# Double parton distributions of the pion

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WB, ERA, arXiv:1910.03707

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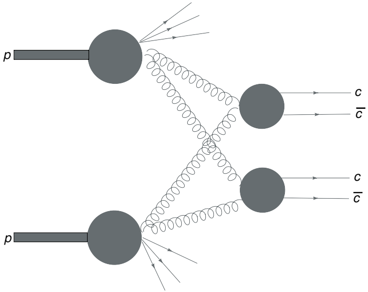
Poland

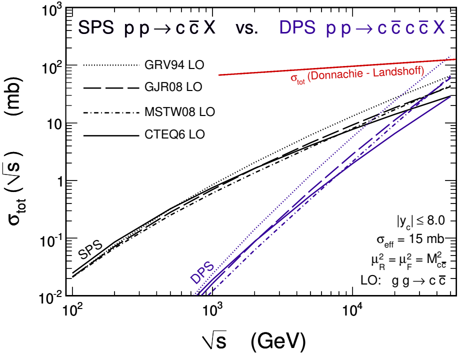
# Double parton distributions

# Motivation for multi-parton distributions

- Old story (Fermilab), renewed interest (e.g., ATLAS measurement for pp$\to$ W+2 jets 2013) [Kuti, Weiskopf 1971, Konishi, Ukwa, Veneziano 1979, Gaunt, Stirling 2010, Diehl, Ostermeier, Schäfer 2012, . . . , reviews: Bartalani et al. 2011, Snigirev 2011, Rinaldi,Ceccopieri 2018]
- Model exploration [MIT bag: Chang, Manohar, Waalewijn 2013], valon [WB+ERA 2013], constituent quarks: Rinaldi, Scopetta, Vento 2013, Rinaldi, Scopetta, Traini, Vento 2018]
- Questions: factorization, interference of single- and double-parton scattering [Manohar, Waalewijn 2012, Gaunt 2013], longitudinal-transverse decoupling, positivity bounds [Diehl, Casemets 2013], transverse momentum dependence [Casemets, Gaunt 2019]
- Gaunt-Stirling sum rules [Gaunt, Stirling 2010, WB+ERA 2013, Diehl, Plößl, Schäfer 2019]

[example from Łuszczak, Maciuła, Szczurek 2011]




DPS can be comparable to SPS at the LHC

Assumption: $D_{gg}(x_1, x_2, \boldsymbol{b}) = g(x_1)g(x_2)F(\boldsymbol{b})$

– no correlations, transverse-longitudinal factorization

# Definition

Intuitive probabilistic definition:

Multi-parton distribution = probability distribution that struck partons have LC momentum fractions $x_i$

Field-theoretic definition of (spin-averaged) sPDF and dPDF [Diehl, Ostermeier, Schaeffer 2012] of a hadron with momentum $p$:

$$D_j(x) = \int \frac{\mathrm{d}z^-}{2\pi}\, e^{i\,x\,z^-p^+} \langle p|\, \mathcal{O}_j(0,z)\,|p\rangle\Big|_{z^+=0\,,\boldsymbol{z}=\mathbf{0}}$$

$$F_{j_1j_2}(x_1,x_2,\boldsymbol{b}) = 2p^+ \int \mathrm{d}y^-\, \frac{\mathrm{d}z_1^-}{2\pi}\, \frac{\mathrm{d}z_2^-}{2\pi}\, e^{i\,(x_1z_1^-+x_2z_2^-)\,p^+}$$

$$\times \langle p|\, \mathcal{O}_{j_1}(y,z_1)\, \mathcal{O}_{j_2}(0,z_2)\,|p\rangle\Big|_{z_1^+=z_2^+=y^+=0\,,\boldsymbol{z}_1=\boldsymbol{z}_2=\mathbf{0}}$$

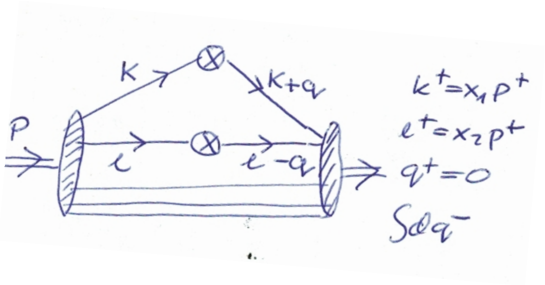
$\mathcal{O}_q(y,z) = \frac{1}{2}\,\bar{q}(y-\frac{z}{2})\gamma^+ q(y+\frac{z}{2}),\ \ldots$ (LC gauge) $\qquad v^{\pm} = (v^0 \pm v^3)/\sqrt{2}$

