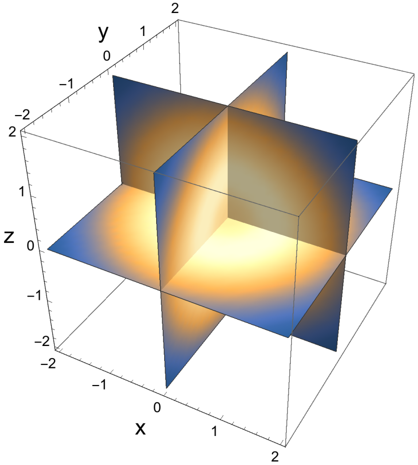
Inalasticty of a collision of two triangle-like (from quarks) protons, if obtained from folding, will not exhibit hollowness

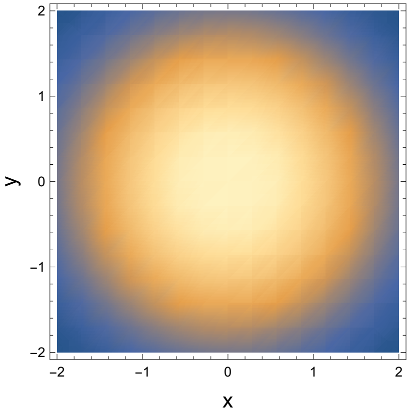
(above statements also hold for the case of correlated wave functions yielding some one-body density $\rho(\vec{b}_{1,2})$)

Hollowness is not a feature of the nucleon itself, but of the scattering process

2D vs 3D opacity – geometric idea

Projection of 3D on 2D covers up the hollow: $f(x,y,z)$ vs $\int_{-\infty}^{\infty} dz\, f(x,y,z)$





The hollow is covered up

Equivalent optical potential – invariant mass method

Phenomenological method [Allen, Payne, Polyzou 2000] introduces the total squared mass operator for the pp system:

$$\mathcal{M}^2 = P^\mu P_\mu \stackrel{CM}{=} 4(p^2 + M_N^2) + \mathcal{V}$$

P^μ – total four-momentum, p – CM three-momentum of each nucleon, M_N – nucleon mass, $\mathcal{V}$ – invariant interaction, determined in the CM frame by matching in the non-relativistic limit to a non-relativistic potential, i.e., $\mathcal{V} = 4U = 4M_N V$. Relativistic Schrödinger equation $\hat{\mathcal{M}}^2\Psi = s\Psi$ transforms into an equivalent non-relativistic Schrödinger equation

$$(-\nabla^2 + U)\Psi = (s/4 - M_N^2)\Psi = p^2\Psi$$

with the reduced potential $U = M_N V = \mathrm{Re}U + i\mathrm{Im}U$ (to be determined by inverse scattering)
p – CM momentum of the proton

No complication of spin/noncentrality (5 complex Wolfenstein amplitudes)

Eikonal limit and optical potential

As in WKB, we solve $-\hbar^2\nabla^2\Psi = (\hbar^2 p^2 - 2mV)\Psi$ with $\Psi = Ae^{iS/\hbar}$

$$(\nabla S)^2 - \cancel{i\hbar\nabla^2 S} = \hbar^2 p^2 - 2mV$$

$$\nabla S/\hbar = \sqrt{p^2 - 2mV/\hbar^2}$$

For $p \gg$ other scales

$$S/\hbar = pz - \frac{m}{\hbar^2 p}\int_{-\infty}^{z} dz' V(z')$$

(transverse dynamics frozen)

Inverse scattering and optical potential

Hence in the eikonal approximation one has

$$\Psi(\vec{b}, z) = \exp\left[ipz - \frac{i}{2p}\int_{-\infty}^{z} U(\vec{b}, z')dz'\right]$$

$$\chi(b) = -\frac{1}{2p}\int_{-\infty}^{\infty} U(\sqrt{b^2 + z^2})dz = -\frac{1}{p}\int_{b}^{\infty} \frac{rU(r)\,dr}{\sqrt{r^2 - b^2}}$$

is the (complex) eikonal phase [Glauber 1959]. This Abel-type equation can be inverted:

$$U(r) = M_N V(r) = \frac{2p}{\pi}\int_{r}^{\infty} db \frac{\chi'(b)}{\sqrt{b^2 - r^2}}$$

On-shell optical potential

From the definition of the inelastic cross section

$$\sigma_{\rm in} = -\frac{1}{p} \int d^3x \, {\rm Im}\, U(\vec{x}) |\Psi(\vec{x})|^2$$

$\rightarrow$ density of inelasticity is proportional to the absorptive part of the optical potential times the square of the modulus of the wave function. One can identify the *on-shell optical potential* (related to Bethe-Salpeter methods)

