Hydrodynamics in phenomenology of heavy ion and proton-ion collisions

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Strong interactions beyond simple factorization:
Collectivity at high energy from initial to final state
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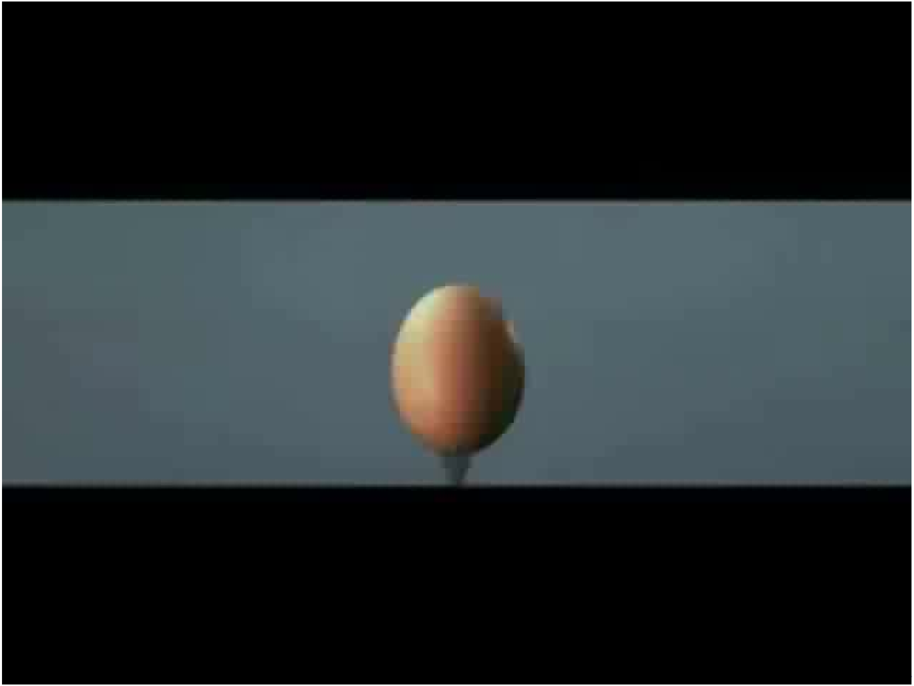
Foreword

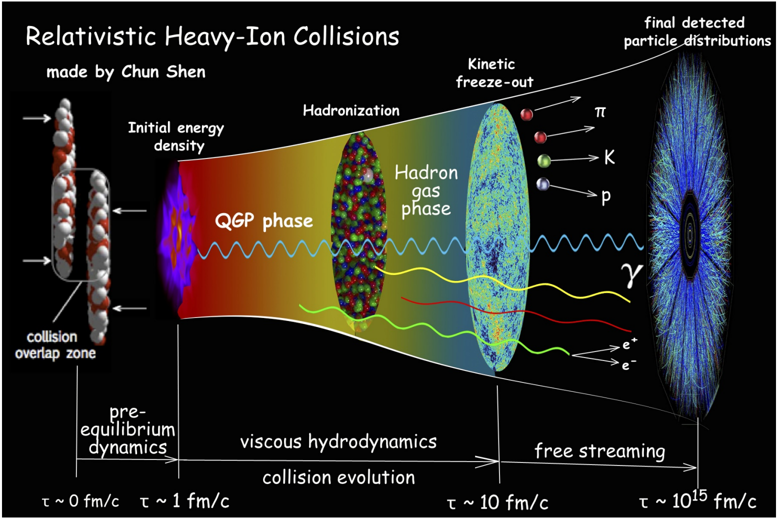
Feynman: Scattering of protons on protons is like colliding Swiss watches to find out how they are built.



Studying the hydrodynamics of water by shooting at a watermelon!

- What is the equation of state, viscosity . . . ?
- What was the shape before destruction?

Little bangs



Three stages of the “Standard Model” of Little Bangs

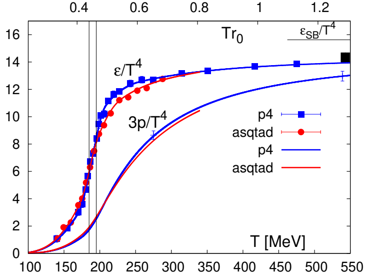
partons hydrodynamization quark-gluon plasma freeze-out hadrons

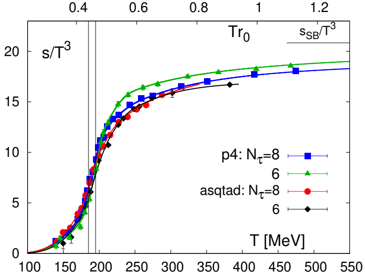
time: $\sim$1 fm/c $\sim$10 fm/c

These lectures focus on the intermediate (hydro) phase and its phenomenological implications

Introduction

At high temperatures the thermal motion is so high, that also the momenta transferred are large. An early expectation was that weakly-interacting quark-gluon plasma (QGP) should be formed. It is not really the case at accessible temperatures!





[Bazavov et al., PRD 80(2009)014504, arXiv:0903.4379]

QGP → sQGP – strongly interacting QGP

Reminder: the Stefan-Boltzmann law

With the grand-canonical ensemble

$$-pV = \Omega(T,V,\mu) = V\gamma T \int \frac{d^3k}{(2\pi)^3} \log\left(1 \pm e^{-(E(k)-\mu)/T}\right)$$

$+$ fermions, $-$ bosons, $E(k) = \sqrt{m^2+k^2}$, V - volume, T - temperature, μ - chemical potential, γ - degeneracy factor.
$m=0$ and $\mu=0$: $p = \gamma\frac{\pi^2}{90}T^4$ for bosons and $p = \gamma\frac{7}{8}\frac{\pi^2}{90}T^4$ for fermions, whereas $\epsilon \equiv E/V = 3p$, $s = 4p/T$.

QGP (gluons and quarks+antiquarks)

$p/T^4 = 8 \times 2(\text{color} \times \text{spin}) + 7/8 \times 2 \times 3 \times 2 \times N_f([q+\bar{q}] \times (\text{color} \times \text{spin} \times \text{flavor})$

$N_f = 2$ and 3: $p/T^4 =\simeq 4.06$ and $\simeq 5.21$, respectively (+ bag constant in some models)
$s \simeq 14/\text{fm}^3$ for $T = 175$ MeV

massive pions at $T \to 0$

$p = \gamma_\pi e^{-m_\pi/T}\frac{m^{3/2}T^{5/2}}{4\sqrt{2}\pi^{3/2}}$, with $\gamma_\pi = 3$

A dramatic growth of the number of degrees of freedom, as seen on the lattice!

sQGP

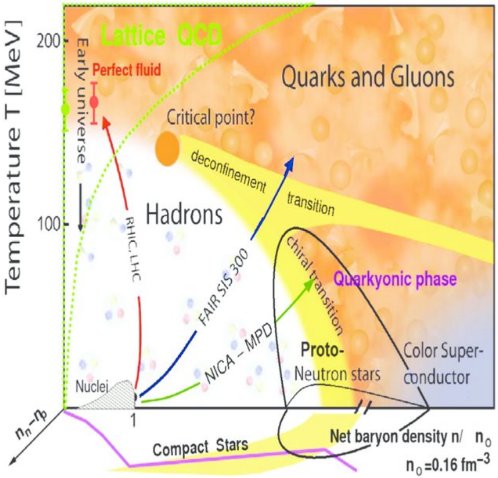
[M. Gyulassy, L. McLerran, NPA 750(2005)30, nucl-th/0405013]:

"Our criteria for the discovery of QGP are

1. *matter at energy densities so large that simple degrees of freedom are quarks and gluons. This energy density is that predicted from lattice gauge theory for the existence of a QGP in thermal systems, and is about 2 GeV/fm*^{3}
2. *the matter must be to a good approximation thermalized*
3. *the properties of the matter . . . must follow QCD computations based on hydrodynamics, lattice gauge theory results, and perturbative QCD for hard processes such as jets.*

All of the above are satisfied from the published data at RHIC ... This leads us to conclude that the matter produced at RHIC is a strongly coupled QGP (sQGP) contrary to original expectations that were based on weakly coupled plasma estimates."

Phase diagram of QCD



[from D. P. Menezes]

Thermal ideas

Strong (soft) interactions $\rightarrow$ so complicated that things become simple again!
It is easier for a system to reach fast the thermal equilibrium when the interactions are strong – shorter mean free path $\rightarrow$ more collisions. How to achieve this from QCD-based approaches is a topic of current active research.

- Isotropization puzzle: how are the pressures in the longitudinal and transverse directions equilibrated
- Early thermalization puzzle: how is thermal equilibrium achieved?
- Relaxed to early hydrodynamization

It is needed that at a short time of the order of 1 fm/c approximate thermal equilibrium in the fireball is reached.

[thermal approach: Fermi 1950, Landau 1953]

What is collectivity?

Groups of objects (particles) move in a similar way

Collectivity = n-body correlations with large n

2-body: $C_2(x_1, x_2) \equiv f_2(x_1, x_2) - f_1(x_1)f_1(x_2)$

3-body: $C_3(x_1, x_2, x_3) \equiv f_3(x_1, x_2, x_3)$
$$-f_1(x_1)C_2(x_2, x_3) - f_1(x_2)C_2(x_1, x_3) - f_1(x_3)C_2(x_1, x_2)$$
$$-f_1(x_1)f_1(x_2)f_1(x_3)$$

. . . and so on

The genuine correlated distribution C_n in nonzero only if there exists a direct physical mechanism correlating n or more particles

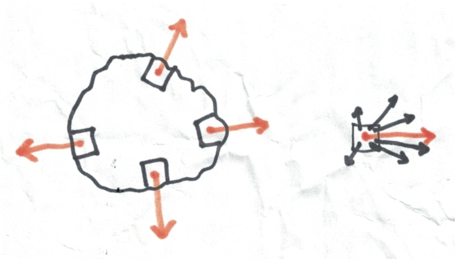
Examples:

- 2-body resonance decays give rise to $C_2(\vec{p}_1, \vec{p}_2)$, and not C_3.
- Bose-Einstein correlations involve all identical bosons in the system

Flow → collectivity

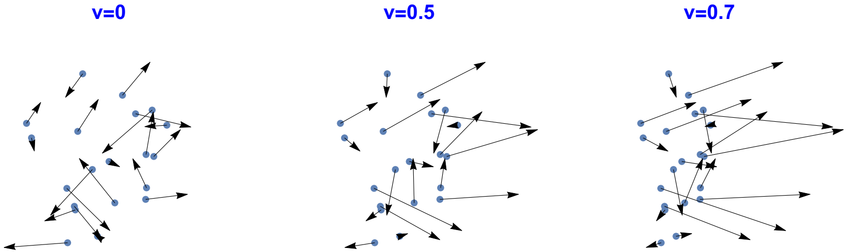
A prominent source of momentum correlations is flow

In the intermediate stage system treated as gas/fluid (see later) - Quark-Gluon Plasma
No container! → the fireball expands and cools down, inevitability of flow



$p_{||} \to p_{||}\cosh\zeta + E\sinh\zeta$, with rapidity $\zeta = \mathrm{arctanh}(v/c)$
Doppler effect → emission of hadrons form a moving (boosted) source

Same space (points) and momentum (arrows) distribution of thermal pions ($T = 160$ MeV) in a fluid element at rest, and moving to the right at a velocity v



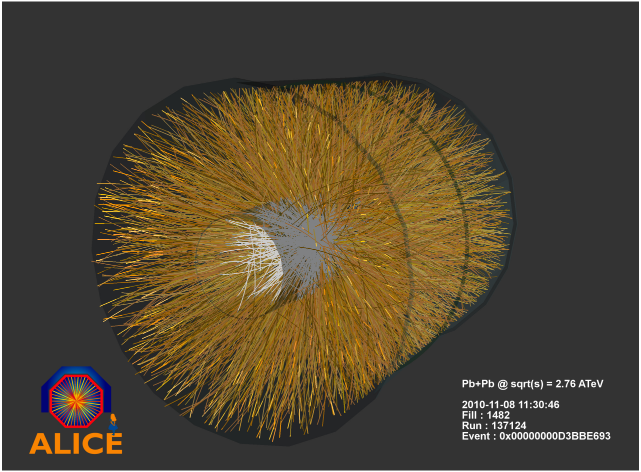
Many elements in modeling, "circumstantial evidence"

Many elements in modeling, “circumstantial evidence”

Fireball

We start with the end of the evolution to show how thermal ideas work

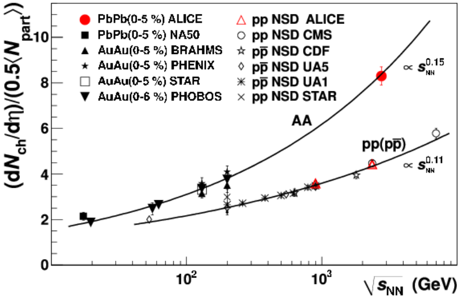
An ALICE event



In relativistic heavy-ion collisions thousands of particles are formed in a single collision

Multiplicities

Growth with $\sqrt{s_{NN}}$, not superposition of p+p



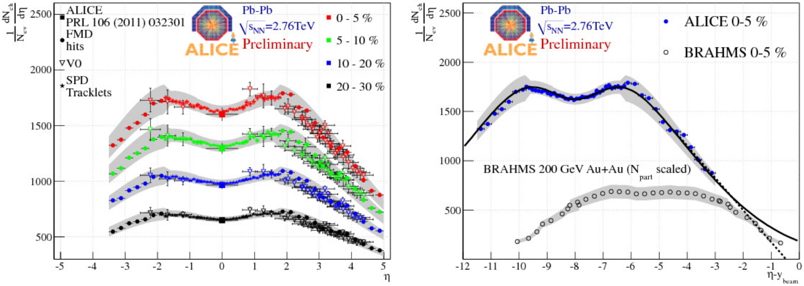
[Aamodt et al. (ALICE) PRL 105(2010)252301]

– $\sqrt{s_{NN}}$ – energy per nucleon pair in their center-of-mass (CM) frame

– pseudorapidity $\eta = \frac{1}{2}\log\frac{p_\parallel + p_\perp}{p_\parallel - p_\perp} = -\log[\mathrm{tg}(\theta/2)]$, rapidity $y = \frac{1}{2}\log\frac{E+p_\perp}{E-p_\perp}$

– $N_{\rm part}$ - number of participating nucleons

Spectra in pseudorapidity



Kinematic range in rapidity is $\sim y_{\text{beam}} = \text{arccosh}[\sqrt{s_{NN}}/(2m_N)]$

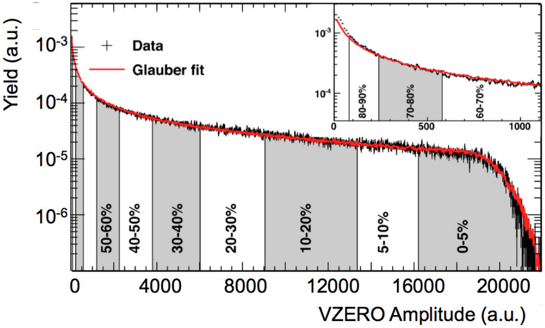
(≃ 8 at 2.76 TeV, ≃5.4 at 200 GeV)

$$y = \frac{1}{2}\log\left(\frac{\sqrt{p_T^2\cosh^2(\eta)+m^2}+p_T\sinh(\eta)}{\sqrt{p_T^2\cosh^2(\eta)+m^2}-p_T\sinh(\eta)}\right) \le \eta, \; \frac{dy}{d\eta} = \frac{p_T\cosh(\eta)}{\sqrt{m^2+p_T^2\cosh^2(\eta)}} \le 1$$

For identified particles y is typically used. Note that η distributions are wider and lower. The central dip is of kinematic origin: $dN/d\eta = dy/d\eta \, dN/dy$

1600 of charged hadrons per unit of η! (~ 2400 for all hadrons)

Centrality



[ALICE, arXiv:1306.3130]

To a very good approximation, in large systems

$$c \simeq \frac{\pi b^2}{\sigma_{\rm inel}^{AB}} \simeq \frac{b^2}{(R_A + R_B)^2},$$

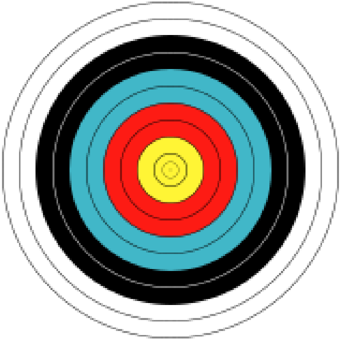
where R_A and R_B are the radii of the nuclei

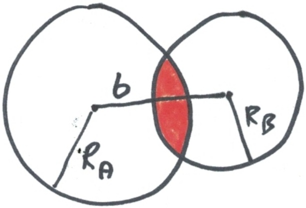
[WB, Florkowski, PRC 65(2002)024905]

Archery competition

probability $\sim 2\pi b db \rightarrow$ cumulative distribution function:

$$c(b) = \int_0^b P(b')db' = \frac{b^2}{b_{\max}^2} = \frac{b^2}{(R_A + R_B)^2}$$





Statistical (thermal) model of hadronization

[Fermi, Pomeranchuk, Hagedorn, Kapusta, Koch, Muller, Rafelski, Sollfrank, Heinz, Becattini, Braun-Munzinger, Stachel, Redlich, Cleymans, Gazdzicki, ...]