$y = (y^+, y^-, \boldsymbol{b})$, ($\boldsymbol{b}$ is the transverse distance between the two quarks

# dPDF in momentum space

Fourier transform in $\boldsymbol{b}$

$$F_{j_1 j_2}(x_1, x_2, \boldsymbol{b}) \to \tilde{F}_{j_1 j_2}(x_1, x_2, \boldsymbol{q})$$




Special case of $\boldsymbol{q} = \boldsymbol{0}$:

$$D_{j_1 j_2}(x_1, x_2) = \tilde{F}_{j_1 j_2}(x_1, x_2, \boldsymbol{q} = \boldsymbol{0})$$

# Gaunt-Stirling sum rules

[Gaunt, Stirling, JHEP (2010) 1003:005, Gaunt, PhD Thesis]

Fock-space decomposition on LC + conservation laws $\rightarrow$

$$|P\rangle = \sum_N \int d[x, \boldsymbol{k}]_N \Phi(\{x_i, \boldsymbol{k}_i\}) |\{x_i, \boldsymbol{k}_i\}\rangle_N$$

$$d[x, \boldsymbol{k}]_N = \prod_{i=1}^{N} \left[ \frac{dx_i d^2 k_i}{\sqrt{2(2\pi)^3 x_i}} \right] \delta\left(1 - \sum_{i=1}^{N} x_i\right) \delta^{(2)}\left(1 - \sum_{i=1}^{N} \boldsymbol{k}_i\right)$$

# Gaunt-Stirling sum rules

[Gaunt, Stirling, JHEP (2010) 1003:005, Gaunt, PhD Thesis]

Fock-space decomposition on LC + conservation laws $\rightarrow$

$$\sum_i \int_0^{1-x_2} dx_1\, x_1 D_{ij}(x_1, x_2) = (1 - x_2) D_j(x_2) \quad \text{(momentum)}$$

$$\int_0^{1-x_2} dx_1\, D_{i_{\rm val} j}(x_1, x_2) = (N_{i_{\rm val}} - \delta_{ij} + \delta_{\bar{i}j}) D_j(x_2) \quad \text{(quark number)}$$

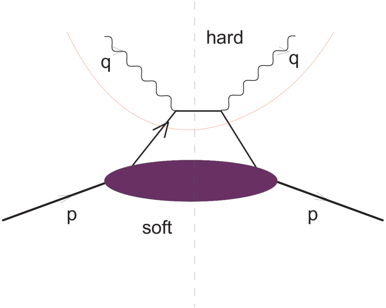
$(A_{i_{\rm val}} \equiv A_i - A_{\bar{i}})$

$$N_{i_{\rm val}} = \int_0^1 dx\, D_{i_{\rm val}}(x)$$

- Preserved by dDGLAP evolution
- Non-trivial to satisfy with the (guessed) function
- Checked in light-front perturbation theory and in lowest-order covariant calculations in [Diehl, Plößl, Schäfer 2019]

Important and fundamental constraints!

# NJL in high-energy processes



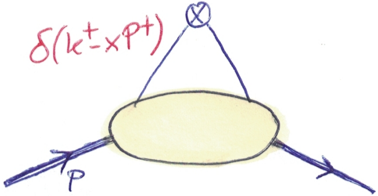

$Q^2 = -q^2, \;\; x = \frac{Q^2}{2p \cdot q}, \;\; Q^2 \to \infty$

Factorization of soft and hard processes, Wilson's OPE

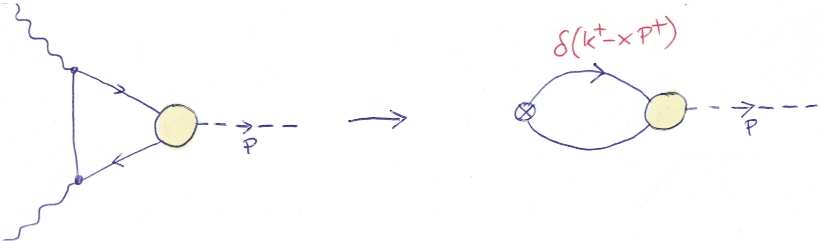
$$\langle J(q) J(-q) \rangle = \sum_i C_i(Q^2; \mu) \langle \mathcal{O}_i(\mu) \rangle$$