$${\rm Im}\, W(\vec{x}) = {\rm Im}\, U(\vec{x}) |\Psi(\vec{x})|^2$$

Upon z integration,

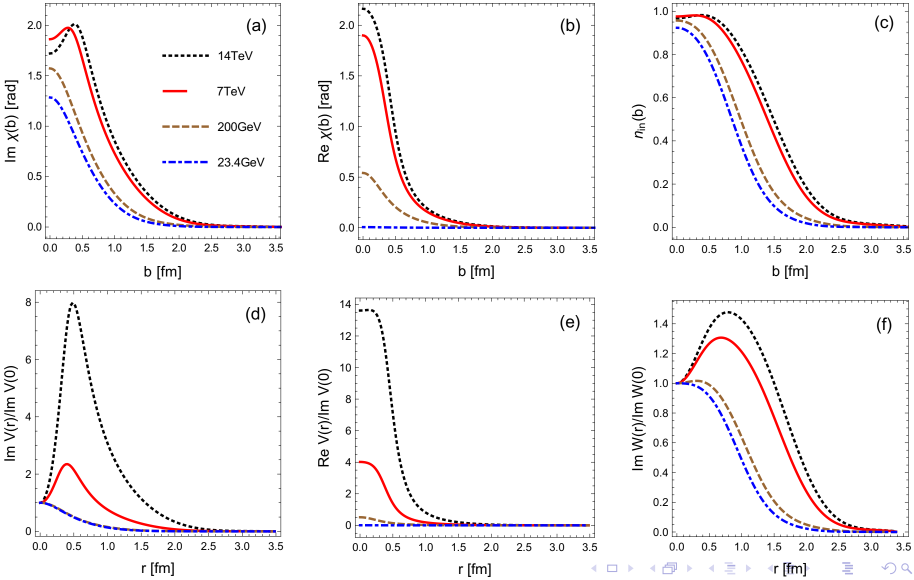
$$-\frac{1}{p} \int dz {\rm Im}\, W(\vec{b}, z) = n_{in}(b)$$

Inversion yields

$${\rm Im} W(r) = \frac{p}{\pi} \int_r^\infty db \frac{n'_{in}(b)}{\sqrt{b^2 - r^2}}$$

Results of the inverse scattering

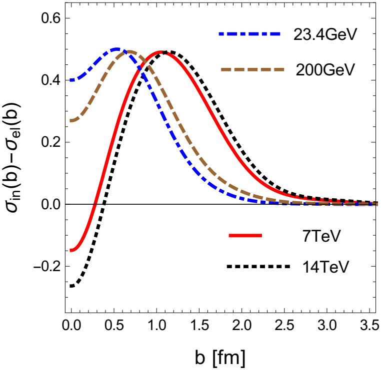
eikonal phase $\rightarrow U(r) = M_N V(r)$, inelasticity profile $\rightarrow \mathrm{Im} W(r)$



The hollow and the edge

The *edge* function is defined as a combination [Block et al. 2014, Rosner 2015] $\sigma_T(b) - 2\sigma_{\rm el}(b) \equiv \sigma_{\rm in}(b) - \sigma_{\rm el}(b)$. In a general case

$$\sigma_{\rm in}(b) - \sigma_{\rm el}(b) = 2e^{-{\rm Im}\chi(b)}\left[\cos({\rm Re}\chi(b)) - e^{-{\rm Im}\chi(b)}\right].$$

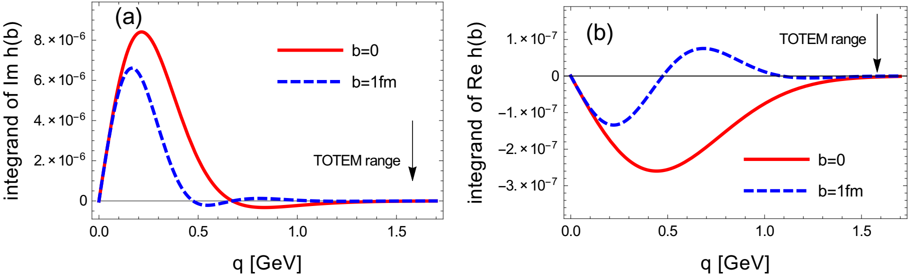


The fact that $\sigma_{\rm in}(b) < \sigma_{\rm el}(b)$ at low b at the LHC stem from the same quantum mechanism as the hollowness effect

Conclusions

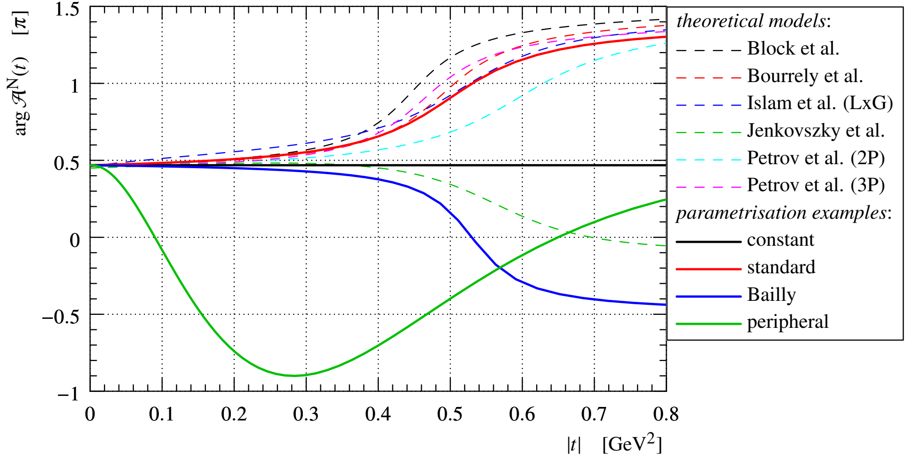
- Hollowness transition inferred from the parametrization of the data, seen in $n_{in}(b)$ for $s > 3$ TeV
- Quantum effect, which can be related to compositeness of the proton and the rise of the real part of the eikonal phase above $\pi/2$. It is a gradual process
- Hot-spot model [Alba Soto+Albacete 2016] – a dynamic realization
- Effect impossible to obtain incoherently by folding the absorptive parts from constituents (uncorrelated or correlated)$\rightarrow$ change of working assumptions in many models
- 2D $\rightarrow$ 3D greatly magnifies the hollowness effect (flat in 2D $\rightarrow$ hollow in 3D), interpretation via optical potential in a relativized problem
- Qualitatively similar hollowness effect appears in low-energy ($\sim$500 keV) n-A scattering – less absorption for head-on collisions than for peripheral! $\mathrm{Im}V(r) \sim d/dr\,\mathrm{Re}V(r)$

Fourier-Bessel transform



(TOTEM extends far enough)

$\rho(s,t)$



from a TOTEM analysis