Large multiplicities $\to$ statistical description – the higher collision energies, the better!

By counting all the particles we cannot obtain the temperature T, as we do not know the volume V. Idea: look at identified hadron multiplicities and take ratios to divide out V.

For the simplified case of the Boltzmann distribution ($\hbar = k_B = c = 1$)

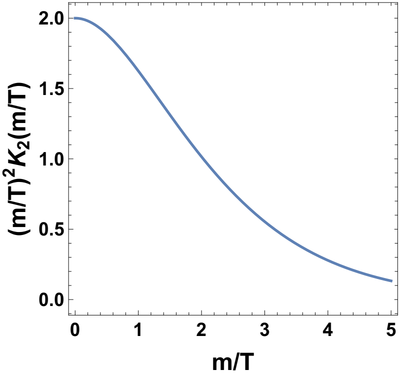
$$N = V\int \frac{d^3p}{(2\pi)^3} e^{-(E-\mu)/T} = V e^{\mu/T} \int \frac{d^3p}{(2\pi)^3} e^{-\sqrt{m^2+p^2}/T} = \frac{VT^3}{2\pi^2} e^{\mu/T} \left(\frac{m}{T}\right)^2 K_2\left(\frac{m}{T}\right)$$

In chemical equilibrium

$$\mu = B\mu_B + S\mu_S + I_3\mu_{I_3}$$

Bessel functions

Modified Bessel function of the second kind



- higher $m \rightarrow$ lower yield of a species

Chemistry

For boost-invariant systems (approximately satisfied at midrapidity) the ratio of abundances of species i and j is

$$\frac{dN_i/dy}{dN_j/dy} = \frac{N_i}{N_j} \simeq \frac{2J_i+1}{2J_j+1} e^{(\mu_i-\mu_j)/T} \frac{m_i^2 K_2(m_i/T)}{m_j^2 K_2(m_j/T)}$$

For instance

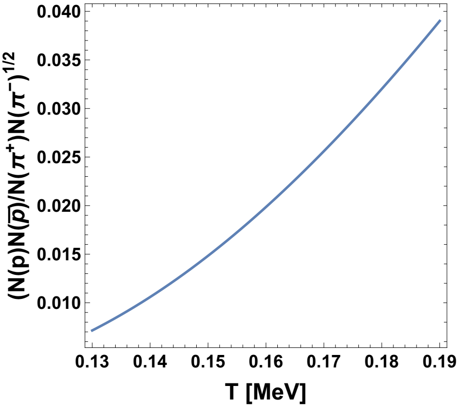
$$\frac{p}{\bar{p}} = e^{2\mu_B/T}, \; \frac{K^+}{K^-} = e^{2\mu_S/T}, \; \frac{\pi^+\pi^-}{p\,\bar{p}} = \left(\frac{1}{2}\frac{m_\pi^2 K_2(m_\pi/T)}{m_p^2 K_2(m_p/T)}\right)^2$$

3 equations allow to find the thermal parameters T, μ_B, μ_S.

In practice μ_S and μ_{I_3} are determined by requiring that the strangeness of the system is zero, and the ratio of the baryon number to the electric charge densities is the same as in the colliding nuclei $\rightarrow$ solve overdetermined system for many ratios in the χ^2 sense

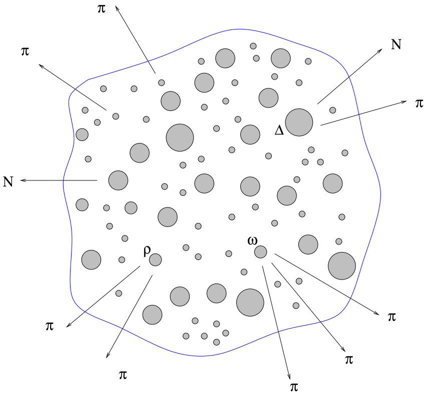
(V should be treated as an independent parameter [Becattini, arXiv:0707.4154])

μ-independent combination



- lower $T \rightarrow$ more difference between species

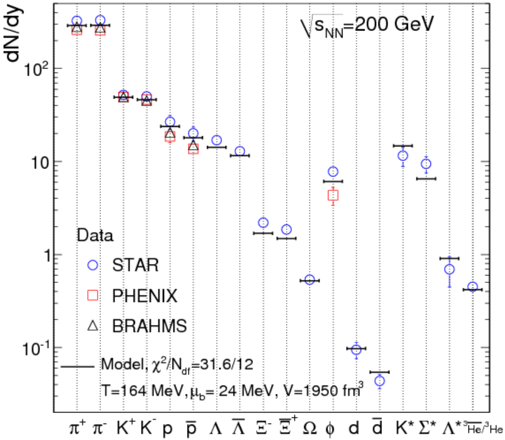
Resonance decays



very important: ∼75% of pions come from resonance decays (!)

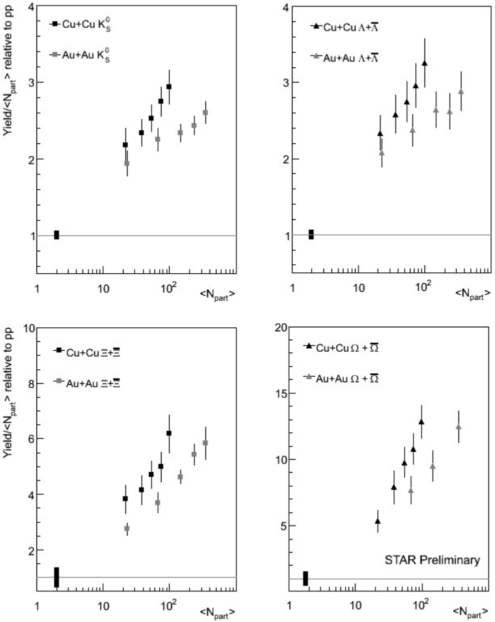
SHARE, THERMUS - publicly available codes carrying out statistical hadronization withe decays of all resonances from Particle Data Tables

RHIC success



[Andronic et al., PLB 697(2011)203, arXiv:1010.2995]

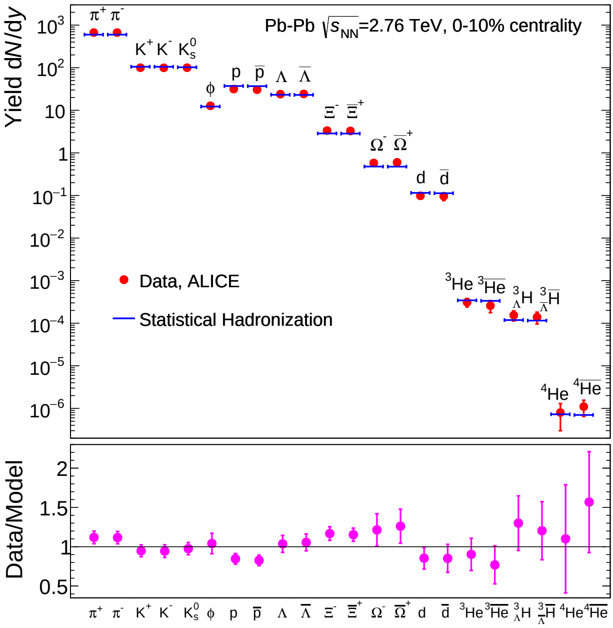
Strangeness production/enhancement



data from NA57

see, e.g., a recent review by Koch, Müller, Rafelski, Int. J. Mod. Phys. A32 (2017) 1730024

LHC



[A. Andronic, P. Braun-Munzinger, K. Redlich, J. Stachel, Nature 561 (2018) 321]

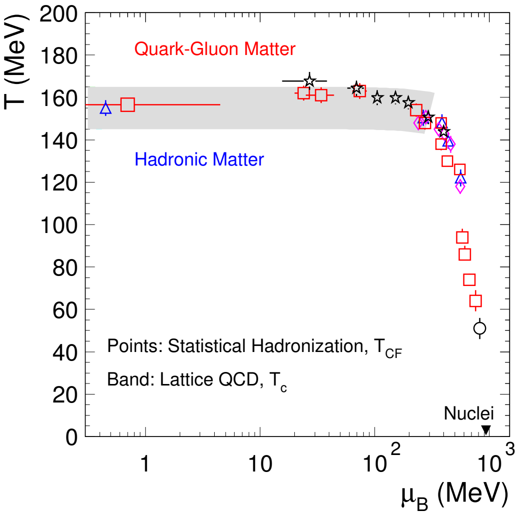
Yields of light nuclei

9 orders of magnitude!

- Fundamentally not possible to understand the production of the light nuclei (albeit described) in the statistical hadronization model. Too weakly bound to achieve thermal equilibrium during the fireball's lifetime. Too large compared to the inter-particle spacing.
- Recent quantitative and detailed discussion: [Y. Cai, T. D. Cohen, B. A. Gelman, Y. Yamauchi, arXiv:1905.02753]
- Alternative approach: coalescence, see [S. Bazak, S. Mrówczyński, Mod. Phys. Lett. A33 (2018) 1850142]

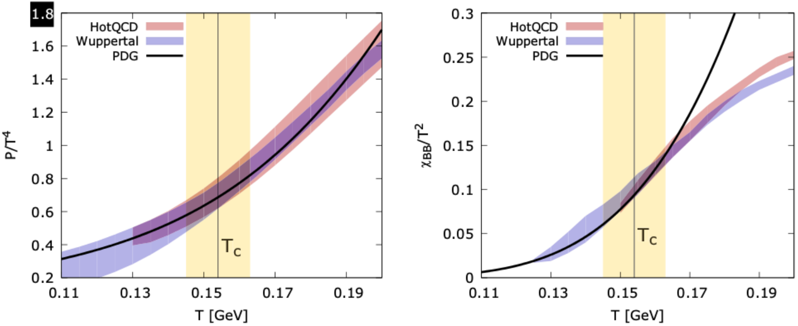
Open problem!

T-μ_B diagram



[A. Andronic, P. Braun-Munzinger, K. Redlich, J. Stachel, Nature 561 (2018) 321]

Hadron resonance gas vs LQCD



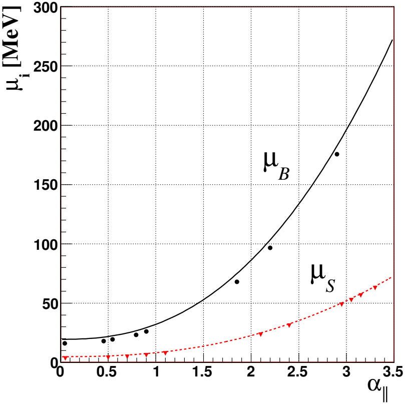
[Lo, Marczenko, Redlich, Sasaki, Eur.Phys. J. A52 (2016) 235]

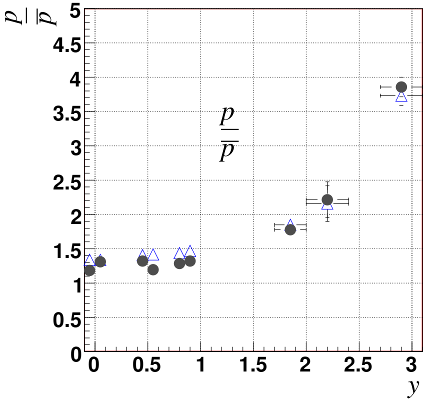
Other effects

- To satisfy the baryon number and strangeness conservation laws $\rightarrow$ canonical ansatz
- To satisfy the energy conservation $\rightarrow$ microcanonical ansatz - relevant for systems with small numbers of particles
- Short-range repulsion, excluded volume
- Incomplete equilibrium (Rafelski's fugacity factors)
- Hierarchy of freeze-outs, based on hierarchy of cross sections

Off mid-rapidity

μ_i depend on the spatial rapidity $\alpha_\parallel = \frac{1}{2}\log\left(\frac{t+z}{t-z}\right)$



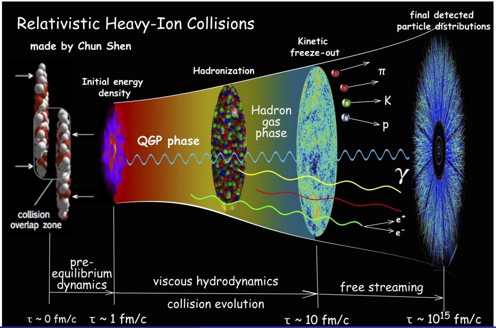


[B. Biedroń, WB, PRC 75(2007)054905]

Summary of thermal approach

- Dense system with numerous collisions
- Estimate: after freeze-out typically one collision per particle (as it should be)
- Thermal and chemical equilibrium (at FO) explain the hadron abundances
- The embarrasing success of light (hyper)nuclei production
- Resonances crucial, HRG
- HRG compares reasonably well to LQCD

The system (at least near the end of the evolution) is close to thermal and chemical equilibrium



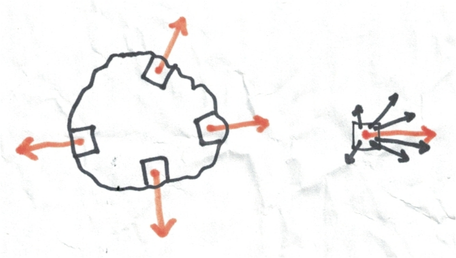
Expansion and flow

The key concept of the approach to collectivity

Flow (and jet quenching) are the two major discoveries of the ultra-relativistic heavy-ion program!

Inevitability of expansion

No container! → the fireball expands (and cools down)
Think in terms of fluid - dense medium, short mean-free path, multiple rescattering



Flow is generic to a system with copious rescattering: hydro, transport, ...