Twist expansion $\rightarrow \quad F(x, Q) = F_0(x, \alpha(Q)) + \frac{F_2(x, \alpha(Q))}{Q^2} + \ldots$

Bjorken limit $\rightarrow$ light-cone momentum is constrained:
$k^+ \equiv k^0 + k^3 = x P^+ \qquad x \in [0, 1]$



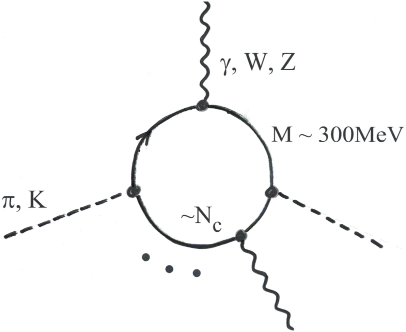
# Distribution amplitude (DA) of the pion




Enters various measures of exclusive processes,
e.g., pion-photon transition form factor

[Anikin, Dorokhov, Tomio 2000]
[Praszałowicz, Rostworowski 2001, + Bzdak 2003, + Kotko 2009]
[ERA, WB 2002]

# Chiral quark models

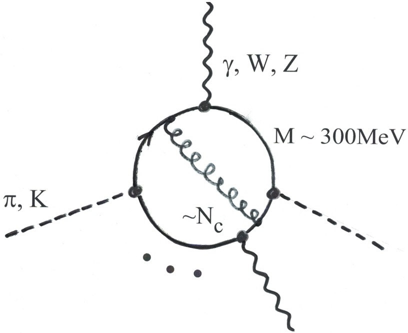



- $\chi$SB breaking $\rightarrow$ massive quarks
- Point-like interactions
- Soft matrix elements with pions (and photons, $W$, $Z$)
- Large-$N_c$ $\rightarrow$ one-quark loop
- Regularization

pion – Goldstone boson of $\chi$SB, fully relativistic $q\bar{q}$ bound state of the Bethe-Salpeter equation

Quantities evaluated at the quark model scale
(where constituent quarks are the only degrees of freedom)

# Chiral quark models

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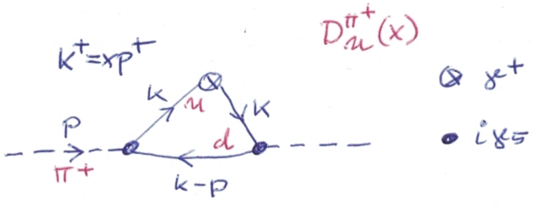
Need for QCD evolution

Gluon dressing, renorm-group improved

# sPDF in NJL

[Davidson, Arriola, 1995]




In the chiral limit of $m_\pi = 0$:

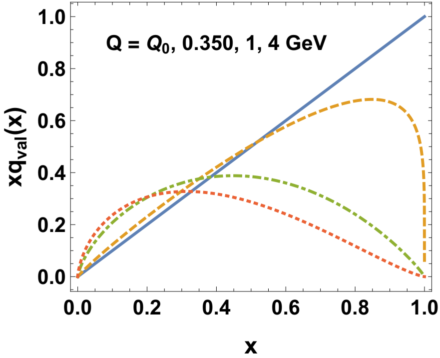
$$q_{\rm val}(x;Q_0) = 1 \times \theta[x(1-x)]$$

Quarks are the only degrees of freedom, hence saturate the PDF sum rules: $\int_0^1 dx\, q_{\rm val}(x;Q_0) = 1$ (valence), $2\int_0^1 dx\, x q_{\rm val}(x;Q_0) = 1$ (momentum)

QM provide non-perturbative result at a low scale $Q_0$

$$F(x,Q_0)|_{\text{model}} = F(x,Q_0)|_{\text{QCD}}\,, \quad Q_0 - \text{the matching scale}$$

Quarks carry 100% of momentum at $Q_0$, adjusted such that when evolved to $Q = 2$ GeV, they carry the experimental value of 47% (radiative generation of gluons and sea quarks)



