Partonic quasi-distributions of the pion in chiral quark models

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details in PLB, arXiv:1707.09588

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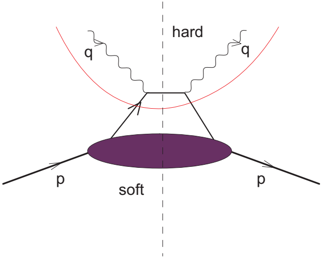
Outline

- Parton distributions – basic properties of hadrons
- Soft matrix elements, accessible from effective low-energy models of QCD
- Chiral quark models of the pion
- Parton quasi-distributions, designed for Euclidean QCD lattices

- Results and predictions for quasi-distributions of the pion from chiral models

Introduction

Parton distribution



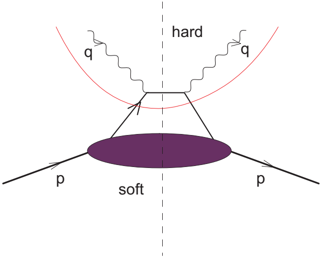
$Q^2 = -q^2, \;\; x = \frac{Q^2}{2p\cdot q}, \;\; Q^2 \to \infty$

Factorization of soft and hard processes, Wilson's OPE

$$\langle J(q)J(-q)\rangle = \sum_i C_i(Q^2;\mu)\langle \mathcal{O}_i(\mu)\rangle$$

Twist expansion $\to \quad F(x,Q) = F_0(x,\alpha(Q)) + \frac{F_2(x,\alpha(Q))}{Q^2} + \ldots$

Parton distribution



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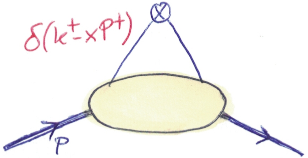
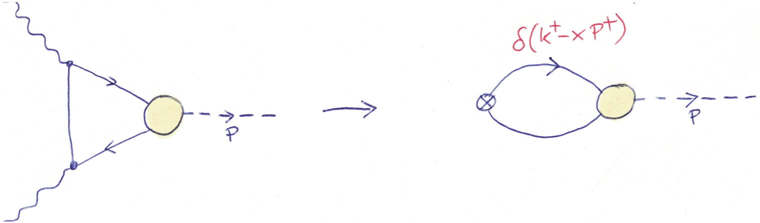
constrained light-cone momentum

$k^+ = k^0 + k^3, \quad x \in [0,1]$



Distribution amplitude (DA) of the pion



Enters various measures of exclusive processes, e.g., pion-photon transition form factor

Field-theoretic definition
(here for quarks in the pion, leading twist)

Parton Distribution Function (DF):

$$V(x) = \int \frac{dz^-}{4\pi} e^{ixP^+z^-} \langle P|\bar{\psi}(0)\gamma^+ [0,z]\psi(z)|P\rangle\Big|_{z^+=0, z^\perp=0}$$

Parton Distribution Amplitude (DA):

$$\phi(x) = \frac{i}{F_\pi} \int \frac{dz^-}{2\pi} e^{i(x-1)P^+z^-} \langle P|\bar{\psi}(0)\gamma^+\gamma_5 [0,z]\psi(z)|\text{vac}\rangle\Big|_{z^+=0, z^\perp=0}$$

(isospin suppressed)

P - pion momentum, $v^\pm \equiv v^0 \pm v^3$ - light-cone basis

$[z_1, z_2] = \exp\left(-ig_s \int_{z_1}^{z_2} d\xi \lambda^a A_a^+(\xi)\right)$ - Wilson's gauge link

x - fraction of the light-cone mom. $P+$ carried by the quark, $x \in [0,1]$

Remarks

- Only indirect experimental information for the pion distributions:
- DF from Drell-Yan in E615, DA from dijets in E791 and from exclusive processes involving pions
- Impossibility to implement PDF or PDA on the euclidean lattices, only lowest moments can be obtained

- However, there exist (largely forgotten) simulations on transverse lattices – discussed later

Quasi-distributions

Parton quasi-distributions (quarks in the pion)

[Ji 2013]

Parton Quasi-Distribution Function (QDF):

$$\tilde{V}(y;P_3) = \int \frac{dz^3}{4\pi} e^{iyP^3 z^3} \, \langle P|\bar{\psi}(0)\gamma^3 \, [0,z]\psi(z)|P\rangle\Big|_{z^0=0,z^\perp=0}$$