Obviously, the expansion affects the momentum spectra, as the velocity of the fluid element yields the Doppler effect

Frye-Cooper formula

One needs to collect particles (hadrons) produced from various fluid elements. For a single element of volume V at rest

$$\frac{d^3N_i}{d^3p} = V\, f_i(E)$$

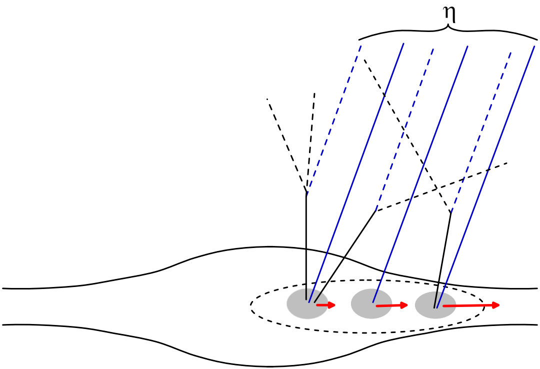
Rewrite invariantly: $u^\mu = \frac{1}{\sqrt{1-v^2}}(1, \vec{v})$, at rest $u^\mu = (1,0,0,0)$
$E = p^0 \to p \cdot u$, E/d^3p – Lorentz invariant $\to$

$$\frac{Ed^3N_i}{d^3p} = \int d^3\Sigma_\mu(x) p^\mu\, f_i[p \cdot u(x)]$$

where $d\Sigma_\mu(x)$ describes the element of a 3D freeze-out hypersurface on the 4D coordinate space

Example

Collecting along pseudorapidity:



Hypersurface

$$d^3\Sigma_\mu(x) = \epsilon_{\mu\alpha\beta\gamma} \frac{\partial x^\alpha}{\partial p} \frac{\partial x^\beta}{\partial q} \frac{\partial x^\gamma}{\partial r} dp\, dq\, dr$$

x^α - coordinates in space-time, p, q, r - parameters of a 3-dim. hypersurface
Examples:

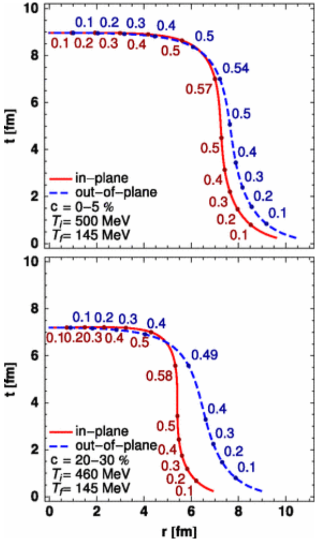
- $x = p$, $y = q$, $z = r \to d^3\Sigma_\mu(x) = dx\, dy\, dz$
- Boost-inv. freeze-out [Schnedermann, Sollfrank, Heinz, PRC48 (1993) 2462]
  $x^\mu = (t, x, y, z) = \left(\tau(\zeta)\cosh\alpha_\parallel, \rho(\zeta)\cos\phi, \rho(\zeta)\sin\phi, \tau(\zeta)\sinh\alpha_\parallel\right) \to$
  $d^3\Sigma^\mu = \left(\frac{d\rho}{d\zeta}\cosh\alpha_\parallel, \frac{d\tau}{d\zeta}\cos\phi, \frac{d\tau}{d\zeta}\sin\phi, \frac{d\rho}{d\zeta}\sinh\alpha_\parallel\right)\rho(\zeta)\tau(\zeta)d\zeta d\alpha_\parallel d\phi$

With a complementary hypothesis for u^μ one may obtain model results without running hydro

Hydrodymanics provides $d^3\Sigma^\mu$ and u_μ when a freeze-out condition is met (typically, $T = T_f$) as a numerical output

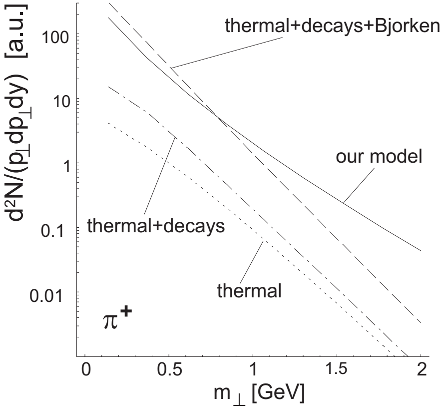
More discussion in [W. Florkowski, WB, Acta Phys.Polon. B35 (2004) 2895]

Freeze-out from one-shot perfect hydrodynamics



boost-inv. case for RHIC@200 GeV, r - transverse radius, t - time, labels - v/c

[WB, M. Chojnacki, W. Florkowski, A. Kisiel, PRL 101 (2008) 022301]



– thermal: pion spectrum from a static fireball

– thermal+decays: initial and secondary pions, which lead to a decrease of the inverse slope

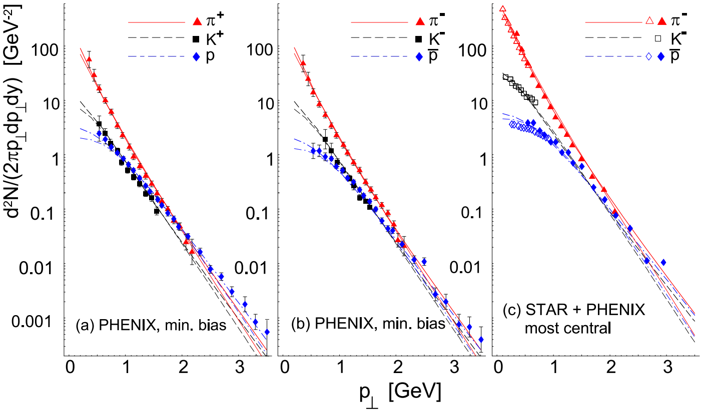
– Bjorken: pure longitudinal expansion → redshift, as all fluid elements move away from the observer → cooling of the spectrum.

– our model: transverse flow added, hence some fluid elements move towards the observer → blueshift

Radial flow → blueshift and redshift → convex

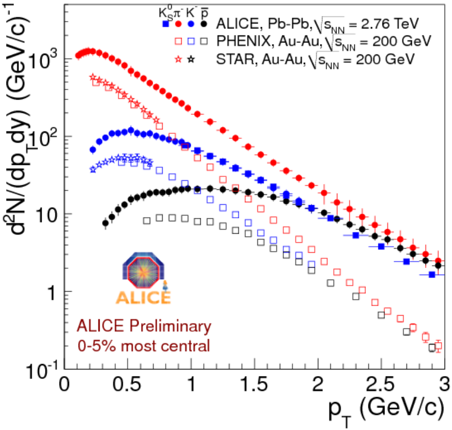
[WB, W. Florkowski, PRL 87(2001)272302]

Example p_T spectra @130 GeV



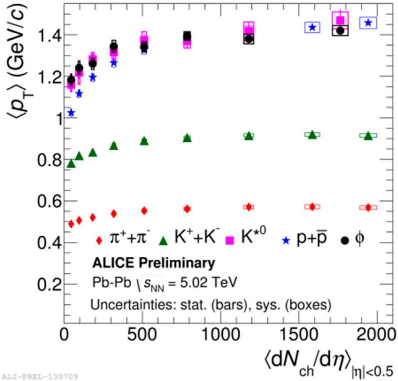
$T_f = 165$ MeV, $\mu_B = 41$ MeV [WB, W. Florkowski, PRL 87(2001)272302]

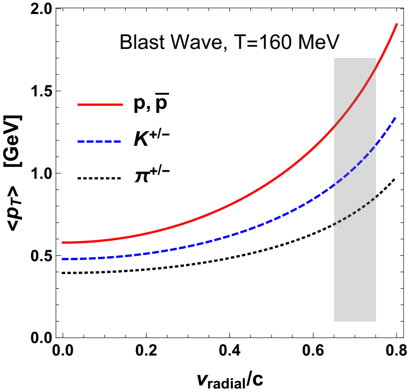
– mass hierarchy (from thermal motion and from transverse flow)



More flow with increasing energy!

Mean transverse momenta





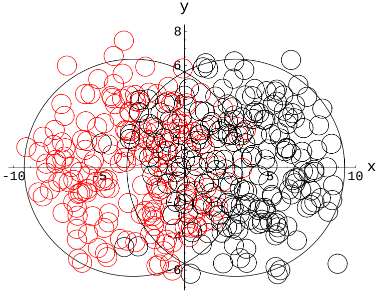
- Thermal component
- Radial flow component

Blast wave model: $\rightarrow$ enhancement of the mass hierarchy

$$\frac{dN}{dy\, d^2 p_T} = \text{const} \times m_T I_0 \left(\frac{p_T \sinh \alpha}{T} \right) K_1 \left(\frac{m_T \cosh \alpha}{T} \right), \quad v_r/c = \tanh \alpha$$

Initial geometry

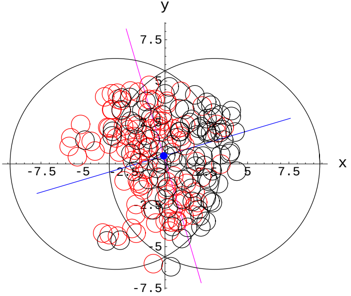
Au+Au collision at RHIC
(view along the beam)



1. Participants determine the geometry of the overlap region
2. Initial entropy distribution in more microscopic approaches (IP Glasma) also follows the geometry of the overlap region
3. Strong radial flow
4. Initial eccentricity → anisotropic flow of hadrons [Ollitrault 1992]

Initial geometry

Au+Au collision at RHIC
(view along the beam)



1. Participants determine the geometry of the overlap region
2. Initial entropy distribution in more microscopic approaches (IP Glasma) also follows the geometry of the overlap region
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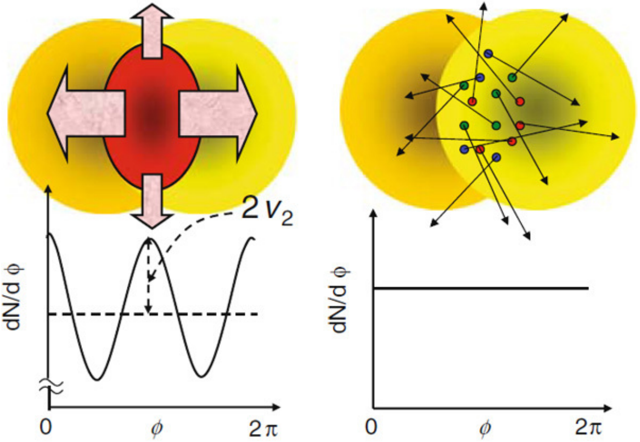
Rescattering/collectivity essential



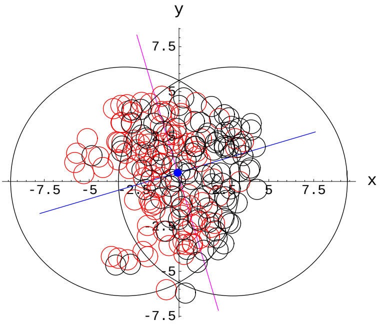
[ALICE]

In each event, define the harmonic flow coefficients and event-plane angles:

$$dN/d\phi \propto 1 + 2\sum_n v_n \cos[n(\phi - \Psi_n)]$$

Fluctuations

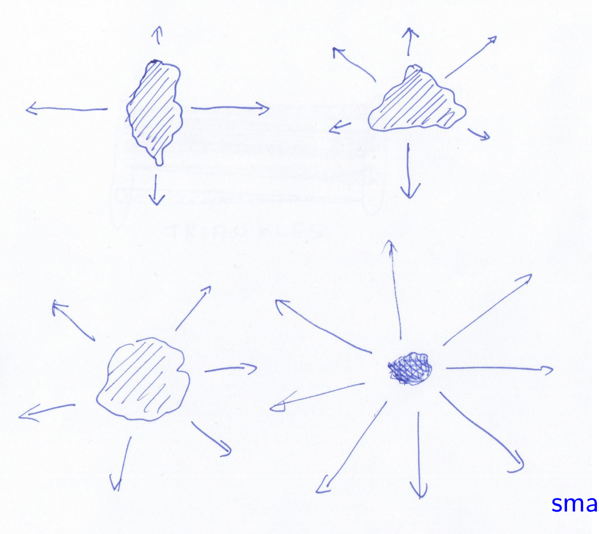
Collapse of the nuclear wave function → each Little Bang different



1. Higher Fourier components appear
2. Odd harmonics also show up, triangular flow
3. Fluctuations dominant for central A+A and for *small systems*, such as p+A (see later on)

New thinking since [Miller and Snellings 2003]

Collectivity: shape/size – flow transmutation

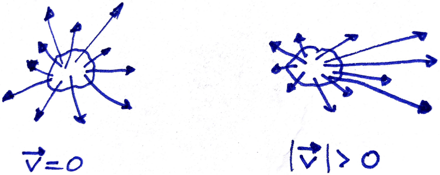


smaller $\rightarrow$ faster

Any rescattering will do!

Collimation from the Doppler effect

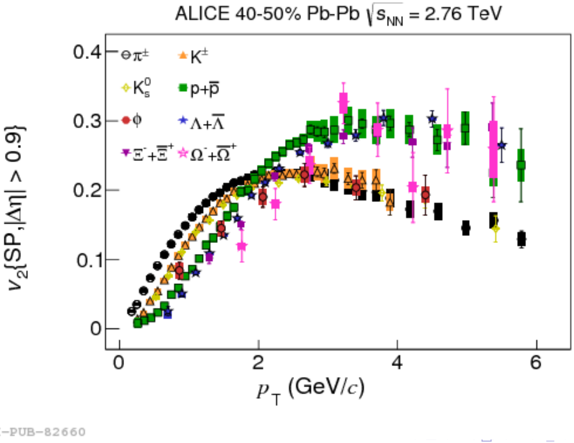
- Emission from a fast moving element of fluid
- Collimation of hadrons (increasing with mass)



Multi-particle correlations in the azimuth are used in the cumulant or other methods to extract the flow coefficients without the non-flow contamination (from jets, resonance decays, . . .)

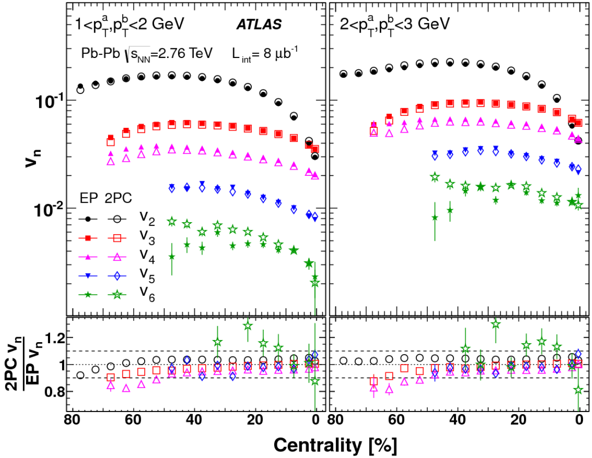
[Borghini, Ollitrault 2001]

1. Mass ordering of harmonic flow coefficients v_n
2. Higher harmonics suppressed
3. Near-side ridge (discussed later on) - considered the "proof" of harmonic flow

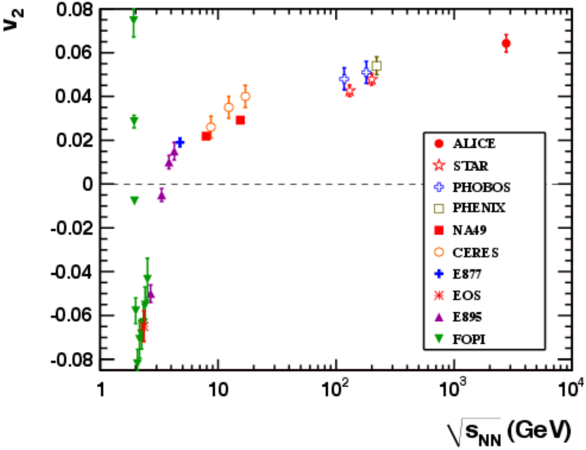


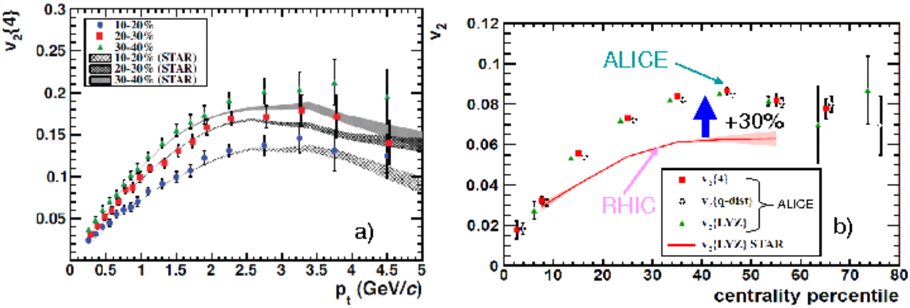
Features of harmonic flow

1. Mass ordering of harmonic flow coefficients v_n
2. Higher harmonics suppressed
3. Near-side ridge (discussed later on) - considered the "proof" of harmonic flow



v_2 vs $\sqrt{s_{NN}}$





[ALICE, PRL 105(2010)252302]

At the LHC the differential elliptic flow is the same as at RHIC, but "sampling" is at higher p_T

Flow (radial and harmonic) leads to correct phenomenology of the p_T spectra and v_n, with proper dependence on the geometry (shape-flow transmutation), collision energy, and mass hierarchy

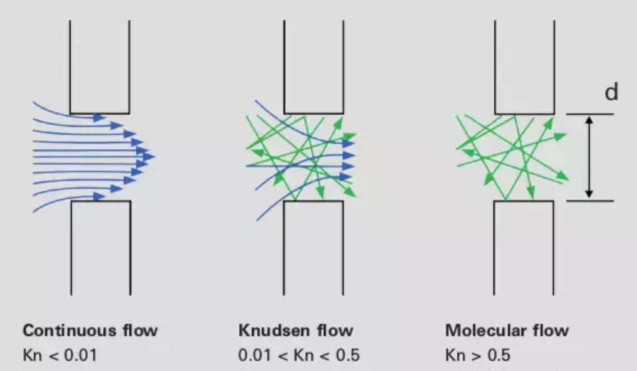
Hydrodynamics

What produces the flow (collectivity)?

Flow (and jet quenching) are the two major discoveries of the ultra-relativistic heavy-ion program!