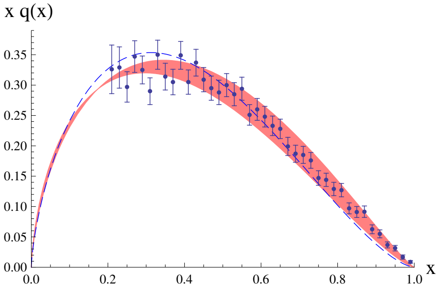

LO DGLAP evolution

$$Q_0 = 313^{+20}_{-10} \text{ MeV}$$

NLO close to LO

$$\sim (1-x)^{p+\frac{4C_F}{\beta_0}\log\frac{\alpha(Q_0)}{\alpha(Q)}}$$

# Pion valence quark sPDF, NJL vs E615



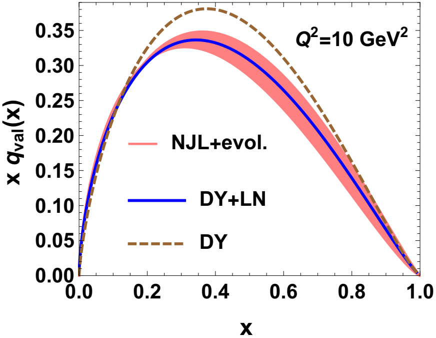

points: Fermilab E615
Drell-Yan, $\pi^{\pm}W \to \mu^{+}\mu^{-}X$

dashed line: 2005 NLO reanalysis [Wijesoorija et al.]

band: QM + LO DGLAP from $Q_0 = 313^{+20}_{-10}$ MeV to $Q = 4$ GeV

# Pion valence quark DF, QM vs JAM analysis

[P. C. Barry, N. Sato, W. Melnitchouk, C.-R. Ji, PRL 121 (2018) 152001, arXiv:1804.01965]

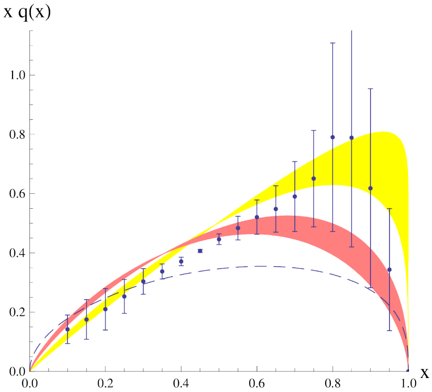



curves: JAM data analysis

band: QM + LO DGLAP from $Q_0 = 313^{+20}_{-10}$ MeV to $Q^2 = 10\ \mathrm{GeV}^2$

Many predictions for related quantities: DA, GPD, TDA, TMD, quasi/pseudo DA/PDF...

# Pion quark DF, NJL vs. transverse lattice



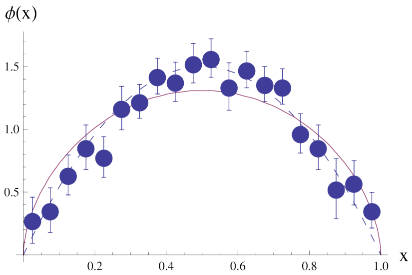

points: transverse lattice
[Dalley, van de Sande 2003]

yellow: QM evolved to 0.35 GeV

pink: QM evolved to 0.5 GeV

dashed: GRS param. at 0.5 GeV

# Pion DA, NJL vs. E791

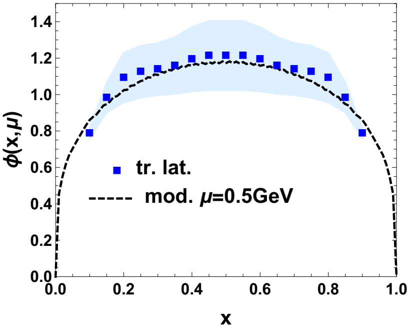



points: E791 data from dijet production in $\pi + A$

solid line: QM at $Q = 2$ GeV

dashed line: asymptotic form $6x(1-x)$ at $Q \to \infty$

# Pion DA, NJL vs. transverse lattice



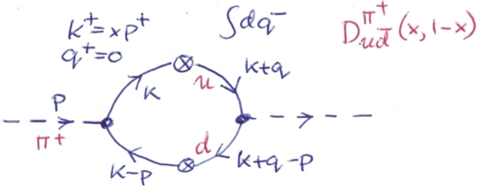

points: transverse lattice data [Dalley, van de Sande 2003]