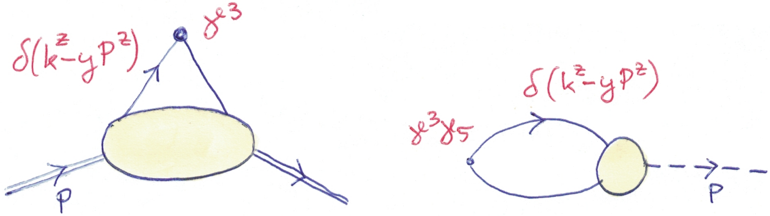
Parton Quasi-Distribution Amplitude (PDA):

$$\tilde{\phi}(y;P_3) = \frac{i}{F_\pi} \int \frac{dz^3}{2\pi} e^{i(y-1)P^3 z^3} \, \langle P|\bar{\psi}(0)\gamma^+\gamma_5 \, [0,z]\psi(z)|\mathrm{vac}\rangle\Big|_{z^0=0,z^\perp=0}$$

y - fraction of P_z carried by the quark
Analogy, but: dependence on P_3, difference in support - y is not constrained
Miracle:

$$\lim_{P_3\to\infty} \tilde{V}(x;P_3) = V(x), \quad \lim_{P_3\to\infty} \tilde{\phi}(x;P_3) = \phi(x)$$

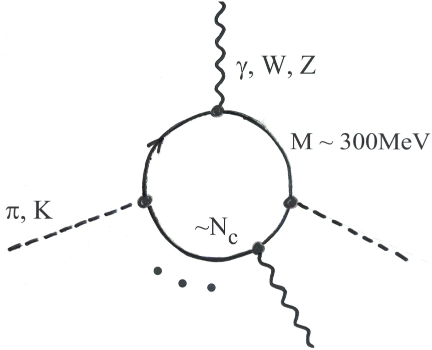
QDF and QDA in the momentum representation



Constrained longitudinal momenta, but $y \in (-\infty, \infty)$
(partons can move "backwards")

Chiral quark models

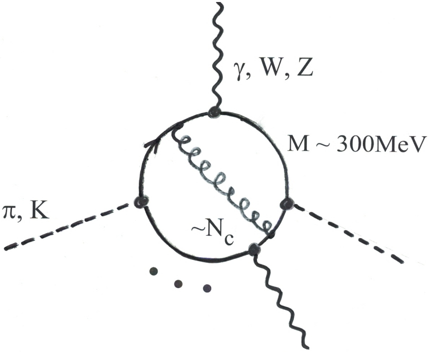
Chiral quark models



- Point-like interactions
- Soft matrix elements with pions (and photons, W, Z)
- One-quark loop, regularization:
  1) Pauli-Villars (PV)
  2) Spectral Quark Model (SQM) - implements VMD

Evaluated at the quark model scale
(where constituent quarks are the only degrees of freedom)

Chiral quark models



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Need for evolution

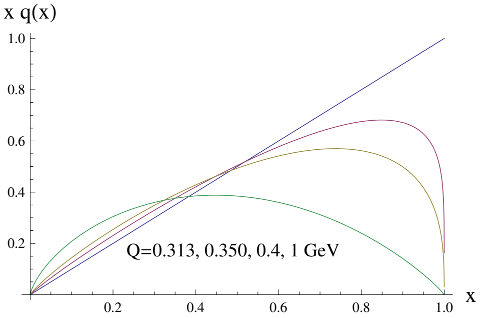
Gluon dressing, renorm-group-improved

Scale and evolution

QM provide non-perturbative result at a low scale Q_0

$$F_i(x, Q_0)|_{\rm model} = F_i(x, Q_0)|_{\rm QCD}\,, \quad Q_0 - \text{the matching scale}$$

Determination of Q_0 via momentum fraction: quarks carry 100% of momentum at Q_0. One adjusts Q_0 in such a way that when evolved to $Q = 2$ GeV, the quarks carry the experimental value of 47%



LO DGLAP evolution:

$$Q_0 = 313^{+20}_{-10}\ \text{MeV}$$

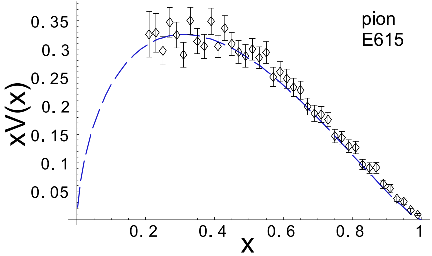
[Davidson, Arriola 1995]:

$$q(x; Q_0) = 1$$

Older results from chiral quark models w/ evolution

Pion non-singlet DF, QM vs. E615

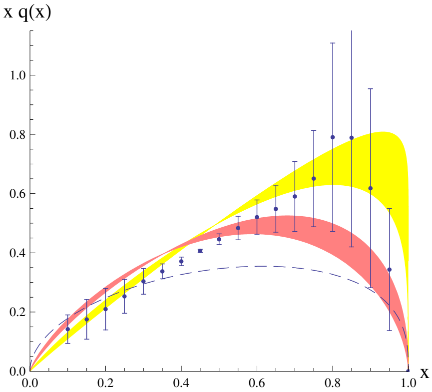
LO DGLAP evolution to the scale $Q^2 = (4 \text{ GeV})^2$:



points: Fermilab E615, Drell-Yan

curve: QM evolved to $Q = 4$ GeV

Pion non-singlet DF, QM vs. transverse lattice



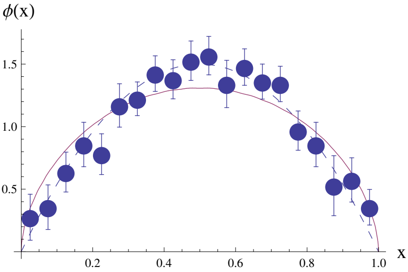
points: transverse lattice [Dalley, van de Sande 2003]

yellow: QM evolved to 0.35 GeV

pink: QM evolved to 0.5 GeV

dashed: GRS param. at 0.5 GeV

Pion DA, QM vs. E791

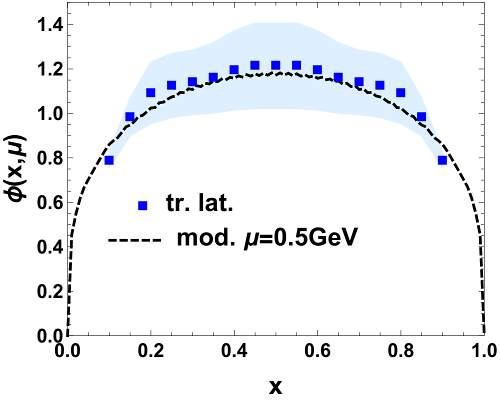


points: E791 data from dijet production in $\pi + A$

solid line: QM at $Q = 2$ GeV

dashed line: asymptotic form $6x(1-x)$ at $Q \to \infty$

Pion DA, QM vs. transverse lattice



points: transverse lattice data [Dalley, van de Sande 2003]

line: QM at $Q = 0.5$ GeV

NEW: Quasi-distributions from QM

Analytic formulas (in the chiral limit)

SQM:

$$\tilde{\phi}(y, P_z) = V(y, P_z) = \frac{1}{\pi}\left[\frac{2m_\rho P_z y}{m_\rho^2 + 4{P_z}^2 y^2} + \mathrm{arctg}\left(\frac{2P_z y}{m_\rho}\right)\right] + (y \to 1 - y)$$

(similar simplicity for PV NJL)

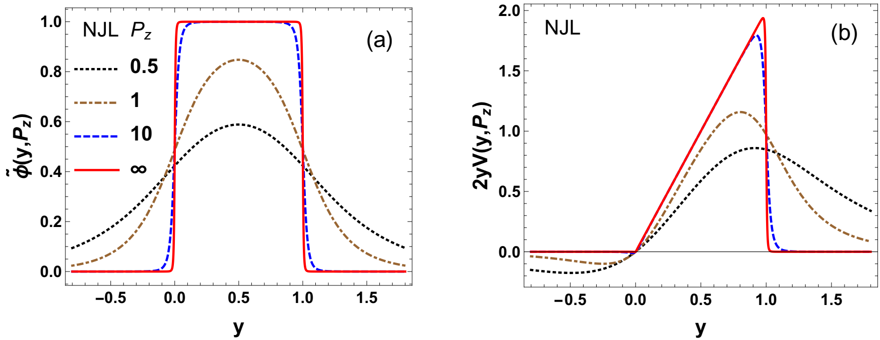
Satisfy the proper normalization

$$\int_{-\infty}^{\infty} dy\, \tilde{\phi}(y, P_z) = \int_{-\infty}^{\infty} dy\, V(y, P_z) = 1, \quad \int_{-\infty}^{\infty} dy\, 2y V(y, P_z) = 1$$

and the limit

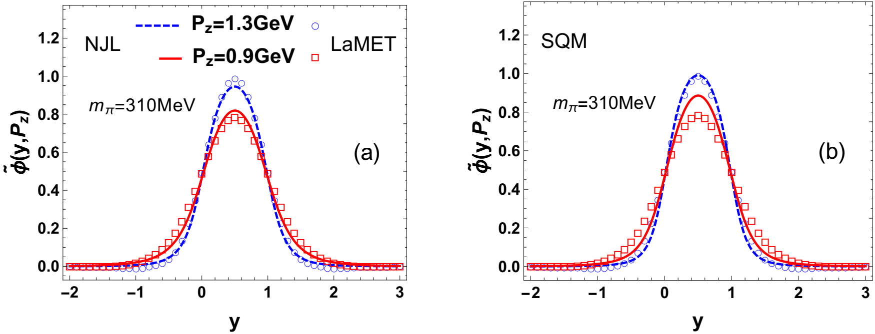
$$\lim_{P_z \to \infty} \tilde{\phi}(y, P_z) = \lim_{P_z \to \infty} V(y, P_z) = \theta[y(1 - y)]$$