Basics

- Fluid $\equiv$ substance that cannot resist any shear force (gas, liquid, plasma), continuously deforms
- size of particles $\ll$ fluid element $\ll$ size of the system
- Knudsen number: $\mathrm{Kn} = \lambda/L$, λ mean free path, L - system's size
- $\mathrm{Kn} \ll 1 \;\; \rightarrow$ fluid description



[Wikipedia]

Perfect hydrodynamics (no viscosity)

Local thermal equilibrium at point x: $T^{\mu\nu}(x) = \int \frac{d^3p}{p^0} p^\mu p^\nu f_{\rm eq}(x, u\cdot p; T, \mu)$

Landau definition of the four-velocity of the fluid

$$T^{\mu\nu}(x)u_\mu(x) = \lambda(x)u^\nu(x)$$

$$u_\mu u^\mu = 1, \quad u^\mu = \gamma(1, v_x, v_y, v_z) = \frac{1}{\sqrt{1-v^2}}(1, v_x, v_y, v_z)$$

The perfect hydro form follows (u^μ and $g^{\mu\nu}$ for disposal):

$$T^{\mu\nu} = (\varepsilon + P)u^\mu u^\nu - P g^{\mu\nu} \quad (\lambda = \varepsilon)$$

In the fluid element's rest frame $u^\mu = (1,0,0,0)$

and $T^{\mu\nu} = \begin{pmatrix} \varepsilon & 0 & 0 & 0 \\ 0 & P & 0 & 0 \\ 0 & 0 & P & 0 \\ 0 & 0 & 0 & P \end{pmatrix}$

The perfect hydro equations

Energy-momentum conservation →

$$\partial_\mu T^{\mu\nu}(x) = 0,$$

4 equations for 5 unknown functions: $\vec{v}$, ε, P – need the equation of state linking ε and P to close the system

- Example: massless particles → $\varepsilon = 3P$

Entropy is conserved

$$\partial_\mu (s u^\mu) = 0$$

Similarly for conserved charges $\partial_\mu (n u^\mu) = 0$

Sound velocity

Consider perturbation on a static background

$$\varepsilon(x) = \varepsilon_0 + \delta\varepsilon(x), \;\; P(x) = P_0 + \delta P(x)$$

and a small velocity $u^\mu = (1, \delta v_x, \delta v_y, \delta v_z)$ (recall the 5 variables). To first order

$$T^{\mu\nu} = \begin{pmatrix} \varepsilon_0 + \delta\varepsilon & (\varepsilon_0+P_0)\delta v_x & (\varepsilon_0+P_0)\delta v_y & (\varepsilon_0+P_0)\delta v_z \\ (\varepsilon_0+P_0)\delta v_x & P_0+\delta P & 0 & 0 \\ (\varepsilon_0+P_0)\delta v_y & 0 & P_0+\delta P & 0 \\ (\varepsilon_0+P_0)\delta v_z & 0 & 0 & P_0+\delta P \end{pmatrix}$$

$$\partial_0 T^{00} + \partial_i T^{i0} \to \partial_t \delta\varepsilon + (\varepsilon_0 + P_0)\vec{\nabla}\cdot\delta\vec{v}$$
$$\partial_0 T^{0j} + \partial_i T^{ij} \to (\varepsilon_0 + P_0)\delta\partial_t v^j + \nabla^j \delta P$$

Combining, $$\partial_t^2 \delta\varepsilon - \nabla^2 \delta P = 0$$

For zero chemical potentials there is only one thermodynamic parameter T. Then $\delta P = \frac{dP}{d\varepsilon}\delta\varepsilon = c_s^2(T)\delta\varepsilon$. We thus arrive at the wave equation

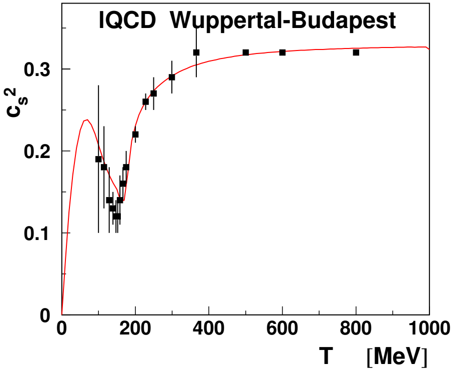
$$\partial_t^2 \delta\varepsilon - c_s^2 \nabla^2 \delta\varepsilon = 0$$

with c_s being the sound velocity, dependent on T (or ϵ)

A simple form hydro for $\mu = 0$

In the case of vanishing chemical potentials one may rewrite the perfect hydro equations in the form, e.g.,

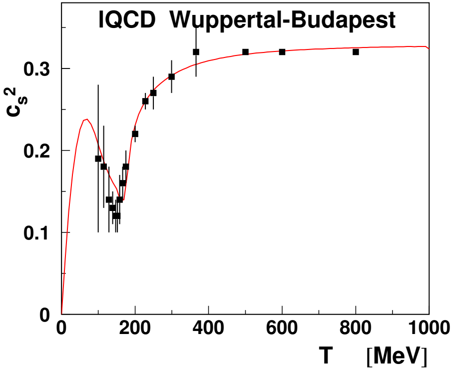
$$s\frac{du^\nu}{d\tau} = c_s^2(s)(g^{\mu\nu} - u^\mu u^\nu)\partial_\mu s, \quad \tau^2 = t^2 - \vec{x}^2$$



- no first order phase transition
- no shock or rarefaction waves (!)
- laminar flow

[M. Chojnacki, W. Florkowski 2007]

As the system cools down, quarks and gluons are gradually replaced with hadrons



- Hadronization is conveniently carried over "behind the back", hidden in the eq. of state
- Fluid changed into particles via the Frye-Cooper mechanism

Bjorken flow

Purely longitudinal expansion $u^\mu = \frac{1}{\tau}(t, 0, 0, z)$, assumed boost invariance involves dependence on the proper time $\tau = \sqrt{t^2 + z^2}$ only
$\partial_\mu u^\mu = \frac{1}{\tau}$, $\partial_\mu \tau = u_\mu$

$$0 = \partial_\mu (s u^\mu) = \frac{ds(\tau)}{d\tau} + \frac{s(\tau)}{\tau} \ \rightarrow\ s(\tau) = s(\tau_0)\frac{\tau_0}{\tau}$$

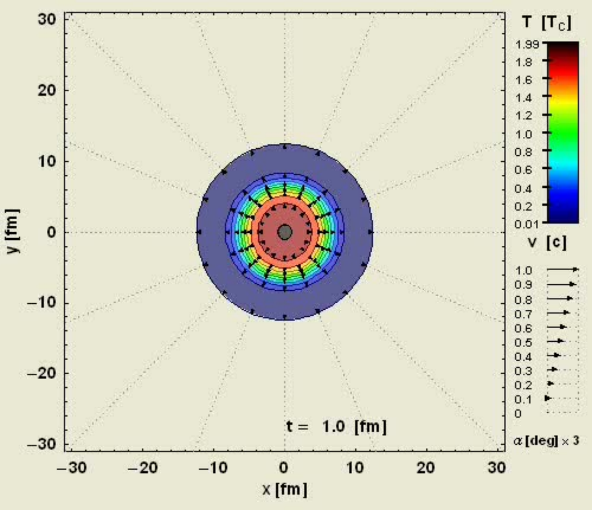
Thermodynamic relations for $\mu = 0$: $\varepsilon + P = Ts$, $d\varepsilon = T\,ds$, $dP = s\,dT$, from where (for ultra-relativistic particles, where $P = c_s^2 \varepsilon$)

$$\varepsilon(\tau) = \varepsilon(\tau_0)\left(\frac{\tau_0}{\tau}\right)^{1+c_s^2}, \ \ T(\tau) = T(\tau_0)\left(\frac{\tau_0}{\tau}\right)^{c_s^2}$$

$\rightarrow$ estimates based on entropy conservation per unit of rapidity. From known experimental hadronic yields one infers $\varepsilon_{\mathrm{QGP}}(\tau_0) \simeq 4$ GeV/fm^3

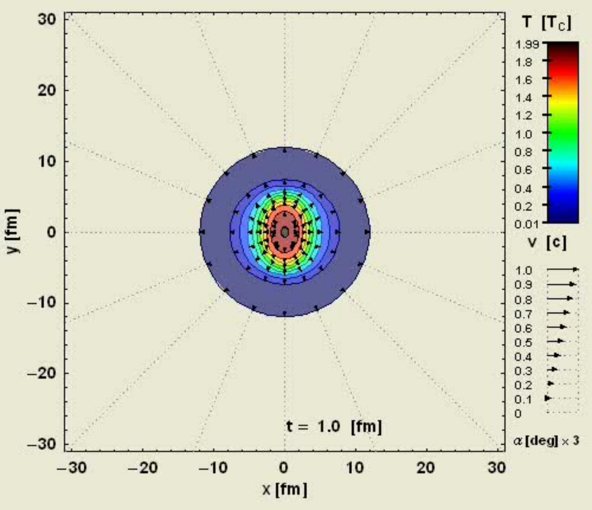
Relativistic 2+1D perfect hydro

central (0-20%)



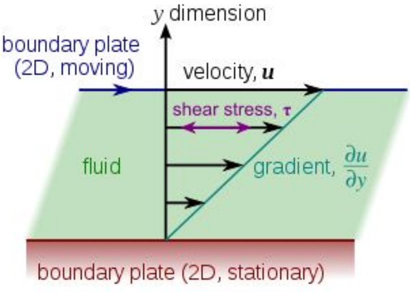
[M. Chojnacki, W. Florkowski]

non-central (40-60%)



[M. Chojnacki, W. Florkowski]

Kinetic arguments for viscosity



[Wikipedia]

$$F/A = \eta \partial_y v_x$$

$$Re = \frac{\rho v L}{\eta}$$

Navier-Stokes equations:

$$\rho \left(\partial_t v_i + \vec{v} \cdot \vec{\nabla} v_i \right) = -\nabla_i P + \eta \nabla^2 v_i$$

one of Millennium Problems!

Various materials

material	η [Pa s]	η/s [$\hbar/k_B$]
water	3×10^{-4}	8
honey	1000	5×10^{7}
superfluid ^{4}He	10^{-6}	2
ultra-cold ^{6}Li	$< 10^{-15}$	< 0.3
QGP	$< 2 \times 10^{11}$	< 0.4
pitch	2×10^{11}	10^{16}

U. of Queensland, 8 drops since 1927, Ig Nobel prize

Bounds on shear viscosity

dilute gas: $\eta = \frac{1}{3} npl$ (density $\times$ momentum $\times$ mean free path)

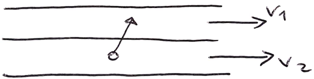
Quantum limit

Heisenberg uncertainty principle: $pl \geq \hbar$ and $s \sim k_B n \quad \rightarrow \quad \eta/s \geq\sim \hbar/k_B$

[P. Danielewicz and M. Gyulassy, PRD 31 (1985) 53]

KSS bound based on AdS/CFT: $\eta/s \geq \frac{1}{4\pi}\hbar/k_B$

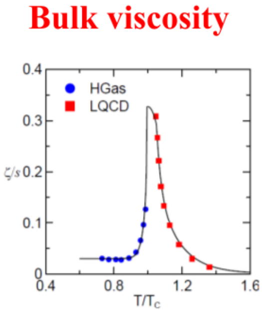
[P. Kovtun, D. T. Son, and A. O. Starinets, PRL 94 (2015) 111601]



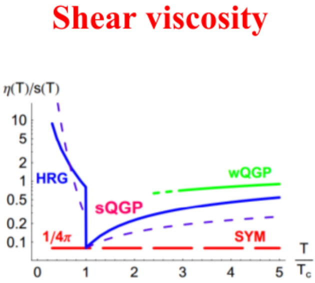
- $l = \frac{1}{n\sigma} \rightarrow \eta = \frac{p}{3\sigma_{\text{el}}}$ – counterintuitive!

Shear and bulk

shear viscosity η – resistance to deformation
bulk viscosity ζ – resistance to expansion (volume change)



Karsh&Kharzeev&Tuchin
Noronha&Noronha&Greiner



Hirano&Gyulassy

[from G. Denicol]

Adding viscosities into relativistic hydro

Recent review: [P. Romatschke, U. Romatschke, arXiv:1712.05816]

Israel-Stewart second-order hydro: perfect fluid

$$T_0^{\mu\nu} = (\epsilon + p)u^\mu u^\nu - p g^{\mu\nu}$$

+ stress corrections from shear π (traceless) and bulk Π viscosities

$$T^{\mu\nu} = T_0^{\mu\nu} + \pi^{\mu\nu} + \Pi\Delta^{\mu\nu}$$

$$\partial_\mu T^{\mu\nu} = 0$$

The viscous corrections are solutions of 6 additional equations:

$$\Delta^{\mu\alpha}\Delta^{\nu\beta}u^\gamma\partial_\gamma\pi_{\alpha\beta} = \frac{2\eta\sigma^{\mu\nu} - \pi^{\mu\nu}}{\tau_\pi} - \frac{4}{3}\pi^{\mu\nu}\partial_\alpha u^\alpha$$

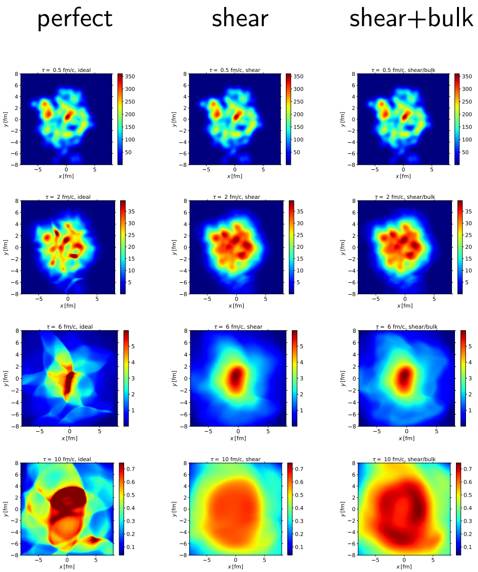
$$u^\gamma\partial_\gamma\Pi = \frac{-\zeta\partial_\gamma u^\gamma - \Pi}{\tau_\Pi} - \frac{4}{3}\Pi\partial_\alpha u^\alpha$$

$$\Delta^{\mu\nu} = g^{\mu\nu} - u^\mu u^\nu, \ \nabla^\mu = \Delta^{\mu\nu}\partial_\nu$$

$$\sigma_{\mu\nu} = \frac{1}{2}\left(\nabla_\mu u_\nu + \nabla_\nu u_\mu - \frac{2}{3}\Delta_{\mu\nu}\partial_\alpha u^\alpha\right)$$

The relaxation time is taken as $\tau_\pi = \tau_\Pi = \frac{3\eta}{Ts}$

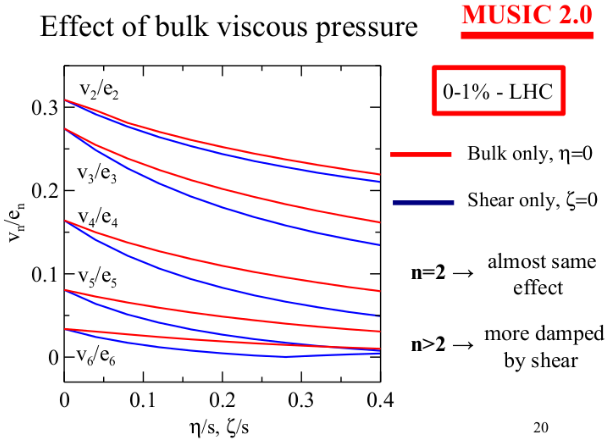
Quenching of flow



- Quenching of flow with viscosity
- Increasing with the Fourier rank
- Sets limits on viscosity, which is close to the KSS bound $\eta/s = 1/4\pi$
- ... but many other model parameters

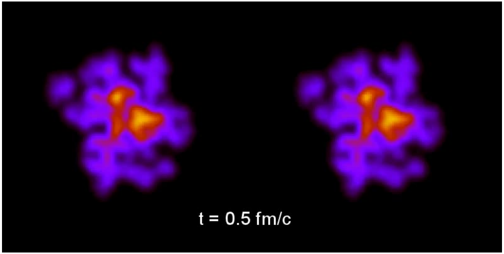
Figure:
[Bazow, Heinz, Strickland 2016]

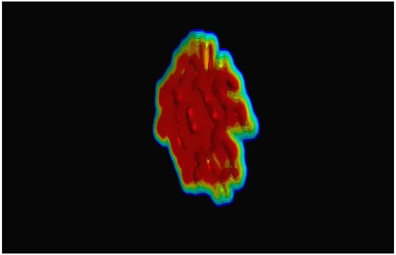
Damping of flow



[from G. Denicol]

3D numerics





[B. Schenke https://quark.phy.bnl.gov/∼bschenke]

[other codes]

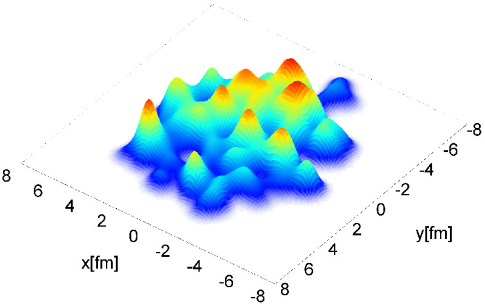
Initial conditions

- Initial value problem for partial differential equations $\rightarrow$ need to choose initial conditions for the functions on a time-like hypersurface, e.g, with constant $\tau = \sqrt{t^2 - z^2}$
- These conditions fluctuate event-by-event ...
- ... and are carried over to freeze-out approximately deterministically
- τ must be short (a fraction of fm) for sufficient flow to develop

However, on the general grounds of the fluctuation-dissipation theorem, hydro must also bring in some fluctuations

[J. I. Kapusta, B. Mueller, M. Stephanov, Phys.Rev. C85 (2012) 054906 – Bjorken flow
L. Yan, H. Grönqvist, JHEP 1603 (2016) 121 – Gubser flow:
"...the effect of hydrodynamical noise on flow harmonics is found to be negligible, especially in the ultra-central Pb-Pb collisions ..."]