# dPDF of the pion in NJL (new stuff)

- WB talk at Light Cone 2019, 16-20 Sep. 2019, Palaiseau, France
- A. Courtoy, S. Noguera, and S. Scopetta, JHEP 12, 045 (2019), arXiv:1909.09530
- WB, ERA, arXiv:1910.03707

# dPDF of the pion in NJL model

In LC kinematics only one diagram:




In the chiral limit of $m_\pi^2 = 0$ a very simple result follows:

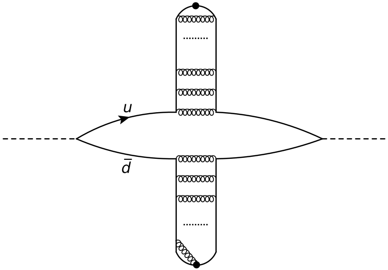
$$D_{u\bar{d}}(x_1, x_2) = 1 \times \delta(1 - x_1 - x_2)\theta[x_1(1 - x_1)]\theta[x_2(1 - x_2)]$$

momentum conservation support

- Factorization (in the chiral limit) of the longitudinal and transverse dynamics
- GS sum rules satisfied (preserved by the evolution)
- Results at the quark-model scale → need for evolution

# dDGLAP evolution in the Mellin space

[Kirschner 1979, Shelest, Snigirev, Zinovev 1982]: method of solving dDGLAP based on the Mellin moments, similarly to sPDF  simplification for valence distributions



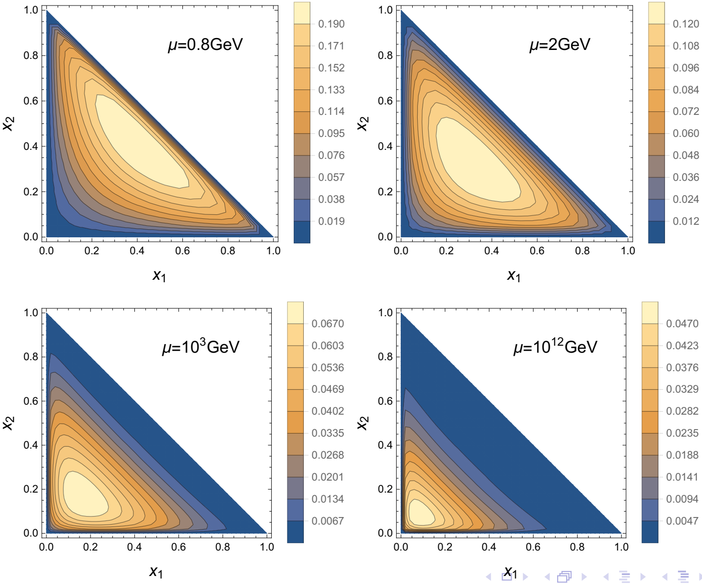

$t = \frac{1}{2\pi\beta} \log\left[1 + \alpha_s(\mu)\beta \log(\Lambda_{\rm QCD}/\mu)\right]$ (single scale for simplicity), $\beta = \frac{11N_c - 2N_f}{12\pi}$

## Valence:

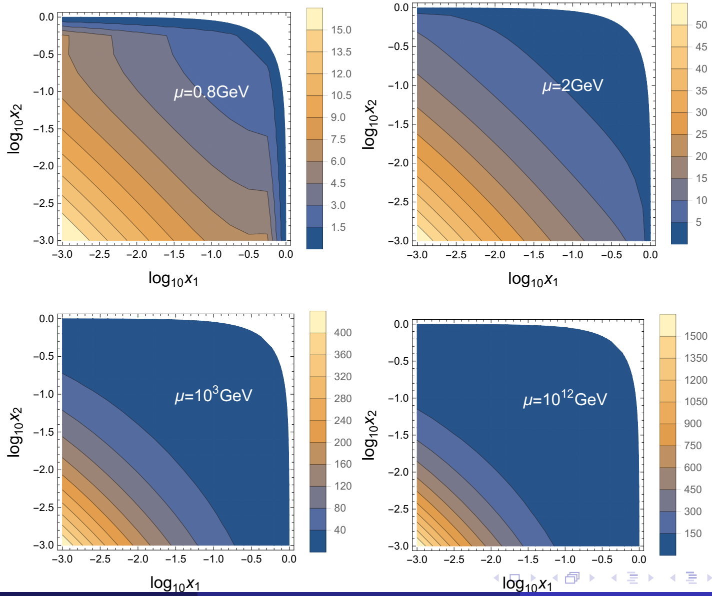
$$\text{dPDF}: \quad \frac{d}{dt} M^{n_1 n_2}_{j_1 j_2}(t) = \left(P^{n_1}_{j_1 \to j_1} + P^{n_2}_{j_2 \to j_2}\right) M^{n_1 n_2}_{j_1, j_2}(t)$$