QDA and QDF from chiral quark models



(a) Quark QDA of the pion in the NJL model (for $m_\pi = 0$) at various values of P_z, plotted vs. the longitudinal momentum fraction y
(b) The same, but for the valence quark QDF multiplied (conventionally) with $2y$

Comparison to lattice



Quark QDA of the pion in NJL (a) and SQM (b) (for $m_\pi = 310$ MeV), plotted vs. the longitudinal momentum fraction y, evaluated with $P_z = 0.9$ and 1.3 GeV and compared to the lattice data at $\mu = 2$ GeV (LaMET)

Evolution of QDF

Relation k_T-unintegrated quantities (TMA, TMD)

Radyushkin's formula [2016]

$$\tilde{\phi}(y, P_z) = \int_{-\infty}^{\infty} dk_1 \int_0^1 dx\, P_z \mathrm{TMA}(x, k_1^2 + (x-y)^2 P_z^2).$$

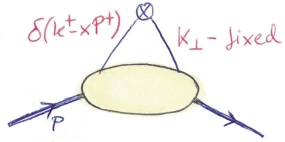
QDA can be obtained from TMA via a double integration!

Analogously

$$\tilde{V}(y, P_z) = \int_{-\infty}^{\infty} dk_1 \int_0^1 dx\, P_z \mathrm{TMD}(x, k_1^2 + (x-y)^2 P_z^2).$$

Evolution of unintegrated DF

UDF or TMD



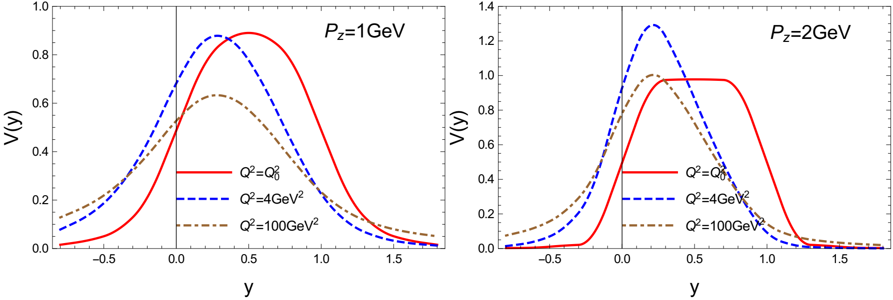
Kwieciński's method [2003], one-loop CCFM

DGLAP-like evolution, diagonal in b-space conjugate to k_T

For the non-singlet case:

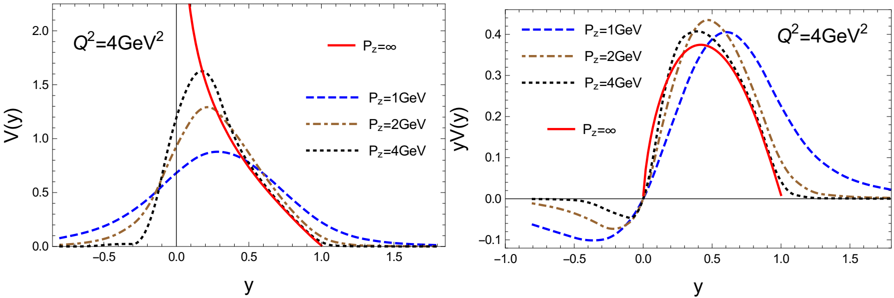
$$Q^2 \frac{\partial f(x,b,Q)}{\partial Q^2} = \frac{\alpha_s(Q^2)}{2\pi} \int_0^1 dz\, P_{qq}(z) \times \left[\Theta(z-x)\, J_0[(1-z)Qb]\, f\left(\frac{x}{z}, b, Q\right) - f(x,b,Q) \right]$$

Results of evolution of pion QDF in Q at fixed P_z



Strength moved to lower y as Q increases

Changing P_z at fixed Q



$P_z \to \infty$ limit achieved fastest at large $y \sim 0.6 - 0.9$

Conclusions

Conclusions

- Model evaluation of quasi-distributions of the pion (at the quark-model scale)
- Very simple analytic results, all consistency conditions met
- Exemplification of definitions and methods
- Results at finite P_z acquire meaning, can be (favorably) compared to QDA from lattice QCD
- For QDF of the pion predictions made for various Q (Kwieciński's evolution) and P_z
- $P_z \sim 1$ GeV, accessible on the lattice, may not be sufficient for assessment of the $P_z \to \infty$ limit
- Convergence fastest for intermediate y, suggesting the domain where lattice may work best

- Recent activity also on related objects: quasi-distributions, Ioffe-time distributions ...