Glasma initial conditions



[Schenke, Tribedy, Venugopalan, PRL 108(2012)252301, arXiv:1202.6646]

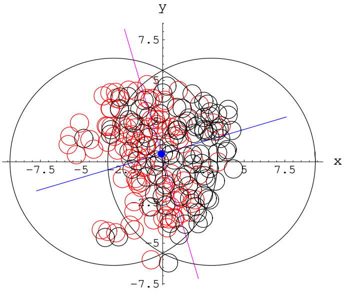
[Białas, Błeszyński, Czyż, NPB 111 (1976) 461]

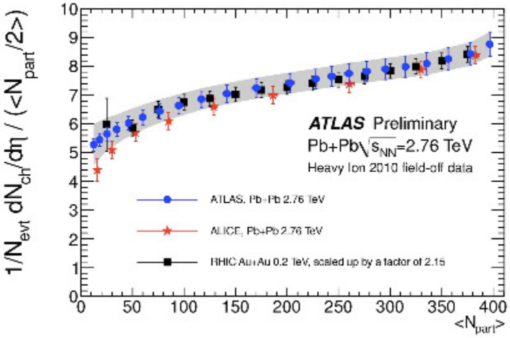
wounded + binary: $N \sim (1-\alpha)N_w/2 + \alpha N_{\rm bin}$, $\alpha \sim 0.14$

[D. Kharzeev, M. Nardi, PLB 507 (2001) 121]

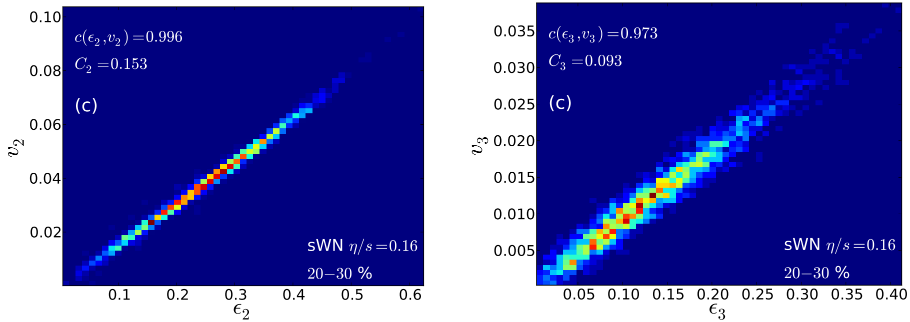
soft – wounded (a nucleon gets wounded only once)

hard – binary





Proportionality of flow to eccentricity



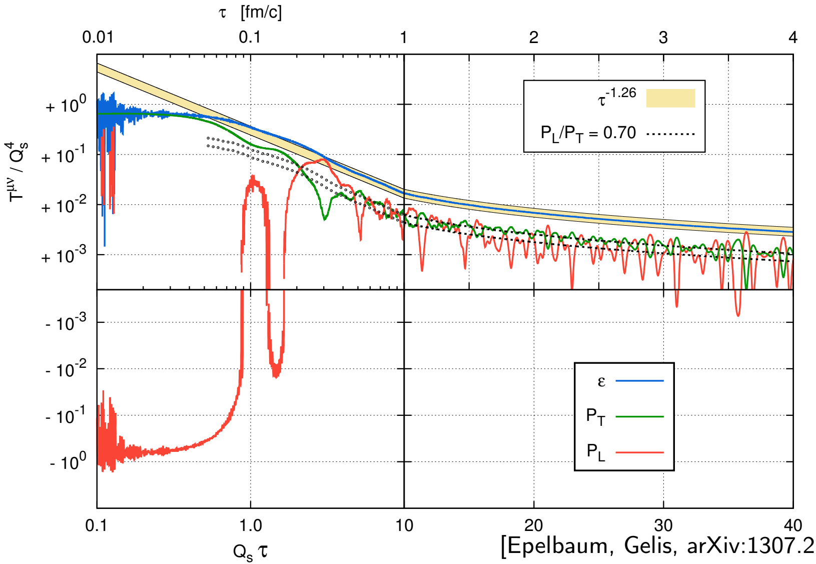
[Niemi, Denicol, Holopainen, Huovinen 2012]

"Hydro without hydro" – linearity of the shape-flow transmutation

$$v_n = \kappa_n \epsilon_n, \quad (n = 2, 3)$$

- κ_n depend on the collision energy, multiplicity, viscosity ...
- Approximate linearity allows us to build scale-less combinations independent of the response coefficient κ_n (see later)

Isotropization in Color Glass Condensate (with $SU_c(2)$)

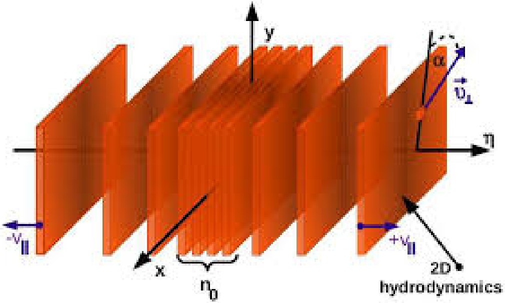


[Epelbaum, Gelis, arXiv:1307.2214]

[Review: Berges, Blaizot, Gelis, J. Phys. G 39(2012)085115]

Longitudinal-transverse anisotropy

[Florkowski, Ryblewski, 2008]

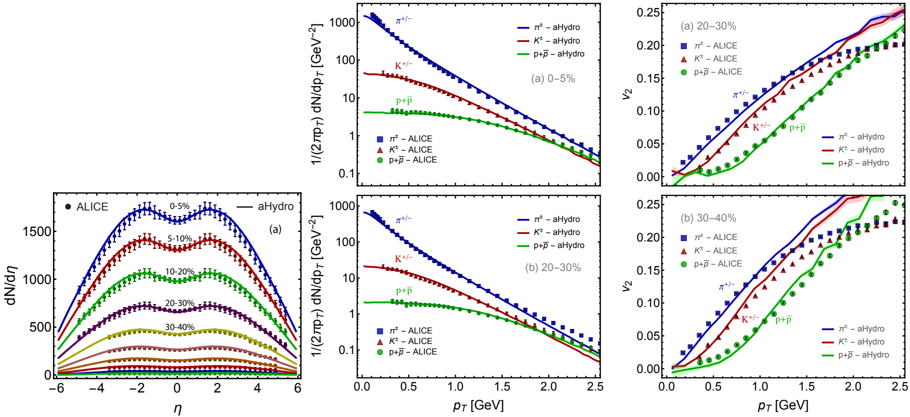


Anisotropic hydro

[see also Babak Kasmaei's talk]

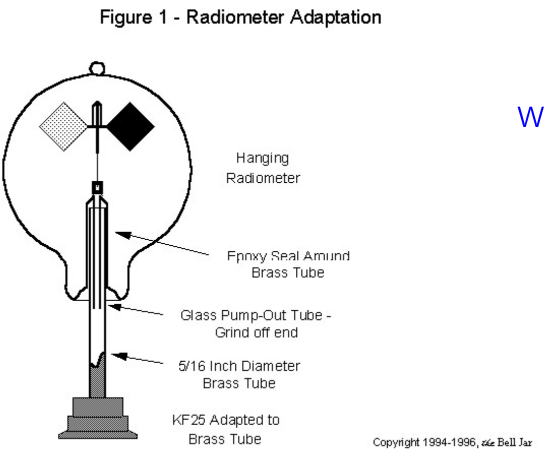
One can obtain satisfactory phenomenology in approaches without isotropization, where $P_T \geq P_L$

[Alqahtani, Nopoush, Ryblewski, Strickland, PRL 119 (2017) 042301]



That things are nontrivial...

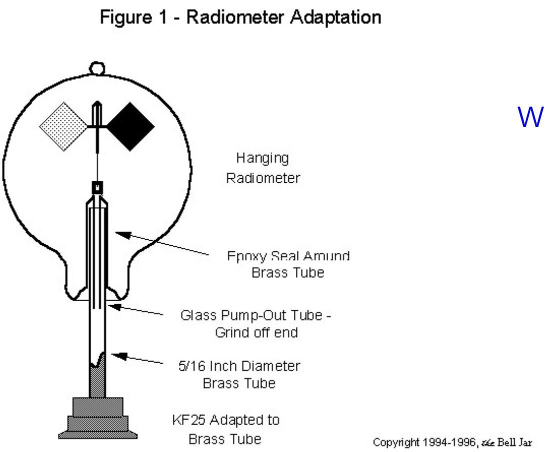
The Crooks radiometer



Which way will it turn?

That things are nontrivial...

The Crooks radiometer



Which way will it turn?

- Not the light pressure!
- Not Navier-Stokes
- The Kortweg equations (capillarity) do it

https://www.quantamagazine.org/famous-fluid-equations-are-incomplete-20150721/

Up to now:

- thermal equilibrium at freeze-out $\rightarrow$ species ratios
- radial flow $\rightarrow$ $\langle p_T \rangle$, mass hierarchy, shape of p_T spectra
- initial anisotropy, shape-flow transmutation from copious rescattering $\rightarrow$ harmonic flow
- viscosity $\rightarrow$ smoothing effect
- early thermalization $\rightarrow$ early hydrodynamization

Up to now:

- thermal equilibrium at freeze-out $\rightarrow$ species ratios
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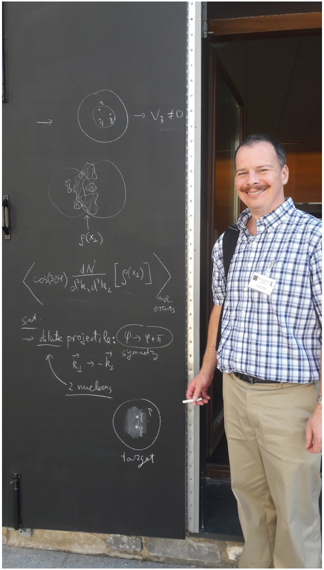
- early thermalization $\rightarrow$ early hydrodynamization

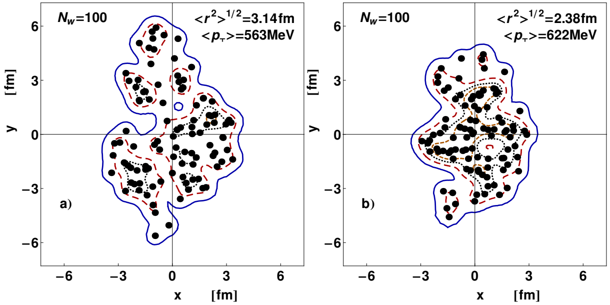
Correlations

Where predominantly generated?



- At the early gluonic stage?
- In hydro/rescattering phase?
- All over?
- Are the early fluctuations destroyed?

Initial fluctuations in the Glauber approach



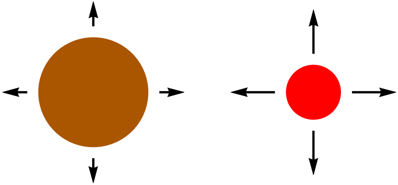
[Bożek, WB 2012]

Two typical configurations of wounded nucleons in the transverse plane generated with GLISSANDO, isentropes at $s = 0.05$, 0.2, and 0.4 GeV^{-3}

Random fluctuations in Color Glass

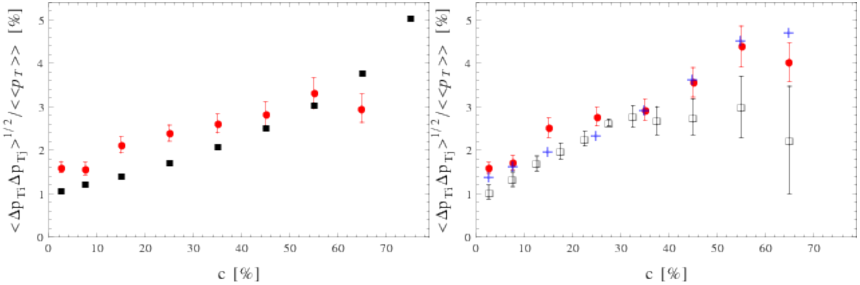
[Giacalone, Guerrero-Rodríguez, Luzum, Marquet, Ollitrault, arXiv:1902.07168]

Size – radial flow transmutation



smaller size → stronger flow
larger size → weaker flow

[WB, Chojnacki, Obara 2009]

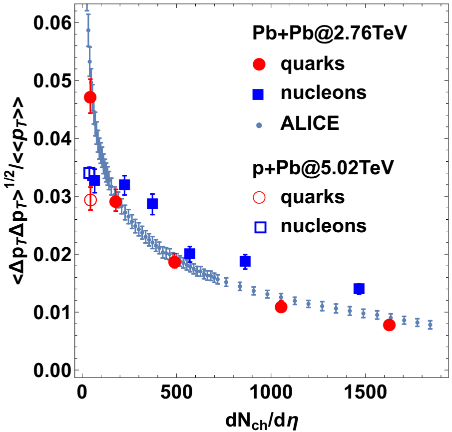


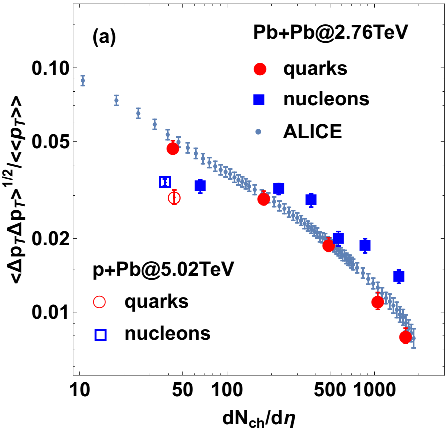
red points – model

[Bożek, WB 2012]

- Measure removes trivial fluctuations from finite sampling
- Model overshoots the data by about 50% for most central collisions, need to decrease initial fluctuations
- Hydro response not modified by viscosity, freeze-out temperature, source smearing, total momentum conservation, ... $\Delta\langle p_T\rangle/\langle\langle p_T\rangle\rangle \simeq 0.4\Delta\langle r\rangle/\langle\langle r\rangle\rangle$

Wounded quark model as implemented in [Bożek, WB, Rybczyński 2016]:
more participants → less fluctuation

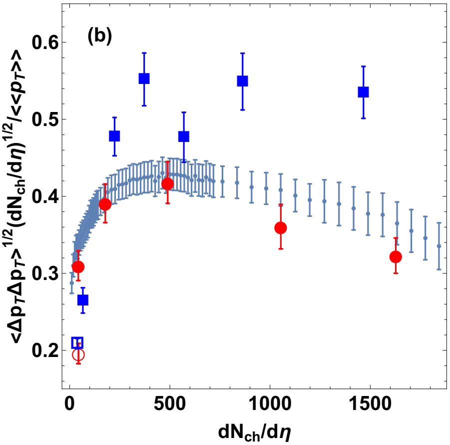




[Bożek, WB 2017]

Transverse momentum fluctuations with wounded quarks

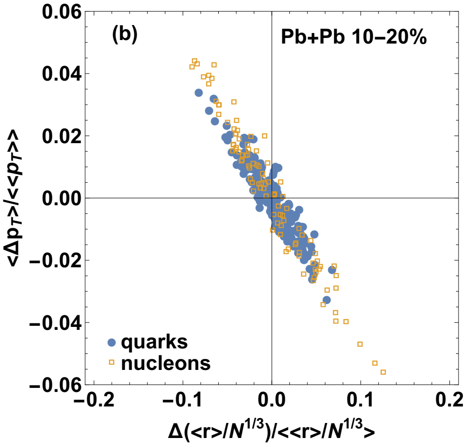
Nontrivial dependence on multiplicity



Excludes independent production from sources (would be flat)

Size – flow anti-correlation

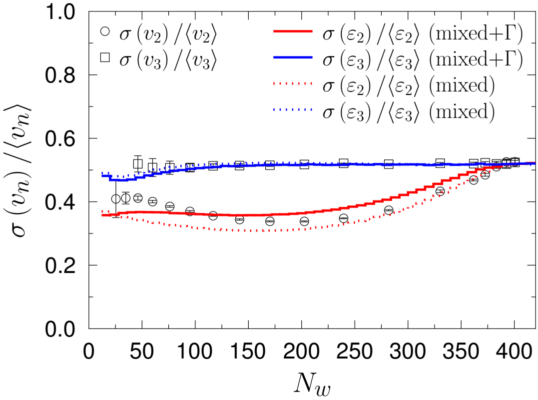
Very strong e-by-e anti-correlation of size and $\langle p_T \rangle$

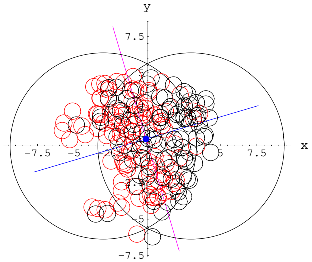


- This is the mechanism for p_T fluctuations!