$$\text{sPDF}: \quad \frac{d}{dt} M^{n}_{j}(t) = P^{n}_{j \to j} M^{n}_{j}(t)$$

# $x_1 x_2 D_{u\bar{d}}^{\pi^+}(x_1, x_2)$

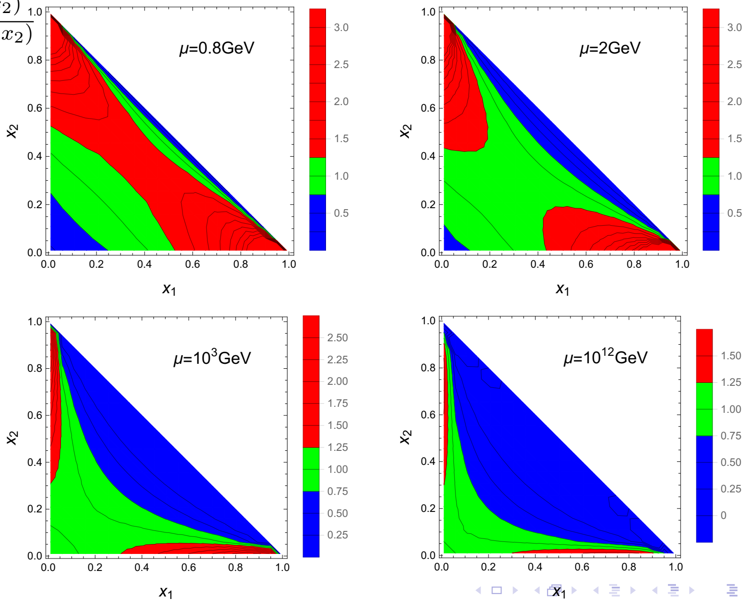


# $D_{u\bar{d}}^{\pi^+}(x_1, x_2)$ – log scale



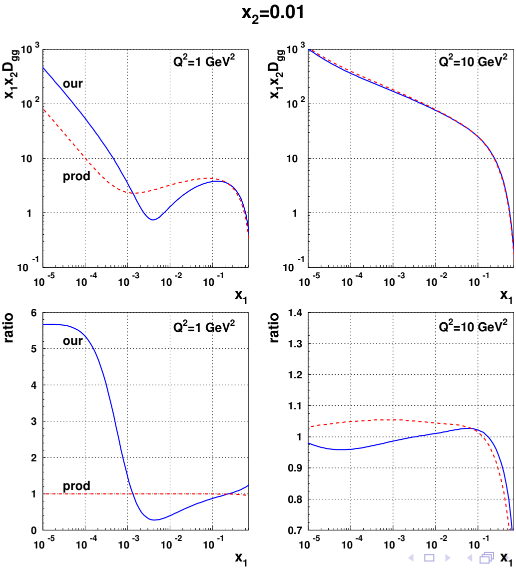
# Correlation

$$\frac{D^{\pi^+}_{u\bar{d}}(x_1,x_2)}{D_u(x_1)D_{\bar{d}}(x_2)}$$



# Gluon correlation in the **nucleon**, "parametric" study

[Golec-Biernat, Lewandowska, Serino, Snyder, Staśto 2015]