Flow fluctuations

Recall $v_n = \kappa_n \epsilon_n \to \sigma(v_n)/\langle v_n \rangle = \sigma(\epsilon_n)/\langle \epsilon_n \rangle$





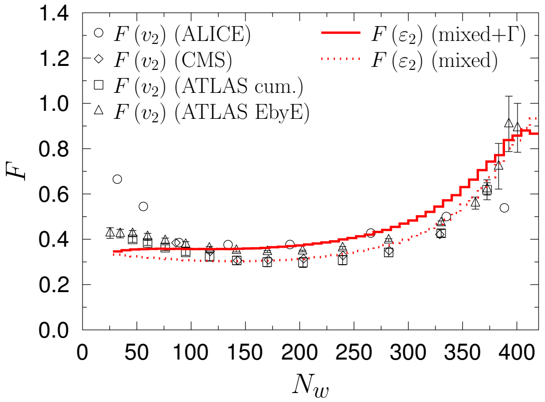
$\leftarrow \quad \sqrt{4/\pi - 1}$

[WB, Rybczyński 2016]

Flow fluctuations

$$F_n = \sqrt{\frac{\varepsilon_n\{2\}^2 - \varepsilon_n\{4\}^2}{\varepsilon_n\{2\}^2 + \varepsilon_n\{4\}^2}}$$

$$\epsilon_n\{2\} = \langle \epsilon_2^2 \rangle^{1/2}, \quad \epsilon_n\{4\} = 2\left(\langle \epsilon_n^2 \rangle^2 - \langle \epsilon_n^4 \rangle\right)^{1/4}$$



[WB, Rybczyński 2016]

Flow fluctuations

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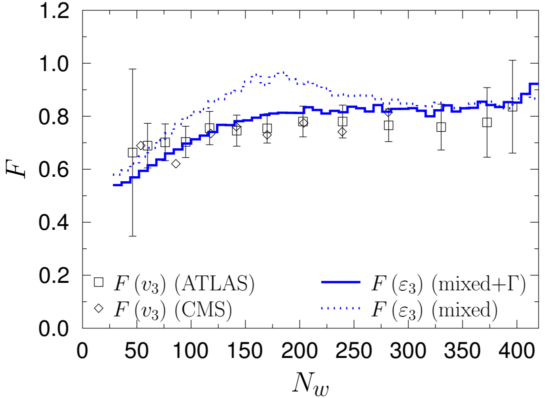
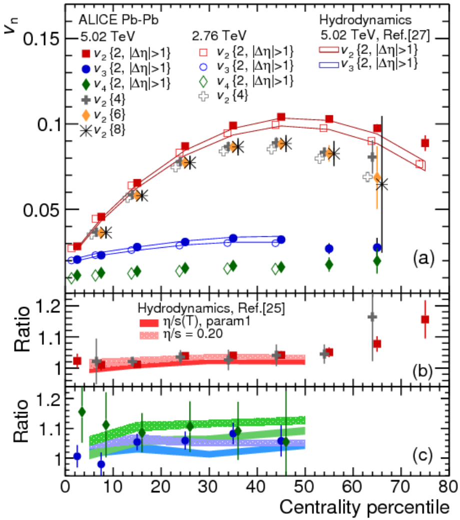
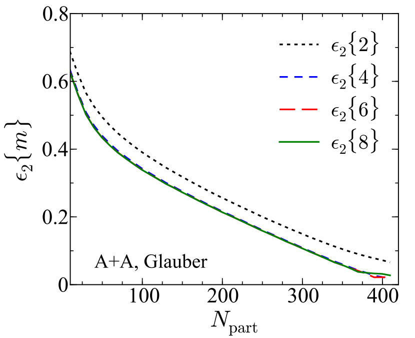


[WB, Rybczyński 2016]

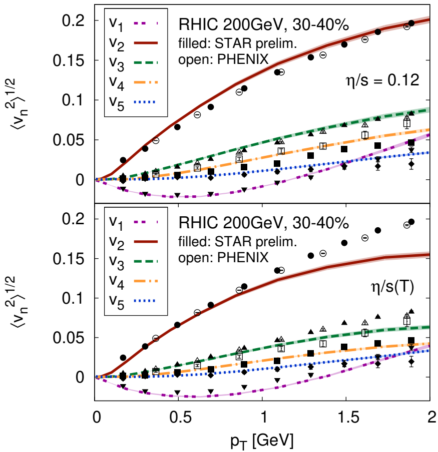
Higher cumulants

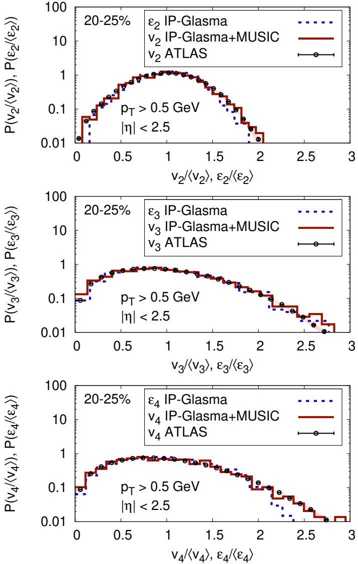




[Bzdak, Bożek, McLerran, Nucl. Phys. A927 (2014) 15]

IP-Glasma initial conditions

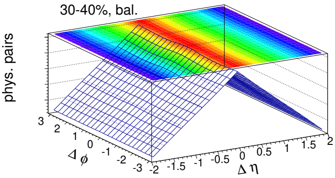




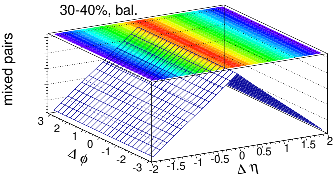
[Gale, Jeon, Schenke, Venugopalan, PRL 110 (2013) 012302]

Modeling in rapidity

$$R_2(\Delta\eta,\Delta\phi) \equiv C(\Delta\eta,\Delta\phi) = \frac{\langle N^{\rm pairs}_{\rm phys}(\Delta\eta,\Delta\phi)\rangle}{\langle N^{\rm pairs}_{\rm mixed}(\Delta\eta,\Delta\phi)\rangle}$$



/



$\rightarrow$

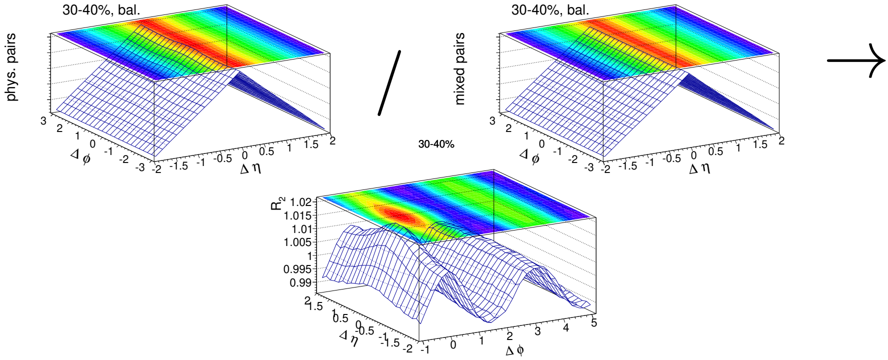
"Tent":

$$\int_{-\eta_a}^{\eta_a} d\eta_1 \int_{-\eta_a}^{\eta_a} d\eta_2\, \delta[\Delta\eta - (\eta_1 - \eta_2)] = \text{triangle in } \Delta\eta \text{ from } -2\eta_a \text{ to } 2\eta_a$$

$$\int_{0}^{2\pi} d\phi_1 \int_{0}^{2\pi} d\phi_2\, \delta[\Delta\phi - (\phi_1 - \phi_2)] = \text{flat in } \Delta\phi$$

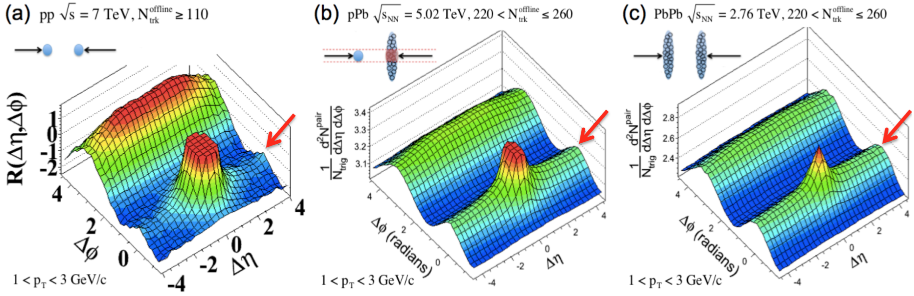
2D two-particle correlations

$$R_2(\Delta\eta,\Delta\phi) \equiv C(\Delta\eta,\Delta\phi) = \frac{\langle N^{\text{pairs}}_{\text{phys}}(\Delta\eta,\Delta\phi)\rangle}{\langle N^{\text{pairs}}_{\text{mixed}}(\Delta\eta,\Delta\phi)\rangle}$$



- free of detector acceptance bias

Near-side ridge

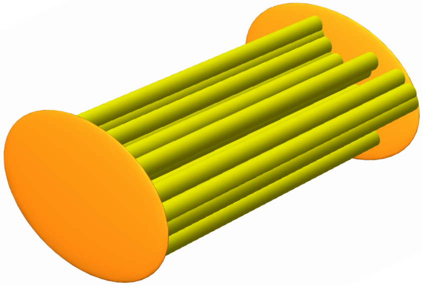


Near-side ridge indicates collectivity

Total surprise in p-p!

Factorization of the transverse and longitudinal distributions

left-moving participants strings right-moving participants



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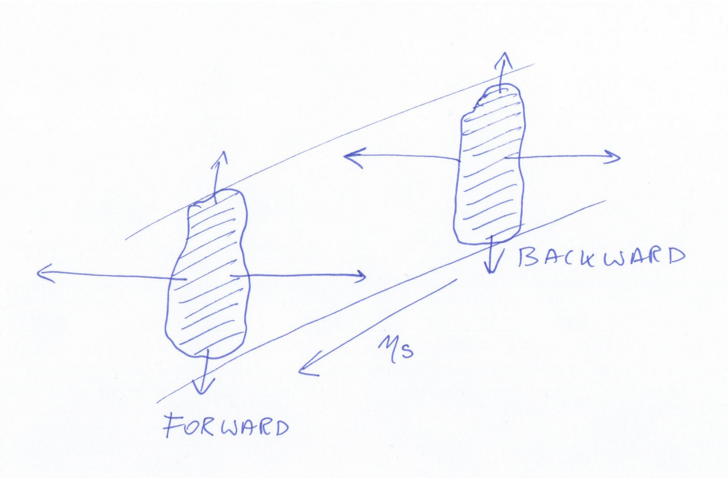
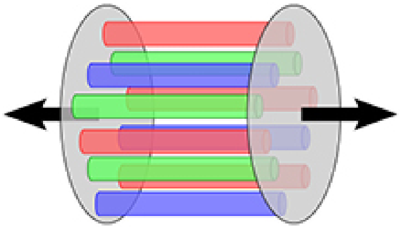
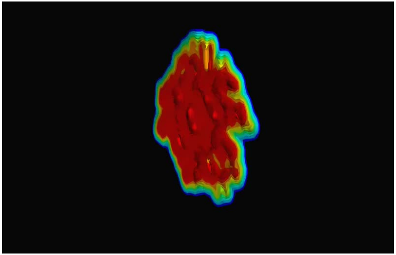


Approximate (up to fluctuations) alignment of F and B event planes

Collimation of flow at very distant longitudinal separations → ridges!

Glasma tubes

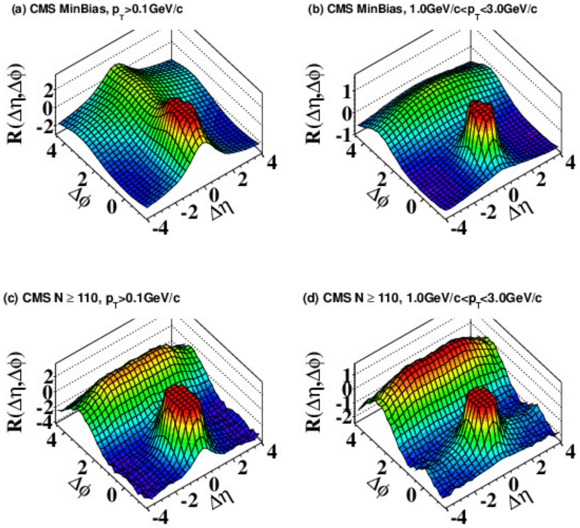




Surfers - the near-side ridge

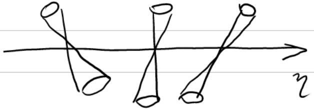
Collimated even if separated by a mile!

p+p – high multiplicity only!



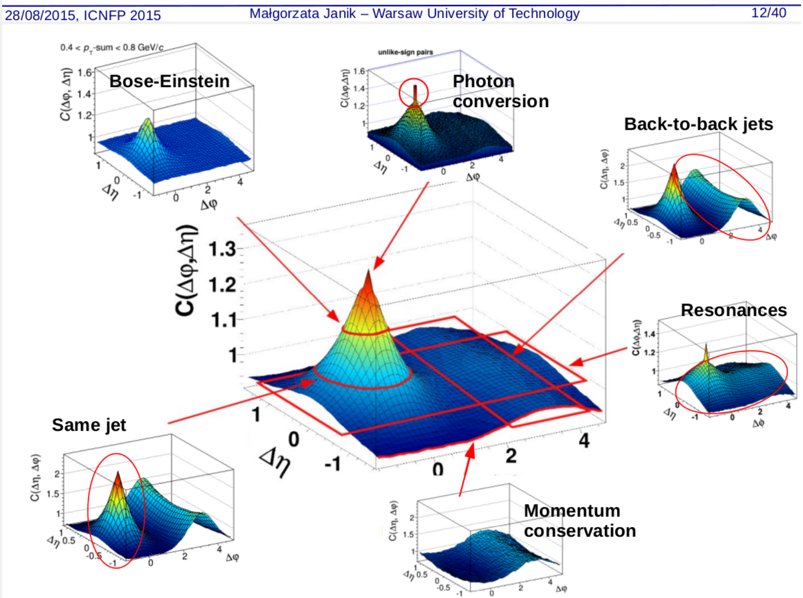
Jets

2 particles from the same jet $\rightarrow$ central peak ($\Delta\phi \sim 0$, $\Delta\eta \sim 0$)



from the opposite jets $\rightarrow$ away ridge ($\Delta\phi \sim \pi$, $\Delta\eta$ - washed out)

Other sources of correlations



Flow measures with rapidity gap

The flow vector in rapidity bin η:

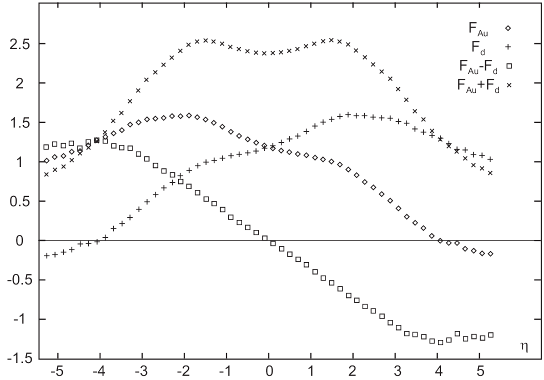
$$q_n(\eta) = \frac{1}{m}\sum_{k=1}^{m} e^{in\phi_k}$$

$$V_{n\Delta} = \langle q_n(\eta_1) q_n^*(\eta_2)\rangle = \langle\langle \cos n(\phi_1 - \phi_2)\rangle\rangle$$

with bins at η_1 and η_2 sufficiently separated

$$\frac{dN^{\rm pair}}{d\Delta\phi} \sim 1 + 2\sum_n V_{n\Delta} \cos n\Delta\phi$$

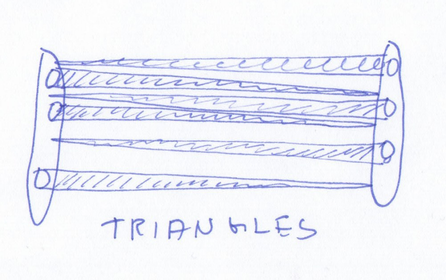
Extracted from the d-Au collisions at RHIC:



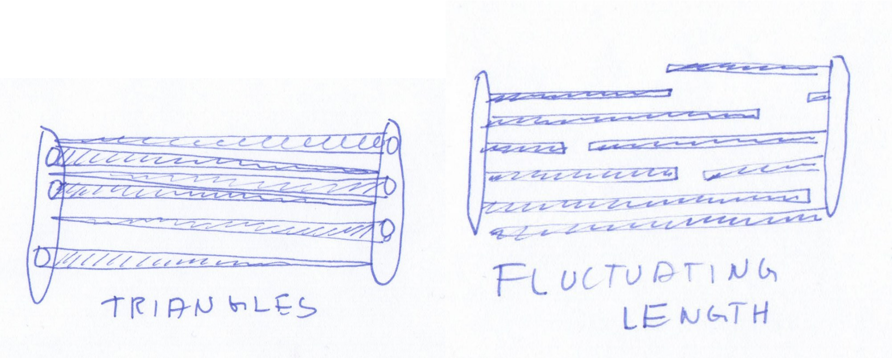
[Białas, Czyż 2004]

Source emits mostly in its own froward hemisphere

Triangles and fluctuating strings



Triangles and fluctuating strings



[...Bierlich, Gustafson, Lönnblad 2016, Monnai, Schenke 2015, Schenke, Schlichting 2016 ...Brodsky, Gunion, Kuhn, 1977]

String models 1970's

Dual Parton Model (Capella et al.)

Dual parton model 229

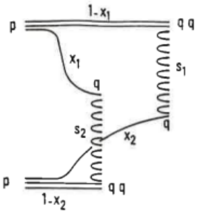
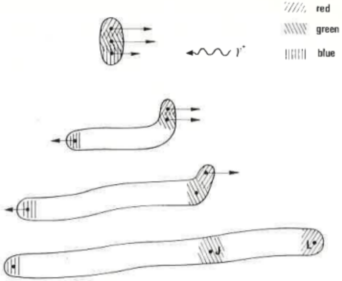


Fig. 1.2. Dominant two-chain diagram describing multiparticle production in high energy proton–proton collisions. The two quark–diquark chain structure results from an s-

Lund model (Anderson et al.)

B. Andersson et al., Parton fragmentation and string dynamics

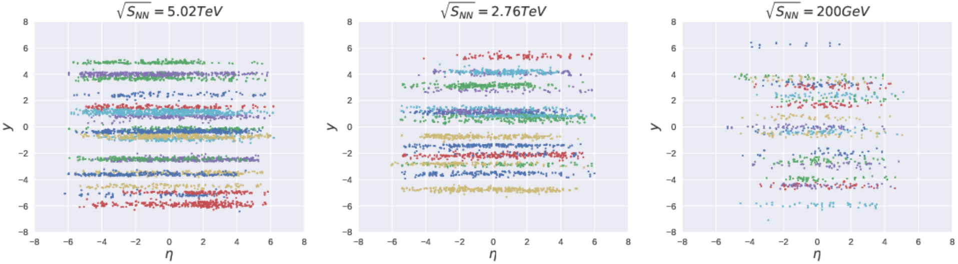


One quark in a proton is hit by a virtual photon (or a W or another hadron), and a colour flux tube is stretched

Basis of many successful codes (Pythia, HIJING, AMPT, EPOS, ...)

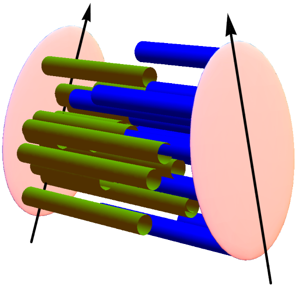
Strings are spatial objects

AMPT [Wu et al. 2018]



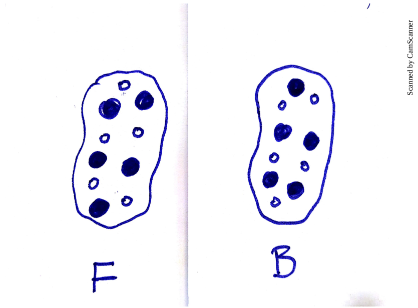
String end-points fluctuate in (here: space-time rapidity) η, uniform production of particles along the string (same thickness)

Fluctuating strings

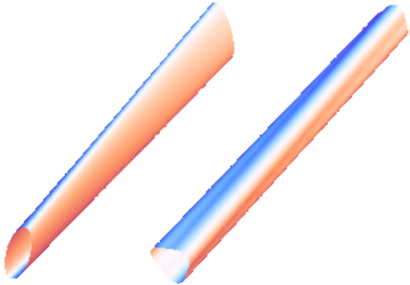


Torque effect (event-by-event)

Transverse sections with triangles



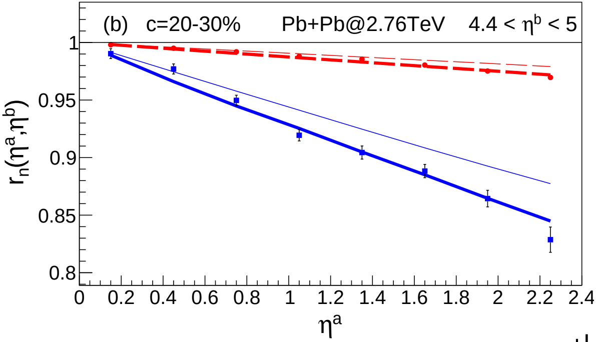
e-by-e longitudinal twist (a few degrees)



- Both e-by-e fluctuations and longitudinal asymmetry of the emission profile needed

[prediction in PB, WB, Moreira 2010 & PB, WB, Olszewski 2015, PB, WB 2016]

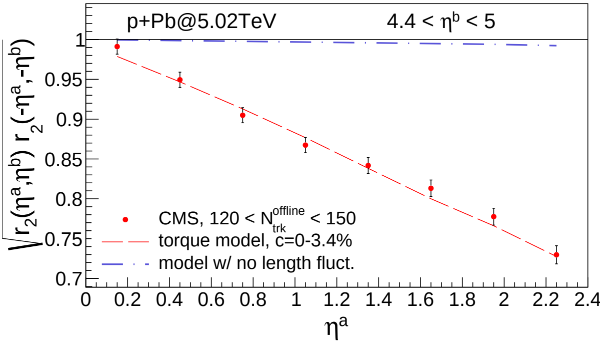
Torque in Pb+Pb



thin - triangles
thick - string breaking
v_2 and v_3

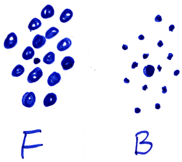
$$r_n(\eta_a, \eta_b) = \frac{\langle\langle \cos(n[\phi_i(-\eta_a) - \phi_j(\eta_b)])\rangle\rangle}{\langle\langle \cos(n[\phi_i(\eta_a) - \phi_j(\eta_b)])\rangle\rangle}$$

Torque in p-Pb

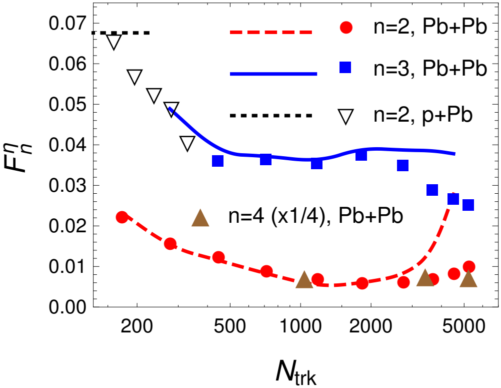


- String breaking essential to describe torque in p-Pb

With triangles:



Slope of r_n

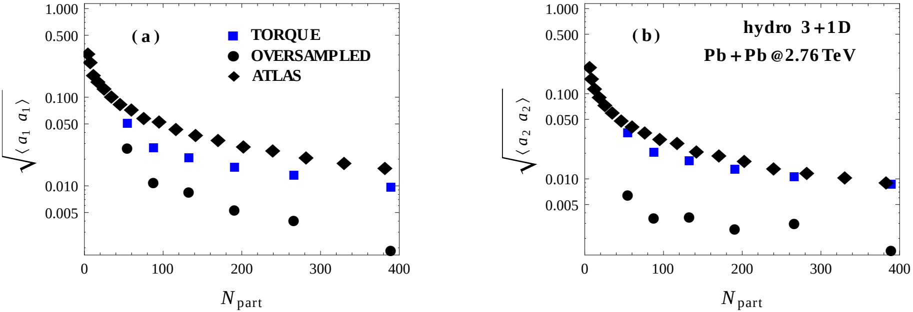


- Fair description of mid-central collisions
- Way too much decorrelation in central collisions
- $F_4 \simeq 4F_2$

η_1-η_2 correlations and a_{nm} coefficients

Method proposed by [Bzdak, Teaney, 2012, Jia, Radhakrishnan, Zhou, 2016]

$$a_{nm} = \int_{-Y}^{Y} \frac{d\eta_1}{Y} \int_{-Y}^{Y} \frac{d\eta_2}{Y} \frac{1}{\mathcal{N}_C} C(\eta_1, \eta_2) T_n\left(\frac{\eta_1}{Y}\right) T_m\left(\frac{\eta_2}{Y}\right)$$



[Bożek, WB, Olszewski, Phys.Rev. C92 (2015) 054913]

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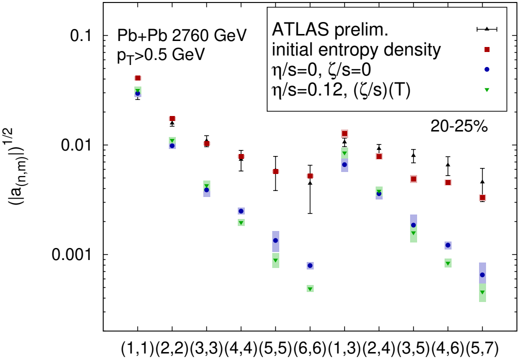
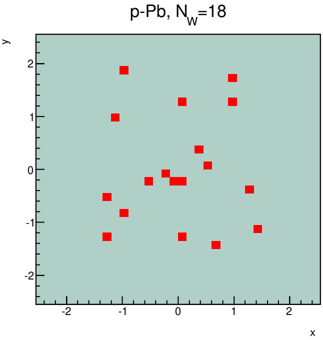


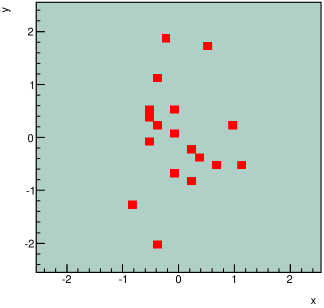
[Monnai, Schenke, PLB 752 (2016) 317]

Small systems

Snapshots of initial Glauber condition in central p-Pb

Typical transverse-plane configuration of centers of the participant nucleons in a p+Pb collision generated with GLISSANDO
5% of collisions have more than 18 participants, rms ~ 1.5 fm – large!

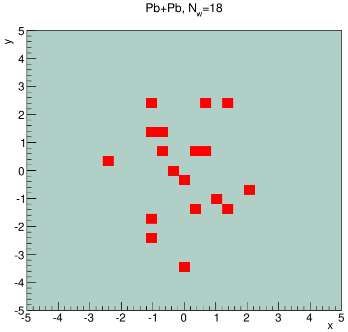




Snapshot of peripheral Pb+Pb

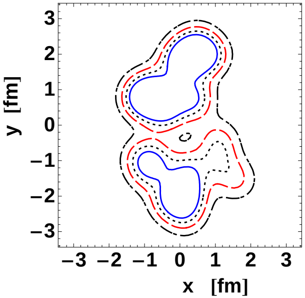
Most central values of N_w in p-Pb would fall into the 60-70% or 70-80% centrality class in Pb+Pb

Pb+Pb: c=60-70% $\equiv 22 \leq N_w \leq 40$, c=70-80% $\equiv 11 \leq N_w \leq 21$



d has an intrinsic dumbbell shape with a large deformation: rms $\simeq 2$ fm

Initial entropy density in a d-Pb collision with $N_{\mathrm{part}} = 24$ [Bożek 2012]



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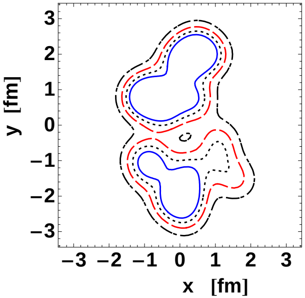
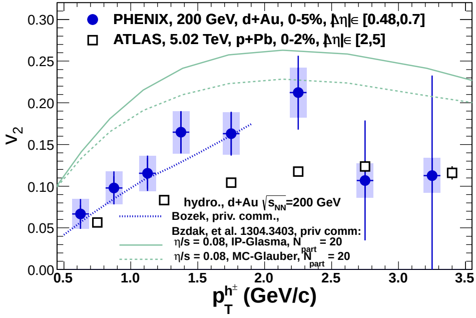
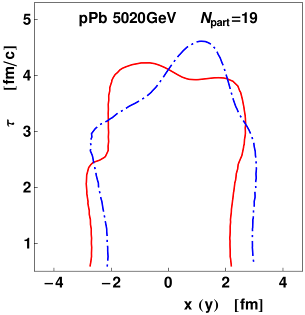




Resulting large elliptic flow confirmed with the later RHIC analysis (geometry + fluctuations)

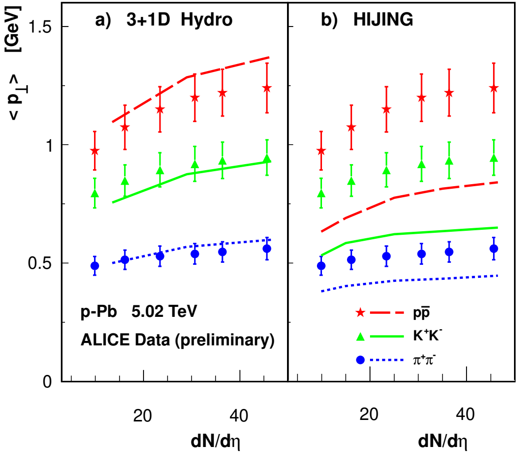
Size of the p-Pb fireball



isotherms at freeze-out $T_f = 150$ MeV
(for two sections in the transverse plane)

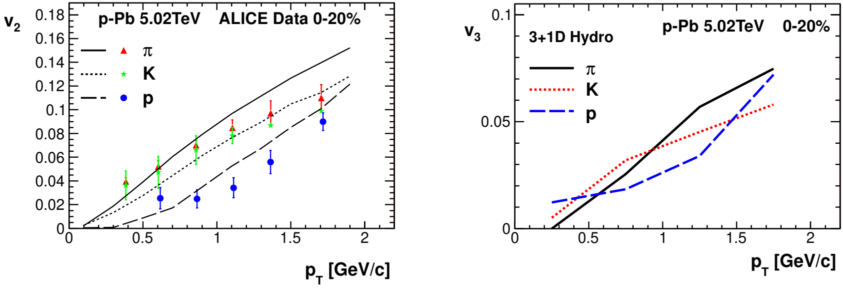
evolution lasts about 4 fm/c – shorter but more rapid than in A+A