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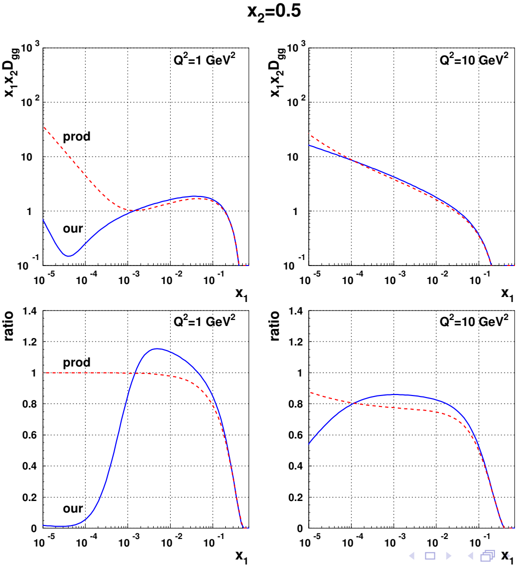
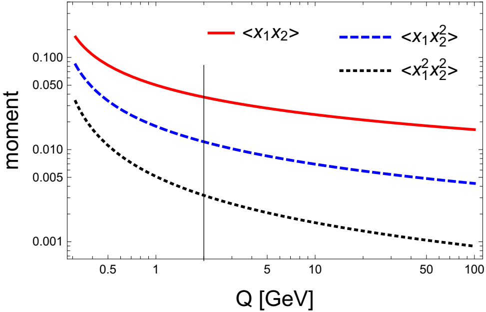


# Valence moments in NJL

$$\frac{\langle x_1^n x_2^m \rangle}{\langle x_1^n \rangle \langle x_2^m \rangle} = \frac{(1+n)!(1+m)!}{(1+n+m)!} \quad \text{(NJL, any scale)}$$

(independent of the evolution scale)

| | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| 1 | $\frac{2}{3}$ | $\frac{1}{2}$ | $\frac{2}{5}$ | $\frac{1}{3}$ | $\frac{2}{7}$ |
| 2 | $\frac{1}{2}$ | $\frac{3}{10}$ | $\frac{1}{5}$ | $\frac{1}{7}$ | $\frac{3}{28}$ |
| 3 | $\frac{2}{5}$ | $\frac{1}{5}$ | $\frac{4}{35}$ | $\frac{1}{14}$ | $\frac{1}{21}$ |
| 4 | $\frac{1}{3}$ | $\frac{1}{7}$ | $\frac{1}{14}$ | $\frac{5}{126}$ | $\frac{1}{42}$ |
| 5 | $\frac{2}{7}$ | $\frac{3}{28}$ | $\frac{1}{21}$ | $\frac{1}{42}$ | $\frac{1}{77}$ |




Double moments reduced compared to product of single moments

[lattice results expected, Zimmermann et al. (?)]

# Transverse structure

# Transverse structure: regularization vs positivity

- Formally (in the chiral limit of $m_\pi^2 = 0$)

$$\begin{aligned} F(\boldsymbol{q}) &= \frac{N_c M^2}{(2\pi)^3 f^2} \int d^2\boldsymbol{k}_1 d^2\boldsymbol{k}_2 \delta(\boldsymbol{k_1} - \boldsymbol{k_2} + \boldsymbol{q}) \\ &\times \left[\Psi(\boldsymbol{k_1})\boldsymbol{k_1} \cdot \Psi(\boldsymbol{k_2})\boldsymbol{k_2} + M^2 \Psi(\boldsymbol{k_1})\Psi(\boldsymbol{k_2})\right] \end{aligned}$$

- Momentum vs coordinate

$$\Psi(\boldsymbol{k}) = \frac{1}{\boldsymbol{k}^2 + M^2} \qquad \Phi(\boldsymbol{b}) = \int \frac{d^2\boldsymbol{k}}{(2\pi)^2} \Psi(\boldsymbol{k}) e^{i\boldsymbol{k}\cdot\boldsymbol{b}} = K_0(bM)$$

- Positive but divergent at low $b$ / high $q$!

$$F(\boldsymbol{q}) = \frac{N_c M^2}{2\pi f^2} \int d^2\boldsymbol{b}\, e^{i\boldsymbol{b}\boldsymbol{q}} \left[\nabla\Phi(\boldsymbol{b})^2 + M^2 \Phi(\boldsymbol{b})^2\right]$$

- Example $\langle 0|\phi^2|0\rangle$ positive but divergent, $:\phi^2 := \phi^2 - \langle 0|\phi^2|0\rangle$ convergent but not positive