Mass hierarchy in p-A

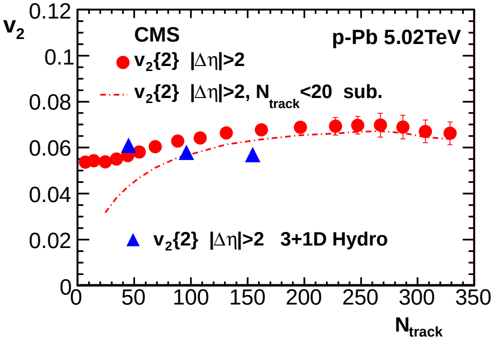


[P. Bożek, WB, G. Torrieri, PRL 111 (2013) 172303]

Mass hierarchy in p-A

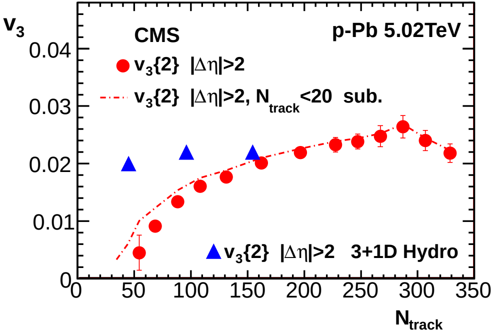


[P. Bożek, WB, G. Torrieri, PRL 111 (2013) 172303] no geometry, only fluctuations

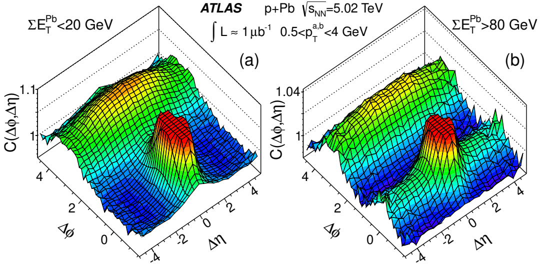


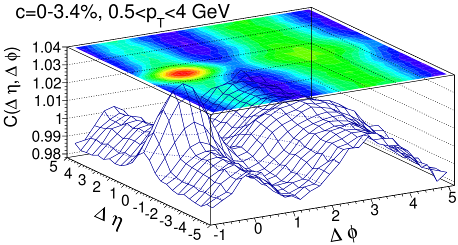
[P. Bożek, WB, PRC 88 (2013) 014903]

no geometry, only fluctuations



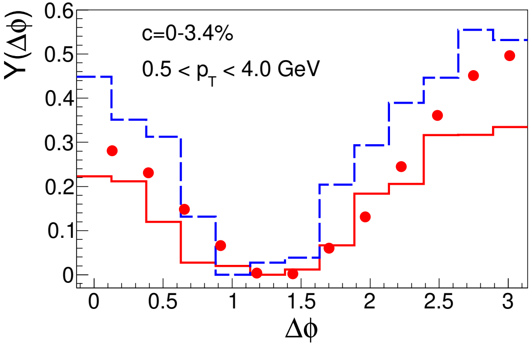
[P. Bożek, WB, PRC 88 (2013) 014903]


ATLAS p+Pb $\sqrt{s_{NN}}$=5.02 TeV
$\int L \approx 1\,\mu b^{-1}$ $0.5<p_T^{a,b}<4$ GeV
$\Sigma E_T^{Pb}<20$ GeV
(a)
$\Sigma E_T^{Pb}>80$ GeV
(b)
$C(\Delta\phi,\Delta\eta)$
$\Delta\phi$
$\Delta\eta$


c=0-3.4%, $0.5<p_T<4$ GeV
$C(\Delta\eta, \Delta\phi)$
$\Delta\eta$
$\Delta\phi$

Near-side ridge, $2 \leq |\Delta\eta| \leq 5$

$$Y(\Delta\phi) = \frac{\int B(\Delta\phi)d(\Delta\phi)}{N} C(\Delta\phi) - b_{\mathrm{ZYAM}}$$

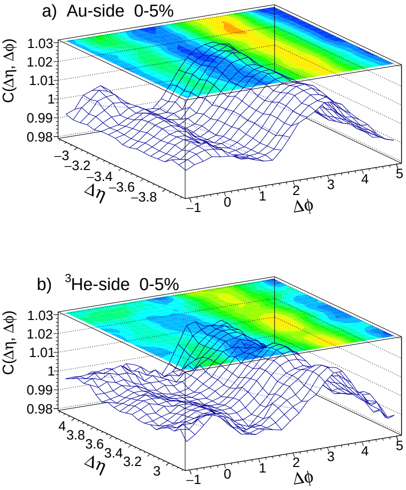


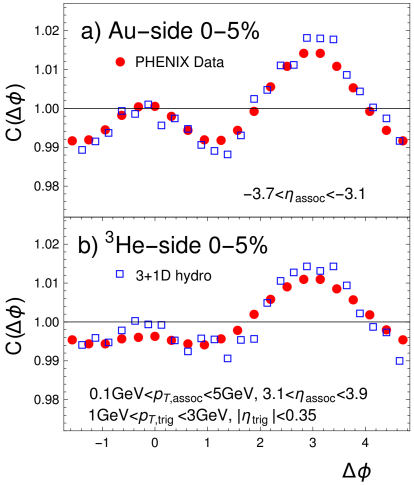
two variants of the Glauber model:

red – $< R^2 >^{1/2} = 1.5$ fm, blue – $< R^2 >^{1/2} = 0.9$ fm, dots – ATLAS

see also CGC-based calculation: [K. Dusling, R. Venugopalan, PRD 87 (2013) 094034]

Ridge in ^{3}He-Au at RHIC



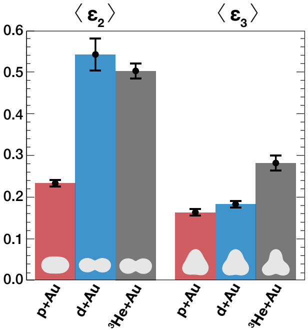


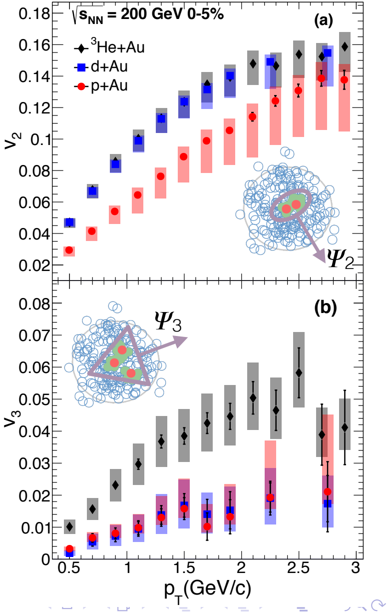
(seen on both pseudorapidity sides)

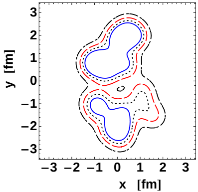
[Bożek, WB 2015]

Flow hierarchy in small systems

[PHENIX, 2018]

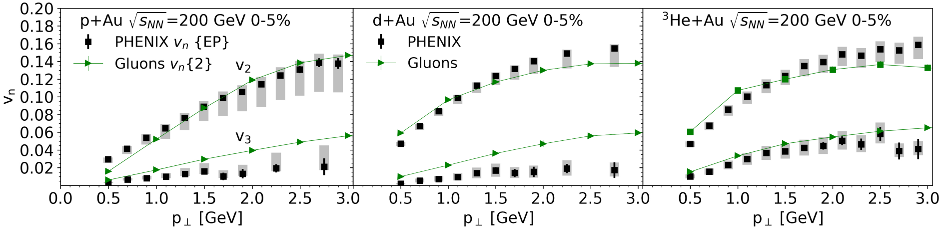






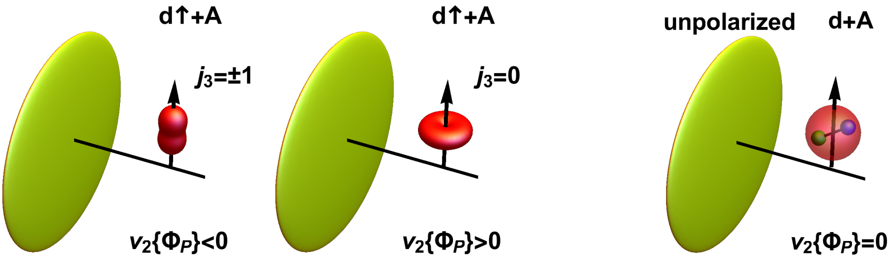
independent sources in d+A →
v_2 in d+A would be smaller than in p+A,
contrary to experiment.

[Mace, Skokov, Tribedy, Venugopalan, 2018]: high multiplicity events have larger saturation scales and specific orientation of the deuteron, with one nucleon behind the other



Questioned in [Nagle, Zajc 2018] → controversy

Polarized d+A collisions



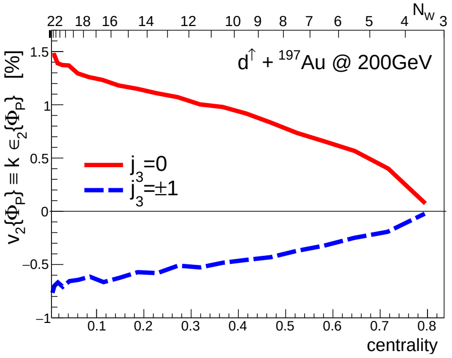
[P. Bożek, WB, PRL 121 (2018) 202301]

Predictions

$$\frac{dN}{d\phi} \propto 1+2v_2 \cos\left[2(\phi - \Phi_P)\right]$$

Φ_P fixed!

$$v_2 \simeq k\epsilon_2, \quad k \sim 0.2$$



For $j = 1$ nuclei the *tensor polarization* is

$$P_{zz} = n(1) + n(-1) - 2n(0)$$
$$v_2\{\Phi_P\} \simeq k\,\epsilon_2^{j_3=\pm1}\{\Phi_P\}P_{zz}$$

$$-0.5\% \lesssim v_2\{\Phi_P\} \lesssim 1\%$$

One-particle distribution - can be measured precisely !

Prospects for AFTER@LHC

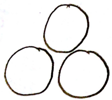
^{12}C-Pb – role of α clusters

Nuclear structure from ultra-relativistic collisions!

Probe to what degree ^{12}C is made of three α's

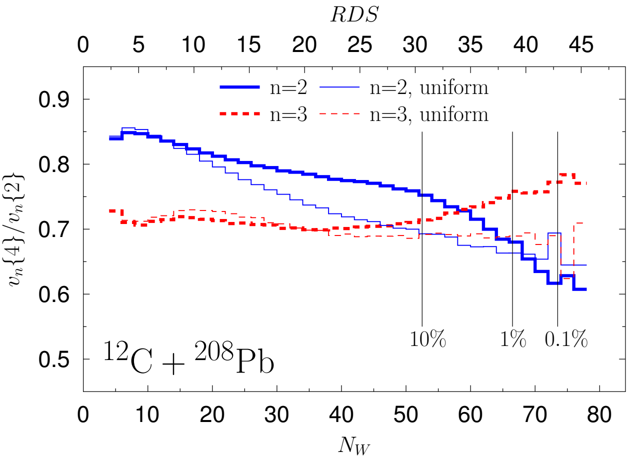
Specific features of the ^{12}C collisions with a "wall":

The cluster plane parallel or perpendicular to the transverse plane:

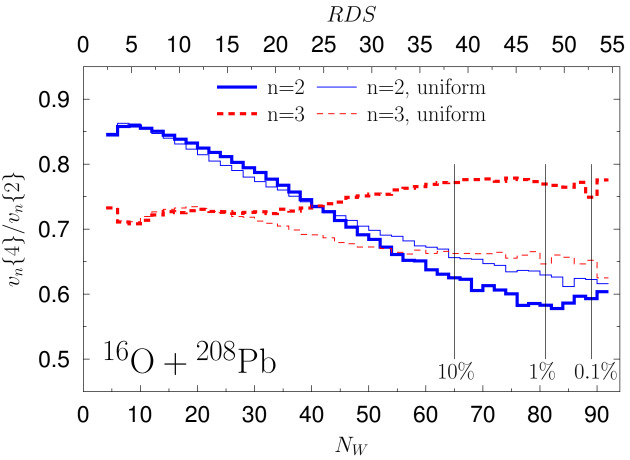


higher multiplicity	lower multiplicity
higher triangularity	lower triangularity
lower ellipticity	higher ellipticity

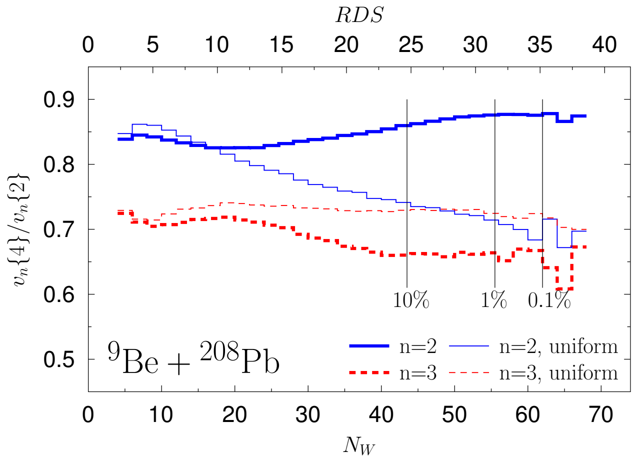
Ellipticity and triangularity vs multiplicity



Ellipticity and triangularity vs multiplicity



Ellipticity and triangularity vs multiplicity



Idea picked up in [Lim, Carlson, Loizides, Lonardoni, Lynn, Nagle, Orjuela Koop, Ouellette, PRC 99 (2019) 044904] with exp. prospects

Summary

Circumstantial evidence

- Multiplicities → thermal parameters
- p_T spectra → radial flow
- harmonic flow → initial geometry and fluctuations
- fluctuations of $< p_T >$ → fluctuations of the initial size
- Ridge → flow
- Interferometry → size and flow (not covered)

Conclusions

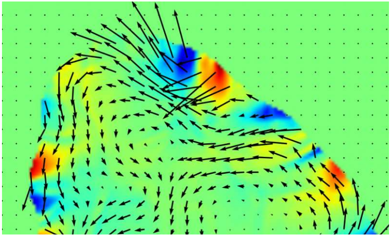
The approach with hydro (copious rescattering in the intermediate stage) works

- Collectivity from rescattering in A+A commonly accepted
- Explanation of the near-side ridge
- Mechanism for p_T fluctuations
- Torque (event-plane angle decorrelation)

- Small systems (p-Pb, d-Pb) not so small
- Torque in p-Pb $\rightarrow$ longitudinal fluctuations (string breaking)
- Shape-flow transmutation in small systems
- Polarized deuteron
- Clustered small nuclei

Not mentioned

- Jet quenching by the medium
- Early probes
- Femtoscopy
- Chiral magnetic effect
- Vorticity and Λ polarization
- ...



[RHIC simulation, Pang et al. 2016]

Recommended literature (and references therein)

- *Phenomenology of Ultra-Relativistic Heavy-Ion Collisions*, Wojciech Florkowski, World Scientific 2010 **(with exercises!)**
- *Ultra-relativistic Heavy-Ion Collisions*, Ramona Vogt, Elsevier 2007
- *Relativistic hydrodynamics for heavy-ion collisions*, Jean-Yves Ollitrault, Lectures given at the Advanced School on Quark-Gluon Plasma, Indian Institute of Technology, Bombay, 3-13 July 2007, Eur.J.Phys. 29(2008)275, arXiv:0708.2433 **(with exercises!)**
- *Nearly perfect fluidity: from cold atomic gases to hot quark gluon plasmas*, Thomas Schäfer, Derek Teaney, Rept. Prog. Phys. 72 (2009) 126001
- *New theories of relativistic hydrodynamics in the LHC era*, Wojciech Florkowski, Michal P. Heller, Michal Spaliński, Rept. Prog. Phys. 81 (2018) 046001
- *Collective flow and viscosity in relativistic heavy-ion collisions*, Ulrich Heinz, Raimond Snellings, Ann. Rev. Nucl. Part. Sci. 63 (2013) 123
- *Initial state fluctuations and final state correlations: Status and open questions*, Andrew Adare, Matthew Luzum, Hannah Petersen, Phys.Scripta 87(2013)048001, Phys.Scripta 04(2013)048001
- ...

THANKS!