# Good regularizations

Lorentz and gauge invariance must be satisfied

- NJL model (Pauli-Villars regularization)

$$
\begin{aligned}
A(M)|_{\rm NJL} &= \sum_i c_i A\left(\sqrt{\Lambda_i^2 + M^2}\right) \\
f_\pi^2 &= -\frac{4N_c^2 M^2}{(4\pi)^2} \sum_i c_i \log(\Lambda_i^2 + M^2)
\end{aligned}
$$

- Spectral Quark Model (implements vector meson dominance)

$$
\begin{aligned}
A(M)|_{\rm SQM} &= \int_C dw \rho(w) A(w) \\
\rho(w) &: \quad F_{\rm em}(q^2) = \frac{m_\rho^2}{m_\rho^2 + q^2}, \quad f_\pi^2 = \frac{N_c m_\rho^2}{24\pi^2}
\end{aligned}
$$

# dPDF form factor in SQM

- Form factor
$$F(\boldsymbol{q}) = \frac{m_\rho^4 - \boldsymbol{q}^2 m_\rho^2}{\left(m_\rho^2 + \boldsymbol{q}^2\right)^2},$$

- ... in coordinate space

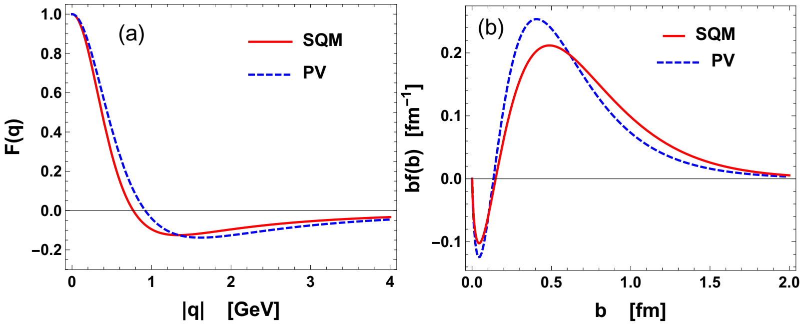
$$f(b) = \int \frac{d^2\boldsymbol{q}}{(2\pi)^2} e^{i\boldsymbol{b}\cdot\boldsymbol{q}} F(\boldsymbol{q}) = \frac{m_\rho^2}{2\pi}\left[b m_\rho K_1\left(b m_\rho\right) - K_0\left(b m_\rho\right)\right]$$

- $\sigma_{\mathrm{DPS}}^{AB} = \frac{1}{C}\sigma_{\mathrm{SPS}}^{A}\sigma_{\mathrm{SPS}}^{B}/\sigma_{\mathrm{eff}}$ ($C = 2$ for $A = B$ and $1$ for $A \neq B$)
- Effective cross section (here coincides with geometric)

$$\sigma_{\mathrm{eff}} = \frac{1}{\int \frac{d^2\boldsymbol{q}}{(2\pi)^2} F(\boldsymbol{q}) F(-\boldsymbol{q})} = \pi \frac{12}{m_\rho^2} = \pi \langle b^2 \rangle_{\mathrm{dPDF}} = 23 \text{ mb}$$

- Rinaldi and Ceccopieri (2018) bounds (derived under plausibility assumptions) $\pi\langle b^2\rangle \leq \sigma_{\mathrm{eff}} \leq 3\pi\langle b^2\rangle$

# Positivity




- Does $F(q)$ have to be positive definite?
- Is it artifact of regularization? Low-$q$ expansion is fine
- Values for large $q$ or small $b$ could be checked on the lattice ($\sim a = 0.1$)
- Similar results in Courtoy et al.

# Summary

- NJL: simplest field theory of the pion in the soft regime; spontaneous chiral symmetry breaking
- Covariant calculations, all symmetries preserved $\to$ good features
- dPDF in NJL = $F(\boldsymbol{q}^2) \times \delta(1 - x_1 - x_2)$ + dDGLAP evolution; factorization (in the chiral limit) of the longitudinal and transverse dynamics
- Correlations decrease with increasing evolution scale and are probably not very important ($\pm 25\%$) in the range probed by experiments, justifying the product ansatz in that limit
- Moments measure the $x_1 - x_2$ factorization breaking; can be verified in future lattice calculations
- Transverse form factor negative at large $q$
- The effective cross section is geometric